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Analysis of the Total (n, p) Cross Sections Around 14 MeV with the Pre-Equilibrium Exciton Model*

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The absolute value of (n, p) reaction cross sections, as given by the pre-equilibrium exciton model is estimated using Fermi gas approximation.

A general agreement with experimental data, particularly in the gross A dependence is obtained. The energy range considered is 10–20 MeV, and the nuclei are those with $A > 100$: In these ranges evaporation is negligible and the analysis is rather easy. The approximate lifetime of one single-particle exciton in nuclear matter is also deduced.

I. INTRODUCTION

It has been shown recently that the pre-equilibrium emission mechanism suggested by Griffin[1] and developed mainly by Williams,[2] Blann,[3,4] Harp and Miller[5] is responsible for proton emission in some (n, p) reactions around 10–20 MeV of incident neutron energies.[6]

In order to study the validity of this model for nuclear reactions, the analysis has been extended to (n, p) reactions in nuclei with $A > 100$. Indeed, it was shown[7] that in this mass and energy range, proton emission cannot be accounted for by the statistical evaporation theory (see Fig. 1). Up to now, the bulk of results on (n, p) reactions on these heavy nuclei, consisting of about 75 cross-section values at a 14.5-MeV neutron energy measured by activation method, and of a few proton spectra around these energies, could not be interpreted.

This paper presents an estimate of the absolute value of the expected cross section for (n, p) reactions based on pre-equilibrium emission theory, as a function of A, using the Fermi gas model; then the results of calculation are compared with experimental data. As a consequence the lifetime of a single-particle exciton in nuclear matter is estimated.

2. CROSS-SECTION CALCULATION

The absolute cross section for precompound emission, integrated over the solid angle, which can be deduced with minor modifications from the treatment given by Williams,[2] is

$$\frac{d\sigma_{pc}}{d\epsilon} = \tfrac{2}{3}(2s+1)\sigma_c \frac{m\epsilon\sigma_{inv}(\epsilon)}{\pi^2\hbar^3} \frac{\hbar}{4\pi} \frac{1}{|M|^2} \frac{1}{g^4E^3} \times \sum_{\substack{n=3 \\ (\Delta n=2)}}^{\bar{n}} \left(\frac{U}{E}\right)^{n-2} (n+1)^2(n-1), \quad (1)$$

where $s = \frac{1}{2}$ = proton spin; σ_c is the optical reaction cross section for the neutron, ϵ is the kinetic energy of the proton; $\sigma_{inv}(\epsilon)$ is the inverse reaction cross section for the proton; E is the excitation energy of the compound nucleus; U is the excitation energy of the residual nucleus; g is the average single-particle level spacing in the Fermi gas model, for compound and residual nuclei; n is the number of excitons (particles + holes) in the precompound nucleus; m is the nucleon mass; $|M|^2$ is the average value of the matrix element square in the intranuclear cascade process $n \rightarrow n+2$; $\bar{n}$ is the limiting value for n (when equilibrium is reached). In the summation the index n varies in steps of 2 from 3 to $\bar{n}$.

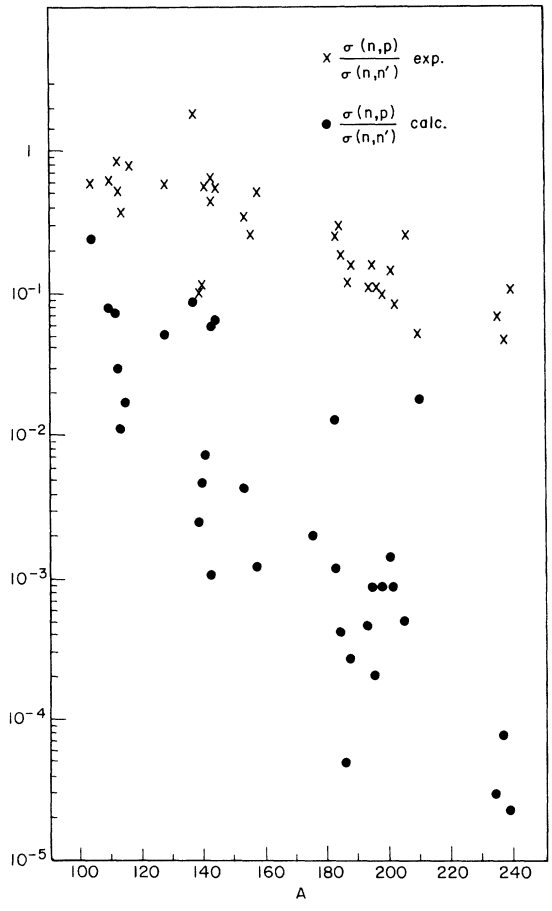

FIG. 1. Plot of experimental (×) and calculated (●) ratios between $\sigma(n,p)$ and $\sigma(n,n')$ as a function of mass number A for $100 < 240$, as given in Ref. 6. The calculation is done on the basis of statistical evaporation at equilibrium. Experimental points are at least an order of magnitude higher than the calculated ones. Evaporation at equilibrium does not seem responsible for proton emission.

The basic assumptions in formula (1) are:
(a) the presence of a pure two-body residual interaction, so that process $n \rightarrow n+2$ is the dominant one in the cascade;
(b) the statistical assumption in matrix elements;
(c) the principle of the detailed balance, taken into account with conservation of energy;
(d) the independence of the behavior of an exciton from the presence of other excitons;
(e) the Fermi gas model for level densities;
(f) negligible depletion due to particle emission;
(g) charge conservation.

It can be observed at this point that (n, p) reactions in these energy and A ranges are well suited for a test of this mechanism, since, after a preliminary examination of formula (1), it can be easily seen that the first term of the sum $(n=3)$ is by far the highest one. Hence the main contribution to proton emission comes from the first step in the nuclear cascade, so that the correction for the depletion[4] due to emission itself is negligible, as requested by point (f). This is physically due to the Coulomb barrier on outgoing protons, which depresses both proton emission from the latest steps of the cascade and proton (but not neutron) evaporation once equilibrium is reached.

As for point (g), it is responsible for the factor $\frac{2}{3}$ appearing in formula (1).

Indeed a more refined treatment of the detailed balance, taking into account charge conservation, requires use of level densities with fixed numbers of protons (particles and holes) and of neutrons (particles and holes) separately. Let p_+, h_+ be the numbers of proton particles and proton holes, and p_-, h_- those of neutrons. Then it can be shown that the corresponding level densities are (at equal neutron and proton single-particle level density $\frac{1}{2}g$)

$$\rho_{p_+h_+p_-h_-}(E) = \frac{g(gE)^{p+h-1}}{2^{p+h}p_+!h_+!p_-!h_-!(p+h-1)!},$$

being $p = p_+ + p_-, h = h_+ + h_-$.

To avoid unnecessary complications, we calculated the resulting correction factor $(\frac{2}{3})$ for the first term $(n=3)$ of the summation, which is dominant and applied the same factor to all terms,

though rigorously they should undergo smaller corrections.

Formula (1) is the combination of the expressions

$$\frac{d\sigma_{pc}}{d\epsilon} = \sigma_c \sum_n \frac{dW^{ph}}{d\epsilon} \frac{1}{\lambda_n^+(E)},$$

$$\frac{dW^{ph}}{d\epsilon} = \frac{m\epsilon\sigma_{inv}(\epsilon)(2S+1)}{\pi^2\hbar^3} \frac{\rho_{p-1,h}(U)}{\rho_{p,h}(E)} \frac{2}{3}, \qquad (2)$$

$$\rho_{p,h}(E) = \frac{g(gE)^{p+h-1}}{p!h!(p+h-1)!},$$

$$\lambda_n^+(E) = \frac{2\pi}{\hbar} |M|^2 \frac{g^3E^2}{p+h+1}, \qquad (3)$$

where p, h are the numbers of particles and holes in the precompound nucleus ($p+h=n$, $p-h=1$), $\rho_{p,h}(E)$ is the level density with p particles and h holes, $\lambda_n^+(E)$ is the rate for process $n \to n+2$ (the cascade process), $dW^{ph}/d\epsilon$ is the specific rate of proton emission from the precompound nucleus. The absolute value of $d\sigma_{pc}/d\epsilon$ depends therefore on $|M|^2$ through $\lambda_n^+(E)$.

The value of $|M|^2$ is not given by Williams and the other quoted authors, and it is implicitly assumed to be independent of E. This value is related to the two-body residual interaction in nuclear matter.

3. CASCADE DESCRIPTION THROUGH TWO-BODY COLLISIONS

An old treatment of the intranuclear cascade was given by Goldberger[8] with Monte Carlo calculations. There the basic assumption was that nucleon collisions inside the nucleus can be treated in a semiclassical way, related to the Fermi gas model, with the same cross section as that of free nucleons collisions, and with conservation of energy and momentum; the only important difference was that the final states forbidden by the Pauli principle were suppressed.

The Goldberger treatment was proposed for an energy range around ~100 MeV, to have short wavelengths of nucleons in the cascade.

Apart from the mathematical treatment, the physical assumptions in the two models (Goldberger and exciton models) are quite similar and, namely, the dominance of the two-body interaction and the effect of the Pauli principle. Goldberger, however, does not take into account hole-hole scattering. He uses free-nucleon cross sections and the Pauli principle to calculate the mean free path of a fast nucleon in nuclear matter. This mean free path is strictly related to our $|M|^2$, and therefore to $\lambda_n^+(E)$.

This procedure has the virtue of *accounting also for the conservation of momentum* in the intranuclear cascade, which *has not been included so far in the exciton model.* Conservation of momentum is useful only for calculation of $\lambda_n^+(E)$, and not for the detailed balance [formula (2)], since in the processes of neutron capture and proton emission a large and variable fraction of momentum is taken by the nucleus as a whole, particularly at energies not too high with respect to Fermi energy E_F.

This line was again followed by Kikuchi and Kawai[9] with a simpler procedure to calculate the phase space integral allowed by the Pauli principle, again ignoring hole-hole scattering and using free-particle collision cross sections. The results are summarized in the following section, with a discussion on the changes required when applying the results around Fermi energy.

4. MEAN FREE PATH OF AN EXCITON

The mean free path of an exciton (for instance a particle) of momentum $P_0 > P$ is calculated in semiclassical approximation assuming that a sphere S of radius P (Fermi momentum) is filled in the momentum space, and that the particle considered is outside this sphere. The average cross section (integrated over the solid angle) for a collision with a particle in S is given, according to Goldberger, by

$$\begin{aligned}\langle\sigma\rangle &= \frac{8\nu}{AP_0} \int d^3P_1 \int d^3P_f \int d^3P_2 \delta(\vec{P}_0 + \vec{P}_1 - \vec{P}_f - \vec{P}_2) \\ &\quad \times \delta(P_0^2 + P_1^2 - P_f^2 - P_2^2) \sigma(p_0, \vec{p}_0 \times \vec{p}_f) \\ &= \frac{\nu}{AP_0} \int d^3g \int d^3P_1 |\vec{P}_0 - \vec{P}_1| \frac{1}{p_0^2} \delta(p_0 - p_f) \\ &\quad \times \sigma(p_0, \vec{p}_0 \times \vec{p}_f) \\ &= \frac{\nu}{AP_0} \int d^3P_1 \int d^3p_f |\vec{P}_0 - \vec{P}_1| \frac{1}{p_0^2} \delta(p_0 - p_f) \\ &\quad \times \sigma(p_0, \vec{p}_0 \times \vec{p}_f), \end{aligned} \qquad (4)$$

where A is the mass number of compound nucleus; $\vec{P}_0$ is the momentum of initial exciton (particle 0); $\vec{P}_1$ is the momentum of target particle 1 ($P_1 < P$); $\vec{P}_f$ is the final momentum of projectile ($P_f > P$); $\vec{P}_2$ is the final momentum of target particle ($P_2 > P$); $\vec{p}_0$ is the momentum of initial particle 0 in the c.m. system of 0 and 1; $\vec{p}_f$ is the final momentum of projectile in the previous c.m. system; $\vec{g} = \vec{p}_f - \vec{p}_0$ = transferred momentum; $\sigma(p_0, \vec{p}_0 \times \vec{p}_f)$ is the differential cross section in nucleon-nucleon collision, indicated also simply as σ; ν is the nucleon density in momentum space $= A/(4/3)P^3$. The integral is extended to the volume in momentum space allowed by the Pauli principle.

We are interested in relatively low energies (unlike Goldberger), and therefore σ is assumed to be isotropic, and to have a value $\sigma_T/4\pi$ corresponding to laboratory nucleon energy $\simeq E_F = p^2/2m$. The nucleus is assumed to be composed of equal numbers ($\frac{1}{2}A$) of protons and neutrons, and the exciton to be with equal probabilities a neutron or proton, particle or hole. The value of σ_T has to be taken $\sim(\frac{5}{8})\sigma_T(n,p) = \frac{5}{8}\times$total (n,p) cross section, to average properly over all types of collisions, as explained in Goldberger's paper.

The calculation of the integral (4) was performed[8] in the case $P_0 > P\sqrt{2}$, with a complicated method. Other authors[9] have calculated the same integral with a simpler method, taking σ constant with energy or in the approximation $\sigma \propto 1/k^2$. Numerical results are almost the same in a wide energy range and we use the analytical result corresponding to σ constant with energy.

In the case $P_0 > P\sqrt{2}$, the result is

$$\langle\sigma\rangle = \frac{1}{A}\,\frac{16\pi^2\nu\sigma}{3}\left[\frac{P^5}{P_0^{\,2}}\left(\frac{P_0^{\,2}}{P^2} - \frac{7}{5}\right)\right].$$

In the case $P < P_0 < \sqrt{2}\,P$, it is

$$\langle\sigma\rangle = \frac{1}{A}\,\frac{16\pi^2\nu\sigma}{3}\left\{\frac{P^5}{P_0^{\,2}}\left[\frac{P_0^{\,2}}{P^2} - \frac{7}{5} + \frac{2}{5}\left(2 - \frac{P_0^{\,2}}{P^2}\right)^{5/2}\right]\right\}. \tag{5}$$

This result can be written as a power series with respect to the exciton energy $u = (P_0^{\,2} - P^2)/2m$. For $P_0 < \sqrt{2}\,P$ the first term was found to be a good approximation; it gives for the bracket the value $3m^2u^2/P$. Furthermore we found that *to take into account only this first term is equivalent to performing calculations in the equidistant spacing model, and as a consequence the hole-hole scattering behaves in a fully symmetric way.*

Therefore

$$\langle\sigma\rangle = \frac{16\pi^2\nu\sigma}{A}\,\frac{m^2u^2}{P}\,. \tag{6}$$

The mean free path of an exciton of energy μ is

$$t = \frac{V}{A\langle\sigma\rangle}\,,$$

V being the nuclear volume. The average lifetime is then

$$\tau(u) = \frac{t}{v} = \frac{V}{A\langle\sigma\rangle v} = \frac{Vm}{A\langle\sigma\rangle P} = \frac{V}{16\pi^2\sigma m u^2 \nu},$$

$v = P/m$ being the velocity around Fermi energy. Since $V/\nu = 2\pi^3\hbar^3$, we have

$$\tau(u) = \frac{\pi\hbar^3}{8\sigma m u^2} = \frac{4\pi^2\hbar^3}{8\times\frac{5}{8}\times\sigma_T(n,p)\,m u^2} = \frac{4\pi^2}{5}\,\frac{\hbar^3}{\sigma_T(n,p)\,m u^2}\,, \tag{7}$$

which is A-independent.

5. A DEPENDENCE OF THE MATRIX ELEMENT

In the treatment by Williams[2] $\tau^{-1}(u) = \lambda_1^+(u)$ is written as

$$\lambda_1^+(u) = \frac{2\pi}{\hbar}\,|M|^2\rho_f(u)\,, \tag{8}$$

where $\rho_f(u)$ is the density of final states which can be reached in a single collision of one specified exciton, and $\rho_f(u) = \frac{1}{2}g(gu)^2$. This ρ_f does not account for momentum conservation.

To account for it, a new ρ_f' is more properly defined as given implicitly by the integral appearing in the expression of $\langle\sigma\rangle$[formula (4), first expression; recall $u = (P_0^{\,2} - P^2)/2m$]:

$$\rho_f'(u) = \nu^2 \int d^3P_1 \int d^3P_f \int d^3P_2\,\delta(\vec{P}_0 + \vec{P}_1 - \vec{P}_f - \vec{P}_2) \times \delta\left(\frac{P_0^{\,2} + P_1^{\,2} - P_f^{\,2} - P_2^{\,2}}{2m}\right)r\,.$$

Here r is a factor taking into account the spin and isospin degeneracy together with charge conservation; it turns out, however, that $r = 1$.

Comparing with (4), we have

$$\langle\sigma\rangle = \frac{8\nu\sigma}{AP_0}\,\frac{1}{2m\nu^2}\rho_f'(u) \simeq \frac{4\sigma}{mAP\nu}\rho_f'(u)\,.$$

Comparing with (6), we obtain

$$\rho_f'(u) = 4\pi^2\nu^2m^3u^2 = \frac{9}{4}\,\frac{A^3m^3u^2}{P^6}\,\frac{1}{A}\,,$$

since $\nu = A/\frac{4}{3}\pi P^3$.

Being that $g = \frac{3}{2}A/E_F = 3mA/P^2$, we have finally

$$\rho_f'(u) = \frac{g^3u^2}{12A} = \frac{1}{6A}\rho_f(u)\,. \tag{9}$$

Hence the effect of momentum and charge conservation is simply equivalent to adding a factor $1/6A$, as far as the density of accessible levels in formula (8) is concerned.

Also $|M|^2$ in (8), however, has to be reconsidered. Indeed our value of τ^{-1} [formula (7)] can be written as

$$\tau^{-1}(u) = \lambda_1^+(u) = \frac{2\pi}{\hbar}\,|M_0|^2\rho_f'(u) = \frac{5\sigma_T(n,p)mu^2}{4\pi^2\hbar^3}\,,$$

$|M_0|^2$ being the square of the newly defined matrix element.

Substituting $\rho_f'(u)$ with (9), we have

$$|M_0|^2 = \frac{15}{2\pi^3}\,\frac{m\sigma_T(n,p)}{\hbar^2}\,\frac{A}{g^3}\,, \tag{10}$$

which is proportional to A^{-2}.

$|M|^2$ in formula (8) (Williams's formula) is then given by

$$|M|^2 = |M_0|^2 \frac{1}{6A} \propto A^{-3}, \tag{11}$$

and it is energy-independent.

This is the final result obtained: In our approximations, $|M|^2$ *appearing in formula (1) is given simply by* $|M_0|^2/6A$ *and is found to depend on A like* A^{-3} *and to be energy-independent.* Of course this energy-independence comes from taking the first term in a power series development, and is expected to hold for incoming energies $\lesssim$20–25 MeV.

Formula (3) can be written now as follows:

$$\lambda_n^+(E) = \lambda_1^+(E) \frac{2}{n+1}.$$

This relationship between λ_n^+ and λ_1^+, which is implicit in Williams's paper[2] and there deduced from an appropriate averaging procedure, is not changed at all by our treatment.

Since our analysis of experimental results will be based on formula (1), we extract from (1) the factor $|M|^2 g^4$ in the denominator, which now we know to be proportional to $A^{-3}A^4 = A$ and estimate the ratio $|M|^2 g^4/A$ in our model. Using (10) and (11) it is obtained:

$$|M|^2 \frac{g^4}{A} = \frac{5}{4\pi^3} \frac{m\sigma_T(n,p)}{\hbar^2} \frac{g}{A}. \tag{12}$$

If a is the level density parameter,[10] we know that

$$g = \frac{6}{\pi^2} a \simeq \frac{6}{\pi^2} \frac{A}{7.5} \text{ MeV}^{-1}$$

from the slow-neutron data,[11] so that $g/A = 0.081$ MeV^{-1}. Proportionality of a and g to A is related to the Fermi gas model.

For $\sigma_T(n,p)$ we take the value of the total neutron-proton cross section[12] at a neutron energy around Fermi energy E_F. To be completely consistent with a Fermi gas model in a square well, the value of E_F should be given by $E_F = (A/g) \times (2/3)$, which is near 20 MeV. However such a model is oversimplified, and we prefer to use independent results from other precompound reactions,[13] based also on the behavior of excitation functions. These results give an E_F value around 40 MeV. Therefore we take[12] $\sigma_T(n,p) = 20 \times 10^{-26}$ cm^2.

At this point a discussion is required on the validity of our approximations. The effective two-body residual interaction in the nuclear matter is expected to be much lower than the free collisions. The average potential seen by a nucleon in the Fermi gas model takes into account the largest fraction of the two-body potential in an average way. It cannot however account for the strong oscillation of the two-body potential, due to the change from attractive to repulsive force, at small distances between nucleons. Therefore the two-body free interaction has to be renormalized to a two-body residual interaction which is expected to give effective cross sections roughly smaller by 1 order of magnitude than free cross sections.

Other less important effects are due to:
(a) the fact that wavelengths/2π inside the nucleus, for incoming energies around 10–20 MeV, are not small as compared with internucleon separation, but of the same order; the effect on (6) is difficult to be estimated, since coherence may give cancellation as well as addition of matrix elements;
(b) the velocity of an exciton in the nuclear matter has been taken equal to that of a free particle with energy E_F.

These considerations may change drastically the value of $|M|^2$ to a smaller value $|M_{\text{eff}}|^2$. The Pauli-principle effect, however, is unaltered; and the value of E_F being quite high and roughly A-independent, the value of $|M_{\text{eff}}|^2 g^4/A$ is still expected to be roughly A-independent and energy-independent for a wide range of exciton energies of the order of 20 MeV. Our formulas for λ_1^+ can indeed be essentially expressed as a first term of a power series with respect to (u/E_F). Density of states, roughly proportional to $(u/E_F)^2$ (owing to the Pauli principle) is separated from the matrix element square $|M|^2$, which for excitons around the Fermi top should have a nearly energy-independent value, even if the effective two-body interaction is quite different from free-nucleon interaction.

The value deduced from (12) being ~0.0035 MeV^{-2}, the comparison between experimental data and formula (1) is expected to give for $|M_{\text{eff}}|^2 g^4/A = \alpha$ a value somewhere between 10^{-4} and 10^{-3} MeV^{-2}, and roughly A-independent.

6. SHELL-EFFECT CORRECTIONS

We have considered the usefulness of taking into account shell effects on level density, relying on g values deduced from experimental data,[11] and distinguishing g_c (compound nucleus) from g_r (residual nucleus). Some doubt, however, can be raised about the correctness of this procedure. Indeed in the exciton model the residual- and compound-nucleus level densities are those described by the model, that is the density of states with definite number of excitons, often having large angular momenta.

This density may be quite different from the "true" level density deduced for each nucleus by slow-neutron measurements, which have in addition low angular momentum, although some rela-

tion with it is to be expected. Therefore this correction has not been made in our analysis, except for one closed-shell nucleus. Nevertheless, the procedure which should be followed to make it, is given here below. In the deduction of formula (1) both g_r and g_c appear. The value of λ_n^+ completely depends on g_c. It is found also for our (n, p) reactions that the first term $(n=3)$ of the series dominates: that is the whole theory is simply a statistical theory of (n, p) knockout. For the first term only, ratio $1/|M|^2g^4$ is to be written as $g_r^2/|M|^2g_c$,[6] where again $|M|^2$ (coming from λ_n^+) is proportional to A^{-3}, and therefore to g_c^{-3}. Hence the appropriate procedure should be to deduce the value of α from the bulk of the experimental data far from the closed shells, and then to replace $|M|^2g^4$ with

$$\alpha\left(\frac{g_c}{g}\right)^4\left(\frac{g}{g_r}\right)^2 A,$$

g being the Fermi gas value of single-particle level density, and g_c, g_r the "true" values.

7. COMPARISON WITH EXPERIMENTAL DATA

A. Proton Spectrum Shape

The starting point of this analysis is the observation that (n, p) cross-section values at 14 MeV are at least by 1 order of magnitude higher than those calculated with the evaporation theory, as shown in Fig. 1. Therefore, statistical evaporation at the equilibrium condition is assumed not to give appreciable contribution, and thus can be disregarded.

We will try now to show that the mechanism responsible for (n, p) reactions in the energy range 10–20 MeV, and for nuclei with mass $A > 100$ is the pre-equilibrium emission mechanism.

The first point concerns the emitted proton spectrum shape. There are only a few measurements of the proton spectrum emitted in (n, p) reactions. One instance is the measurement made on a CsI crystal between 12 and 22 MeV by Langkau,[14] already analyzed in a preceding paper[6] on the basis of pre-equilibrium emission. The calculation was done using Williams's formula, which does not give the absolute cross-section value and which is essentially the same thing as our formula (1) as to the emitted particle spectrum shape. A very good agreement was found with the spectrum shape for the three highest-energy points (18, 19.6, 21.5 MeV) while in the other cases an important contribution of pre-equilibrium emission, certainly exceeding 50%, is evident; it is to be noted that in the lowest-energy points the experimental errors are important. The incident energy dependence of the relative

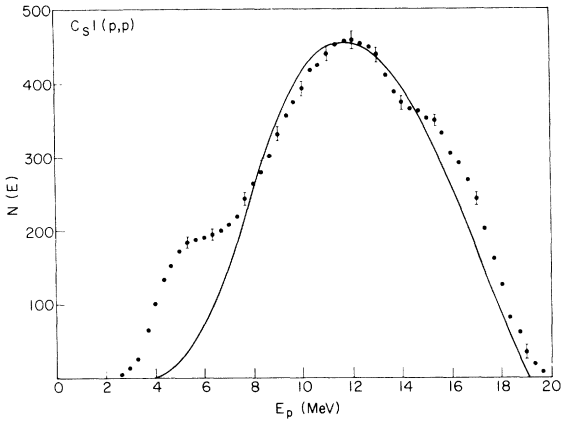

FIG. 2. Experimental proton energy spectrum at 21.5-MeV neutron energy for CsI. The solid line represents the precompound emission calculated in relative units by means of Williams's formula (Ref. 2).

cross-section values was also found in agreement with the pre-equilibrium emission hypothesis. Figs. 2 and 3 show an example of CsI(n, p) spectrum and the experimental and calculated cross-section values as a function of energy.

There are three other proton spectra in the mass range here considered, and, namely, those of ^{103}Rh(n, p), ^{181}Ta(n, p), and ^{197}Au(n, p) measured in our laboratory many years ago.[15,16] The measurement on Rh was made in the best experimental conditions, that is with mass discrimination of the emitted particles, so that the contribution (rather important, as it was found) of deuterons was subtracted. In the two other cases this was not done, so that some deuteron contributions must be expected in the lower-energy part of the spectrum. Figure 4 shows these three experimental spectra, together with the calculated ones

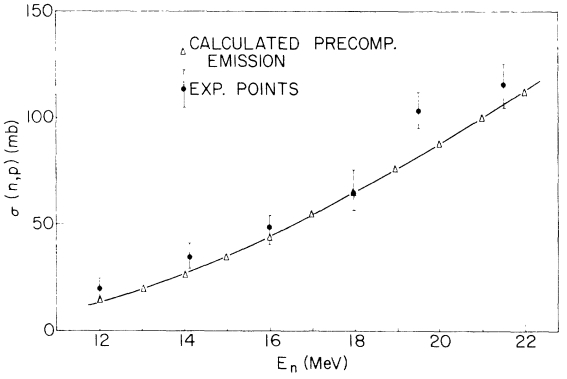

FIG. 3. Energy variation of cross section for the (n, p) reaction on CsI as a function of neutron energy, compared with calculated precompound emission in relative units (Williams's formula).

(Williams's formula). The agreement is quite good, except for the lower-energy part of the two heaviest nuclei, where deuterons are certainly present.

B. Absolute Cross-Section Value

In order to study the absolute (n, p) cross-section values, we have examined cross-section experimental determinations made by the activation method on about 75 nuclei with mass ≥ 100, at a neutron energy around 14.5 MeV.

In the case of the activation method measurements, there is certainly no contribution by other reactions (such as n, d), the nuclide under study is always one and certainly known, and the cross-section value refers to the emission in the total solid angle. The last point is generally not fulfilled in other kinds of measurements.

For our analysis use was made of the experimental $\sigma(n, p)$ values as given in the collection by Csikai *et al.*[17] integrated with more recent values.[18]

Unfortunately, as pointed out by Csikai, these experimental results seem to be affected by errors much more important than those given by the authors. Indeed, even when the errors reported together with the measurement are quite reasonable, that is ±10 or 20%, the same measurement made in a different laboratory very often gives a result falling definitely out of the reported uncertainty. In extreme cases, the disagreement can be of 1 order of magnitude or even more, but a factor of 2 is very common.

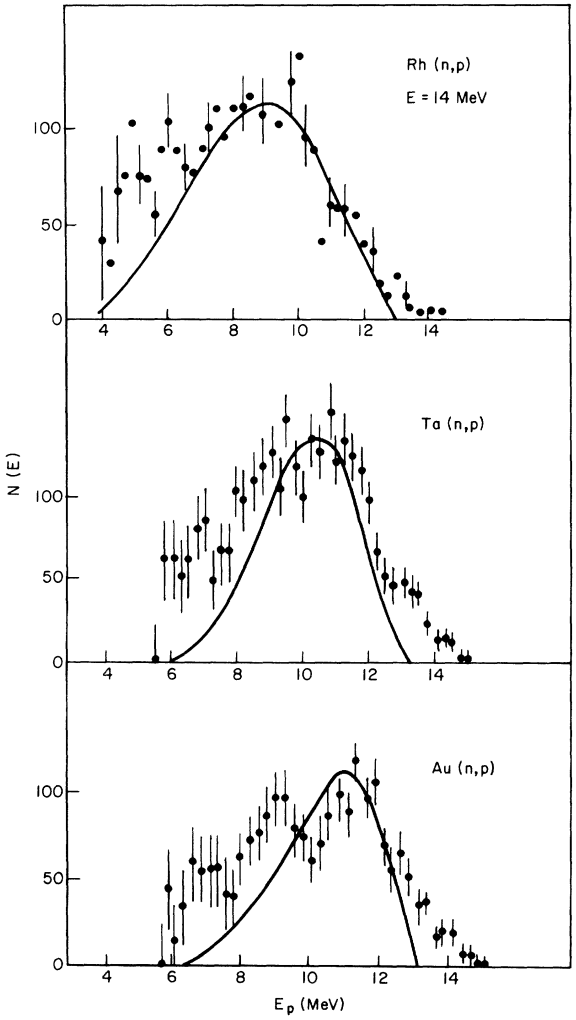

FIG. 4. Comparison between experimental proton spectra and calculated precompound emission (solid line) in relative units for (n, p) reactions on ^{103}Rh, ^{181}Ta, and ^{197}Au.

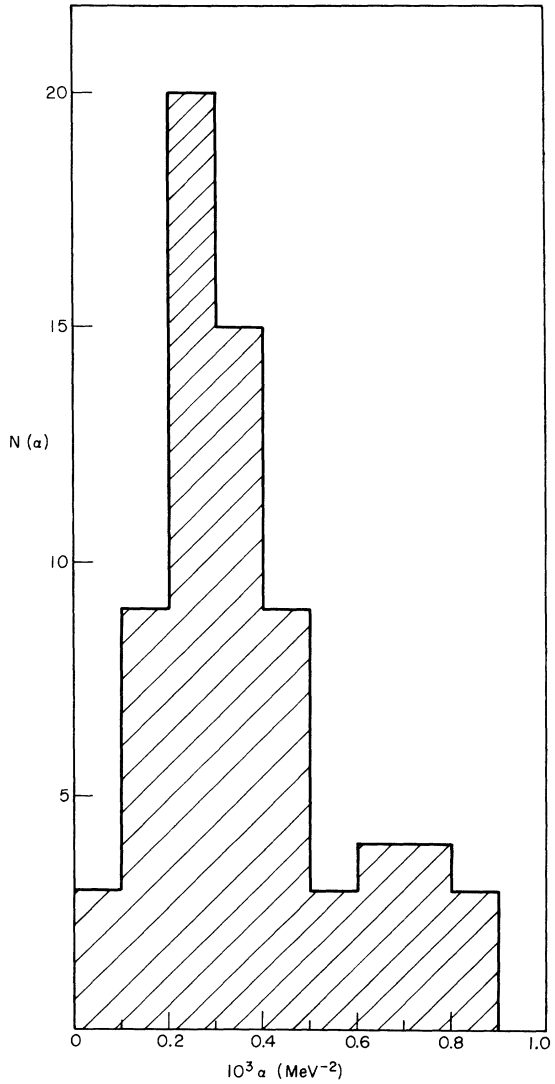

FIG. 5. Distribution of α values extracted from the (n, p) reaction cross-section value. A marked peak at 3.3×10^{-4} MeV^{-2} is shown.

The analysis was conducted as follows. Starting from formula (1) integrated over the emitted proton energy, the result of the calculations developed in the preceding section was expressed as

$$|M|^2 g^4 = \alpha A ,$$

α being a constant to be determined. So that

$$\sigma(n, p) = \frac{2}{3} \frac{(2s+1)\sigma_c m}{4\pi^3 \hbar^2 \alpha A E^3} \int \epsilon \sigma_{\text{inv}}(\epsilon) \times \sum_{\substack{n=3 \\ (\Delta n = 2)}}^{\bar{n}} n(\Delta n = 2) \left(\frac{U}{E}\right)^{n-2} (n+1)^2 (n-1) d\epsilon . \tag{13}$$

Giving $\sigma(n, p)$ its experimental value, all the other parameters being known, the value of α can be easily extracted. The values of σ_c and $\sigma_{\text{inv}}(\epsilon)$ were taken by the optical-model calculations by Lindner[19] for neutrons and by Mani, Melkanoff, and Iori[20] for protons.

It is to be observed at this point that the excitation energy values of residual and compound nuclei are calculated taking into account the pairing energy,[21] which is subtracted, when the numbers of protons or neutrons (or of both) are even.

The results of the described analysis are shown in Fig. 5 which shows the distribution of the α values extracted from all the 75 (n, p) cross-section values. This distribution shows a prominent peak at $\alpha \sim 3.3 \times 10^{-4}$ MeV^{-2}; the width of α distribution is what can be expected from the described experimental situation.

This value is in fair agreement with the theoretical prediction, which gives for α the approximate limitation:

$$10^{-4} \leq \alpha \leq 10^{-3} \text{ MeV}^{-2} .$$

The value of α extracted by this analysis can now be used to calculate the (n, p) cross-section value, using formula (13).

Applying the remark of Sec. 6, we can try to introduce the shell-model effect on level density. This is important for nuclei near closed shells and for residual nucleus level density. Indeed, the residual nucleus is left at rather low excitation energies, where the shell-model effect is important.[11]

The only case where it seems worthwhile applying this correction, is the case of Bi(n, p). This cross section was measured in several laboratories, so that its value is most likely to be correct, and the residual nucleus ^{209}Pb has $Z = 82$ and $N = 127$, that is, it is very close to being doubly magic. Hence, the value of a_r (=27.8) given by the $A/7.5$ (Fermi gas model) law is very different from the value obtained by slow-neutron measurements,[11] that is $a_r = 9.5$. Indeed, the value of cross section calculated by formula (1) gives $\sigma_{\text{calc}} = 4.5$ mb, which is much higher than the experimental value $\sigma_{\text{exp}} = 0.85$ mb. Correcting σ_{calc} by multiplying it by $(9.5/27.8)^2$, we obtain $\sigma_{\text{calc}} = 0.39$ mb, which is much closer to the experimental

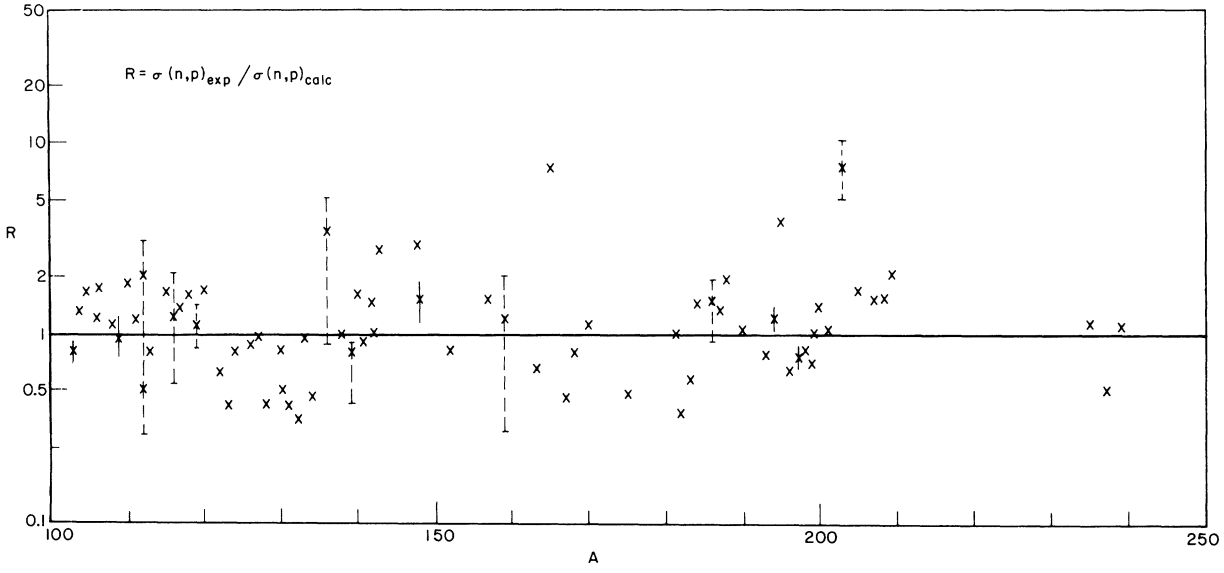

FIG. 6. Plot of $\sigma_{\text{exp}}(n, p)/\sigma_{\text{calc}}(n, p)$ ratio, as a function of mass number A. The vertical bars show in some instances the experimental errors. This figure shows that the discrepancies between calculations and experiments are of the order of the experimental errors and that the gross behavior of the α value as a function of A is that of a constant, within experimental uncertainty and within the approximations implied by the statistical nature of the model. This fact is supported also by the markedly peaked histogram of Fig. 5.

value.

In all other cases the correction is not so important, and practically not out of experimental uncertainty.

From the Bi analysis, it does not seem necessary to make a correction to a_c, the level density parameter of compound nucleus. As shown in (6) the dependence on a_c is very strong, $\sigma \propto 1/a_c^4$, so that if $a_c = 9.5$ a much greater disagreement than before is obtained. Therefore, it seems that at a 20-MeV excitation energy, the shell-model effect is already diminished.

Figure 6 shows the ratio $\sigma_{exp}(n, p)/\sigma_{calc}(n, p)$, where only the Bi value is corrected for the shell-model effect. It is here shown that the gross behavior of the α value seems to be A-independent, as the theory predicts, within the rather large experimental errors and within the approximations implied in the statistical nature of the model.

It can be concluded that n, p reaction cross sections are represented by formula (1) within the experimental errors, with only a few exceptions. The shell-model corrections to the Fermi gas model probably contribute to the dispersion of α values around their average, but probably to a much lesser extent than the uncertainty in the experimental values does.

8. LIFETIME OF A SINGLE EXCITON

The value found experimentally for α can now be used to estimate lifetime $\tau(u)$ and mean free path t of an exciton in nuclear matter, using formulas (7) and (8) with $|M_{eff}|^2$ instead of $|M|^2$.

The value of t has to be interpreted as the mean free path of a "localized" exciton, whose wave function is a superposition of quasidegenerate single-particle states in the average potential, and whose position and momentum are sufficiently defined. This is conceptually possible in the limiting case of nuclear matter: In our approximations, τ and t do not depend on A.

Ratio $|M_{eff}|^2/|M|^2$ has been found to be ~0.0035/0.000 33 $\simeq$ 10 so that, from formula (7), $\tau_{eff}(u) \simeq 10 \times \tau(u) = 10 \times 4.7 \times 10^{-21} \times u_{MeV}^2$ sec, and therefore

$$\tau_{eff}(u) \times u_{MeV}^2 = 4.7 \times 10^{-20} \text{ sec MeV}^2 .$$

The relationship between τ_{eff} and α is in general

$$\tau_{eff} = \frac{\hbar}{2\pi} \frac{3}{E_F} \frac{1}{\alpha u^2} .$$

An exciton of energy $u \simeq 10$ MeV lasts ~4.7 $\times 10^{-22}$ sec. The corresponding mean free path $t = v \times \tau$, with v roughly corresponding to Fermi energy, is about 30 fm, ~4 times larger than the nuclear radii for the heaviest nuclei. The wavelength/2π of the nucleons around Fermi energy is of the same order as the internucleon separation, and therefore somewhat smaller than the nuclear radius for $A > 100$. Owing to the Pauli principle, τ and t are inversely proportional to u^2.

One may wonder whether there is some relationship between τ_{eff} and the true lifetime of some real eigenstates of the nucleus: The answer is that there is no relationship. Indeed the width $\Gamma = \hbar/\tau_{eff}$ represents only the amplitude of mixing of the single-particle eigenstates (eigenstates of the Hamiltonian H_0 associated with the average single-particle potential) due to the residual two-body interaction. The true eigenstates of the total Hamiltonian $H_0 + V_{two\text{-}body}$ are different. The eigenstates of H_0 represent dynamical states, coherent superpositions of width Γ of the true eigenstates: The model of pre-equilibrium emission is based on these dynamical states. Therefore, these considerations cannot be applied to deduce the lifetime of specific states, such as isobaric analog states, nor the lifetime of residual nucleus, which could be even ∞ for particle emission, when $u <$ binding energy of particle.

It has been often stressed that the approximate validity of the shell-model implies long lifetimes of single-particle states. This is true for very low excitation only: Indeed we have seen that Γ is proportional to u^2, and that for $u = 10$ MeV one has $\tau_{eff} \simeq 4.7 \times 10^{-22}$ sec, and hence $\Gamma \simeq 1.3$ MeV.

9. CONCLUSIONS

The qualitatively successful comparison between the experimental (n, p) cross sections and the absolute value predicted by the model, together with the previously found agreement between theory and experiments for some nuclei as for proton spectra and excitation function,[6] strongly suggest that the precompound emission as described by the exciton model is the mechanism through which (n, p) reactions actually proceed in the energy range 10–20 MeV for $A > 100$.

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New High-Spin Isomer 2.3-Day ^{198m}Au and the ^{198}Au Level Structure*

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In 18-MeV deuteron bombardments of enriched ^{200}Hg, we have produced a new isomer 2.27 ± 0.05-day ^{198m}Au, which is believed to be analogous to the $(h_{11/2}\pi, i_{13/2}\nu)12^-$ isomer in ^{196}Au. No β^- branching is observed in the decay of ^{198m}Au. The isomeric transition is followed by a cascade of 204.10- and 180.31-keV γ rays to a 123-nsec level at 312.10 keV which deexcites to ground by a sequence of 97.21-keV ($E1$) and 214.89-keV ($E2$) transitions. A revised interpretation of the results of earlier (n, γ) and $(d, p\gamma)$ studies of the ^{198}Au level structure is presented and shell-model assignments for the levels populated in the isomeric decay are proposed. It is argued that the $E1$ deexcitation of the 123-nsec isomer involves the l-forbidden single-particle transition $(d_{3/2}\pi, i_{13/2}\nu)5^+ \rightarrow (d_{3/2}\pi, f_{5/2}\nu)4^-$.

I. INTRODUCTION

Most of the information now available on the ^{198}Au level structure has been obtained in extensive neutron-capture γ-ray studies.[1-3] The level spectrum, including a negative-parity first excited state at 55.2 keV, has been established mainly on the basis of energy sums and transition intensities.[4] A 123-nsec isomer, deexciting by a sequence of 97.2-keV ($E1$) and 214.9-keV ($E2$) transitions, has been studied in ^{197}Au$(d, p\gamma)$ by Bonitz[5] and in ^{197}Au(n, γ) by Löbner *et al.*[6] Since the latter workers obtained some indication that the isomer deexcites through the 55.2-keV level, they placed the 123-nsec level at 367.3 keV, thereby accommodating a known primary capture γ ray; J^π values of 1^+ or 3^+ were inferred.

In ^{196}Au, a 12^- isomer, arising from the coupling of an $h_{11/2}$ proton and an $i_{13/2}$ neutron, has been known for several years.[7-10] More recently, we discovered a 19-h isomer in ^{200}Au, which decays predominantly by β^- emission to high-spin levels in ^{200}Hg[11]; very recent NMR measurements with oriented ^{200m}Au nuclei have shown that this isomer is analogous to the 12^- isomer in ^{196}Au.[12] In the course of a more detailed investigation of