Penetration Matrix Elements in the Internal Conversion of the 150.4-keV Transition in ^{177}Lu

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The K- and L-subshell internal-conversion coefficients of the anomalous 150.4-keV transition in ^{177}Lu have been measured with a double-focusing electron spectrometer and a Ge(Li) γ-ray spectrometer. The analysis of the results gave the $M2$ content, $Q=(0.92\pm0.88)\%$, and the penetration parameters, $\lambda_1=-21.4\pm2.3$, and $\lambda_2=-660\pm900$. Using these results along with the known lifetime of the 150.4-keV level, the $E1$ and $M2$ γ-ray matrix elements and the $\int \vec{J}_n\cdot\vec{r}$ and the $\int \vec{J}_n\cdot\vec{\nabla}$ type penetration matrix elements were evaluated and compared with those of similar transitions in ^{175}Lu and ^{181}Ta and with the estimates of the Nilsson model.

INTRODUCTION

Dynamic penetration effects in the internal conversion of γ rays are more easily observable in highly retarded electromagnetic transitions.[1,2] It has been established from the studies[3-6] in the decay of 1.9-h ^{177}Yb that the 150.4-keV transition is a retarded $E1$ transition with a hindrance factor $H_W\approx2.9\times10^6$ with respect to the single-particle estimate.[7]

Figure 1 shows the partial decay scheme[6,8] of 1.9-h ^{177}Yb. The 150.4-keV transition takes place between the Nilsson orbits[9,10] $\frac{9}{2}^-[514]\uparrow$ and $\frac{7}{2}^+[404]\downarrow$. Since the hindrance of this transition is due to violation of the selection rules in the asymptotic quantum numbers[10,11] and not due to the K selection rule, large penetration effects could be expected[1,2] in its internal conversion. Such effects are indeed observed in the 396.3-keV transition[12,13] in ^{175}Lu and in the 6.2-keV transition[14,15] in ^{181}Ta between the same Nilsson levels and the penetration matrix elements have been obtained. Previous measurements[16,17] have indicated that the 150.4-keV transition has anomalous K conversion coefficient and L-subshell ratios. It is of interest to evaluate the penetration matrix elements in this case also in order to study the systematics of such transitions. In the present work the penetration matrix elements and the $M2/E1$ mixing ratio of the 150.4-keV transition have been evaluated from the measurement of the absolute conversion coefficients in the K shell and the L subshells.

The K conversion coefficient and L-subshell ratios of the 121.6- and 138.6-keV transitions were also measured in order to determine their mixing ratios.

EXPERIMENTAL

The samples for the conversion-electron measurements were made by vacuum evaporation of Yb_2O_3 enriched to 96.5% in ^{176}Yb onto ≈1-mg/cm^2 pure aluminum foil. The thickness of the deposit was of the order of 10 μg/cm^2. Pieces of this foil, 18 mm×1 mm in size, were irradiated in the pneumatic facility at the CIRUS reactor at Trombay for 1 h. Nearly 1 h after the irradiation the foil was stuck on a ≈0.8-mg/cm^2 aluminized Mylar film for the conversion-electron and γ ray measurements.

The γ spectrum was measured by a Ge(Li) detector having a resolution of ≈4 keV at 662 keV. The low-energy part of the γ-ray spectrum is shown in Fig. 2. The relative efficiency of the Ge(Li) detector as a function of energy was determined[18] by using ^{160}Tb and ^{152}Eu sources having γ rays of known relative intensity and also by using standard International Atomic Energy Agency sources. The relative intensities of the γ rays of ^{177}Yb were obtained from the analysis of the spectrum.

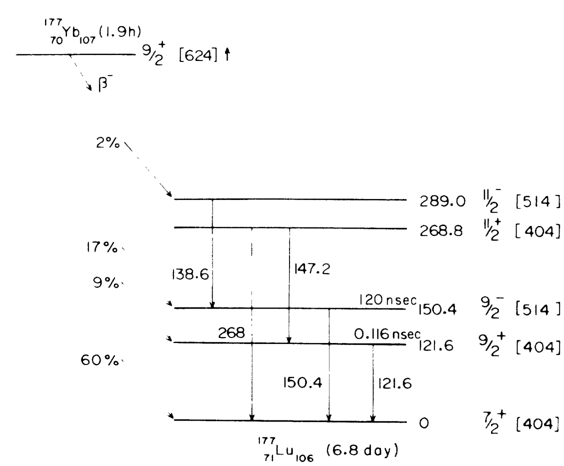

FIG. 1. Partial decay scheme of ^{177}Yb to ^{177}Lu.

The internal-conversion-electron spectra were observed using an iron-yoke double-focusing spectrometer at a momentum resolution of better than 0.1%. The K, L_{I}, L_{II}, and L_{III} conversion lines of the 150.4-, 121.6-, and 138.6-keV transitions and the K line of the 147.2-keV transition were measured. Since the source was short-lived (1.9 h), several runs had to be made. A typical L-subshell spectrum of the 150.4-keV transition is shown in Fig. 3. The decay of each conversion line was followed for several half-lives of the source. Any contribution of long-lived impurities and the 6.8-day ^{177}Lu daughter activity under each line was measured after complete decay of ^{177}Yb and was corrected for. Corrections for the background due to the β continuum in ^{177}Yb and for the absorption of electrons by the $\approx$1-mg/cm^2 counter window were applied. The spectra were analyzed in the manner explained in Ref. 18. For each run the ratios of the relative intensities of the individual lines with the K 150.4-keV line were determined and averaged over several runs. A typical L-subshell spectrum of the 121.6-keV transition is shown in Fig. 4. The long-lived background shown by crosses in the figure includes any contribution of the M lines of the 113-keV transition in the decay of the ^{177}Lu daughter activity. The normalized relative intensities of the electrons and γ rays are given in Table I. The errors in the relative intensities of electron lines shown in Table I were estimated from the statistical errors and the uncertainties in the background, the β continuum, the low-energy tails of the lines, the decay correction, and the absorption correction.

The absolute conversion coefficient of the 150.4-keV transition was determined by using a source of ^{169}Yb prepared in a similar way as the ^{177}Yb. The ratio of the electron intensity of the K 307.7-keV conversion line of ^{169}Yb to that of K 150.4-keV line of ^{177}Yb was measured using the electron spectrometer. Similarly the ratio of the 307.7- and 150.4-keV γ rays of ^{169}Yb and ^{177}Yb, respectively, was determined using a Ge(Li) detector, in the same geometry for both the sources. The 307.7-keV transition in ^{169}Tm is known to be a pure $E2$ transition from L-subshell ratios.[18,19] Its theoretical[20] K conversion coefficient is 0.049, which has been verified by experiment.[19] Using this as a standard the absolute K conversion coefficient of the 150.4-keV transition is obtained as $\alpha_K^{150.4} = 0.383 \pm 0.030$. The conversion coefficient was also measured in a similar way using the 411-keV transition of ^{198}Au as standard. The result was in agreement with the above value. The conversion coefficients of the 150.4-keV transition as determined above are shown in Table II and compared with the theoretical values of Hager and Seltzer calculated without taking penetration effects into account.[20] The errors in the absolute values of conversion coefficients were estimated by compounding the errors in the relative intensities of the electrons and γ rays and the estimated error in the intensity of the standard line due to possible effects of source thickness.

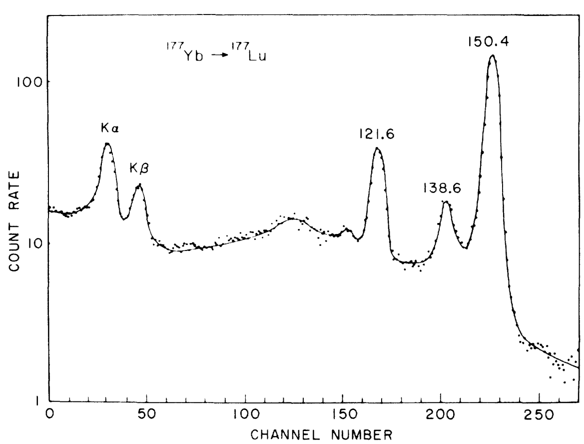

FIG. 2. The low-energy part of the γ-ray spectrum of ^{177}Yb taken with a 20-cm^3 Ge(Li) detector.

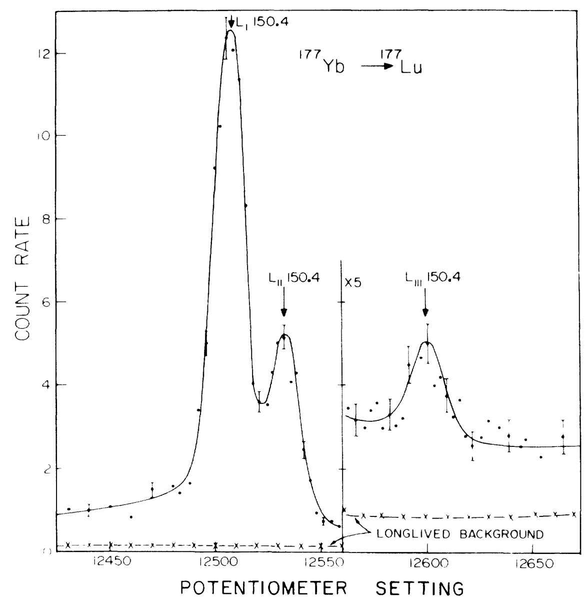

FIG. 3. A typical L-subshell conversion-electron spectrum of the 150.4-keV transition taken with a double-focusing electron spectrometer. The long-lived background shown by crosses was measured after the complete decay of the ^{177}Yb activity. In addition to this the background due to the β continuum also was taken into account in the analysis of the spectrum.

TABLE I. Relative γ and conversion-electron intensities. Intensities of γ rays and conversion electrons are normalized separately.

E_γ (keV)	I_γ	I_K	I_{L_I}	$I_{L_{II}}$	$I_{L_{III}}$	Multipolarity
150.4	100	100	22.9 ± 1.1	9.3 ± 0.54	1.19 ± 0.15	$E1$ [a] $+M2$
138.6	7.4 ± 0.7	22.7 ± 1.3	3.8 ± 0.4	0.54 ± 0.14	0.28 ± 0.14	$M1+6\%E2$
121.6	16.2 ± 0.9	53.9 ± 2.8	15.1 ± 0.9	7.52 ± 0.55	5.0 ± 0.4	$M1+22\%E2$
147.2	1.0 ± 0.3 [b]	3.2 ± 1.0				$M1+30\%E2$

[a] Anomalous $E1$ conversion.
[b] From Ref. 6.

ANALYSIS OF THE DATA

The experimental values of the conversion coefficients of the 150.4-keV transition were analyzed using the formulation of Hager and Seltzer.[21] The internal-conversion coefficient in the ith shell including the penetration effects in the lowest order is given by

$$\alpha^i(\text{Expt}) = Q\beta^i + (1-Q)\alpha^i(1 + A_1^i\lambda_1 + A_2^i\lambda_1^2 + A_3^i\lambda_2 + A_4^i\lambda_2^2 + A_5^i\lambda_1\lambda_2)\,, \quad (1)$$

where α^i(Expt) is the experimentally determined conversion coefficient, α^i is the theoretical value for $E1$, Q the possible $M2$ admixture in the transition, and β^i is the theoretical $M2$ conversion coefficient. The A coefficients, which depend on the electron radial wave functions, are tabulated by Hager and Seltzer.[21] The λ_1 and λ_2 are the penetration parameters containing the $\int \vec{J}_n \cdot \vec{r}$ and the $\int \vec{J}_n \cdot \vec{\nabla}$ type matrix elements, respectively.[21]

The A coefficients for the 150.4-keV transition in ^{177}Lu were obtained by interpolation from the tables.[21] In order to obtain approximate values for the parameters λ_1 and λ_2, plots of λ_2 against λ_1 for an assumed value of the parameter Q consistent with each of the measured conversion coefficients α^i(Expt) were made. A set of such plots for $Q = 0.01$ is shown in Fig. 5. From this figure a region of agreement among all the measured conversion coefficients is observed at $\lambda_1 \approx -21$ and $\lambda_2 \approx 0$. From similar plots for $Q = 0.013$ another set of solutions was indicated at $\lambda_1 \approx 17$ and $\lambda_2 \approx 2600$. A nonlinear least-squares fit was made for λ_1, λ_2, and Q with initial values from these two regions which gave the two sets of solutions A and B given in Table III.

According to the Nilsson model[9] the $E1$ γ-ray matrix element G_{E1} and the $\int \vec{J}_n \cdot \vec{\nabla}$ type penetra-

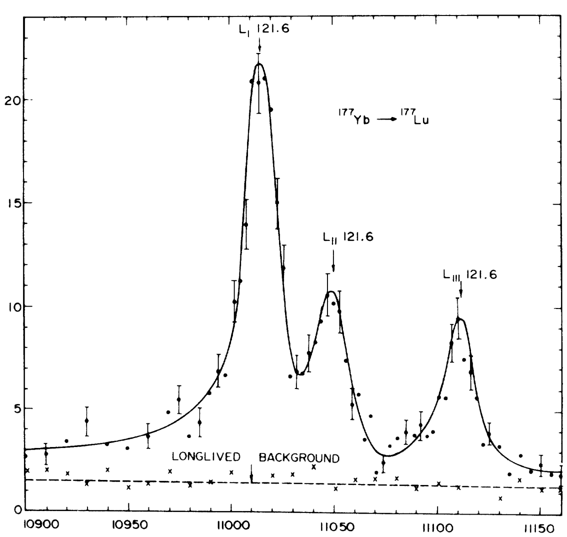

FIG. 4. A typical L-subshell spectrum of the 121.6-keV transition. (See also caption for Fig. 3.)

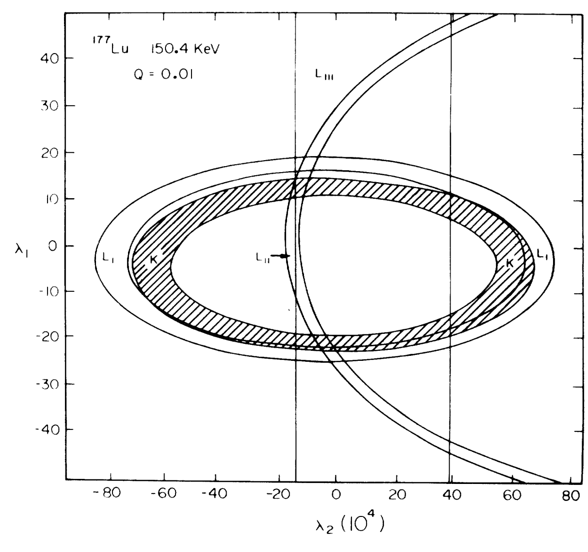

FIG. 5. Plots of λ_1 vs λ_2 for the 150.4-keV transition for an assumed $M2$ admixture, $Q = 0.01$ consistent with the measured α_K, α_{L_I}, $\alpha_{L_{II}}$, and $\alpha_{L_{III}}$. A region of agreement at $\lambda_1 \approx -21$ and $\lambda_2 \approx 0$ is obtained from this. From similar plots for $Q \approx 0.013$ another possible solution was indicated at $\lambda_1 \approx 17$, and $\lambda_2 \approx 2600$. The actual solutions shown in Table III are obtained by least-squares fit using these two sets of initial values.

TABLE II. Absolute conversion coefficients for the 150.4-keV transition.

α_{Expt}	Theoretical (Hager and Seltzer, Ref. 20) $E1$	$M2$
$\alpha_K = 0.383 \pm 0.030$	0.100	5.80
$\alpha_{L_{\mathrm{I}}} = 0.088 \pm 0.008$	0.011	1.12
$\alpha_{L_{\mathrm{II}}} = 0.036 \pm 0.005$	0.00228	0.145
$\alpha_{L_{\mathrm{III}}} = 0.0046 \pm 0.0012$	0.00245	0.175

tion matrix element are forbidden by the asymptotic quantum numbers, while the $M2$ and the $\int \vec{J}_n \cdot \vec{r}$ type penetration matrix elements are allowed.[10, 11, 22] With this prediction one can choose the set of solutions A which allows a small value or zero for λ_2 within error limits, and reject the Set B. We have made a least-squares fit of the experimental quantities for λ_1 and Q keeping $\lambda_2 = 0$, and the resulting solutions are given as Set C in Table III. The values of Q and λ_1 obtained in Set C are essent ally in agreement with the set of solutions A.

EVALUATION OF THE MATRIX ELEMENTS

The half-life of the 150.4-keV level is known[23] to be 0.12 μsec. Taking the total conversion coefficient to be $\alpha_T = 0.55$ and the $M2$ admixture as $Q = 0.92\%$ from the present work, we have calculated the partial $E1$ γ-ray transition probability of the 150.4-keV transition. The experimental value of the $E1$ γ-ray matrix element is obtained as $|G_{E1}| = 1.32 \times 10^{-3}$. From the set of solutions A from Table III, the penetration matrix elements are obtained as $|\int \vec{J}_n \cdot \vec{r}| = 1.70 \pm 0.19$ and $|\int \vec{J}_n \cdot \vec{\nabla}| = 0.87 \pm 1.2$. The Nilsson estimate of the $\int \vec{J}_n \cdot \vec{r}$ penetration matrix element[15] is $\approx +0.95$. Since this value agrees with the experimental value within a factor of 2, we may assume that the phase is positive as predicted.

The $E1$ and $M2$ γ-ray matrix elements and the $E1$ penetration matrix elements of the $\frac{9}{2}^- \rightarrow \frac{7}{2}^+$ transitions in ^{175}Lu, ^{177}Lu, and ^{181}Ta are compared in Table IV. For the three transitions the values of the $\int \vec{J}_n \cdot \vec{r}$ matrix element which is allowed by the asymptotic quantum numbers[22] are in agreement to within ±20% with each other and within a factor of 2 with the Nilsson estimate. Except for the difference in the pairing factors and the deformations these values should be constant for all the three transitions. The G_{M2} values for the three transitions are also in fair agreement with each other and also with the Nilsson estimate.[9] In the case of the $E1$ γ-ray matrix element and the $\int \vec{J}_n \cdot \vec{\nabla}$ type penetration matrix element, which are forbidden in the asymptotic limit, pairing and Coriolis coupling[24] affect the transition matrix elements very sensitively and are not expected to be constant for the three transitions.

TABLE III. Least-squares-fit solutions for Q, λ_1, and λ_2 for the 150.4-keV transition (see text).

Set	Q (%)	λ_1	λ_2	χ^2
A	0.92 ± 0.88	-21.4 ± 2.3	-659 ± 897	1.93
B	1.3 ± 1.5	17.5 ± 2.7	2608 ± 976	5.09
C	0.92 ± 0.80	-22.3 ± 1.6	···	2.95

TABLE IV. Comparison of the properties of the $\frac{9}{2}^- \rightarrow \frac{7}{2}^+$ transitions in ^{175}Lu, ^{177}Lu, and ^{181}Hf.

Quantity	^{175}Lu	^{177}Lu	^{181}Hf
E_γ (keV)	396.3	150.4	6.25
$t_{1/2}$ (level)	3.3 nsec [a]	0.12 μsec [a]	6.8 μsec [a]
$H_W(E1)$ [b]	1.5×10^6	2.9×10^6	3.5×10^5
Q (%$M2$)	1.15 ± 1.27 [c]	0.92 ± 0.88	$\lesssim 4 \times 10^{-4}$ [d]
λ_1	-13.8 ± 0.14 [e]	-21.4 ± 2.3	-9 ± 1 [d]
λ_2	$-(<165)$ [f]	-659 ± 897	···
G_{E1}	-1.86×10^{-3}	-1.32×10^{-3}	-3.8×10^{-3}
$H_N(E1)$ [g]	145	278	31
$\lvert G_{M2} \rvert$ (expt)	$6.4^{+2.9}_{-6.4}$	$7.9^{+3.2}_{-6.2}$	$\lesssim 3.5$
G_{M2}(Nilsson)	-7.15	-7.20	-7.25
$\int \vec{J}_n \cdot \vec{r}$	1.54 [d] (0.98 ± 0.07) [c]	1.70 ± 0.19	2.03 [h]
$\int \vec{J}_n \cdot \vec{\nabla}$	0.3 [f] (0.5 ± 1.8) [c]	0.87 ± 1.19	···

[a] Reference 8.
[b] Hindrance factor with respect to single-particle estimate of Ref. 7.
[c] Reference 13.
[d] Reference 15.
[e] References 12 and 15.
[f] Estimated from Ref. 12.
[g] Hindrance factor with respect to Nilsson estimate, Ref. 9.
[h] Recalculated using results of Ref. 15.

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Analysis of the Total (n, p) Cross Sections Around 14 MeV with the Pre-Equilibrium Exciton Model*

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The absolute value of (n,p) reaction cross sections, as given by the pre-equilibrium exciton model is estimated using Fermi gas approximation.

A general agreement with experimental data, particularly in the gross A dependence is obtained. The energy range considered is 10–20 MeV, and the nuclei are those with $A > 100$: In these ranges evaporation is negligible and the analysis is rather easy. The approximate lifetime of one single-particle exciton in nuclear matter is also deduced.

I. INTRODUCTION

It has been shown recently that the pre-equilibrium emission mechanism suggested by Griffin[1] and developed mainly by Williams,[2] Blann,[3,4] Harp and Miller[5] is responsible for proton emission in some (n, p) reactions around 10–20 MeV of incident neutron energies.[6]

In order to study the validity of this model for nuclear reactions, the analysis has been extended to (n, p) reactions in nuclei with $A > 100$. Indeed, it was shown[7] that in this mass and energy range, proton emission cannot be accounted for by the statistical evaporation theory (see Fig. 1). Up to now, the bulk of results on (n, p) reactions on these heavy nuclei, consisting of about 75 cross-section values at a 14.5-MeV neutron energy measured by activation method, and of a few proton spectra around these energies, could not be interpreted.

This paper presents an estimate of the absolute value of the expected cross section for (n, p) reactions based on pre-equilibrium emission theory, as a function of A, using the Fermi gas model; then the results of calculation are compared with experimental data. As a consequence the lifetime of a single-particle exciton in nuclear matter is estimated.