Estimate of the Alpha-Nucleus Spin-Orbit Potential*

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The spin-orbit part $V_{\vec{I}\cdot\vec{L}}$ of the α-nucleus interaction is estimated on the basis of the nucleon-α spin-orbit potential. Here $\vec{I}$ is the spin of the target nucleus and $\vec{L}$ is the orbital angular momentum of α-nucleus relative motion. The calculation assumes that the target nucleus undergoes no virtual excitation during the collision and that core polarization effects are absent. Numerical results are obtained for the nuclei of ^{9}Be, ^{59}Co, and ^{14}N. The result for the two first cases is found to be an order of magnitude smaller than the phenomenological values of $V_{\vec{I}\cdot\vec{L}}$ quoted in the literature. For ^{14}N the value of $V_{\vec{I}\cdot\vec{L}}$ is nearly 10 times larger than for ^{9}Be, a result which depends on the fact that for the valence nucleon $j=l-\frac{1}{2}$ in the former case, and $j=l+\frac{1}{2}$ in the latter case. For ^{14}N no phenomenological values of $V_{\vec{I}\cdot\vec{L}}$ appear to be known.

I. INTRODUCTION

The spin-orbit part $\vec{s}\cdot\vec{\mathcal{L}}$ of the nucleon-nucleus interaction is well known. Its origin is attributed to the spin-orbit part of the nucleon-nucleon interaction, as is discussed, for example, in the book by Bohr and Mottelson.[1] Here $\vec{s}$ denotes the spin operator $\frac{1}{2}\hbar\sigma$ of the nucleon, and $\vec{\mathcal{L}}$ is the nucleon-nucleus orbital angular momentum of relative motion. If the target nucleus has a spin $\vec{I}$ different from zero, other potentials can also arise. One, the spin-spin interaction $\vec{s}\cdot\vec{I}$, has been suggested some time ago by Feshbach.[2] Its effect on the elastic scattering of polarized projectiles, has been discussed by Stamp[3] amongst others, and the presence of this term has been seen experimentally in the absorption of neutrons on polarized nuclei[4] and in the splitting of energies in isobaric analog resonances.[5]

Another term which can arise in either the nucleon-nucleus or the α-nucleus optical potential, and which is the subject of the present discussion, is an $\vec{I}\cdot\vec{L}$ term.[6] The origin of this term can be either the spin-orbit part $\vec{S}\cdot\vec{L}'$ or the $\vec{L}'\cdot\vec{L}'$ part of the nucleon-nucleon interaction. Here $\vec{S}$ denotes the sum of the spins of the two interacting nucleons, $\vec{S}=\vec{s}_1+\vec{s}_2$, and $\vec{L}'$ is the orbital angular momentum of their relative motion. The resulting $\vec{I}\cdot\vec{L}$ term arises even if the spin of the projectile is zero and a term of this type has been looked for in the elastic scattering of both ^{4}He and ^{3}He projectiles from nuclei.[7-9] The effect of the $\vec{I}\cdot\vec{L}$ potential appears to be small. Nevertheless, it is possible that it could play a nonnegligible role in inelastic or rearrangement reactions, in which the nucleus in the final state has a nonzero spin. In inelastic reactions the spin of the final nucleus is likely to be polarized along the normal to the scattering plane and hence the $\vec{I}\cdot\vec{L}$ potential will give an effect to first order. By contrast, for the elastic scattering on unpolarized target nuclei the effect of the $\vec{I}\cdot\vec{L}$ potential is of second order, and hence its effect will be less noticeable.

The estimate of the strength of the $\vec{I}\cdot\vec{L}$ potential presented here is intended to give only an indication of the magnitude, in order to provide a theoretical orientation for phenomenological calculations[7,10] employing such terms. The derivation is straightforward, the only difficulty being the transformation of coordinates required in order to appropriately define the various orbital angular momenta involved. The orbital angular momentum $\vec{L}'$ describing the relative motion of two nucleons, required for the nucleon-nucleon $\vec{S}\cdot\vec{L}'$ potential, has to be related to the orbital angular momentum $\vec{L}$ of motion of the center of mass of the projectile relative to that of the target nucleus. The derivation consists in averaging the spin-orbit part of the nucleon-nucleon interaction over the nucleons of the projectile nucleus. The latter is taken to be the α particle. The result is a spin-orbit potential $\vec{s}_i\cdot\vec{L}$ between a nucleon i in the target nucleus and the orbital angular momentum of the projectile. For subsequent applications this potential is replaced by the phenomenological optical-model spin-orbit nucleon-α-particle potential and the final nucleus-α $\vec{I}\cdot\vec{L}$ potential is obtained by averaging the above-mentioned potential over the nucleons in the target nucleus. It is assumed that only the valence nucleons which have unpaired spins contribute to the averaging process, thereby disregarding core polarization effects. In Appendix B an expansion of the α-nucleus interaction in terms of general tensors[11] is presented, but for the numerical evaluation only the central part and the $\vec{I}\cdot\vec{L}$ part of the potential is kept. Numerical results for the

$\vec{l}\cdot\vec{L}$ potential for the nuclei of ^{9}Be and ^{59}Co are presented. These nuclei were chosen since phenomenological values are available.[7,8] The present estimate turns out to be an order of magnitude smaller than the phenomenological result. For the nucleus of ^{14}N, on the other hand, the $\vec{l}\cdot\vec{L}$ potential is found to be considerably larger than for the two other nuclei, which makes this nucleus a more promising candidate for studying this potential.

II. THEORY

The nucleon-nucleon spin-orbit potential between nucleons i and j is given by

$$V_{so}(i,j)=\frac{\hbar}{i}V_{LS}(r_{ij})(\vec{x}_i-\vec{x}_j)\times(\vec{\nabla}_{\vec{x}_i}-\vec{\nabla}_{\vec{x}_j})\cdot(\vec{s}_i+\vec{s}_j)\,. \tag{1}$$

Here $\vec{x}_i$ and $\vec{x}_j$ are the respective coordinate vectors of the nucleons i and j as measured relative to an inertial frame, and $\vec{s}_i$ and $\vec{s}_j$ denote the respective spin operators. The distance between nucleons i and j is r_{ij}, the gradients act on the coordinates indicated by the subscripts, all other variables being held constant. If the number of nucleons in the target and projectile nucleus is denoted by A and B, respectively, and if i is restricted to vary from 1 to A and j from $A+1$ to $A+B$ then Eq. (1) denotes the spin-orbit interaction between two nucleons in the target and projectile nuclei, respectively.

The spin-orbit potential between nucleon i in the target averaged over all nucleons j in the projectile is given by

$$V_{so}(i,B)=\langle\phi_B|\sum_j V_{so}(i,j)|\phi_B\rangle\,, \tag{2}$$

where $|\phi_B\rangle$ denotes the internal wave function of nucleus B. It is shown in Appendix A that an approximation to $V_{so}(i,B)$ is given by

$$V_{so}(i,B)\simeq\frac{\hbar}{i}V_{ls}^B(\xi_i)\vec{\xi}_i\times\left[\vec{\nabla}_{\vec{r}_i}-\left(\sum_{s=1}^{A-1}\vec{\nabla}_{\vec{r}_s}\right)\Big/A-\frac{\vec{\nabla}_{\vec{R}}}{\mu_{AB}}\right]\cdot\vec{s}_i\,, \tag{3}$$

where $\vec{\xi}_i$ is the vector pointing from the center of nucleus B to nucleon i and $V_{ls}^B(\xi_i)$ is such that

$$V_{ls}^B(\xi_i)\vec{l}_i\cdot\vec{s}_i$$

represents the nucleon-α spin-orbit potential, $\vec{l}_i$ being the orbital angular momentum of nucleon i relative to the α particle. In Eq. (3) above $\vec{r}_i$ is the distance of nucleon i to the center of mass of the target nucleus, $\vec{R}$ is the distance between the centers of mass of target and projectile nuclei, $\vec{R}=\vec{R}_B-\vec{R}_A$, and μ_{AB} is the reduced mass $AB/(A+B)$ of the target projectile system, in atomic mass units. The quantities inside the square brackets are due to the transformation from nucleon coordinates measured relative to an inertial system to coordinates measured relative to the center of mass of one of the nuclei. The appearance of the term $\vec{\nabla}_{\vec{R}}/\mu_{AB}$ can be understood classically from the relation between the momenta $\vec{p}_A$ and $\vec{p}_B$ of a mass point of mass m measured relative to two different centers A and B moving relative to each other. If the masses of centers A and B are M_A and M_B, respectively, and if $\vec{P}$ is the momentum of mass M_B relative to the center of mass of the system, then

$$\vec{p}_A-\vec{P}m/\mu_{AB}=\vec{p}_B\,.$$

The above expression is very similar to the square bracket in Eq. (3) and shows that $V_{so}(i,B)$ as given by Eq. (3) is quite reasonable in that it represents the spin-orbit potential of nucleon i relative to the projectile B.

The result of Eq. (3) is derived under the assumption that nucleus B can be represented in the $\vec{L}\cdot\vec{S}$ coupling scheme, with the orbital and spin parts of the wave function each having zero angular momentum. If the spin part has nonvanishing angular momentum, as would be the case in ^{3}He, the spin operator $\vec{s}_i$ in Eq. (3) should be replaced by $\vec{s}_i+\vec{S}_BN$, where $\vec{S}_B$ is the total spin operator of nucleus B, and N is a reduction factor, as is discussed in Appendix A. The desired expression for the $\vec{l}\cdot\vec{L}$ potential (where $\vec{L}$ is the orbital angular momentum of the relative α-nucleus motion) is obtained by averaging the potential $V_{so}(i,B)$ over all nucleons in the target nucleus,

$$V_{so}(A,B)=\langle\psi_A|\sum_i V_{so}(i,B)|\psi_A\rangle\,. \tag{4}$$

Here ψ_A represents the internal wave function of the nucleus A.

In order to obtain an estimate of V_{so} as given by Eq. (4), several assumptions are made which will now be discussed.

It will be assumed that the presence of the projectile does not polarize the target wave function, and also that the "free" nucleon-α spin-orbit potential can be used for $V_{ls}(\xi_i)$ in Eq. (3). Both assumptions are similar to those made in obtaining the real part of the α-nucleus central potential by folding an effective "free" nucleon-α potential into the target nucleus matter distribution. This folding procedure has been found to be quite successful[12] thus justifying the approximate validity of these assumptions. For nucleon-nucleus scattering a similar assumption has been made by Pyle and Greenlees,[13] but, as shown by Owen and Satchler,[14] neglect of exchange terms due to antisymmetrization does produce a sizable effect in the nucleon-nucleus case.

It is quite possible that the polarization of the target nucleus by the incident projectile can give rise to a nonnegligible contribution to the $\vec{I}\cdot\vec{L}$ potential. Indeed, a recent investigation of deuteron-nucleus elastic scattering which takes into account part of the deuteron distortion by means of a coupled-channel treatment[15] shows that approximately half of the phenomenological deuteron-nucleus spin-orbit potential can be caused by such distortion effects. Also, an investigation of the effect of the presence of an $\vec{I}\cdot\vec{L}$ potential in the final channel in inelastic α-nucleus scattering shows that reorientation terms included in the coupled-channel treatment do give rise to additional $\vec{I}\cdot\vec{L}$-like effects.[10] On the other hand, Satchler[16] has shown that core polarization effects do quench the $\vec{s}\cdot\vec{I}$ potential in nucleon-nucleus scattering. The effects discussed above may be quite important but are outside the scope of the present estimate which limits itself to the "static" estimate of the $\vec{I}\cdot\vec{L}$ potential. Among effects of the type mentioned above are also the effects due to the tensor part of the nucleon-nucleon potential. Since this potential is velocity independent it cannot give rise to a $\vec{I}\cdot\vec{L}$ potential in first order. Terasawa[17] shows that the tensor force can increase the spin-orbit splitting of single-particle bound energy levels through higher-order core polarization effects. On the other hand, Elliott, *et al.*[18] calculate spin-orbit splittings for bound-state energies for several nuclei directly from free nucleon-nucleon scattering phase shifts, with excellent results. The authors point out that in their calculations the tensor part of the nucleon-nucleon force does not contribute to the bound-state spin-orbit splitting. The evaluation of Eq. (4) within the "folding" assumptions discussed above will now be described. The expression for $V_{so}(i, B)$ given by Eq. (3) is inserted into Eq. (4). The quantity $\vec{\xi}_i$ is replaced by $\vec{r}_i - \vec{R}$. The term $\sum_s \vec{\nabla}_{\vec{r}_s}/A$ in the square brackets in Eq. (3) is dropped, since it represents the sum of the momenta of all nucleons in nucleus A relative to its center of mass and should vanish. The equation (3) is thus replaced by

$$V_{so}(i,B) \simeq \frac{\hbar}{i} V_{ls}^{B}(\xi_i)\{\vec{r}_i\times\vec{\nabla}_{\vec{r}_i} + \vec{R}\times\vec{\nabla}_{\vec{R}}\,\mu_{AB}{}^{-1} - \vec{r}_i\times\vec{\nabla}_{\vec{R}}\,\mu_{AB}{}^{-1} - \vec{R}\times\vec{\nabla}_{\vec{r}_i}\}\cdot\vec{s}_i\,. \tag{5}$$

In order to carry out the integrals over d^3r_i, it is also convenient to expand the potential $V_{ls}^{B}(|\vec{r}_i - \vec{R}|)$ in spherical harmonics around the center of the target nucleus

$$V_{ls}^{B}(\xi_i) = \sum_{L,\lambda} v_L(r_i, R) Y_{L\lambda}^{*}(\hat{r}_i) Y_{L\lambda}(\hat{R}), \tag{6}$$

with the expansion coefficients v_L given by

$$v_L(r,R) = \frac{2\pi}{Rr}\int_{|R-r|}^{R+r} r' V_{ls}^{B}(r') P_L(\cos u)\, dr', \tag{7}$$

where

$$\cos u = \frac{R^2 + r^2 - r'^2}{2Rr} \tag{8}$$

and where $\hat{r}_i$ and $\hat{R}$ denote the directions of the vectors $\vec{r}_i$ and $\vec{R}$, respectively. In Appendix B this expansion is carried out in detail in the form of general tensor expressions $T_K(i)$ and $P_k(R)$, the former operating on the coordinates of nucleon i in the target nucleus, the latter operating on the coordinates of relative motion of the α particle. The lowest-order terms of interest are given in Eq. (A7) and (A6) of the Appendix. Making use of Eqs. (7) and (8), the central and spin-orbit parts of the α-nucleon interaction $V_{so}(i, B)$ can be written, respectively,

$$\left[\frac{v_0(r_i,R)}{4\pi} - \frac{R\, v_1(r_i,R)}{4\pi r_i}\right]\vec{s}_i\cdot\vec{l}_i \tag{9}$$

and

$$\mu_{AB}{}^{-1}\left[\frac{v_0}{4\pi}\vec{s}_i - \frac{r_i v_1}{4\pi R}(\vec{s}_i + \sqrt{2\pi}\,\vec{T}_i)\right]\cdot\vec{L}\,. \tag{10}$$

Here the vector $\vec{T}_1$ represents the tensor of rank 1 (Ref. 19) formed from the spherical harmonic $Y_2(\hat{r}_i)$ and the operator $\vec{s}_i$:

$$T_{1\mu} = \sum_m \langle 1\mu|21m\,\sigma\rangle\, Y_{2m}(\hat{r}_i)(\vec{s}_i)_\sigma\,.$$

The corresponding central and spin-orbit parts of the α-nucleus interaction are obtained by averaging the expressions above over the internal wave function of nucleus A, $|\psi_A\rangle$,

$$V_{so}(A,B)|_{\text{CENTRAL}} = \sum_i \int R_A{}^2(r_i)\left(\frac{v_0}{4\pi} - \frac{R}{r_i}\frac{v_1}{4\pi}\right) r_i{}^2 dr_i \langle\phi_A|\vec{l}_i\cdot\vec{s}_i|\phi_A\rangle, \tag{11}$$

$$V_{so}(A,B)|_{IL} = V_{IL}(R)\,\vec{I}\cdot\vec{L}, \tag{12a}$$

$$V_{IL} = \mu_{AB}{}^{-1}\int R_A{}^2(r_i)\left[\frac{v_0}{4\pi}g_i - \frac{r_i}{R}\frac{v_1}{4\pi}(g_i + \sqrt{2\pi}\,h_i)\right] r_i{}^2 dr, \tag{12b}$$

where R_A is the radial part of $|\psi_A\rangle$ and $|\phi_A\rangle$ the angle, and spin part. The numbers g_i and h_i are projection factors

$$g_i = \frac{\langle\psi_A\| s_i \|\psi_A\rangle}{\langle\psi_A\| I \|\psi_A\rangle}, \tag{13}$$

$$h_i = \frac{\langle \psi_A \| T_1 \| \psi_A \rangle}{\langle \psi_A \| I \| \psi_A \rangle}, \qquad (14)$$

which arise through the use of the projection theorem,[19] and I is the spin operator for the whole target nucleus, which is understood to be operating on the target wave function $|\psi_A\rangle$. It is interesting that the signs of both the central and the spin-orbit parts of the potentials given by Eqs. (10) and (11) depend on the total spin of the nucleus. This feature results from the presence $\vec{1}_i \cdot \vec{s}_i$ and of g_i and h_i in the respective matrix elements. For example in the case of a nucleus with one valence nucleon outside a closed core, the sign of V_{IL} depends on whether the spin of the valence nucleon is parallel or antiparallel to j. In this case the g and h factors are given explicitly in Table I.

The dependence of the $\vec{I} \cdot \vec{L}$ potential on the j value of the nuclear state appears to have been observed by Weller[20] in his interpretation of the spectra of ^{15}N in terms of a ^{11}B + α particle states. According to his interpretation one set of levels in ^{15}N is due to the interaction of an α particle in a 1^- state with ^{11}B in a $j=\frac{3}{2}$ state. The sequence of levels is $\frac{5}{2}^+$, $\frac{3}{2}^+$, and $\frac{1}{2}^+$. The next set of levels corresponds to the α particle in a 1^- state interacting with ^{11}B in a $j=\frac{1}{2}^-$ state. There the order of the levels is $\frac{1}{2}^+$, $\frac{3}{2}^+$, which is the inverse of the one for the first set, in accordance with the sign dependence of the g and h factors discussed above. For the case of two valence nucleons coupled according to the j-j coupling scheme

$$\psi_A = |((l_1, s_1)j_1, (l_2, s_2)j_2)IM_I\rangle,$$

the projection factors g_i become

$$g_1 = \frac{1}{2}\,\frac{j_1(j_1+1)+I(I+1)-j_2(j_2+1)}{I(I+1)}\, g_{l_1 j_1}, \qquad (15a)$$

$$g_2 = \frac{1}{2}\,\frac{j_2(j_2+1)+I(I+1)-j_1(j_1+1)}{I(I+1)}\, g_{l_2 j_2}, \qquad (15b)$$

where the single-nucleon projection factors g_{lj} are given in Table I. A similar result holds for the h's if in Eqs. (15) all letters g are replaced by h, with $h_{l_i j_i}$ given in Table I. The phenomenological nucleon-nucleus spin-orbit potential is frequently given in the form

$$V_{ls} = 2\left(\frac{\hbar}{m_\pi c}\right)^2 V_{l\sigma} r^{-1} \frac{d}{dr}\left[1+\exp\left(\frac{r-R_{l\sigma}}{a_{l\sigma}}\right)\right]^{-1} \vec{1}\cdot\vec{s}\,. \qquad (16)$$

The factor 2 allows for the use of $\vec{1}\cdot\vec{s}$ rather than $\vec{1}\cdot\vec{\sigma}$. Inserting Eq. (16) into Eq. (12b) results in

$$V_{IL}(R) = \mu_{AB}^{-1}(\hbar/m_\pi c)^2 V_{l\sigma} R^{-1} f_{IL}(R), \qquad (17)$$

$$f_{IL}(R) = \sum_i g_i f_i^{(0)}(R) - (g_i + \sqrt{2\pi}\, h_i) f_i^{(1)}(R), \qquad (18)$$

with

$$f_i^{(k)}(R) = \int_0^\infty \psi_A{}^2(r)\, r\, dr \int_{|R-r|}^{|R+r|} u^{(k)}(r,r') w(r')\, dr', \qquad (19)$$

and

$$w(r') = \frac{d}{dr'}\left[1+\exp\left(\frac{r'-R_{l\sigma}}{a_{l\sigma}}\right)\right]^{-1} \qquad (20a)$$

and

$$u^{(k)}(r,r') = \begin{cases} 1 & \text{for } k=0 \\ (R^2+r^2-r'^2)/(2R^2) & \text{for } k=1 \\ (R^2+r^2-r'^2)/(2r^2) & \text{for } k=2\,. \end{cases} \qquad (20b)$$

For $k=0$ the integral in Eq. (19) can of course be done explicitly by parts.

The central potential is given by the expression

$$V_{\text{CENTRAL}}(R) = (\hbar/m_\pi c)^2 V_{l\sigma} R^{-1} f_c(R), \qquad (21)$$

$$f_c(R) = \sum_i \left[f_i^{(0)}(R) - f_i^{(2)}(R)\right] \langle \psi_A | \vec{1}_i \cdot \vec{s}_i | \psi_A \rangle. \qquad (22)$$

It is interesting to note that the $\vec{I}\cdot\vec{L}$ spin-orbit potential given by Eq. (17) is inversely proportional to the reduced mass of the projectile and target nucleus. However, in addition to the factor $1/\mu$, there is a further reduction due to the cancellation between the $f_i^{(0)}$ and $f_i^{(1)}$ in Eq. (18). This cancellation is very severe, as the numerical results in Sec. III will show, with the result, that the ^{4}He-nucleus $\vec{I}\cdot\vec{L}$ potential is much smaller than one quarter of the nucleon-^{4}He spin-orbit potential. As the following discussion will show, the deuteron-nucleus spin-orbit potential is formally similar

TABLE I. The projection factors g and h defined in Eqs. (13) and (14) for the case of a valence nucleon of orbital momentum l and total spin j.

	$j=l+\frac{1}{2}$	$j=l-\frac{1}{2}$
g_{lj}	$(2l+1)^{-1}$	$-(2l+1)^{-1}$
$\sqrt{2\pi}\, h_{lj}$	$l(2l+1)^{-1}(2l+3)^{-1}$	$-(l+1)(2l-1)^{-1}(2l+1)^{-1}$
$1+\sqrt{2\pi}\, h_{lj}/g_{lj}$	$3(l+1)(2l+3)^{-1}$	$3l(2l-1)^{-1}$

to Eqs. (17) to (20) but the cancellation between $f^{(0)}$ and $f^{(2)}$ is much less severe in this case.

Deuteron-Nucleus Spin-Orbit Potential

The formula derived in the Appendix also can be used to obtain the deuteron-nucleus or the ^{3}He-nucleus spin-orbit $\vec{S}\cdot\vec{L}$ potential. In this case B is interpreted as the target nucleus (assumed to have spin 0) and i refers to the nucleon contained in either the deuteron or the ^{3}He projectile. The nucleon-nucleus spin-orbit potential given by Eq. (A14), with $S_B=0$, then has to be averaged over the internal wave function of the deuteron or ^{3}He. The operator $\vec{c}_i$ is given by the expression in square brackets in Eq. (3). For the case of the deuteron one can proceed by expressing the coordinates $\vec{\xi}_1$ and $\vec{\xi}_2$ of the neutron and proton in terms of the relative coordinate $\vec{\xi}=\vec{\xi}_1-\vec{\xi}_2$ and the c.m. coordinate $\vec{R}_A=\frac{1}{2}(\vec{\xi}_1+\vec{\xi}_2)+\vec{R}_B$, one then obtains for $\vec{c}_1$ and $\vec{c}_2$ the expressions

$$\vec{c}_1=\vec{\nabla}_{\vec{\xi}}-\mu_{AB}{}^{-1}\vec{\nabla}_{\vec{R}},$$

$$\vec{c}_2=-\vec{\nabla}_{\vec{\xi}}-\mu_{AB}{}^{-1}\vec{\nabla}_{\vec{R}},$$

with $\vec{R}=\vec{R}_B-\vec{R}_A$. After averaging the result over the internal deuteron wave function, $|\phi_D\rangle$, one obtains for the deuteron-nucleus spin-orbit potential the result

$$V_{LS}(R)=\mu^{-1}\vec{L}\cdot\vec{S}\left\langle\phi_D\left|\frac{v_0(R,\frac{1}{2}\xi)}{4\pi}-\frac{1}{2}\frac{\xi}{R}\frac{v_1(R,\frac{1}{2}\xi)}{4\pi}\right|\phi_D\right\rangle, \tag{23}$$

where

$$\vec{L}=(\hbar/i)\vec{R}\times\vec{\nabla}_{\vec{R}}$$

and

$$\vec{S}=\tfrac{1}{2}\hbar(\vec{\sigma}_1+\vec{\sigma}_2).$$

If the D-state contribution to the internal deuteron wave function is ignored, $|\phi_D\rangle$ can be replaced by $[U(\xi)/\xi]|1,M_D\rangle$ (where $|1,M_D\rangle$ represents the deuteron spinor), and the equations analogous to V_{IL} given by Eq. (17) are

$$V_{SL}(R)=\vec{S}\cdot\vec{L}\,\frac{2}{\mu}\left(\frac{\hbar}{m_\pi c}\right)^2 V_{l\sigma}\frac{1}{R}\left[f_D^{(0)}(R)-f_D^{(1)}(R)\right], \tag{24}$$

with

$$f_D^{(k)}=\int_0^\infty r^{-1}[U(2r)]^2\int_{|(R-r)|}^{R+r}u^{(k)}(r,r')w(r')\,dr' \tag{25}$$

and where $u^{(k)}$ and w have been defined in Eqs. (20).

The above result for the $f_D^{(k)}$ is formally similar to that given in Eq. (19) for $f_i^{(k)}$, with the exception that $r\psi_A{}^2(r)$ is replaced by $[U(2r)]^2/r$. Expressions equivalent to the above have also been given by Keaton *et al.*[21] for the deuteron and ^{3}He-nucleus interaction. These authors also observe the reduction of the nucleon-nucleus spin-orbit potential by the reduced mass μ. However, contrary to what was the case for the ^{4}He-nucleus $\vec{I}\cdot\vec{L}$ potential discussed above, the additional reduction due to the cancellation between $f_D^{(0)}$ and $f_D^{(1)}$ is much smaller here. The reason is that the relative spatial extents of U and w in the deuteron $\vec{S}\cdot\vec{L}$ case, Eq. (25), is different from that of ψ_A and w, Eq. (19), in the ^{4}He $\vec{I}\cdot\vec{L}$ case. For example, for the case of the D-^{40}Ca spin-orbit potential, numerical calculations show that the maximum in $f_D^{(0)}-f_D^{(1)}$ occurs near $R=3.5$ F, and $f^{(1)}$ is only about 8% of $f^{(0)}$ at that distance. For other distances, $f^{(1)}$ may be larger, but is usually less than 30% of $f^{(0)}$. The numerical results quoted above were obtained with a modification of the code used for the calculation of the α-nucleus $\vec{I}\cdot\vec{L}$ potential and the good agreement with the results of Keaton *et al.* served as a check of the code. Further results will not be given here since extensive numerical values for the deuteron-nucleus and ^{3}He-nucleus central, spin-orbit and tensor potentials are contained in the work by Keaton *et al.*[21]

Nucleon-Nucleus Spin-Orbit Potential

Although it is not the purpose of the present paper to give numerical results for the nucleon-nucleus spin-orbit potential, it is of some interest to note that Eqs. (A8) to (A13) in Appendix A do provide the microscopic description required for such a calculation.[22] For this purpose it is sufficient to take B as the target nucleus, and assume only one nucleon (nucleon i) present in nucleus A. For a spherical target with zero spin, the spherically symmetric part of Eq. (A8) given by Eq. (A12) would be the appropriate expression to be evaluated, as is indicated by Eq. (A13).

If the nucleons in nucleus B have their spins coupled pairwise to zero, then neither $\vec{R}$ nor the portion due to $\vec{s}_j$ in $\vec{S}_{ij}$ ($\vec{S}_{ij}=\vec{s}_i+\vec{s}_j$) will give any contribution to Eq. (A8). In this case the first term in Eq. (A8) and the $\vec{s}_i$ part of the last term should provide the microscopic description for the full Thomas deformed spin-orbit potential, recently discussed by Sherif and Blair,[23] and used for inelastic transitions $|\phi_B\rangle\to|\phi_B'\rangle$.

If, for some excited state of nucleus B, the nucleon spins are not coupled pairwise to zero, then in addition to the deformed full Thomas $\vec{s}\cdot\vec{L}$ potential mentioned above, the remaining terms in Eq. (A8) do provide the microscopic basis for a deformed $\vec{I}\cdot\vec{L}$ potential. Since experiment appears to favor a deformation parameter of the spin-orbit potential (used in the full Thomas term) which is larger than the deformation required of the central potential,[24] it would be of interest to investigate whether inclusion of the additional $\vec{I}\cdot\vec{L}$ potential would remove this discrepancy. It should also be remembered that the $\vec{L}\cdot\vec{L}$ part of the nucleon-nucleon potential will give rise to an additional $\vec{I}\cdot\vec{L}$ potential, as is mentioned in the Introduction.

III. NUMERICAL EXAMPLES

The α-nucleus $\vec{I}\cdot\vec{L}$ potential given by Eq. (12) and Eqs. (17) through (20) will be used for the present numerical examples. The nuclear wave function ψ_A which appears in these equations is replaced by the single-particle radial shell-model wave function $\varphi_j(r)$. The parameters for the shell-model potential are taken from a study by Batty and Greenlees.[25] For a proton the radius and diffuseness of the central potential is $1.28\times A^{1/3}$ F and 0.60 F, respectively, and the strength is 7 MeV. The Coulomb potential is that of a uniform charge distribution with radius $1.28\times A^{1/3}$ F. The nucleon separation energy is taken for the single-particle energy.

The nucleon-α spin-orbit potential parameters are taken from a study by Morgan and Walter.[26] The parameters $V_{l\sigma}$, $R_{l\sigma}$, and $a_{l\sigma}$, defined in Eq. (16), have the values $(3.95+0.144E_n)$ MeV, 1.956, and 0.435 F, respectively, for the neutron-α case. There are other sets of parameters for the nucleon-α spin-orbit potential. The one arrived at

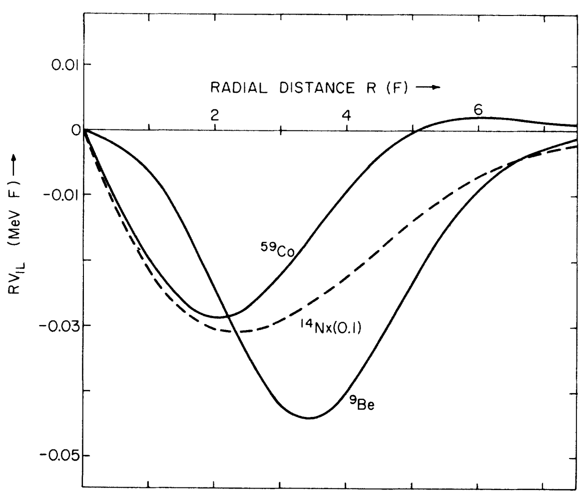

FIG. 1. The α-nucleus spin-orbit potential $V_{IL}(R)$ times the α-nucleus distance R. The $\vec{I}\cdot\vec{L}$ potential is defined in Eq. (12) and V_{IL} is evaluated according to Eqs. (17) to (20) utilizing shell-model wave functions for the valence nucleons. The origin of V_{IL} is the spin-orbit part of the nucleon-nucleon potential. The potential V_{IL} can be parametrized according to Eq. (26) and the corresponding parameters are given in Table II.

by Satchler *et al.*[27] for neutrons is $(3.0+0.1E_n)$ MeV, 1.585 F and 0.25 F for $V_{l\sigma}$, $R_{l\sigma}$, and $a_{l\sigma}$, respectively. The latter values are also used and give somewhat smaller results for V_{IL}. The proton or neutron energy E_p or E_n is taken as the incident α-particle energy divided by 4 so as to resemble the energies of a nucleon in the nucleus relative to the α particle. The incident energy of the α particle is taken equal to 10 MeV, which then gives for $V_{l\sigma}$ the α value 4.31 MeV. The results for R times $V_{IL}(R)$, are illustrated in Fig. 1 for three nuclei: ^{59}Co, ^{9}Be, and ^{14}N. For ^{59}Co the nucleon is taken as a $f_{7/2}$ proton hole bound to ^{60}Ni. The corresponding g factor is $-\frac{1}{7}$, the $-$ sign reflecting the absence of this nucleon.

The potential V_{LI} contains the difference of the functions $f^{(0)}$ and $f^{(1)}$. Both functions are nearly equal and hence they nearly cancel. For example, for ^{59}Co the value of $f^{(0)}$ and $f^{(1)}$ at $R=4$ F is 0.229 F^{-1} and 0.193 F^{-1}, respectively. The corresponding value of $f_{LI}=\frac{1}{7}(f^{(0)}-1.333f^{(1)})$ is -0.004 F, i.e., the cancellation produces a reduction of nearly an order of magnitude. In a preliminary report,[28] the term $f^{(1)}$ had not yet been included and hence the results stated there are considerably larger. This cancellation is responsible for a change in sign for $R>5$. On the other hand, for ^{14}N the cancellation does occur to a much smaller extent because the coefficients g and $g+\sqrt{2\pi}h$, which are to be multiplied into $f^{(0)}$ and $f^{(1)}$, are quite different from each other. The ratio of the coefficients of $f^{(0)}$ and $f^{(1)}$ is $1+\sqrt{2\pi}h/g$ and is given in the last line of Table I.

This ratio has the value of 1.5 in the limit of large l for both j values, but the ratio becomes progressively different for the two j values as l decreases. When $l=1$, this ratio is 1.2 for ^{9}Be and 3.0 for ^{14}N, which explains the lack of cancellation in ^{14}N, and hence the occurrence of the comparatively large spin-orbit potential in this case.

As can be seen from Fig. 1, the function $RV_{IL}(R)$ has a maximum and falls off at large distances. Disregarding the change in sign which takes place in ^{59}Co, it is useful to approximate $RV_{IL}(R)$ by an expression of the form

$$RV_{IL}(R)\sim\left(\frac{\hbar}{m_\pi c}\right)^2 V''_{LI}\frac{d}{dR}\{1+\exp[(R-R'')/a'']\}^{-1},$$

since in this case the spin-orbit potential V_{IL} can be written in the conventional phenomenological form

$$V_{IL}=\left(\frac{\hbar}{m_\pi c}\right)^2 V''_{LI}R^{-1}\frac{d}{dR}\{1+\exp[(R-R'')/a'']\}^{-1}. \tag{26}$$

The results for the parameters V''_{LI}, R'', and a'' are given in Table II.

TABLE II. Parameters [a] for the $\vec{I}\cdot\vec{L}$ potential defined in Eq. (26).

	^{9}Be		^{59}Co		^{14}N
	Theory	Phenom [b]	Theory	Phenom [c]	Theory
V''_{IL} (MeV)	0.078	2	0.045	0.2→0.65	0.75
R'' (F)	3.5	3.87	2.0	5.76	2.5
a'' (F)	0.89	0.60	0.83	0.586	1.27

[a] The nucleon-α spin-orbit potentials are taken from Ref. 26.
[b] Reference 8.
[c] Reference 7. The $\vec{\sigma}\cdot\vec{L}$ potential used by these authors is converted to a $\vec{I}\cdot\vec{L}$ potential according to Eq. (27), as is explained in the text.

The central potential is also calculated on the basis of Eqs. (21) and (22), and the results for ^{9}Be, ^{14}N, and ^{59}Co are shown in Fig. 2. The phenomenological values given in Table II have been determined from the elastic (α, α) scattering on randomly oriented nuclei.[7,8] In this case the $\vec{I}\cdot\vec{L}$ potential gives no contribution to first order in V_{IL} and the sign of this potential is therefore not determined. The phenomenological analysis of α-^{59}Co scattering has been carried out[7] assuming a spin $\frac{1}{2}$ rather than $\frac{7}{2}$ for ^{59}Co. An analysis in terms of first-order Born approximation of the effect of a small $\vec{I}\cdot\vec{L}$ potential on the elastic cross section shows that the correction to the cross section is proportional to

$$I(I+1)(V''_{IL})^2 . \tag{27}$$

Assuming the value of I to be alternately $\frac{1}{2}$ and $\frac{7}{2}$ and equating both results of Eq. (27) provides a relation between V_{IL} and the equivalent $V_{\frac{1}{2}L}$. Although the interference of the second-order Born-approximation terms with the unperturbed scattering amplitude which are of the same order have not been included in the expression above, the above recipe appears to be reasonable. For example Taylor *et al.*[7] find that a spin-$\frac{1}{2}$ $\vec{I}\cdot\vec{L}$ potential of 4.4 MeV is equivalent to a 2.4-MeV spin-$\frac{3}{2}$ potential. The conversion procedure based on Eq. (27) gives the value 5.4 MeV instead of 4.4 MeV, which is acceptable for our present purposes.

Comparison of the theoretical and phenomenological values of the parameters listed in Table I shows that the phenomenological value of V''_{LI} is larger by nearly an order of magnitude, that the radius is also larger but the diffuseness is smaller. The phenomenological determination of these parameters is based on the requirement that the addition of the spin-orbit potential improves the fit to the elastic cross section. It is therefore subject to various uncertainties inherent in the use of the optical-model formalism. On the other hand, the present theoretical estimate leaves out the effects of core polarization and virtual excitation of intermediate states, which may be quite large. Comparison of the theoretical and phenomenological values in Table II is thus an indication that more work needs to be done both in the theoretical calculation of V_{IL} as well as in the phenomenological determination of its value.

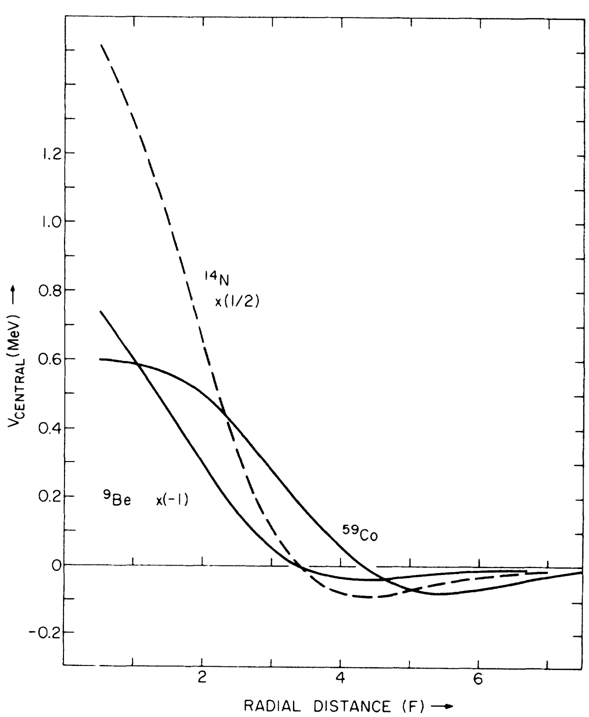

FIG. 2. The central part of the α-nucleus potential due to the spin-orbit portion of the nucleon-nucleon potential. This potential is defined in Eq. (11) and is evaluated by means of Eqs. (19) to (22) in the text.

IV. DISCUSSION AND CONCLUSIONS

A portion of the interaction between an α particle and a nucleon in the target nucleus is due to the spin-orbit part of the nucleon-nucleon potential. This interaction, described by Eq. (5), involves the spin $\vec{s}_i$ of the individual nucleons i in the target nucleus dotted into various operators. One of these operators is the orbital angular momentum $\vec{L}$ of the α-target nucleus relative motion. This operator, when averaged over all nucleons i in the target nucleus gives rise to a α-nucleus $\vec{I}\cdot\vec{L}$ potential, which is the object of the present investigation. Here I represents the spin of the target nucleus. Another operator is the internal orbital angular momentum $\vec{l}_i$ of the same nucleon i, and this part of the interaction gives rise to central α-nucleus potentials whose magnitude depends on the spin $\vec{I}$ of the nucleus. All these potentials can give rise to inelastic as well as to elastic processes and although small, may have to be included in future more precise analyses of α-nucleus collisions.

In the present discussion emphasis is placed on the elastic α-nucleus interaction, and estimates for both the $\vec{I}\cdot\vec{L}$ and the above-mentioned central part of the α-nucleus potential are given for the nuclei of ^{9}Be, ^{14}N, and ^{59}Co, using a simple shell-model description of the ground states of these nuclei. Core polarizations are ignored in the present estimate. The r-dependent coefficient the "static" $\vec{I}\cdot\vec{L}$ potential is found to be nearly 2 orders of magnitude smaller than the conventional spin-orbit potential which occurs in nucleon-nucleus interactions.

An application envisaged is the inclusion of the $\vec{I}\cdot\vec{L}$ potential in the optical-model potential of the exit channel in (α, α') reactions.[10] As already mentioned in the Introduction, the spin of the nucleus excited in the reaction is usually polarized relative to the normal to the scattering plane, and hence the $\vec{I}\cdot\vec{L}$ potential has an effect on the inelastic cross section already to first order in the strength of this potential. It has been found[10] that if the $\vec{I}\cdot\vec{L}$ potential is as large as what the phenomenological analyses indicate then it has a nonnegligible effect on the degree of polarization of the excited nucleus in (α, α') reactions at energies above 30 MeV. It would be of some interest to investigate to what extent this potential can also affect the determination of nuclear deformation parameters in (α, α') reactions, particularly at high energies and whether it can introduce significant j-dependent effects in rearrangement collisions.[29] It is interesting that the sign of the $\vec{I}\cdot\vec{L}$ potential is itself I-dependent, in view of the fact that the projection factors g and h, defined in Eqs. (13) and (14) are positive or negative depending on whether the spin of the nucleon is parallel or antiparallel to the total spin I of the nucleus.

In conclusion, the strength of the "static" $\vec{I}\cdot\vec{L}$ potential in α-nucleus collisions was estimated and for the nuclei of ^{9}Be and ^{59}Co it was found to be considerably smaller than the corresponding phenomenological value observed in elastic scattering.[7-9] The discrepancy may point to the importance of the role played by the virtual excitation of nuclear intermediate states during the collision, neglected in the present treatment. Further, in the case of nuclei with a valence nucleon whose j equals $l-\frac{1}{2}$ the spin-orbit potential is expected to be larger than when $j=l+\frac{1}{2}$. A particularly good example appears to be that of ^{14}N. The $\vec{I}\cdot\vec{L}$ potential is unlikely to play a significant role in the description of elastic α scattering, but its effect on inelastic reactions may turn out to be of some importance.

ACKNOWLEDGMENTS

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APPENDIX A

It will now be shown that $V_{so}(i, B)$ is given by Eq. (3). In order to carry out the average over the nucleons in nucleus B, it is convenient to make a transformation of coordinates to the center of mass of nucleus B, $\vec{R}_B$. Denoting, respectively, by $\vec{\xi}_i$ and $\vec{\eta}_j$ the distances of particles in nucleus A and B to $\vec{R}_B$, the transformation of coordinates

$$\vec{x}_1\vec{x}_2\cdots\vec{x}_A\vec{x}_{A+1}\cdots\vec{x}_{A+B}\rightarrow\vec{\xi}_1\cdots\vec{\xi}_A\vec{\eta}_1\cdots\vec{\eta}_{B-1}\vec{R}_B \tag{A1}$$

is given by

$$\vec{R}_B=\sum_{j=A+1}^{A+B}\vec{x}_j/B,$$

$$\vec{\xi}_i=\vec{x}_i-\sum_{A+1}^{A+B}\vec{x}_s/B,$$

and

$$\vec{\eta}_j=\vec{x}_j-\sum_{A+1}^{A+B}\vec{x}_s/B.$$

The gradient difference $\vec{\nabla}_{\vec{x}_i}-\vec{\nabla}_{\vec{x}_j}$ is then given by

$$\vec{\nabla}_{\vec{x}_i}-\vec{\nabla}_{\vec{x}_j}=\vec{c}_i+\vec{c}_j-\vec{d}_j, \tag{A2}$$

with

$$\vec{c}_i = \vec{\nabla}_{\vec{\xi}_i} + \sum_1^A \vec{\nabla}_{\vec{\xi}_s}/B - \vec{\nabla}_{\vec{R}_B}/B, \quad (A3)$$

$$\vec{c}_j = \sum_{s \neq j} \vec{\nabla}_{\vec{\eta}_s}/B,$$

$$\vec{d}_j = (1 - B^{-1})\vec{\nabla}_{\vec{\eta}_j}. \quad (A4)$$

Substituting Eq. (1) (see text) for $V_{so}(i,j)$ into Eq. (2), remembering that $\vec{x}_i - \vec{x}_j = \vec{r}_{ij} = \vec{\xi}_i - \vec{\eta}_j$, and replacing $\vec{s}_i + \vec{s}_j$ by $\vec{S}_{ij}$, one obtains for Eq. (2) the expression

$$V_{so}(i,B) = \langle \phi_B | \sum_j V_{ls}(r_{ij})(\vec{\xi}_i - \vec{\eta}_j) \times [\vec{c}_i + \vec{c}_j' + \vec{d}_j] \cdot \vec{S}_{ij} | \phi_B \rangle. \quad (A5)$$

The terms $\vec{c}_j'$ and $\vec{d}_j$ do not contain a gradient relative to the position of particle i, and hence they cannot give rise to a spin-orbit potential of the form $\vec{\xi}_i \times \vec{\nabla}_{\vec{\xi}_i} \cdot \vec{s}_i$ involving particle i. Nevertheless it will now be shown that the terms involving c_j' and d_j give no contribution of any type for the case that $|\phi_B\rangle$ is composed of $l=0$ (s-state) nucleons as in ^{3}He and ^{4}He. The terms due to $\vec{c}_j'$ vanish for the ^{3}He or ^{4}He case, since the reduced matrix element of the momentum $\vec{\nabla}_{\vec{\eta}_s}$ vanishes for each $s \neq j$. The terms $\vec{\eta}_j \times \vec{d}_j \cdot \vec{S}_{ij}$ also vanish in this case since $\vec{\eta}_j \times \vec{d}_j$ is proportional to the angular momentum $\vec{\eta}_j \times \vec{p}_j = \vec{L}_j$ of particle j relative to the center of mass of nucleus B, and $L_j \cdot S_{ij} | \phi_B\rangle$ vanishes for s-state nucleons.

The terms $\vec{\xi}_i \times \vec{d}_j \cdot \vec{S}_{ij}$ also give no contribution as will now be shown. This term gives rise to the matrix element

$$\vec{s}_i \times \vec{\xi}_i \cdot \langle \phi_B | V_{ls}(r_{ij}) \vec{d}_j | \phi_B \rangle + \vec{\xi}_i \cdot \langle \phi_B | V_{ls} \vec{d}_j \times \vec{s}_j | \phi_B \rangle. \quad (A6)$$

If the nucleus B can be described in the $L \cdot S$ coupling scheme with the total orbital angular momentum $\vec{L}$ equal to zero, then the first matrix element in (A6) can be shown to be parallel to $\vec{\xi}_i$ and hence is orthogonal to $\vec{s}_i \times \vec{\xi}_i$, and vanishes. The second matrix element in (A6) is perpendicular to $\vec{\xi}_i$, so that its dot product with $\vec{\xi}_i$ also vanishes. Hence the two terms in (A6) give no contribution. The quantity to be evaluated is

$$V_{so}(i,B) = \sum_j \langle \phi_B | V_{ls}(r_{ij})(\vec{\xi}_i - \vec{\eta}_j) \times \vec{c}_i \cdot \vec{S}_{ij} | \phi_B \rangle, \quad (A7)$$

where it will at first be assumed that $|\phi_B\rangle$ represents the wave function of a general nucleus B, and later the result will be specialized to the case of the ^{4}He nucleus.

By using the vectors $\vec{R}(i,j)$ and $\vec{y}(i,j)$ to be defined below, Eq. (A7) can be written in the form

$$V_{so}(i,B) = \vec{\xi}_i \times \vec{c}_i \cdot \langle \phi_B | \sum_j [\vec{s}_i V_{ls}(r_{ij}) + \vec{R}(i,j)] | \phi_B \rangle + \vec{c}_i \cdot \langle \phi_B | \sum_{j,L} \eta_j v_L(\xi_i \eta_j) \vec{y}_{(\mathcal{L}L)}(i,j) \times \vec{S}_{ij} | \phi_B \rangle. \quad (A8)$$

The first line in Eq. (A8) arises from the terms in ξ_i in Eq. (A7). The v_L's arise from the spherical harmonic expansion of the nucleon-nucleon potential $V_{ls}(r_{ij})$ given by Eqs. (6) and (7) and $\vec{R}(i,j)$ is the vector whose spherical projection σ is given by

$$R_\sigma(i,j) = \sum_{KL} (-)^{1-K} [\tfrac{1}{3}(2K+1)]^{1/2} v_L(\xi_i \eta_j) \sum_{MQ} \langle 1\sigma | LKMQ \rangle Y_{LM}(\hat{\xi}_i) T_{KQ}[Y_L(\hat{\eta}_j), s_j], \quad (A9)$$

where the tensor T_{KQ} is defined in the usual way,[19]

$$T_{KQ} = \sum_M \langle KQ | L1M\rho \rangle Y_{LM}(\hat{\eta}_j)(s_j)_\rho. \quad (A10)$$

The second line in Eq. (A8) arises from the η_j term in (A7). Here the vector $\vec{y}_{\mathcal{L}L}$, written as a tensor of rank 1 and projection σ, is given as

$$y_{(\mathcal{L}L)1\sigma}(i,j) = -\begin{pmatrix} L & 1 & \mathcal{L} \\ 0 & 0 & 0 \end{pmatrix} (3)^{-1/2} [(2L+1)(2\mathcal{L}+1)]^{1/2} \sum_{mM} \langle 1\sigma | \mathcal{L}LmM \rangle Y_{\mathcal{L}m}(\hat{\eta}_j) Y_{LM}(\hat{\xi}_i). \quad (A11)$$

The contributions to Eq. (A8) which arise from the spherical harmonics of zero order in the direction $\hat{\eta}_j$, $Y_{00}(\hat{\eta}_j)$ are given by

$$[V_{so}(i,B)]_{00} = \vec{\xi}_i \times \vec{c}_i \cdot \langle \phi_B | \sum_j \{[V_0(\xi_i \eta_j) - (\eta_j/\xi_i) v_1(\xi_i \eta_j)]/(4\pi)\} (\vec{s}_i + \vec{s}_j) | \phi_B \rangle. \quad (A12)$$

If nucleus A only contains nucleon i, then $\vec{\xi}_i \times \vec{c}_i = \mu_{1B}^{-1} \vec{L}_i$, where μ_{1B} is the reduced mass of nucleon i relative to nucleus B and $\vec{L}_i$ is the corresponding angular momentum operator. In this case the $\vec{s}_i$ part of Eq. (A12) represents the nucleon-i nucleus-B spin-orbit potential

$$V_{so}(i,B) = \mu_{1B}^{-1} V_{l_i s_i}^B(\xi_i) \vec{L}_i \cdot \vec{s}_i, \quad (A13)$$

with $V^B_{l_i s_i}$ given as the matrix element over $|\phi_B\rangle$ of the curly brackets (summed over j) in Eq. (A12). An approximation to $V^B_{l_i s_i}$ has been stated in the book by Bohr and Mottelson[1] as

$$V^B_{l_i s_i} \sim -\tfrac{1}{3}\left[\int V_{ls}(r')\, r'^2 d^3r'\right](\partial\rho/\partial\eta)_{\eta=\xi_i}\,,$$

where $\rho(\eta)$ is the matter distribution density of nucleus B normalized such that $\int\rho(\eta)\eta^2 d\eta = B$. The above expression will however not be used here.

If the wave function of nucleus B can be expressed in the $\vec{L}\cdot\vec{S}$ coupling scheme and if the total orbital angular momentum is zero, then (A12) represents the only contribution to $V_{so}(i,B)$. Replacing the matrix element of the curly brackets, taken over the spatial part of $|\phi_B\rangle$, by $V^B_{l_i s_i}(\xi_i)$, Eq. (A12) can be further simplified into

$$V_{so}(i,B) = \vec{\xi}_i \times \vec{c}_i \cdot (\vec{s}_i + N\vec{S}_B) V^B_{l_i s_i}(\xi_i)\,. \tag{A14}$$

The total angular momentum operator of nucleus B is $\vec{S}_B$, and the factor N makes allowance for the fact that in Eq. (A12) the matrix elements of $\vec{s}_j$ cancel pairwise in the sum over j and only the last unpaired s_j survives.

For the discussion which follows, B represents the nucleus of ^4He. In this case $N\vec{S}_B$ vanishes and $V^B_{l_i s_i}(\xi_i)$ is taken as the phenomenological nucleon-^4He spin-orbit potential, as defined in Eq. (A13). Further, nucleus A is assumed to contain more than one nucleon i.

In order to bring the result given by Eq. (A14) the form of Eq. (3) in the text, two successive transformation of coordinates are made

$$\vec{\xi}_1\vec{\xi}_2\cdots\vec{\xi}_A\vec{R}_B \to \vec{r}_1\vec{r}_2\cdots\vec{r}_{A-1}\vec{R}_A\vec{R}_B \to \vec{r}_1\cdots\vec{r}_{A-1}\vec{R}\vec{R}_{c.m.}\,.$$

The first reexpresses the $\vec{\xi}_i$'s in terms of the center of mass $\vec{R}_A$ of nucleus A, and the second introduces the position of the center of mass of nucleus A and B, $B\vec{R}_B + A\vec{R}_A = (A+B)\vec{R}_{c.m.}$ and the relative coordinate $\vec{R} = \vec{R}_B - \vec{R}_A$. The expression for $\vec{c}_i$ in these coordinates then becomes equal to the expression in square brackets in Eq. (3).

APPENDIX B

The multipole expansion required for the evaluation of $V_{so}(A,B)$, given by Eq. (4), will now be discussed. The procedure consists of inserting the expression for $V_{so}(i,B)$ given by Eq. (5) into Eq. (4), and at the same time expanding $V^B_{ls}(|\vec{r}_i - \vec{R}|)$ into spherical harmonics of the directions of $\vec{r}_i$ and $\vec{R}$ as given in Eq. (6). The quantity to be evaluated is thus $V_{so}(A,B)$, given by

$$\frac{\hbar}{i}\sum_i \langle\psi_A|\sum_{L\Lambda} v_L(r_i,R) Y_{L\Lambda}(\hat{\vec{r}}_i) Y^*_{L\Lambda}(\hat{\vec{R}})\{\vec{r}_i\times\vec{\nabla}_{\vec{r}_i} + \vec{R}\times\vec{\nabla}_{\vec{R}}\mu_{AB}^{-1} - \vec{r}_i\times\vec{\nabla}_{\vec{R}}\mu_{AB}^{-1} - \vec{R}\times\vec{\nabla}_{\vec{r}_i}\}\cdot\vec{s}_i|\psi_A\rangle\,. \tag{B1}$$

The contribution from each multipole moment L in the first line of Eq. (B1) is a scalar. After multiplication by each of the scalar terms in curly brackets the result can be regrouped into the scalar product[19] $T\cdot P = \sum_\mu T_{K\mu}P_{K-\mu}(-)^\mu$ of two tensors of rank K. The first, $T_{K\mu}(i)$, acts on the coordinates of particle i, the second, $P_{K\mu}(R)$, acts on the coordinates of the α particle.

The two terms in curly brackets in Eq. (B1) which give rise to $\vec{l}\cdot\vec{L}$ terms are the second and the third. They will now be discussed in terms of the tensor notation introduced above. In this notation[19] the tensor T_{KQ} formed out of tensors U_k and V_l is given by

$$T_{KQ}(U,V) = \sum_{m\mu}\langle KQ|klm\mu\rangle U_{km}V_{l\mu}\,.$$

The term $\vec{r}_i\times\vec{\nabla}_{\vec{R}}\cdot\vec{s}_i$ in Eq. (B1) gives rise to

$$\begin{aligned}\mu_{AB}^{-1}\frac{\hbar}{i}V_{ls}\,\vec{r}_i\times\vec{\nabla}_{\vec{R}}\cdot\vec{s}_i = {}&+\mu_{AB}^{-1}(v_1/4\pi)(r_i/R)[\vec{s}_i + (2\pi)^{1/2}\vec{T}_1]\cdot\vec{\mathfrak{L}}\\ &+\mu_{AB}^{-1}\sum_{LkK} v_L i\sqrt{6}\, r_i(2L+1)^{1/2}(2k+1)^{1/2}\begin{pmatrix}L&1&k\\0&0&0\end{pmatrix}\\ &\times\begin{Bmatrix}1&L&K\\k&1&1\end{Bmatrix}(-)^K T_K(Y_k(\hat{r}_i),s_i)\cdot P_K(Y_L(\hat{\vec{R}}),(\hbar/i)\vec{\nabla}_{\vec{R}})\,.\end{aligned} \tag{B2}$$

Here $\vec{\mathfrak{L}}$ represents the operator $(\hbar/i)\vec{R}\times\vec{\nabla}_{\vec{R}}$. In the text, $\mathfrak{L}$ is written as L. The summation in the second line of the above equation starts with $L=2$. The terms with $L=0$ vanish, and the terms with $L=1$ have been written out explicitly in the first line. There is an additional term with $L=1$, $k=2$, and $K=2$ which is also present among the terms in the second line. However it does not give rise to an operator of the $\vec{I}\cdot\vec{L}$ type, and hence was not written explicitly. The vector $\vec{T}_1$ in the first line in Eq. (B2) corresponds to the tensor of first rank $T_{1\mu}(Y_2(\hat{r}_i), \vec{s}_i)$. By using the projection theorem, both operators $\vec{s}_i$ and $\vec{T}_1$ can be replaced by the total spin operator $\vec{I}_A$ of nucleus A, times a reduced matrix element. Hence the first line in Eq. (B2) gives rise to an operator $\vec{I}_A\cdot\vec{\mathfrak{L}}$, while the terms in the second line give rise to tensors more complicated than $\vec{I}_A\cdot\vec{\mathfrak{L}}$.

The other term of interest for the discussion in the text arises from the second term in curly brackets in Eq. (B1) $\vec{R}\times\vec{\nabla}_R\mu_{AB}{}^{-1}$:

$$(\hbar/i)V_{ls}\vec{R}\times\vec{\nabla}_{\vec{R}}\cdot\vec{s}_i\mu_{AB}{}^{-1}=\mu_{AB}{}^{-1}(1/4\pi)v_0\vec{s}_i\cdot\vec{\mathfrak{L}}+\mu_{AB}{}^{-1}\sum_{\substack{KLQ\\L\geq 1}}(-)^{K-L+1+Q}v_L(r_iR)T_{KQ}(Y_L(\vec{r}_i),\vec{s}_i)P_{K-Q}(Y_L(\vec{R}),\vec{\mathfrak{L}})\,. \tag{B3}$$

The $L=0$ term has been written explicitly in the first line of (B3); the $L=1$ term will vanish by parity considerations when integrated over the wave function ψ_A. The other terms, which appear in the second line of Eq. (B3) give rise to tensors of rank higher than first and are too complicated to be considered here. They represent the deformed parts of the $\vec{I}\cdot\vec{\mathfrak{L}}$-type spin-orbit potential since they depend on the direction of $\vec{R}$ through $Y_L(\hat{R})$.

The term in $V_{so}(i,B)$, Eq. (B1) due to $\vec{R}\times\vec{\nabla}_{\vec{r}_i}\cdot\vec{s}_i$ is of the form

$$(\hbar/i)V_{ls}\vec{R}\times\vec{\nabla}_{\vec{r}_i}\cdot\vec{s}_i=(R/r_i)(v_1/4\pi)\vec{l}_i\cdot\vec{s}_i-i\sqrt{6}\sum_{LKkQ}Rv_LT_{KQ}[T_k(Y_L(\vec{r}_i),\vec{\nabla}_{\vec{r}_i}),\vec{s}_i]\,Y_{K-Q}(\vec{R})(-)^{Q+L}$$
$$\times\begin{pmatrix}L&1&K\\0&0&0\end{pmatrix}\begin{Bmatrix}k&1&K\\1&L&1\end{Bmatrix}(2L+1)^{1/2}(2K+1)^{1/2}\,. \tag{B4}$$

Again, the most important term has been written in the first line. Included in the second line is another term with $L=1$ and $K=2$, which however gives rise to a noncentral $Y_2(\vec{R})$ type potential, and will thus not be considered explicitly.

The term due to $\vec{r}_i\times\vec{\nabla}_{\vec{r}_i}\cdot\vec{s}_i$ in Eq. (B1) is of the form

$$(\hbar/i)V_{ls}\vec{r}_i\times\vec{\nabla}_{\vec{r}_i}\cdot\vec{s}_i=(v_0/4\pi)\vec{l}_i\cdot\vec{s}_i+\sum_{LkQ}(-)^{k+L}v_L[(2k+1)/(2L+1)]^{1/2}T_{LQ}[T_k(Y_L(\hat{r}_i),\vec{l}_i),\vec{s}_i]Y_{L-Q}(\vec{R})(-)^Q\,. \tag{B5}$$

Here $\vec{l}_i$ is the operator $\hbar\vec{\xi}_i\times\vec{\nabla}_{\vec{\xi}}/i$. None of the terms in the two last equations give rise to a $\vec{I}\cdot\vec{\mathfrak{L}}$ term. However, they give rise to central, α-nucleus potentials which, since they lack the factor $\mu_{AB}{}^{-1}$, can be of some importance. In view of the fact that these terms contain the factor $\langle\psi_A|\vec{l}_i\cdot\vec{s}_i|\psi_A\rangle$ their magnitude and sign depends on the total spin of the target nucleus, a feature which may contribute to their identification in future optical-model analyses. The magnitude of this potential is estimated in Sec. III.

In summary the terms in $V_{so}(i,B)$ which give rise to a $\vec{I}\cdot\vec{\mathfrak{L}}$ interaction are given by the first lines in Eqs. (B2) and (B3). They are collected below

$$\mu_{AB}{}^{-1}[(v_0/4\pi)\vec{s}_i-(r_iv_1/4\pi R)(\vec{s}_i+\sqrt{2\pi}\,\vec{T}_1)]\cdot\vec{\mathfrak{L}}\,. \tag{B6}$$

The terms in $V_{so}(i,B)$ which give rise to a central α-nucleus interaction arise in Eqs. (B4) and (B5) and they are

$$[(v_0/4\pi)-(Rv_1/4\pi r_i)]\vec{l}_i\cdot\vec{s}_i\,. \tag{B7}$$

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