E. Almquist, in *Proceedings of the Third Conference on Reactions between Complex Nuclei,* edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (Univ. of California Press, Berkeley, 1963), p. 11.

[31] P. M. Endt and C. Van der Leun, Nucl. Phys. 34, 1 (1962).

[32] H. L. Harney and A. Richter, Phys. Rev. C 2, 421 (1970).

[33] S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat. – Fys. Medd. 32, No. 16 (1960).

[34] J. J. Griffin, in *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960,* edited by D. A. Bromley and E. W. Vogt (Univ. of Toronto Press, Toronto, 1960); Phys. Rev. 132, 2204 (1963).

[35] P. A. Moldauer, Phys. Rev. 157, 907 (1967).

[36] M. G. B. Marcazzan and L. M. Colli, Progr. Nucl. Phys. 11, 145 (1969).

[37] L. G. Moretto, Phys. Letters 34B, 191 (1971); L. G. Moretto and R. Stella, *ibid.* 32B, 558 (1970).

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Energy Dependence of Elastic α-Particle Scattering: Microscopic Model

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The energy dependence of elastic α scattering from ^{40}Ca between 39.6 and 115.4 MeV is determined using a microscopic optical model. The agreement between theory and experiment improves as the energy increases. We find that both the real and imaginary parts of the optical potential are well described by a strength depending linearly on energy. For the real part the slope is about half the corresponding value for proton scattering; no theoretical estimates of this quantity are presently available.

The α-particle optical potential, $U(r_\alpha)$, has been related to the nucleon density distribution, $\rho(r)$, by the equation[1,2]

$$U(r_\alpha) = \lambda \int \rho(r) V_{\text{eff}}(\vec{r} - \vec{r}_\alpha) d\tau , \qquad (1)$$

where

$$\lambda \equiv \lambda_R + i\lambda_I \qquad (2)$$

is an empirically determined complex constant. V_{eff} is the effective α-particle–bound-nucleon interaction derived by folding the α-particle form factor with a nucleon-nucleon interaction which fits the low-energy data.[3] It is found to be approximately a Gaussian whose strength is 37 MeV and range is 2.0 fm.[4,5] Because elastic α scattering is sensitive only to the potential over a relatively small region near the diffraction radius, it is possible to absorb the uncertainties in V_{eff} into λ.[5,6]

This model has been applied to the scattering of α particles from 40 to 166 MeV[1,2,5,7-9] and good agreement with experiment has been obtained. Once λV_{eff} has been determined, Eq. (1) can be used to predict the scattering of α particles (in the diffraction region) given a model for $\rho(r)$. Alternatively, the procedure may be inverted to empirically determine $\rho(r)$ in the surface region.[2,5,7,8]

The method used to determine λ (at a fixed value of E, the center of mass energy) is from α scattering by ^{40}Ca. This is the heaviest $T=0$ nucleus, and one can assume $\rho_n \approx \rho_p$ which is obtained from electron scattering. This approximation has been checked using Hartree-Fock densities.[5] It has been assumed that λ is a smooth function of E and independent of A. The latter assumption has been confirmed from $A=16$ to $A=40$ by successfully predicting α scattering from several $T=0$ nuclei at 79 and 104 MeV.[2,7] Although an analysis of existing elastic α-particle-scattering data from ^{40}Ca over the range 31 to 166 MeV[9] confirmed the expected behavior of λ with energy, a gradual decrease of λ_R and increase of λ_I, quantitative information could not be obtained because of systematic differences between data sets obtained in different laboratories.

We have measured elastic α scattering by ^{40}Ca with the Texas A & M cyclotron from 39.6 to 115.4 MeV using an essentially fixed experimental setup. Two 3-mm Si(Li) detectors separated by 5.5° were placed on a single mount. At the higher energies the same 3-mm detectors were turned 60°. Angular acceptance ranged from $\frac{1}{2}$° at 39.6 MeV to $\frac{1}{4}$° at 115.4 MeV; energy resolution was 0.2 to 0.3%. Self-supporting, weighed Ca metal and CaO targets of about 0.5 mg/cm^2 were used and different targets were always cross-checked against a CaO standard. Absolute normalizations were also checked by comparing forward angle, low-energy α scattering with the Rutherford formula and are believed accurate to 5%. The zero angle of the beam and the Faraday cup efficiency were checked by small-angle scattering on both sides of the beam using Au or Pb targets of known thickness. Two monitors were used on either side of the beam out of the scattering plane.

The nuclear (centers) density for ^{40}Ca was obtained by unfolding the proton size (an exponential

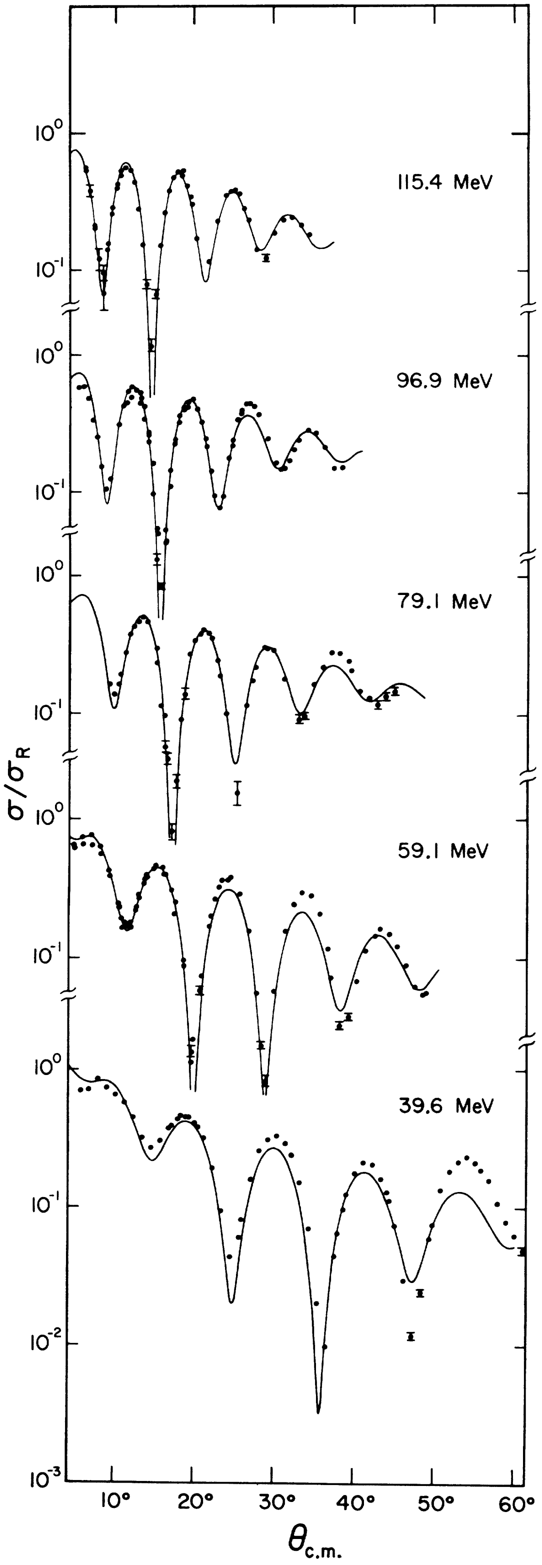

FIG. 1. The experimental ^{40}Ca(α, α)^{40}Ca cross sections. Theoretical curves are the best two parameter fits.

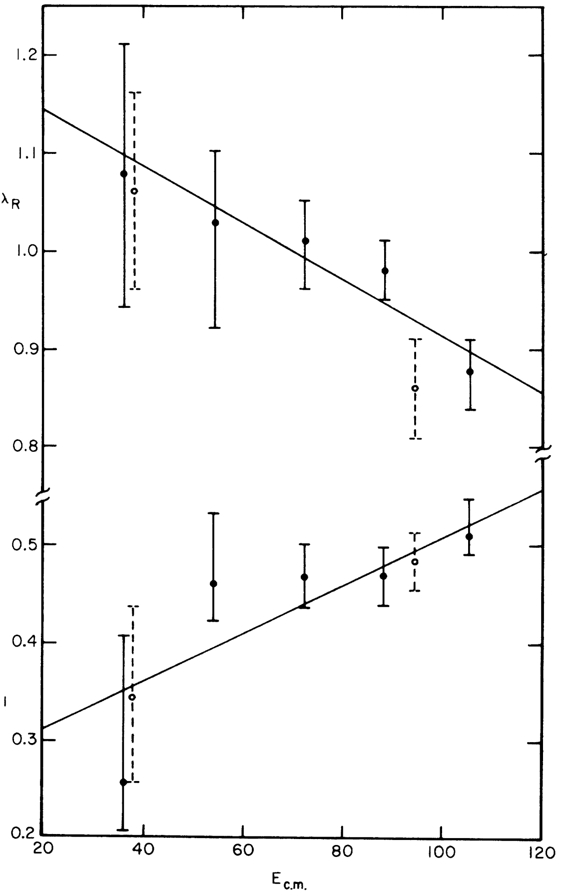

FIG. 2. The energy dependence of optical-potential parameters. Errors are uncorrelated changes which increase χ^2 by 50%. The points with dashed error bars are from Refs. 12 and 13.

TABLE I. Energy dependence of optical-potential parameters. Errors are uncorrelated changes which increase χ^2 by 50%.

E_{cm} (MeV)	λ_R	λ_I	χ^2 per point
35.9	1.08 ± 0.14	$0.26^{+0.15}_{-0.05}$	45
53.7	$1.03^{+0.07}_{-0.10}$	$0.46^{+0.07}_{-0.04}$	11
71.9	$1.01^{+0.04}_{-0.05}$	$0.46^{+0.04}_{-0.03}$	4.1
88.1	$0.96^{+0.03}_{-0.04}$	0.47 ± 0.03	4.7
104.9	$0.88^{+0.03}_{-0.04}$	$0.51^{+0.03}_{-0.02}$	2.3

shape of rms radius 0.8 fm[10] was assumed) from the parabolic Fermi charge distribution obtained by Frosch *et al.*[11] The parameters used were $C = 3.6758$ fm, $a = 0.5851$ fm, and $W = -0.1017$. This yields an rms charge radius of 3.487 fm, and an rms proton (=matter) radius of 3.394 fm.

Figure 1 shows the best two parameter fits to the data. The quality of the fits improves with energy in agreement with the results of Seidler[9] who concluded that the model was most applicable at energies above 60 MeV. The energy dependence of λ_R and λ_I is displayed in Fig. 2 and Table I. Quoted errors are the uncorrelated changes in λ (i.e., λ_R is changed independently of λ_I and vice versa) which increase χ^2 by 50%. Data points at 38 and 94 MeV are from Fernandez and Blair[12] and Hauser *et al.*,[13] respectively. The lines (in Fig. 2) represent weighted least-squares fits to the real and imaginary parts of λ, obtained by using our data only.

Several other ^{40}Ca mass distributions from electron scattering[11] and Hartree-Fock[14] calculations were used to ascertain the sensitivity of our parameters to the particular density used. The basic correction can be expressed as $d\lambda_R(0)/dR \approx -2\ \text{fm}^{-1}$, where R is the rms matter radius. Uncertainties in R for ^{40}Ca are of the order of ± 0.05 fm, which implies a systematic uncertainty of ∓ 0.1 for $\lambda_R(0)$.

For the real potential, the linear dependence on center-of-mass energy is written as:

$$\lambda_R(E) = \lambda_R(0)[1 - a_R(\alpha)E], \tag{3}$$

in analogy with the expression for protons,

$$V_R(E) = V_R(0)[1 - a_R(p)E]. \tag{4}$$

We obtain $\lambda_R(0) = 1.2 \pm 0.1$ and $a_R(\alpha) = 0.0024 \pm 0.0010$ MeV^{-1}, which may be compared with the value $a_R(p) = 0.0052$ MeV^{-1} found by van Oers[15] in his analysis of 10- to 180-MeV elastic proton scattering from ^{40}Ca. It is interesting that $a_R(p)/a_R(\alpha) = 2.2 \pm 1.0$. If one assumes that the energy dependence arises solely from approximating a nonlocal, energy-independent potential of the Perey-Buck[16] form with an effective local, energy-dependent potential, then a_R is given by

$$a_R = M\beta^2/2\hbar^2, \tag{5}$$

where M is the mass of the projectile and β is the range of the nonlocality.

If β were the same for protons and α particles then $a_R(p)/a_R(\alpha) = 0.25$. The experimental value is about 10 times higher than this. If we evaluate β from Eq. (5), we obtain $\beta \approx 0.7$ fm for protons and $\beta \approx 0.2$ fm for α particles. It is possible that this small value is a consequence of the localized nature of the α-nucleus interaction. A more likely explanation is that the cause of the energy dependence in α-particle scattering is not the result of an energy-independent, nonlocal potential but has another physical origin. At the present time we know of no theoretical predictions for this quantity.

A linear energy dependence also appears adequate for the imaginary potential (see Fig. 2). This contrasts with the result for protons where a mixture of decreasing surface and increasing volume absorption is needed to fit the data.[15] We find:

$$\lambda_I(E) = \lambda_I(0)[1 + a_I(\alpha)E], \tag{6}$$

where

$$\lambda_I(0) = 0.27 \pm 0.08,$$

$$a_I(\alpha) = 0.009 \pm 0.004\ \text{MeV}^{-1}.$$

Quoted uncertainties correspond to a confidence limit of 1 standard deviation in fitting the data in Fig. 2. These errors are larger [except for $\lambda_R(0)$ as noted above] than systematic errors resulting from an assumed rms matter radius for ^{40}Ca of 3.39 ± 0.05 fm.

We conclude that elastic α scattering in the diffraction region is adequately described by equations (1), (2), (3), and (6) for scattering energies above 40 MeV. This model is useful for predicting scattering cross sections for nuclei of known density or empirically determining nuclear densities in the surface from elastic α data.[2, 5, 7, 8] We have shown that this model works better with increasing energy and that the parameters λ_R and λ_I vary approximately linearly with energy. The physical origin of the energy dependence, particularly of the real potential, is not known at the present time.

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[1] D. F. Jackson, Phys. Letters 32B, 233 (1970); C. G. Morgan and D. F. Jackson, Phys. Rev. 188, 1758 (1969); D. F. Jackson and V. K. Kembhavi, *ibid.* 178, 1 26 (1969).

[2] A. M. Bernstein and W. A. Seidler, II, Phys. Letters 34B, 569 (1971); 39B, 583 (1972).

[3] I. Reichstein and Y. C. Tang, Nucl. Phys. A139, 144 (1969).

[4] A. M. Bernstein, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum Press, New York, 1969), Vol. 3.

[5] A. M. Bernstein and W. A. Seidler, II, to be published.

[6] C. J. Batty, E. Friedman, and D. F. Jackson, Nucl. Phys. A175, 1 (1971).

[7] G. M. Lerner, J. C. Hiebert, and L. Rutledge, Bull. Am. Phys. Soc. 17, 536 (1972).

[8] B. Tatischeff, I. Brissaud, and L. Bimbot, Phys. Rev. C 5, 234 (1972).

[9] W. A. Seidler, II, Ph.D. thesis, Massachusetts Institute of Technology, 1972 (unpublished).

[10] R. Hofstader, F. Bumiller, and M. R. Yearian, Rev. Mod. Phys. 30, 482 (1958).

[11] R. F. Frosch *et al.*, Phys. Rev. 174, 1380 (1968).

[12] B. Fernandez and J. S. Blair, Phys. Rev. C 1, 523 (1969).

[13] G. Hauser *et al.*, Nucl. Phys. A128, 81 (1969); G. Schatz, private communication.

[14] J. W. Negele, Phys. Rev. C 1, 1260 (1970).

[15] W. T. H. van Oers, Phys. Rev. C 3, 1550 (1971).

[16] F. G. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

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Study of (^{3}He, t) Reactions at 70 MeV to Isobaric Analog States of ^{50}Cr, ^{62}Ni, and ^{90}Zr†

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The analysis of (^{3}He, t) reactions at 70 MeV to isobaric analog ground states of ^{50}Cr, ^{62}Ni, and ^{90}Zr have shown an energy dependence in the extracted isospin-dependent interaction strengths consistent with results at lower energies; the interaction strengths are approximately 50% smaller than at lower bombarding energies. The shapes of the form factors in a macroscopic analysis are nuclei-dependent. A mass-three optical potential with a real strength of about 110 MeV and a volume imaginary term is strongly preferred in the (^{3}He, t) calculations.

I. INTRODUCTION

Charge-exchange reactions to isobaric analog states (IAS) of target ground states have been studied quite extensively in recent years with both $(p, n)^{1,2}$ and $(^{3}\mathrm{He}, t)^{3-6}$ reactions at a variety of bombarding energies. The analysis of the differential cross sections for such reactions primarily has used the distorted-wave Born approximation (DWBA) in a macroscopic (generalized optical-potential) or microscopic (nucleon-nucleon interaction) framework, and has yielded information on the strength and form of the isospin-dependent interaction. Recent studies by Fadner, Kraushaar, and Hayakawa[6] of (^{3}He, t) transitions to IAS in several nuclei at bombarding energies between 21.4 and 37.5 MeV have shown a marked energy dependence in the extracted strength of this isospin interaction and a variation in the extracted shapes of the isospin term (for the macroscopic analysis) for different nuclei. We have extended the study of (^{3}He, t) reactions to 70 MeV (the highest reported bombarding energy has been 50 MeV[7]) by examining transitions to IAS of the ground states of ^{50}Cr, ^{62}Ni, and ^{90}Zr to provide more information on the energy dependence of the charge-exchange interaction.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The reactions ^{50}Cr, ^{62}Ni, ^{90}Zr(^{3}He, t) were studied at a bombarding energy of 70 MeV using ^{3}He ions accelerated in the Michigan State University sector-focused cyclotron. The experiment was conducted in a 40-in. scattering chamber with the tritons detected in a 1-cm stack of three Si(Li) detectors. A ΔE-E particle identification program was used in conjunction with the Sigma-7 computer. An over-all resolution of 150 keV was obtained, which was detector limited. The targets were all 1-mg/cm^2 rolled foils. An energy spectrum for the reaction ^{62}Ni(^{3}He, t)^{62}Cu is shown in Fig. 1. At all angles the 0^+ IAS was populated