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Three-Body Clusters in Nuclear Matter

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An extended derivation of the three-body cluster energy is given which clarifies the presentation in a previously published paper.

Our paper on the subject of three-body clusters in nuclear matter[1] contains an inconsistency due, essentially, to a deficient notation. The main problem is that the quantity ΔT appearing in Eq. (38) of Ref. 1 is not the same quantity as that defined in Eq. (33) of that reference. In order to resolve this inconsistency we recall that we had defined

$$|\Psi_{ijk}\rangle=(1+f_{12}+f_{13}+f_{23}+f_{123})|ijk\rangle_A, \tag{1}$$

$$f_{12}|ij\rangle_A=\sum_{mn}|mn\rangle\langle mn|f_{12}|ij\rangle_A, \tag{2}$$

$$f_{123}|ijk\rangle_A=\sum_{mnp}|mnp\rangle\langle mnp|f_{123}|ijk\rangle_A, \tag{3}$$

where $m, n, p, \ldots$ are unoccupied single-particle states and $i, j, k, \ldots$ are occupied single-particle states. From Eqs. (1)–(3), the necessity of defining the following two types of Q operators becomes clear,

$$Q_{12}=\sum_{mn}|mn\rangle\langle mn|, \tag{4}$$

$$Q_{123}=\sum_{mnp}|mnp\rangle\langle mnp|. \tag{5}$$

We then have

$$\begin{aligned}|\Psi_{ijk}\rangle&=(1+f_{12}+f_{13}+f_{23})|ijk\rangle_A+Q_{123}|\Psi_{ijk}\rangle\\&=(1+f_{12}+f_{13}+f_{23})|ijk\rangle_A\\&\quad-\sum_{mnp}\frac{|mnp\rangle\langle mnp|(v_{12}+v_{13}+v_{23})|\Psi_{ijk}\rangle}{t_m+t_n+t_p-\epsilon_i-\epsilon_j-\epsilon_k},\end{aligned} \tag{6}$$

where use has been made of Eq. (25) of Ref. 1.

We introduce a shortened notation,

$$F^{(2)}=f_{12}+f_{13}+f_{23}, \tag{7}$$

$$V=v_{12}+v_{13}+v_{23}, \tag{8}$$

and

$$e_{123}=t_m+t_n+t_p-\epsilon_i-\epsilon_j-\epsilon_k. \tag{9}$$

Then we may write,

$$|\Psi_{ijk}\rangle=(1+F^{(2)})|ijk\rangle_A-\frac{Q_{123}}{e_{123}}V|\Psi_{ijk}\rangle. \tag{10}$$

We further define,

$$T'|ijk\rangle_A=V|\Psi_{ijk}\rangle, \tag{11}$$

and obtain the following equation for T'

$$T'|ijk\rangle_A=V(1+F^{(2)})|ijk\rangle_A-V\frac{Q_{123}}{e_{123}}T'|ijk\rangle_A. \tag{12}$$

The quantity T' coincides with the T of Rajaraman and Bethe[2,3] only as far as the matrix elements of the type $\langle mnp|T'|ijk\rangle$ are concerned.

In order to cast Eq. (12) in a form similar to Eq. (32) of Ref. 1, we write,

$$\begin{aligned}T'|ijk\rangle_A&=[v_{12}(1+f_{12})+v_{13}(1+f_{13})+v_{23}(1+f_{23})]|ijk\rangle_A\\&\quad+[v_{12}(f_{23}+f_{13})+v_{23}(f_{12}+f_{13})+v_{13}(f_{12}+f_{23})]|ijk\rangle_A-(v_{12}+v_{13}+v_{23})\frac{Q_{123}}{e_{123}}T'|ijk\rangle_A\\&=(T'^{(1)}+T'^{(2)}+T'^{(3)})|ijk\rangle_A.\end{aligned} \tag{13}$$

Note that $v_{12}(1+f_{12})|ijk\rangle_A = K_{12}|ijk\rangle_A$ etc. The $T'^{(1)}$, $T'^{(2)}$, and $T'^{(3)}$ are defined through the relations, e.g.,

$$T'^{(3)}|ijk\rangle_A = \left\{K_{12} + v_{12}\left[(f_{23}+f_{12}) - v_{12}\frac{Q_{123}}{e_{123}}(T'^{(1)}+T'^{(2)}+T'^{(3)})\right]\right\}|ijk\rangle_A \tag{14}$$

or

$$\left(1+v_{12}\frac{Q_{123}}{e_{123}}\right)T'^{(3)}|ijk\rangle_A = \left[K_{12}+v_{12}(f_{23}+f_{12}) - v_{12}\frac{Q_{123}}{e_{123}}(T'^{(1)}+T'^{(2)})\right]|ijk\rangle_A . \tag{15}$$

Thus

$$T'^{(3)}|ijk\rangle_A = \left(1+v_{12}\frac{Q_{123}}{e_{123}}\right)^{-1}\left\{K_{12}+v_{12}\left[(f_{23}+f_{13}) - \frac{Q_{123}}{e_{123}}(T'^{(1)}+T'^{(2)})\right]\right\}|ijk\rangle_A . \tag{16}$$

Now

$$\left(1+v_{12}\frac{Q_{123}}{e_{123}}\right)^{-1}K_{12}|ijk\rangle_A = K_{12}|ijk\rangle_A \tag{17}$$

because $Q_{123}K_{12}|ijk\rangle_A = 0$. Thus far we have defined K_{12} by $K_{12}|ij\rangle_A = v_{12}|\Psi_{ij}\rangle$. We must specify the effect of the application of K_{12} to $|imn\rangle_A$ or $|mnp\rangle_A$. We define, therefore,

$$K_{12} = \left(1+v_{12}\frac{Q_{123}}{e_{123}}\right)^{-1}v_{12} . \tag{18}$$

Then we have,

$$T'^{(3)}|ijk\rangle_A = \left\{K_{12}+K_{12}\left[f_{23}+f_{13} - \frac{Q_{123}}{e_{123}}(T'^{(1)}+T'^{(2)})\right]\right\}|ijk\rangle_A . \tag{19}$$

Note also that $f_{23} = -(Q_{23}/e_{23})K_{23}$, etc. Therefore, we may also write

$$T'^{(3)}|ijk\rangle_A = \left\{K_{12} - K_{12}\left[\frac{Q_{23}}{e_{123}}K_{23} + \frac{Q_{13}}{e_{13}}K_{13} + \frac{Q_{123}}{e_{123}}(T'^{(2)}+T'^{(1)})\right]\right\}|ijk\rangle_A . \tag{20}$$

Finally, this equation may be written in a shortened notation as,

$$T'^{(3)}|ijk\rangle_A = \left[K_{12} - K_{12}\frac{Q}{e}(T'^{(1)}+T'^{(2)})\right]|ijk\rangle_A \tag{21}$$

with the convention $(Q/e)T'^{(1)} = (Q_{23}/e_{23})T'^{(1)} = (Q_{23}/e_{23})K_{23}$, if particle 1 is not excited by $T'^{(1)}$, and $(Q/e)T'^{(1)} = (Q_{123}/e_{123})T'^{(1)}$ if all particles (1, 2, 3) are excited by $T'^{(1)}$. The foregoing discussion has clarified the deviation of Eq. (32) of Ref. 1, except for the fact that the T's appearing in that reference are now denoted as T''s.

From Eq. (20) we have, clearly,

$$\langle ijk|(T'^{(1)}+T'^{(2)}+T'^{(3)})|ijk\rangle_A = \langle ijk|(K_{12}+K_{13}+K_{23})|ijk\rangle_A , \tag{22}$$

and if one defines $\Delta T'$ through,

$$T' = K_{12}+K_{13}+K_{23}+\Delta T' , \tag{23}$$

Eq. (22) implies $\langle ijk|\Delta T'|ijk\rangle_A = 0$. The expression for $K_{ijk,\,ijk}$, Eq. (34) of Ref. 1, is correct and may be written (with T' replacing T) as,

$$\begin{aligned} K_{ijk,\,ijk} &= \langle ijk|(f_{12}^\dagger+f_{13}^\dagger+f_{23}^\dagger)(T'^{(1)}+T'^{(2)}+T'^{(3)}-K_{12}-K_{13}-K_{23})|ijk\rangle_A , \\ &= \langle ijk|(f_{12}^\dagger+f_{13}^\dagger+f_{23}^\dagger)\Delta T'|ijk\rangle_A . \end{aligned} \tag{24}$$

As stated in Ref. 1, this result is in agreement with the results of Rajaraman and Bethe.[3] *Now Eq. (38) of Ref. 1 reads correctly if we define the* ΔT *appearing there as,*

$$\begin{aligned} \Delta T &\equiv (f_{12}^\dagger+f_{13}^\dagger+f_{23}^\dagger)(T'-K_{12}+K_{23}+K_{13}) , \\ &\equiv (f_{12}^\dagger+f_{13}^\dagger+f_{23}^\dagger)(\Delta T') , \end{aligned} \tag{25}$$

and also keep in mind that the T, $T^{(1)}$, $T^{(2)}$, $T^{(3)}$, *and* ΔT *in Eqs. (31)–(37) of Ref. 1 have been changed to primed quantities.*

Finally the *last* identity in Eq. (37) should be deleted and an extra factor of $(t_1 + t_2)$ should be removed in the last line of Eq. (B3). These corrections and the extended discussion given above should provide a significant clarification of the derivation presented in Ref. 1.

[1]J. da Providência and C. M. Shakin, Phys. Rev. C 5, 53 (1972).

[2]H. A. Bethe, Phys. Rev. 138, B804 (1965).

[3]R. Rajaraman and H. A. Bethe, Rev. Mod. Phys. 39, 745 (1967).