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Faddeev Equations for Realistic Three-Nucleon Systems. II. Bound-State Wave Functions*

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Starting from our previous work on the complete angular momentum reduction of the Faddeev equations, general formulas are developed for constructing the bound-state wave function from the Faddeev amplitudes. The $\mathcal{L}$-$\mathcal{S}$ coupling scheme is used. For trinucleon systems with nucleon-nucleon interactions in the ${}^1S_0$ and ${}^3S_1$-${}^3D_1$ states, the complete set of homogeneous Faddeev equations and the formulas for constructing the wave functions are given in detail. The wave function is also given in terms of the Derrick-Blatt classification of states.

I. INTRODUCTION

In a previous paper[1] (hereafter referred to as I), we carried out a complete angular momentum reduction of the Faddeev equations for the case of realistic nonrelativistic trinucleon systems with (local or nonlocal) interactions having general spin, isospin, and velocity dependence.[2]

In I, the construction of the completely antisymmetric trinucleon bound-state wave function from the Faddeev amplitudes was briefly described. The purpose of this paper is to provide general formulas for this construction. The $\mathcal{L}$-$\mathcal{S}$ coupling classification of trinucleon basis states is used because it is closer than the J-j coupling scheme[1] to the conventional Derrick-Blatt classification.[3] The formulas given in this paper may be easily transformed to corresponding ones in the J-j coupling scheme by a unitary transformation.[1]

In Sec. II, we summarize the results of I which are relevant to the subject of this paper. General formulas for the bound-state wave function are given in Sec. III. In Sec. IV, we consider the important special case of local nucleon-nucleon interactions in the ${}^1S_0$ and ${}^3S_1$-${}^3D_1$ states. The complete set of homogeneous Faddeev equations and formulas for the bound-state wave function are presented in detail, with numerical values for the angular momentum coupling factors. Several workers[2,4,5] have obtained solutions of truncated versions of these equations in which one approximates the nucleon-nucleon t matrices by separable forms,[4] or one neglects the part of the trinucleon wave function in which the spectator nucleon is in a D state relative to the center of mass of the interacting pair.[2,5] We have recently solved the complete set of Faddeev equations for the case of the Reid potential and will discuss our results in another paper.[6] In Sec. V, an expansion of the wave function of Sec. III is given in terms of the Derrick-Blatt classification of states,[3] and the usefulness of this expansion for checking the consistency of numerical calculations is discussed.

II. FADDEEV EQUATIONS, KINEMATIC VARIABLES, AND $\mathcal{L}$-$\mathcal{S}$ COUPLING BASIS STATES

As in I, we work with the linear momentum combinations $\vec{p}_j$, $\vec{q}_j$, and $\vec{P}$, where

$$\vec{p}_1=\frac{m_3\vec{k}_2-m_2\vec{k}_3}{[2m_2m_3(m_2+m_3)]^{1/2}},$$
$$\vec{q}_1=\frac{m_1(\vec{k}_2+\vec{k}_3)-(m_2+m_3)\vec{k}_1}{[2m_1(m_2+m_3)(m_1+m_2+m_3)]^{1/2}}, \tag{2.1}$$
$$\vec{P}=\frac{\vec{k}_1+\vec{k}_2+\vec{k}_3}{[2(m_1+m_2+m_3)]^{1/2}}.$$

m_i is the mass of particle i and $\vec{k}_i$ is the momentum of particle i in the space-fixed coordinate system. The definitions of $(\vec{p}_2,\vec{q}_2)$ and $(\vec{p}_3,\vec{q}_3)$ follow from (2.1) by cyclic permutation of the indices 1, 2, and 3. The total kinetic energy is given by

$$H_0=\sum_{i=1}^{3}\frac{(\vec{k}_i)^2}{2m_i}=(\vec{P})^2+(\vec{p}_j)^2+(\vec{q}_j)^2. \tag{2.2}$$

All of our analysis will be done in the center-of-mass system ($\vec{P}=\vec{0}$). The linear relations between $(\vec{p}_i,\vec{q}_i)$ and $(\vec{p}_j,\vec{q}_j)$ are

$$\left.\begin{aligned}\vec{p}_i&=-\alpha_{ij}\vec{p}_j-\beta_{ij}\vec{q}_j\\ \vec{q}_i&=\beta_{ij}\vec{p}_j-\alpha_{ij}\vec{q}_j\end{aligned}\right\}\quad(i\neq j), \tag{2.3}$$

where

$$\left.\begin{aligned}\alpha_{ij}&=\left(\frac{m_im_j}{(m_i+m_k)(m_j+m_k)}\right)^{1/2}=\alpha_{ji}\\ \beta_{ij}&=(1-\alpha_{ij}{}^2)^{1/2}=-\beta_{ji},\end{aligned}\right\}\quad(ijk\text{ cyclic}). \tag{2.4}$$

We neglect the neutron-proton mass difference, so that $\alpha_{ij}=\frac{1}{2}$, $\beta_{ij}=\frac{1}{2}\sqrt{3}$. $P(i)=P_{jk}$ (ijk cyclic) is defined as the operator which interchanges particles j and k. Thus, for the equal-mass case, we have

$$\left.\begin{aligned} P(i)\vec{p}_i &= -\vec{p}_i \\ P(i)\vec{q}_i &= \vec{q}_i \end{aligned}\right\}\quad (i=1,2,3); \tag{2.5}$$

$$\left.\begin{aligned} P(i)\vec{p}_j &= -\vec{p}_k \\ P(i)\vec{q}_j &= \vec{q}_k \\ P(j)\vec{p}_i &= -\vec{p}_k \\ P(j)\vec{q}_i &= \vec{q}_k \end{aligned}\right\}\quad (ijk \text{ cyclic}). \tag{2.6}$$

The $\mathcal{L}$-$\mathcal{S}$ basis state[1] $|p,q,\alpha\rangle_i=|pq,\alpha(i,jk)\rangle_i$, ($ijk$ cyclic), is defined to be an eigenstate of the operators: $(\vec{p}_i)^2$, $(\vec{q}_i)^2$, $(\vec{L}_i)^2$, $(\vec{l}_i)^2$, $(\vec{\mathcal{L}})^2=(\vec{L}_i+\vec{l}_i)^2$, $(\vec{S}_i)^2=(\vec{s}_j+\vec{s}_k)^2$, $(\vec{s}_j)^2$, $(\vec{s}_k)^2$, $(\vec{s}_i)^2$, $(\vec{\mathcal{S}})^2=(\vec{S}_i+\vec{s}_i)^2$, $(\vec{\mathcal{J}})^2=(\vec{\mathcal{L}}+\vec{\mathcal{S}})^2$, $\mathcal{J}_z$, $(\vec{t}_j)^2$, $(\vec{t}_k)^2$, $(\vec{T}_i)^2=(\vec{t}_j+\vec{t}_k)^2$, $(\vec{t}_i)^2$, $(\vec{\mathcal{T}})^2=(\vec{T}_i+\vec{t}_i)^2$, $\mathcal{T}_z$ with corresponding quantum numbers p, q, L, l, $\mathcal{L}$, S, $s_j=\frac{1}{2}$, $s_k=\frac{1}{2}$, $s=\frac{1}{2}$, $\mathcal{S}$, $\mathcal{J}$, $\mathcal{J}_z$, $t_j=\frac{1}{2}$, $t_k=\frac{1}{2}$, T, $t=\frac{1}{2}$, $\mathcal{T}$, and $\mathcal{T}_z$, respectively. $\vec{L}_i$ is the relative orbital angular momentum of the jk pair; $\vec{l}_i$ is the orbital angular momentum of nucleon i in the c.m. system; $\vec{s}_i$ is the spin angular momentum of nucleon i; and $\vec{t}_i$ is the isospin of nucleon i. The explicit construction of $|p,q,\alpha\rangle_i$ is given by

$$|p,q,\alpha\rangle_i=|[pq(Ll)\mathcal{L},(Ss)\mathcal{S}]\mathcal{J}\mathcal{J}_z;(Tt)\mathcal{T}\mathcal{T}_z\rangle_i=\sum_{m_{\mathcal{L}}m_{\mathcal{S}}}\langle\mathcal{L}m_{\mathcal{L}}\mathcal{S}m_{\mathcal{S}}|\mathcal{J}\mathcal{J}_z\rangle\,|pq(Ll)\mathcal{L}m_{\mathcal{L}}\rangle_i|(Ss)\mathcal{S}m_{\mathcal{S}}\rangle_i|(Tt)\mathcal{T}\mathcal{T}_z\rangle_i\,, \tag{2.7}$$

where

$$|pq(Ll)\mathcal{L}m_{\mathcal{L}}\rangle_i=\sum_{m_L,m_l}\langle Lm_Llm_l|\mathcal{L}m_{\mathcal{L}}\rangle\,|pLm_L;qlm_l\rangle_i=\sum_{m_L,m_l}\langle Lm_Llm_l|\mathcal{L}m_{\mathcal{L}}\rangle\int d\hat{p}\int d\hat{q}\,Y_{Lm_L}(\hat{p})Y_{lm_l}(\hat{q})|\vec{p},\vec{q}\rangle_i\,, \tag{2.8}$$

$$|(Ss)\mathcal{S}m_{\mathcal{S}}\rangle_i=\sum_{m_Sm_s}\langle Sm_Ssm_s|\mathcal{S}m_{\mathcal{S}}\rangle\,|Sm_S\rangle_i|sm_s\rangle_i\,, \tag{2.9}$$

and $|(Tt)\mathcal{T}\mathcal{T}_z\rangle_i$ has an expansion analogous to (2.9). Antisymmetry with respect to jk interchange gives the restriction $(-1)^{L+S+T}=-1$. The states (2.7) satisfy the orthonormality relations

$${}_i\langle p,q,\alpha|p'q',\alpha'\rangle_i=\frac{\delta(p-p')}{p^2}\,\frac{\delta(q-q')}{q^2}\,\delta_{\alpha,\alpha'}\,. \tag{2.10}$$

The Faddeev equations for realistic trinucleon systems in the $\mathcal{L}$-$\mathcal{S}$ basis are given by Eqs. (4.6), (4.28), and (5.7) of I:

$$\begin{aligned}
{}_i\langle p,q,\alpha|T^{(1)}(s)|\psi\rangle_A&=\psi_s^{(1)}(p,q,\alpha)=\varphi_s^{(1)}(p,q,\alpha)+\frac{2}{\pi}\sum_{S_1,J_1}\delta_{SS_1}\delta_{\mathcal{T}_z\mathcal{T}_{2z}}(-1)^{t_2+T_2-\mathcal{T}_2}\hat{T}\hat{T}_2W(t_2t_3\mathcal{T}_2t_1;T_2T)\\
&\quad\times\sum_{T_z,t_z}\langle TT_ztt_z|\mathcal{T}\mathcal{T}_z\rangle\langle TT_ztt_z|\mathcal{T}_2\mathcal{T}_{2z}\rangle\sum_{L_1}\bar{\delta}_{L_1,L}\sum_{\lambda\Lambda rr_1r_2}\binom{2l+1}{2\lambda}^{1/2}\binom{2L_1+1}{2\Lambda}^{1/2}(\alpha_{12})^{l-\lambda+\Lambda-1}\\
&\quad\times(\beta_{12})^{\lambda+L_1-\Lambda-1}(-1)^{L_1+l-\lambda}(2L_2+1)^{1/2}(2l_2+1)^{1/2}(2r_1+1)(2r_2+1)[2(L_1-\Lambda)+1]^{1/2}\\
&\quad\times[2(l-\lambda)+1]^{1/2}\hat{r}^2\begin{pmatrix}L_2&r&r_1\\0&0&0\end{pmatrix}\begin{pmatrix}\Lambda&\lambda&r_1\\0&0&0\end{pmatrix}\begin{pmatrix}r&l_2&r_2\\0&0&0\end{pmatrix}\begin{pmatrix}L_1-\Lambda&l-\lambda&r_2\\0&0&0\end{pmatrix}G_{\mathcal{L}-\mathcal{S}}\frac{1}{q^{l+1}}\\
&\quad\times\int_0^\infty dq_2\,q_2^{L_1-\Lambda+l-\lambda+1}\int_{|\alpha_{12}q_2-q|/\beta_{12}}^{(\alpha_{12}q_2+q)/\beta_{12}}p_2^{\Lambda+\lambda+1}\,t_{L,L_1}^{J_1STT_z}(p,(p_2^2+q_2^2-q^2)^{1/2};(s-q^2)^{1/2})\\
&\quad\times P_r\left(\frac{\beta_{12}^2p_2^2+\alpha_{12}^2q_2^2-q^2}{2\alpha_{12}\beta_{12}p_2q_2}\right)\frac{\psi_s^{(1)}(p_2,q_2,\alpha_2)}{(p_2^2+q_2^2-s)(p_2^2+q_2^2-q^2)^{L_1/2}}\,dp_2\,.
\end{aligned} \tag{2.11}$$

s is the total energy of the trinucleon system. The off-shell t matrices are normalized so that

$$t_{L,L}^{L,0,T,T_z}(k,k;k)=\frac{e^{i\delta_L}\sin\delta_L}{k}\,, \tag{2.12}$$

where δ_L is the partial-wave phase shift. Since only the homogeneous Faddeev equations are relevant for the bound-state problem, we will not give the explicit form of the $\varphi_s^{(1)}(p,q,\alpha)$. The geometrical factor $G_{\mathcal{L}-\mathcal{S}}$ is given by Eq. (5.7) of I.

III. BOUND-STATE WAVE FUNCTION

The components of the bound-state wave function $\langle pq,\alpha|\psi_B\rangle$ are obtained from the homogeneous solution $\psi^{(1)}_{s(=E_B)}(p,q,\alpha)$ of (2.11):

$$\begin{aligned}
{}_1\langle p,q,\alpha|\psi_B\rangle &= N\frac{1}{E_B-p^2-q^2}\langle(e+P_{132}+P_{123})(p,q,\alpha)_1|\tilde{T}^{(1)}(E_B)|\psi_A\rangle \\
&= N\frac{1}{p^2+q^2-E_B}\langle[P(1)+P(2)+P(3)](p,q,\alpha)_1|\tilde{T}^{(1)}(E_B)|\psi_A\rangle, \\
{}_1\langle p,q,\alpha|\tilde{T}^{(1)}(E_B)|\psi_A\rangle &= \tilde{\psi}^{(1)}_{E_B}(p,q,\alpha)=\lim_{s\to E_B}(s-E_B)\langle p,q,\alpha|T^{(1)}(s)|\psi_A\rangle.
\end{aligned} \tag{3.1}$$

N is a normalization constant.

We now outline the procedure for calculating $P(i)|p,q,\alpha\rangle_1$. For the space-spin parts of the basis states, we have

$$P(1)|[pq(Ll)\mathcal{L},(Ss)\mathcal{S}]\mathcal{J}\mathcal{J}_z\rangle_1=(-1)^{L+1-S}|[pq(Ll)\mathcal{L},(Ss)\mathcal{S}]\mathcal{J}\mathcal{J}_z\rangle_1, \tag{3.2}$$

$$P(2)|[pq(Ll)\mathcal{L},(Ss)\mathcal{S}]\mathcal{J}\mathcal{J}_z\rangle_1=\sum_{\substack{m_\mathcal{L},m_\mathcal{S}\\ m_L,m_l}}\langle\mathcal{L}m_\mathcal{L}\mathcal{S}m_\mathcal{S}|\mathcal{J}\mathcal{J}_z\rangle\langle Lm_Llm_l|\mathcal{L}m_\mathcal{L}\rangle[P(2)|pLm_L;qlm_l\rangle_1][P(2)|(Ss)\mathcal{S}m_\mathcal{S}\rangle_1], \tag{3.3}$$

$$P(2)|(Ss)\mathcal{S}m_\mathcal{S}\rangle_1=\sum_{S_1',S_2'}(-1)^{s+s_3-S_2'}\hat{S}\hat{S}_1'(2S_2'+1)W(s_2s_3\mathcal{S}s;SS_2')W(s_2s_3\mathcal{S}s;S_1'S_2')|(S_1's)\mathcal{S}m_\mathcal{S}\rangle_1, \tag{3.4}$$

where the W's are the usual Racah coefficients, $\hat{S}_1=(2S_1+1)^{1/2}$, and

$$\begin{aligned}
P(2)|pLm_L;qlm_l\rangle_1 &= P(2)\int d\hat{p}\int d\hat{q}\,Y_{Lm_L}(\hat{p})Y_{lm_l}(\hat{q})|\vec{p},\vec{q}\rangle_1 \\
&= \int d\hat{p}\int d\hat{q}\,Y_{Lm_L}(\hat{p})Y_{lm_l}(\hat{q})|-\vec{p}_3,\vec{q}_3\rangle_1 \\
&= \int d\hat{p}\int d\hat{q}\,Y_{Lm_L}(\hat{p})Y_{lm_l}(\hat{q})\sum_{L_3,m_{L_3}}Y^*_{l_3m_{l_3}}(-\hat{p}_3)Y^*_{l_3m_{l_3}}(\hat{q}_3)|p_3L_3m_{L_3};q_3l_3m_{l_3}\rangle_1,
\end{aligned} \tag{3.5}$$

where

$$-\vec{p}_3=\tfrac{1}{2}\vec{p}+\tfrac{1}{2}\sqrt{3}\,\vec{q},\quad \vec{q}_3=\tfrac{1}{2}\sqrt{3}\,\vec{p}-\tfrac{1}{2}\vec{q}. \tag{3.6}$$

The spherical harmonics of $\hat{p}_3$ and $\hat{q}_3$ in (3.5) are expanded in spherical harmonics of $\hat{p}$ and $\hat{q}$ [7]:

$$\begin{aligned}
Y^*_{L_3m_{L_3}}(-\hat{p}_3) &= p_3^{-L_3}\sum_{\Lambda=0}^{L_3}\sum_{m_\Lambda=-\Lambda}^{\Lambda}\frac{4\pi}{\hat{\Lambda}}\binom{2L_3+1}{2\Lambda}^{1/2}(\tfrac{1}{2}p)^\Lambda(\tfrac{1}{2}\sqrt{3}\,q)^{L_3-\Lambda} \\
&\quad\times\langle\Lambda m_\Lambda L_3-\Lambda m_{L_3}-m_\Lambda|L_3m_{L_3}\rangle Y^*_{\Lambda m_\Lambda}(\hat{p})Y^*_{L_3-\Lambda,\,m_{L_3}-m_\Lambda}(\hat{q})
\end{aligned} \tag{3.7}$$

and

$$\begin{aligned}
Y^*_{l_3m_{l_3}}(\hat{q}_3) &= q_3^{-l_3}\sum_{\lambda=0}^{l_3}\sum_{m_\lambda=-\lambda}^{\lambda}\frac{\sqrt{4\pi}}{\hat{\lambda}}\binom{2l_3+1}{2\lambda}^{1/2}(\tfrac{1}{2}\sqrt{3}\,p)^\lambda(-\tfrac{1}{2}q)^{l_3-\lambda} \\
&\quad\times\langle\lambda m_\lambda l_3-\lambda m_{l_3}-m_\lambda|l_3m_{l_3}\rangle Y^*_{\lambda m_\lambda}(\hat{p})Y^*_{l_3-\lambda,\,m_{l_3}-m_\lambda}(\hat{q}),
\end{aligned} \tag{3.8}$$

where $\binom{2L_3+1}{2\Lambda}$ is the binomial coefficient. We then combine the $p_3^{-L_3}$ and $q_3^{-l_3}$ factors in (3.7) and (3.8) with $|p_3L_3m_{L_3};\ q_3l_3m_{l_3}\rangle_1$ and expand the result in partial waves:

$$\frac{1}{p_3^{L_3}q_3^{l_3}}|p_3L_3m_{L_3};q_3l_3m_{l_3}\rangle_1=\sum_{r,m_r}|r,pq;L_3m_{L_3}l_3m_{l_3}\rangle Y^*_{r,m_r}(\hat{p})Y_{r,m_r}(\hat{q}), \tag{3.9}$$

where

$$|r,pq;L_3m_{L_3}l_3m_{l_3}\rangle=2\pi\int_{-1}^{1}d(\cos\theta)\frac{P_r(\cos\theta)}{p_3^{L_3}q_3^{l_3}}|p_3L_3m_{L_3};q_3l_3m_{l_3}\rangle_1, \tag{3.10}$$

with $\cos\theta=\hat{q}\cdot\hat{p}$. Finally, we use (3.7)–(3.10) and the expansion

$$|p_3L_3m_{L_3};q_3l_3m_{l_3}\rangle_1|(S_1's)\mathcal{S}m_{\mathcal{S}}\rangle_1=\sum_{\mathcal{L}_3,m_{\mathcal{L}_3}}\sum_{\mathcal{J}_3,\mathcal{J}_{3z}}\langle L_3m_{L_3}l_3m_{l_3}|\mathcal{L}_3m_{\mathcal{L}_3}\rangle\langle\mathcal{L}_3m_{\mathcal{L}_3}\mathcal{S}m_{\mathcal{S}}|\mathcal{J}_3\mathcal{J}_{3z}\rangle$$
$$\times|[p_3q_3(L_3l_3)\mathcal{L}_3(S_1's_1)\mathcal{S}]\mathcal{J}_3\mathcal{J}_{3z}\rangle,\tag{3.11}$$

in (3.5), do the $\hat{p}$ and $\hat{q}$ integrations, and obtain

$$P(2)|[pq(Ll)\mathcal{L},(Ss)\mathcal{S}]\mathcal{J}\mathcal{J}_z\rangle_1=\sum_{S_1'S_2'}(-1)^{s+s_3-S_2'}\hat{S}\hat{S}_1'(2S_2'+1)W(s_2s_3\mathcal{S}s;S_1'S_2')W(s_2s_3\mathcal{S}s;SS_2')$$
$$\times\sum_{L_3,l_3}\sum_{\Lambda,\lambda,r}(\tfrac{1}{2}p)^{\Lambda}(\tfrac{1}{2}\sqrt{3}q)^{L_3-\Lambda}(\tfrac{1}{2}\sqrt{3}p)^{\lambda}(-\tfrac{1}{2}q)^{l_3-\lambda}\binom{2L_3+1}{2\Lambda}^{1/2}\binom{2l_3+1}{2\lambda}^{1/2}\hat{L}\hat{l}(2r+1)\hat{L}_3\hat{l}_3$$
$$\times[2(L_3-\Lambda)+1]^{1/2}[2(l_3-\lambda)+1]^{1/2}$$
$$\times\sum_{r_1,r_2}(2r_1+1)(2r_2+1)\begin{pmatrix}L&r&r_1\\0&0&0\end{pmatrix}\begin{pmatrix}\Lambda&\lambda&r_1\\0&0&0\end{pmatrix}\begin{pmatrix}r&l&r_2\\0&0&0\end{pmatrix}\begin{pmatrix}L_3-\Lambda&l_3-\lambda&r_2\\0&0&0\end{pmatrix}$$
$$\times\sum_{\mathcal{L}_3}G_{\rm ex}\frac{1}{2}\int_{-1}^{1}d(\cos\theta)\,\frac{P_r(\cos\theta)}{p_3^{L_3}q_3^{l_3}}\,|[p_3q_3(L_3l_3)\mathcal{L}_3(S_1's)\mathcal{S}]\mathcal{J}\mathcal{J}_z\rangle_1\,.\tag{3.12}$$

The geometric factor $G_{\rm ex}$ in (3.12) is given by

$$G_{\rm ex}=\sum_{\text{all }m}\langle\mathcal{L}m_{\mathcal{L}}\,\mathcal{S}m_{\mathcal{S}}|\mathcal{J}\mathcal{J}_z\rangle\langle Lm_L\,lm_l|\mathcal{L}m_{\mathcal{L}}\rangle\langle\mathcal{L}_3m_{\mathcal{L}_3}\mathcal{S}m_{\mathcal{S}}|\mathcal{J}\mathcal{J}_z\rangle\langle L_3m_{L_3}l_3m_{l_3}|\mathcal{L}_3m_{\mathcal{L}_3}\rangle(-1)^{m_r}\delta_{m_\Lambda+m_\lambda,\,m_L-m_r}$$
$$\times\delta_{m_r+m_l,\,m_{L_3}+m_{l_3}-m_\Lambda-m_\lambda}\langle\Lambda m_\Lambda L_3-\Lambda m_{L_3}-m_\Lambda|L_3m_{L_3}\rangle\langle\lambda m_\lambda l_3-\lambda m_{l_3}-m_\lambda|l_3m_{l_3}\rangle$$
$$\times\begin{pmatrix}L&r&r_1\\m_L&-m_r&m_r-m_L\end{pmatrix}\begin{pmatrix}\Lambda&\lambda&r_1\\m_\Lambda&m_\lambda&-m_\Lambda-m_\lambda\end{pmatrix}\begin{pmatrix}r&l&r_2\\m_r&m_l&-m_r-m_l\end{pmatrix}$$
$$\times\begin{pmatrix}L_3-\Lambda&l_3-\lambda&r_2\\m_{L_3}-m_\Lambda&m_{l_3}-m_\lambda&m_\Lambda+m_\lambda-m_{L_3}-m_{l_3}\end{pmatrix}.\tag{3.13}$$

Using diagram techniques,[1,8] we find

$$G_{\rm ex}=(-1)^{r+2\mathcal{S}+\mathcal{L}+l+L+2\mathcal{J}}\{\mathcal{L}\mathcal{S}\mathcal{J}\}\begin{Bmatrix}L_3&\Lambda&L_3-\Lambda\\l_3&\lambda&l_3-\lambda\\\mathcal{L}&r_1&r_2\end{Bmatrix}\begin{Bmatrix}r_1&r_2&\mathcal{L}\\l&L&r\end{Bmatrix}\delta_{\mathcal{L}\mathcal{L}_3},\tag{3.14}$$

where $\{\mathcal{L}\mathcal{S}\mathcal{J}\}$ denotes the triangular relation among $\mathcal{L}$, $\mathcal{S}$, and $\mathcal{J}$, and the last two factors are the 9-j and 6-j symbols, respectively. In deriving (3.14), we have used the fact that $G_{\rm ex}$ is independent of $\mathcal{J}_z$ and that $\mathcal{J}$ and $\mathcal{J}_z$ are invariant under pair-exchange operations.

A similar calculation gives $P(3)|[pq(Ll)\mathcal{L},(Ss)\mathcal{S}]\mathcal{J}\mathcal{J}_z\rangle_1$ equal to the right-hand side of (3.12) except for an extra factor $(-1)^{l_3+l+S-S_1'}$ in the summations. $P(3)|(Ss)\mathcal{S}m_{\mathcal{S}}\rangle_1$ is the same as the right-hand side of (3.4) except for the factor $(-1)^{S-S_1'}$ in the summations. $P(i)|(Tt)\mathcal{T}\mathcal{T}_z\rangle_1$ is the same as $P(i)|(Ss)\mathcal{S}m_{\mathcal{S}}\rangle_i$ with isospin quantum numbers replacing corresponding spin quantum numbers.

Using the results just presented, we may write (3.1) in the form

$${}_1\langle p,q,\alpha|\psi_B\rangle=\frac{N}{p^2+q^2-E_B}\Bigg((-1)^{L+S+T}\tilde{\psi}^{(1)}_{E_B}(p,q,\alpha)$$
$$+\sum_{\Lambda,\lambda,r}\sum_{L_3,l_3}\sum_{S_1'}\sum_{T_1'}A^{\Lambda\lambda r}(Ll\mathcal{L}|L_3l_3\mathcal{L}_3(=\mathcal{L}))B_{\mathcal{S}}(S|S_1')C_{\mathcal{T}}(T|T_1')(p)^{n_p}(q)^{n_q}$$
$$\times[1+(-1)^{l_3+l+S-S_1'+T-T_1'}]\frac{1}{2}\int_{-1}^{1}d(\cos\theta)\frac{P_r(\cos\theta)}{p_3^{L_3}q_3^{l_3}}\tilde{\psi}^{(1)}_{E_B}(p_3,q_3,\alpha')\Bigg),$$
$$n_p=\Lambda+\lambda\,,\quad n_q=L_3-\Lambda+l_3-\lambda\,,\tag{3.15}$$

where

$$\alpha=[(Ll)\mathcal{L},(Ss)\mathcal{S}]\mathcal{J},\mathcal{J}_z;(Tt)\mathcal{T}\mathcal{T}_z\,,$$
$$\alpha'=[(L_3l_3)\mathcal{L}_3,(S_1's)\mathcal{S}]\mathcal{J}\mathcal{J}_z;(T_1't)\mathcal{T}\mathcal{T}_z\,, \tag{3.16}$$

$\vec{p}_3$, $\vec{q}_3$ are given by (3.6), $\cos\theta=\hat{p}\cdot\hat{q}$, and the coefficient matrices $A^{\Lambda\lambda r}$, $B_{\mathcal{S}}$, and $C_{\mathcal{T}}$ may be easily read off from (3.12) and (3.4).

IV. FADDEEV KERNEL AND TRINUCLEON WAVE FUNCTION FOR THE CASE OF LOCAL NUCLEON-NUCLEON INTERACTIONS IN THE $^1S_0$ AND $^3S_1$-$^3D_1$ STATES

With most of the presently available computer facilities, the case of local nucleon-nucleon interactions in the $^1S_0$ and $^3S_1$-$^3D_1$ states is about the most complicated one for which an *exact* solution of the Faddeev equations can be obtained. As was mentioned in the Introduction, the Faddeev equations for this case have heretofore only been solved in truncated form. The purpose of this section is to present explicitly for this case the relevant equations with numerical values for the various angular momentum coupling parameters so that other workers might be spared from this rather tedious task.

The eight trinucleon $\mathcal{L}$-$\mathcal{S}$ coupling states involved in the Faddeev equations (2.11) are listed in Table I. The even parity of the trinucleon bound state and the dynamical assumption concerning the nucleon-nucleon interaction restrict L and l to the values 0, 2. Only five of the eight Faddeev equations are independent. This may be easily seen by considering the eight J-j coupling states in Table II, which are related to the $\mathcal{L}$-$\mathcal{S}$ states in Table I by the unitary transformation[1]:

$$|[pq(Ll)\mathcal{L},(Ss)\mathcal{S}]\mathcal{J}\mathcal{J}_z;(Tt)\mathcal{T}\mathcal{T}_z\rangle_1 =\sum_{J,j}\hat{J}\hat{j}\hat{\mathcal{L}}\hat{\mathcal{S}}\begin{Bmatrix}L & l & \mathcal{L}\\ S & s & \mathcal{S}\\ J & j & \mathcal{J}\end{Bmatrix} \times|[p(LS)J,q(ls)j]\mathcal{J}\mathcal{J}_z;(Tt)\mathcal{T}\mathcal{T}_z\rangle_1\,. \tag{4.1}$$

Only the first five of the listed J-j states enter the Faddeev equations, since there is assumed to be no nucleon-nucleon interaction in partial waves with $J>1$. The first four $\mathcal{L}$-$\mathcal{S}$ states are identical, respectively, to the first four J-j states. The last four $\mathcal{L}$-$\mathcal{S}$ states are linear combinations of the last four J-j states. Thus, with the states in Table I labeled as $|p,q,\alpha\rangle_1$ with $\alpha=1,2,\ldots,8$, we have

$$\begin{aligned}&{}_1\langle pq,6|T^{(1)}(s)|\psi\rangle_A=\psi_s^{(1)}(p,q,6)=\alpha\psi_s^{(1)}(p,q,5)\,,\\ &\psi_s^{(1)}(p,q,7)=\beta\psi_s^{(1)}(p,q,5)\,,\\ &\psi_s^{(1)}(p,q,8)=\gamma\psi_s^{(1)}(p,q,5)\,,\end{aligned} \tag{4.2}$$

with

$$\alpha=\frac{\hat{1}\hat{\frac12}\hat{1}\hat{\frac32}}{\hat{0}\hat{\frac12}\hat{1}\hat{\frac32}}\begin{Bmatrix}2 & 1 & 1\\ 2 & \frac12 & \frac32\\ 1 & \frac12 & \frac12\end{Bmatrix}\Bigg/\begin{Bmatrix}2 & 1 & 1\\ 2 & \frac12 & \frac32\\ 0 & \frac12 & \frac12\end{Bmatrix}=\sqrt{\tfrac32}\,,$$

and similarly, $\beta=\frac12\sqrt3$ and $\gamma=\frac12\sqrt7$.

We may write the homogeneous Faddeev

TABLE I. Trinucleon $\mathcal{L}$-$\mathcal{S}$ coupling states with $\mathcal{J}=|\mathcal{J}_z|=\mathcal{T}=|\mathcal{T}_z|=\frac12$ and positive parity, which are involved in the Faddeev equations (2.11) for the case of local nucleon-nucleon interactions in the $^1S_0$ and $^3S_1$-$^3D_1$ states.

State	$(Ll)\mathcal{L}$	$(Ss)\mathcal{S}$	(Tt)
1	(00)0	$(0\,\frac12)\frac12$	$(1\,\frac12)$
2	(00)0	$(1\,\frac12)\frac12$	$(0\,\frac12)$
3	(20)2	$(1\,\frac12)\frac32$	$(0\,\frac12)$
4	(02)2	$(1\,\frac12)\frac32$	$(0\,\frac12)$
5	(22)0	$(1\,\frac12)\frac12$	$(0\,\frac12)$
6	(22)1	$(1\,\frac12)\frac12$	$(0\,\frac12)$
7	(22)1	$(1\,\frac12)\frac32$	$(0\,\frac12)$
8	(22)2	$(1\,\frac12)\frac32$	$(0\,\frac12)$

TABLE II. Trinucleon J-j coupling states which are related to the $\mathcal{L}$-$\mathcal{S}$ coupling states in Table I by the unitary transformation (4.1).

State	$(LS)J$	$(ls)j$	(Tt)
1	(00)0	$(0\,\frac12)\frac12$	$(1\,\frac12)$
2	(01)1	$(0\,\frac12)\frac12$	$(0\,\frac12)$
3	(21)1	$(0\,\frac12)\frac12$	$(0\,\frac12)$
4	(01)1	$(2\,\frac12)\frac32$	$(0\,\frac12)$
5	(21)1	$(2\,\frac12)\frac32$	$(0\,\frac12)$
6	(21)2	$(2\,\frac12)\frac32$	$(0\,\frac12)$
7	(21)2	$(2\,\frac12)\frac52$	$(0\,\frac12)$
8	(21)3	$(2\,\frac12)\frac52$	$(0\,\frac12)$

TABLE III. Numerical values for Faddeev kernel factors $\Gamma^{\alpha,\alpha_2}_{L_1,\Lambda,\lambda,r}$ in (4.4), for $\alpha=1-5$, $\alpha_2=1-8$.

$\alpha\alpha_2$	L_1	Λ	l	λ	r	n_q	n_p	Γ
11	0	0	0	0	0	1	1	0.577 350 27
12	0	0	0	0	0	1	1	−1.732 050 81
13								
14								
15	0	0	0	0	2	1	1	−3.872 983 35
16								
17								
18								
21	0	0	0	0	0	1	1	−1.732 050 81
22	0	0	0	0	0	1	1	0.577 350 27
23	2	0	0	0	2	3	1	0.866 025 40
	2	1	0	0	1	2	2	1.000 000 00
	2	2	0	0	0	1	3	0.288 675 13
24	2	0	0	0	0	3	1	0.866 025 40
	2	1	0	0	1	2	2	1.000 000 00
	2	2	0	0	2	1	3	0.288 675 13
25	0	0	0	0	2	1	1	1.290 994 45
26								
27								
28	2	0	0	0	2	3	1	−1.035 098 34
	2	1	0	0	1	2	2	−0.836 660 03
	2	1	0	0	3	2	2	−0.358 568 58
	2	2	0	0	2	1	3	−0.345 032 78
31	0	0	0	0	0	1	1	1.732 050 81
32	0	0	0	0	0	1	1	−0.577 350 27
33	2	0	0	0	2	3	1	−0.866 025 40
	2	1	0	0	1	2	2	−1.000 000 00
	2	2	0	0	0	1	3	−0.288 675 13
34	2	0	0	0	0	3	1	−0.866 025 40
	2	1	0	0	1	2	2	−1.000 000 00
	2	2	0	0	2	1	3	−0.288 675 13
35	0	0	0	0	2	1	1	−1.290 994 45
36								
37								
38	2	0	0	0	2	3	1	1.035 098 34
	2	1	0	0	1	2	2	0.836 660 03
	2	1	0	0	3	2	2	0.358 568 58
	2	2	0	0	2	1	3	0.345 032 78
41	2	0	2	0	0	5	1	0.324 759 53
	2	0	2	1	1	4	2	−1.125 000 00
	2	0	2	2	2	3	3	0.974 278 58
	2	1	2	0	1	4	2	0.375 000 00
	2	1	2	1	0	3	3	−1.082 531 75
	2	1	2	1	2	3	3	−0.216 506 35
	2	1	2	2	1	2	4	1.125 000 00
	2	2	2	0	2	3	3	0.108 253 18
	2	2	2	1	1	2	4	−0.375 000 00
	2	2	2	2	0	1	5	0.324 759 53
42	2	0	2	0	0	5	1	−0.108 253 18
	2	0	2	1	1	4	2	0.375 000 00
	2	0	2	2	2	3	3	−0.324 759 53
	2	1	2	0	1	4	2	−0.125 000 00
	2	1	2	1	0	3	3	0.360 843 92
	2	1	2	1	2	3	3	0.072 168 78
42	2	1	2	2	1	2	4	0.375 000 00
	2	2	2	0	2	3	3	−0.036 084 39
	2	2	2	1	1	2	4	0.125 000 00
	2	2	2	2	0	1	5	−0.108 253 18
43	0	0	2	0	2	3	1	−0.288 675 13
	0	0	2	1	1	2	2	1.000 000 00
	0	0	2	2	0	1	3	−0.866 025 40
	2	0	2	0	2	5	1	−0.153 093 11
	2	0	2	1	1	4	2	0.371 231 06
	2	0	2	1	3	4	2	0.159 099 03
	2	0	2	2	2	3	3	−0.459 279 33
	2	1	2	0	1	4	2	−0.123 743 69
	2	1	2	0	3	4	2	−0.053 033 01
	2	1	2	1	0	3	3	0.357 217 25
	2	1	2	1	2	3	3	0.255 155 18
	2	1	2	2	1	2	4	−0.530 330 09
	2	2	2	0	2	3	3	−0.051 031 04
	2	2	2	1	1	2	4	0.176 776 70
	2	2	2	2	0	1	5	−0.153 093 11
44	0	0	2	0	0	3	1	−0.288 675 13
	0	0	2	1	1	2	2	1.000 000 00
	0	0	2	2	2	1	3	−0.866 025 40
	2	0	2	0	0	5	1	−0.153 093 11
	2	0	2	1	1	4	2	0.530 330 09
	2	0	2	2	2	3	3	−0.459 279 33
	2	1	2	0	1	4	2	−0.176 776 70
	2	1	2	1	0	3	3	0.357 217 25
	2	1	2	1	2	3	3	0.255 155 18
	2	1	2	2	1	2	4	−0.371 231 06
	2	1	2	2	3	2	4	−0.159 099 03
	2	2	2	0	2	3	3	−0.051 031 04
	2	2	2	1	1	2	4	0.123 743 69
	2	2	2	1	3	2	4	0.053 033 01
	2	2	2	2	2	1	5	−0.153 093 11
45	2	0	2	0	2	5	1	−0.242 061 46
	2	0	2	1	1	4	2	0.335 410 20
	2	0	2	1	3	4	2	0.503 115 29
	2	0	2	2	0	3	3	−0.145 236 88
	2	0	2	2	2	3	3	−0.207 481 25
	2	0	2	2	4	3	3	−0.373 466 25
	2	1	2	0	1	4	2	−0.111 803 40
	2	1	2	0	3	4	2	−0.167 705 10
	2	1	2	1	0	3	3	0.032 274 86
	2	1	2	1	2	3	3	0.852 978 48
	2	1	2	1	4	3	3	0.082 992 50
	2	1	2	2	1	2	4	−0.335 410 20
	2	1	2	2	3	2	4	−0.503 115 29
	2	2	2	0	0	3	3	−0.016 137 43
	2	2	2	0	2	3	3	−0.023 053 47
	2	2	2	0	4	3	3	−0.041 496 25
	2	2	2	1	1	2	4	0.111 803 40
	2	2	2	1	3	2	4	0.167 705 10
	2	2	2	2	2	1	5	−0.242 061 46
46	2	0	2	1	1	4	2	−0.308 093 94
	2	0	2	1	3	4	2	0.308 093 94
	2	0	2	2	0	3	3	0.177 878 12
	2	0	2	2	2	3	3	0.127 055 80
	2	0	2	2	4	3	3	−0.304 933 92

TABLE III *(Continued)*

$\alpha\alpha_2$	L_1	Λ	l	λ	r	n_q	n_p	Γ
46	2	1	2	0	1	4	2	−0.102 697 98
	2	1	2	0	3	4	2	0.102 697 98
	2	1	2	2	1	2	4	0.308 093 94
	2	1	2	2	3	2	4	−0.308 093 94
	2	2	2	0	0	3	3	−0.019 764 24
	2	2	2	0	2	3	3	−0.014 117 31
	2	2	2	0	4	3	3	0.033 881 55
	2	2	2	1	1	2	4	0.102 697 98
	2	2	2	1	3	2	4	−0.102 697 98
47	2	0	2	1	1	4	2	0.435 710 63
	2	0	2	1	3	4	2	−0.435 710 63
	2	0	2	2	0	3	3	−0.251 557 65
	2	0	2	2	2	3	3	−0.179 684 03
	2	0	2	2	4	3	3	0.431 241 68
	2	1	2	0	1	4	2	0.145 236 88
	2	1	2	0	3	4	2	−0.145 236 88
	2	1	2	2	1	2	4	−0.435 710 63
	2	1	2	2	3	2	4	0.435 710 63
	2	2	2	0	0	3	3	0.027 950 85
	2	2	2	0	2	3	3	0.019 964 89
	2	2	2	0	4	3	3	−0.047 915 74
	2	2	2	1	1	2	4	−0.145 236 88
	2	2	2	1	3	2	4	0.145 236 88
48	0	0	2	0	2	3	1	0.345 032 78
	0	0	2	1	1	2	2	−0.836 660 03
	0	0	2	1	3	2	2	−0.358 568 58
	0	0	2	2	2	1	3	1.035 098 34
	2	0	2	0	2	5	1	0.182 981 26
	2	0	2	1	1	4	2	−0.348 626 13
	2	0	2	1	3	4	2	−0.285 239 56
	2	0	2	2	0	3	3	0.384 260 65
	2	0	2	2	2	3	3	−0.117 630 81
	2	0	2	2	4	3	3	0.282 313 95
	2	1	2	0	1	4	2	0.116 208 71
	2	1	2	0	3	4	2	0.095 079 85
	2	1	2	1	0	3	3	0.085 391 26
	2	1	2	1	2	3	3	−0.880 052 74
	2	1	2	1	4	3	3	0.062 736 43
	2	1	2	2	1	2	4	0.348 626 13
	2	1	2	2	3	2	4	0.285 239 56
	2	2	2	0	0	3	3	0.042 695 63
	2	2	2	0	2	3	3	−0.013 070 09
	2	2	2	0	4	3	3	0.031 368 22
	2	2	2	1	1	2	4	−0.116 208 71
	2	2	2	1	3	2	4	−0.095 079 85
	2	2	2	2	2	1	5	0.182 981 26
51	2	0	2	0	0	5	1	−0.145 236 88
	2	0	2	1	1	4	2	0.503 115 29
	2	0	2	2	2	3	3	−0.435 710 63
	2	1	2	0	1	4	2	−0.167 705 10
	2	1	2	1	0	3	3	0.484 122 92
	2	1	2	1	2	3	3	0.096 824 58
	2	1	2	2	1	2	4	−0.503 115 29
	2	2	2	0	2	3	3	−0.048 412 29
	2	2	2	1	1	2	4	0.167 705 10
	2	2	2	2	0	1	5	−0.145 236 88
52	2	0	2	0	0	5	1	0.048 412 29
	2	0	2	1	1	4	2	−0.167 705 10
52	2	0	2	2	2	3	3	0.145 236 88
	2	1	2	0	1	4	2	0.055 901 70
	2	1	2	1	0	3	3	−0.161 374 31
	2	1	2	1	2	3	3	−0.032 274 86
	2	1	2	2	1	2	4	0.167 705 10
	2	2	2	0	2	3	3	0.016 137 43
	2	2	2	1	1	2	4	−0.055 901 70
	2	2	2	2	0	1	5	0.048 412 29
53	0	0	2	0	2	3	1	0.129 099 44
	0	0	2	1	1	2	2	−0.447 213 60
	0	0	2	2	0	1	3	0.387 298 33
	2	0	2	0	2	5	1	0.068 465 32
	2	0	2	1	1	4	2	−0.166 019 58
	2	0	2	1	3	4	2	−0.071 151 25
	2	0	2	2	2	3	3	0.205 395 96
	2	1	2	0	1	4	2	0.055 339 86
	2	1	2	0	3	4	2	0.023 717 08
	2	1	2	1	0	3	3	−0.159 752 41
	2	1	2	1	2	3	3	−0.114 108 87
	2	1	2	2	1	2	4	0.237 170 82
	2	2	2	0	2	3	3	0.022 821 77
	2	2	2	1	1	2	4	−0.079 056 94
	2	2	2	2	0	1	5	0.068 465 32
54	0	0	2	0	0	3	1	0.129 099 44
	0	0	2	1	1	2	2	−0.447 213 60
	0	0	2	2	2	1	3	0.387 298 33
	2	0	2	0	0	5	1	0.068 465 32
	2	0	2	1	1	4	2	−0.237 170 82
	2	0	2	2	2	3	3	0.205 395 96
	2	1	2	0	1	4	2	0.079 056 94
	2	1	2	1	0	3	3	−0.159 752 41
	2	1	2	1	2	3	3	−0.114 108 87
	2	1	2	2	1	2	4	0.166 019 58
	2	1	2	2	3	2	4	0.071 151 25
	2	2	2	0	2	3	3	0.022 821 77
	2	2	2	1	1	2	4	−0.055 339 86
	2	2	2	1	3	2	4	−0.023 717 08
	2	2	2	2	2	1	5	0.068 465 32
55	2	0	2	0	2	5	1	0.108 253 18
	2	0	2	1	1	4	2	−0.150 000 00
	2	0	2	1	3	4	2	−0.225 000 00
	2	0	2	2	0	3	3	0.064 951 91
	2	0	2	2	2	3	3	0.092 788 44
	2	0	2	2	4	3	3	0.167 019 19
	2	1	2	0	1	4	2	0.050 000 00
	2	1	2	0	3	4	2	0.075 000 00
	2	1	2	1	0	3	3	−0.014 433 76
	2	1	2	1	2	3	3	−0.381 463 57
	2	1	2	1	4	3	3	−0.037 115 37
	2	1	2	2	1	2	4	0.150 000 00
	2	1	2	2	3	2	4	0.225 000 00
	2	2	2	0	0	3	3	0.007 216 88
	2	2	2	0	2	3	3	0.010 309 83
	2	2	2	0	4	3	3	0.018 557 69
	2	2	2	1	1	2	4	−0.050 000 00
	2	2	2	1	3	2	4	−0.075 000 00
	2	2	2	2	2	1	5	0.108 253 18
56	2	0	2	1	1	4	2	0.137 783 80

TABLE III *(Continued)*

$\alpha\alpha_2$	L_1	Λ	l	λ	r	n_q	n_p	Γ
56	2	0	2	1	3	4	2	−0.137 783 80
	2	0	2	2	0	3	3	−0.079 549 51
	2	0	2	2	2	3	3	−0.056 821 08
	2	0	2	2	4	3	3	0.136 370 59
	2	1	2	0	1	4	2	0.045 927 93
	2	1	2	0	3	4	2	−0.045 927 93
	2	1	2	2	1	2	4	−0.137 783 80
	2	1	2	2	3	2	4	0.137 783 80
	2	2	2	0	0	3	3	0.008 838 83
	2	2	2	0	2	3	3	0.006 313 45
	2	2	2	0	4	3	3	−0.015 152 29
	2	2	2	1	1	2	4	−0.045 927 93
	2	2	2	1	3	2	4	0.045 927 93
57	2	0	2	1	1	4	2	−0.194 855 72
	2	0	2	1	3	4	2	0.194 855 72
	2	0	2	2	0	3	3	0.112 500 00
	2	0	2	2	2	3	3	0.080 357 14
	2	0	2	2	4	3	3	−0.192 857 14
	2	1	2	0	1	4	2	−0.064 951 91
	2	1	2	0	3	4	2	0.064 951 91
	2	1	2	2	1	2	4	0.194 855 72
	2	1	2	2	3	2	4	−0.194 855 72
	2	2	2	0	0	3	3	−0.012 500 00
	2	2	2	0	2	3	3	−0.008 928 57
	2	2	2	0	4	3	3	0.021 428 57
57	2	2	2	1	1	2	4	0.064 951 91
	2	2	2	1	3	2	4	−0.064 951 91
58	0	0	2	0	2	3	1	−0.154 303 35
	0	0	2	1	1	2	2	0.374 165 74
	0	0	2	1	3	2	2	0.160 356 75
	0	0	2	2	2	1	3	−0.462 910 05
	2	0	2	0	2	5	1	−0.081 831 71
	2	0	2	1	1	4	2	0.155 910 35
	2	0	2	1	3	4	2	0.127 563 01
	2	0	2	2	0	3	3	−0.171 846 59
	2	0	2	2	2	3	3	0.052 606 10
	2	0	2	2	4	3	3	−0.126 254 64
	2	1	2	0	1	4	2	−0.051 970 12
	2	1	2	0	3	4	2	−0.042 521 00
	2	1	2	1	0	3	3	−0.038 188 13
	2	1	2	1	2	3	3	0.393 571 55
	2	1	2	1	4	3	3	−0.028 056 59
	2	1	2	2	1	2	4	−0.155 910 35
	2	1	2	2	3	2	4	−0.127 563 01
	2	2	2	0	0	3	3	−0.019 094 07
	2	2	2	0	2	3	3	0.005 845 12
	2	2	2	0	4	3	3	−0.014 028 29
	2	2	2	1	1	2	4	0.051 970 12
	2	2	2	1	3	2	4	0.042 521 00
	2	2	2	2	2	1	5	−0.081 831 71

equations as

$$\psi_s^{(1)}(p,q,\alpha)=\int_0^\infty dq_2\int_{|\alpha_{12}q_2-q|/\beta_{12}}^{(\alpha_{12}q_2+q)/\beta_{12}}dp_2\sum_{\alpha_2=1}^{8}K(\alpha\,|\,\alpha_2)\psi_s^{(1)}(p_2,q_2,\alpha)\,,\qquad \alpha=1,2,\ldots,8\,, \tag{4.3}$$

where

$$K(\alpha\,|\,\alpha_2)=\sum_{L_1,\Lambda,\lambda,r}\frac{t_{L,L_1}^{J_1(=S)STT_z}(p,(p_2^2+q_2^2-q^2)^{1/2};(s-q^2)^{1/2})}{q^l(p_2^2+q_2^2-s)(p_2^2+q_2^2-q^2)^{L_1/2}}q_2^{n_q}p_2^{n_p}P_r\left(\frac{\beta_{12}^2p_2^2+\alpha_{12}^2q_2^2-q^2}{2\alpha_{12}\beta_{12}p_2q_2}\right)\Gamma^{\alpha,\alpha_2}_{L_1,\Lambda,\lambda,r}\,, \tag{4.4}$$

$$n_q=L_1-\Lambda+l-\lambda+1\,,\qquad n_p=\Lambda+\lambda+1\,.$$

$\Gamma^{\alpha,\alpha_2}_{L_1,\Lambda,\lambda,r}$ may be easily read off from (2.11), and is tabulated numerically in Table III for $\alpha=1,2,\ldots,5$; $\alpha_2=1,2,\ldots,8$. We need not list values of $\Gamma^{\alpha,\alpha_2}_{L_1,\Lambda,\lambda,r}$ for $\alpha=6$, 7, 8, since $K(6\,|\,\alpha_2)=\sqrt{\tfrac{3}{2}}K(5\,|\,\alpha_2)$, $K(7|\,\alpha_2)=\tfrac{1}{2}\sqrt{3}K(5\,|\,\alpha_2)$, and $K(8|\,\alpha_2)=\tfrac{1}{2}\sqrt{7}K(5|\,\alpha_2)$, in agreement with the results stated in (4.2). The five independent Faddeev equations may be taken to be (4.3), with $\alpha=1,2,\ldots,5$, with (4.2) used to relate $\psi_s^{(1)}(p,q,\alpha)$, $(\alpha=6,7,8)$ to $\psi_s^{(1)}(p,q,5)$.

Equation (3.15) gives the components of the bound-state wave function in terms of the homogeneous solution $\psi^{(1)}_{s(=E_B)}(p,q,\alpha)$, $(\alpha=1,2,\ldots,8)$ of (4.3). Table IV gives the numerical values of $B_{\mathcal{S}}(S\,|\,S_1')$ and $C_{\mathcal{T}}(T\,|\,T_1')$, and Table V gives values of $A^{\Lambda\lambda r}(Ll\mathcal{L}\,|\,L_3l_3\mathcal{L}_3(=\mathcal{L}))$ for $0\leq L$, $l\leq 2$, $\mathcal{L}=0$, 2. The wave-function components for which $\mathcal{L}=1$ for L, $l>2$ are negligible for realistic nucleon-nucleon interactions.[6]

TABLE IV. Numerical values for the spin particle-exchange coefficients $B_{1/2}(S\,|\,S_1')$ in (3.15). The isospin particle-exchange coefficients $C_{\mathcal{T}}(T\,|\,T_1')$ are obtained by letting $S\to T$, $S_1'\to T_1'$, $\mathcal{S}\to\mathcal{T}$.

$S \backslash S_1'$	0	1
0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$
1	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$

V. EXPANSION OF THE BOUND-STATE WAVE FUNCTION IN DERRICK-BLATT BASIS STATES

Derrick and Blatt[3] have derived a general classification of trinucleon states based on the properties of the rotation and symmetric groups of degree 3.

The spin-isospin states in their scheme, with

TABLE V. Numerical values of $A^{\Lambda\lambda r}(Ll\mathcal{L}|L_3 l_3 \mathcal{L}_3(=\mathcal{L}))$ in (3.15) for $0<(L, l, \mathcal{L})<2$, $\mathcal{L}\neq 1$, for the case of nucleon-nucleon interactions in the $^1S_0$ and $^3S_1-{}^3D_1$ states.

L	l	$\mathcal{L}$	L_3	l_3	Λ	λ	r	n_p	n_q	A
0	0	0	0	0	0	0	0	0	0	1.000 000 00
1	1	0	0	0	0	0	1	0	0	−1.732 050 81
2	2	0	0	0	0	0	2	0	0	2.236 067 98
0	2	2	2	0	0	0	0	0	2	0.750 000 00
			2	0	1	0	1	1	1	0.866 025 40
			2	0	2	0	2	2	0	0.250 000 00
1	1	2	2	0	0	0	1	0	2	0.821 583 84
			2	0	1	0	0	1	1	0.790 569 42
			2	0	1	0	2	1	1	0.158 113 88
			2	0	2	0	1	2	0	0.273 861 28
2	0	2	2	0	0	0	2	0	2	0.750 000 00
			2	0	1	0	1	1	1	0.866 025 40
			2	0	2	0	0	2	0	0.250 000 00
2	2	2	2	0	0	0	2	0	2	−0.896 421 46
			2	0	1	0	1	1	1	−0.724 568 84
			2	0	1	0	3	1	1	−0.310 529 50
			2	0	2	0	0	2	0	0.000 000 00
			2	0	2	0	2	2	0	−0.298 807 15
0	2	2	0	2	0	0	0	0	2	0.250 000 00
			0	2	0	1	1	1	1	−0.866 025 40
			0	2	0	2	2	2	0	0.750 000 00
1	1	2	0	2	0	0	1	0	2	0.273 861 28
			0	2	0	1	0	1	1	−0.790 569 42
			0	2	0	1	2	1	1	−0.158 113 88
			0	2	0	2	1	2	0	0.821 583 84
2	0	2	0	2	0	0	2	0	2	0.250 000 00
			0	2	0	1	1	1	1	−0.866 025 40
			0	2	0	2	0	2	0	0.750 000 00
2	2	2	0	2	0	0	2	0	2	−0.298 807 15
			0	2	0	1	1	1	1	0.724 568 84
			0	2	0	1	3	1	1	0.310 529 50
			0	2	0	2	0	2	0	0.000 000 00
			0	2	0	2	2	2	0	−0.896 421 46
0	2	2	2	2	0	0	0	0	4	−0.224 105 36
			2	2	0	1	1	1	3	0.776 323 75
			2	2	0	2	2	2	2	−0.672 316 09
			2	2	1	0	1	1	3	−0.258 774 58
			2	2	1	1	0	2	2	0.522 912 52
			2	2	1	1	2	2	2	0.373 508 94
			2	2	1	2	1	3	1	−0.543 426 63
			2	2	1	2	3	3	1	−0.232 897 13
			2	2	2	0	2	2	2	−0.074 701 79
			2	2	2	1	1	3	1	0.181 142 21
			2	2	2	1	3	3	1	0.077 632 38
0	2	2	2	2	2	2	0	4	0	0.000 000 00
			2	2	2	2	2	4	0	−0.224 105 36
1	1	2	2	2	0	0	1	0	4	−0.245 495 13
			2	2	0	1	0	1	3	0.496 078 37
			2	2	0	1	2	1	3	0.354 341 69
			2	2	0	2	1	2	2	−0.515 539 77
			2	2	0	2	3	2	2	−0.220 945 61
			2	2	1	0	0	1	3	−0.165 359 46
			2	2	1	0	2	1	3	−0.118 113 90
			2	2	1	1	1	2	2	1.031 079 53
			2	2	1	1	3	2	2	−0.049 099 03
			2	2	1	2	0	3	1	−0.496 078 37
			2	2	1	2	2	3	1	−0.354 341 69
			2	2	2	0	1	2	2	−0.057 282 20
			2	2	2	0	3	2	2	−0.024 549 51
			2	2	2	1	0	3	1	0.165 359 46
			2	2	2	1	2	3	1	0.118 113 90
			2	2	2	2	1	4	0	−0.245 495 13
2	0	2	2	2	0	0	2	0	4	−0.224 105 36
			2	2	0	1	1	1	3	0.543 426 63
			2	2	0	1	3	1	3	0.232 897 13
			2	2	0	2	0	2	2	0.000 000 00
			2	2	0	2	2	2	2	−0.672 316 09
			2	2	1	0	1	1	3	−0.181 142 21
			2	2	1	0	3	1	3	−0.077 632 38
			2	2	1	1	0	2	2	0.522 912 52
			2	2	1	1	2	2	2	0.373 508 94
			2	2	1	2	1	3	1	−0.776 323 75
			2	2	2	0	0	2	2	0.000 000 00
			2	2	2	0	2	2	2	−0.074 701 79
			2	2	2	1	1	3	1	0.258 774 58
			2	2	2	2	0	4	0	−0.224 105 36
2	2	2	2	2	0	0	2	0	4	0.267 857 14
			2	2	0	1	1	1	3	−0.510 336 40
			2	2	0	1	3	1	3	−0.417 547 96
			2	2	0	2	0	2	2	0.562 500 00
			2	2	0	2	2	2	2	−0.172 193 88
			2	2	0	2	4	2	2	0.413 265 31
			2	2	1	0	1	1	3	0.170 112 13
			2	2	1	0	3	1	3	0.139 182 65
			2	2	1	1	0	2	2	0.125 000 00
			2	2	1	1	2	2	2	−1.288 265 31
			2	2	1	1	4	2	2	0.091 836 73
			2	2	1	2	1	3	1	0.510 336 40
			2	2	1	2	3	3	1	0.417 547 96
			2	2	2	0	0	2	2	0.062 500 00
			2	2	2	0	2	2	2	−0.019 132 65
			2	2	2	0	4	2	2	0.045 918 37
			2	2	2	1	1	3	1	−0.170 112 13
			2	2	2	1	3	3	1	−0.139 182 65
			2	2	2	2	0	4	0	0.000 000 00
			2	2	2	2	2	4	0	0.267 857 14

$\mathcal{T}=\mathcal{T}_z=\frac{1}{2}$, are:

$$
\begin{aligned}
W^{\mathrm{Sym}}_{1/2\ m_{\mathcal{S}}} &= \frac{1}{\sqrt{2}}(\chi_1\eta_1+\chi_2\eta_2)\,,\\
W^{A}_{1/2\ m_{\mathcal{S}}} &= \frac{1}{\sqrt{2}}(\chi_2\eta_1-\chi_1\eta_2)\,,\\
W^{+}_{1/2\ m_{\mathcal{S}}} &= \frac{1}{\sqrt{2}}(\chi_2\eta_2-\chi_1\eta_1)\,,\\
W^{-}_{1/2\ m_{\mathcal{S}}} &= \frac{1}{\sqrt{2}}(\chi_1\eta_2+\chi_2\eta_1)\,,\\
W^{-}_{3/2\ m_{\mathcal{S}}} &= \chi_3\eta_1\,,\\
W^{+}_{3/2\ m_{\mathcal{S}}} &= -\chi_3\eta_2\,,
\end{aligned}
\tag{5.1}
$$

where

$$
\begin{aligned}
\chi_1 &= \chi_{1m_{\mathcal{S}}} = |(Ss)\mathcal{S}m_{\mathcal{S}}\rangle_1\,, \quad S=0\,, \quad s=\mathcal{S}=\tfrac{1}{2}\,;\\
\chi_2 &= \chi_{2m_{\mathcal{S}}} = |(Ss)\mathcal{S}m_{\mathcal{S}}\rangle_1\,, \quad S=1\,, \quad s=\mathcal{S}=\tfrac{1}{2}\,;\\
\chi_3 &= \chi_{3m_{\mathcal{S}}} = |(Ss)\mathcal{S}m_{\mathcal{S}}\rangle_1\,, \quad S=1\,, \quad s=\tfrac{1}{2}\,, \quad \mathcal{S}=\tfrac{3}{2}\,;
\end{aligned}
\tag{5.2}
$$

with isospin states η_1 and η_2 obtained by the replacements $\chi_{1,2}\to\eta_{1,2}$, $S\to T$, $s\to t$, $\mathcal{S}\to\mathcal{T}$, and $m_{\mathcal{S}}\to\mathcal{T}_z$. The superscripts Sym, A, +, and − denote, respectively, complete symmetry, complete antisymmetry, mixed symmetry with symmetry under $P(1)$, and mixed symmetry with antisymmetry under $P(1)$. For the mixed-symmetry states, we have

$$
\begin{aligned}
P(2)W^+ &= -\tfrac{1}{2}W^+ - \tfrac{1}{2}\sqrt{3}\,W^-\,,\\
P(2)W^- &= -\tfrac{1}{2}\sqrt{3}\,W^+ + \tfrac{1}{2}W^-\,,\\
P(3)W^+ &= -\tfrac{1}{2}W^+ + \tfrac{1}{2}\sqrt{3}\,W^-\,,\\
P(3)W^- &= \tfrac{1}{2}\sqrt{3}\,W^+ + \tfrac{1}{2}W^-\,,
\end{aligned}
\tag{5.3}
$$

with subscripts $\mathcal{S}m_{\mathcal{S}}$ suppressed.

The trinucleon bound-state vector with $\mathcal{J}=\mathcal{J}_z=\mathcal{T}=\mathcal{T}_z=\frac{1}{2}$ may be represented as

$$
\begin{aligned}
|\psi_B\rangle =\;& W^{\mathrm{Sym}}_{1/2\ 1/2}|\psi^A_{1/2}(^2S_{1/2})\rangle + W^A_{1/2\ 1/2}|\psi^{\mathrm{Sym}}_{1/2}(^2S_{1/2})\rangle + W^+_{1/2\ 1/2}|\psi^-_{1/2}(^2S_{1/2})\rangle - W^-_{1/2\ 1/2}|\psi^+_{1/2}(^2S_{1/2})\rangle\\
&+\sum_{m_{\mathcal{L}},m_{\mathcal{S}}}\langle 1m_{\mathcal{L}}\tfrac{1}{2}m_{\mathcal{S}}|\tfrac{1}{2}\tfrac{1}{2}\rangle[W^{\mathrm{Sym}}_{1/2\ m_{\mathcal{S}}}|\psi^A_{m_{\mathcal{S}}}(^2P_{1/2})\rangle + W^A_{1/2\ m_{\mathcal{S}}}|\psi^{\mathrm{Sym}}_{m_{\mathcal{S}}}(^2P_{1/2})\rangle + W^+_{1/2\ m_{\mathcal{S}}}|\psi^-_{m_{\mathcal{S}}}(^2P_{1/2})\rangle - W^-_{1/2\ m_{\mathcal{S}}}|\psi^+_{m_{\mathcal{S}}}(^2P_{1/2})\rangle]\\
&+\sum_{m_{\mathcal{L}},m_{\mathcal{S}}}\langle 1m_{\mathcal{L}}\tfrac{3}{2}m_{\mathcal{S}}|\tfrac{1}{2}\tfrac{1}{2}\rangle[W^+_{3/2\ m_{\mathcal{S}}}|\psi^-_{m_{\mathcal{S}}}(^4P_{1/2}) - W^-_{3/2\ m_{\mathcal{S}}}|\psi^+_{m_{\mathcal{S}}}(^4P_{1/2})\rangle]\\
&+\sum_{m_{\mathcal{L}},m_{\mathcal{S}}}\langle 2m_{\mathcal{L}}\tfrac{3}{2}m_{\mathcal{S}}|\tfrac{1}{2}\tfrac{1}{2}\rangle[W^+_{3/2\ m_{\mathcal{S}}}|\psi^-_{m_{\mathcal{S}}}(^4D_{1/2})\rangle - W^-_{3/2\ m_{\mathcal{S}}}|\psi^+_{m_{\mathcal{S}}}(^4D_{1/2})\rangle]\,,
\end{aligned}
\tag{5.4}
$$

where the superscripts Sym, A, and ± denote the same particle-exchange properties for the spacial states as they do for the spin-isospin states.

The components of the spacial states are easily expressed in terms of the components of $|\psi_B\rangle$ given by (3.15):

$$
{}_1\langle pq(LL)00|\psi^A_{1/2}(^2S_{1/2})\rangle = \frac{1}{\sqrt{2}}\,{}_1\langle[pq(LL)0,(0\tfrac{1}{2})\tfrac{1}{2}]\tfrac{1}{2}\tfrac{1}{2};(0\tfrac{1}{2})\tfrac{1}{2}\tfrac{1}{2}|\psi_B\rangle + \frac{1}{\sqrt{2}}\,{}_1\langle[pq(LL)0,(1\tfrac{1}{2})\tfrac{1}{2}]\tfrac{1}{2}\tfrac{1}{2};(1\tfrac{1}{2})\tfrac{1}{2}\tfrac{1}{2}|\psi_B\rangle\,,
$$

L odd; etc.; (5.5)

$$
{}_1\langle pq(LL)1m_{\mathcal{L}}|\psi^A_{m_{\mathcal{S}}}(^2P_{1/2})\rangle = \langle 1m_{\mathcal{L}}\tfrac{1}{2}m_{\mathcal{S}}|\tfrac{1}{2}\tfrac{1}{2}\rangle\frac{1}{\sqrt{2}}\{{}_1\langle[pq(LL)1,(0\tfrac{1}{2})]\tfrac{1}{2}\tfrac{1}{2};(0\tfrac{1}{2})\tfrac{1}{2}\tfrac{1}{2}|\psi_B\rangle + {}_1\langle[pq(LL)1,(1\tfrac{1}{2})]\tfrac{1}{2}\tfrac{1}{2};(1\tfrac{1}{2})\tfrac{1}{2}\tfrac{1}{2}|\psi_B\rangle\}\,,
$$

L odd; etc. (5.6)

From the exchange properties of the spacial mixed-symmetry states, it follows that

$$
\langle\psi^+|\psi^+\rangle = \langle P_{13}\psi^+|P_{13}\psi^+\rangle = \langle -\tfrac{1}{2}\psi^+ - \tfrac{1}{2}\sqrt{3}\,\psi^-|-\tfrac{1}{2}\psi^+ - \tfrac{1}{2}\sqrt{3}\,\psi^-\rangle = \tfrac{1}{4}\langle\psi^+|\psi^+\rangle + \tfrac{3}{4}\langle\psi^-|\psi^-\rangle\,. \tag{5.7}
$$

Thus,

$$
\langle\psi^+|\psi^+\rangle = \langle\psi^-|\psi^-\rangle\,. \tag{5.8}
$$

The relation (5.8) is very useful for checking the consistency of the complicated numerical calculations involved in the determination of $|\psi_B\rangle$.[6]

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Coulomb Forces in the Nuclear $1p$ Shell*

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Correlated wave functions obtained by solving the Bethe-Goldstone equation with realistic nuclear interactions are employed to calculate Coulomb shifts, isospin mixing, Coulomb energies, and coefficients of the isobaric mass formula in $1p$-shell nuclei. Improved agreement with experiment is obtained, particularly for the Coulomb shifts and isospin mixing which are not sensitive to the size parameter. No evidence is found favoring a charge-dependent component in the nuclear force.

I. INTRODUCTION

The concept of charge-independent nuclear forces is very nearly as old as the discovery of the neutron.[1,2] It is very well established that any charge-dependent component of the nuclear force must be quite weak compared with the basic interactions which bind atomic nuclei. A definitive evaluation of this component is hampered by the presence of the Coulomb interaction. Charge dependent effects clearly exist in nuclei; can they be *precisely* attributed to Coulomb forces?

To answer this question, one obviously requires precise knowledge of nuclear wave functions. Thus, the theoretical investigation of charge-dependent effects in nuclei requires a twofold approach. First one tries to calculate charge-dependent effects from known electromagnetic interactions with a trial wave function, then one must determine if any remaining discrepancies are to be attributed to additional charge-dependent interactions or an inadequate wave function.

The first nuclear p shell ($4<A\leq 16$) provides a wealth of charge-dependent data. The differences in binding energy for a mirror pair,

$$-\Delta(Z)\equiv \mathrm{B.E.}(Z,N)-\mathrm{B.E.}(Z-1,N+1)\,, \qquad (1.1)$$

have received extensive attention in the literature,[3-5] and have proved useful in the investigation of nuclear size. Likewise the alternation of second differences,

$$\Delta\Delta(Z)\equiv\Delta(Z)-\Delta(Z-1)\,, \qquad (1.2)$$

with odd-even Z has been useful in establishing the pairing correlation.[3,6]

More recently there has been considerable interest in the isospin mass formula[7]:

$$E(A;T,T_3)=a+bT_3+cT_3{}^2\,. \qquad (1.3a)$$

This formula relates the energies of isobaric analog levels in neighboring isobars. It is valid so long as the charge-dependent part of the interaction between nucleons is strictly of a two-body character and isospin mixing is negligible.

Sufficient data are now available on several multiplets, three of which ($A=7,\ 9,\ 13$) are in the first p shell.[8,9] Usually an empirical fit to the