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†Present address: Argonne National Laboratory, Argonne, Illinois 60439.

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PHYSICAL REVIEW C VOLUME 6, NUMBER 4 OCTOBER 1972

Test of Single-Channel Strength-Function Limit*

Joseph J. Devaney

University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544

(Received 16 June 1972)

Using the exact solution of a particle decaying from a square well, it is shown that the strength function, (Γ/D), is unbounded and has the form $(2\pi\Gamma/D) = -\ln(1-T)$ suggested by Moldauer (for average parameters), where T is the transmission.

I. INTRODUCTION

In the use of, especially the extrapolation of, average widths Γ_c and spacings D_c for a channel c, it is of critical interest whether the channel-strength function Γ_c/D_c is bounded. In the past it seemed evident that Γ_c/D_c was in fact bounded for one expected[1] that $2\pi\Gamma_c/D_c = T_c$, where T_c is the transmission and thus limited to a maximum of 1, so that $\Gamma_c/D_c \leq 1/2\pi$. A slightly different formula[2] has also been suggested leading to the bound $1/\pi$. Moldauer[3] has emphasized that the foregoing formulas are valid only in the limit of very small values of T_c, and has proposed rather that

$$T_c = 1 - e^{-2\pi\Gamma_c/D_c}. \qquad (1)$$

In Eq. (1) the limit $T_c = 1$ does *not* imply any bound on the channel-strength function Γ_c/D_c. Moldauer demonstrates the validity of (1) for a number of simple analytic unitary models of the S matrix.[3] Ullah and Moldauer[4] have offered a proof of (1) assuming the plausible simple-pole expansion (Mittag-Leffler) of the statistical collision matrix. However, there remain some (hopefully minor) questions about the validity of the proofs,[4] and, more important, the generality of the assumed collision matrix has not been established by these authors.[5]

Accordingly, it seems useful to subject (1) to a test by means of a simple precise calculation. For such a trial we choose the spherically symmetric

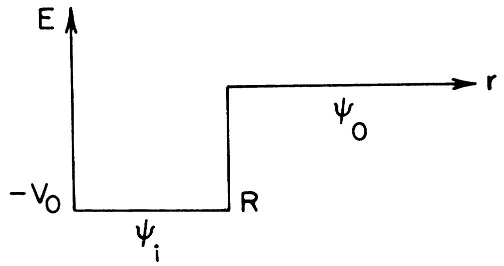

FIG. 1. Potential energy diagram.

decay of a particle from a square well. We find that (dropping the channel or state index c):

(1) Γ/D is unbounded.

(2) Moldauer's formula (1) gives precisely the correct functional form for T as a function of the strength function.

In this latter remark is incorporated the resolution of a certain ambiguity, since T is defined at an energy E, and D is defined by two energies, $E_{n+1}-E_n = \Delta E = D$, and then averaged over many energy spacings. We argue below that our resolution is the most logical, and should be equal to a reasonable choice of average.

II. SQUARE-WELL DECAYING STATE

Consider therefore the zero angular momentum decay of a particle of energy E from a square well of depth V_0 and radius R. (See Fig. 1.) The boundary condition at infinity of a decaying state is an exponentially increasing outgoing wave, since the "earlier" wave is farther out. We therefore take for the wave function outside of R, $\psi_o = e^{(ik+\lambda)(r-R)}$, $\lambda > 0$. In order to match boundary conditions at $r=R$, the inner wave number must also be complex and we choose for the inner wave function, (i refers to inner, o to outer),

$$\psi_i = Ae^{(iK+\Lambda)(r-R)} + Be^{-(iK+\Lambda)(r-R)} \tag{2a}$$

for $r \leq R$; A, B are constants to be determined.

The energy of a decaying state must be complex so our radial wave equations are

$$\frac{-\hbar^2}{2\mu}\frac{d^2}{dr^2}\psi_i = \left(E + V_0 - i\frac{\Gamma}{2}\right)\psi_i\,, \quad r \leq R\,, \tag{2b}$$

with

$$\psi_i(r=0) = 0$$

and

$$\frac{-\hbar^2}{2\mu}\frac{d^2}{dr^2}\psi_o = \left(E - i\frac{\Gamma}{2}\right)\psi_o\,, \quad r \geq R\,, \tag{2c}$$

μ being the (reduced) mass and $\Gamma \geq 0$ chosen to give a time decay of $e^{-\Gamma t/\hbar}$ for $|\psi|^2$. Matching solutions and their derivatives at $r=R$, setting $\psi_i(r=0) = 0$, and rearranging, we arrive at the following equations:

$$\tanh \Lambda R = k/K$$

or

$$2\Lambda R = \ln\left|\frac{k+K}{k-K}\right|\,, \tag{3}$$

$$K^2 = (\mu/\hbar^2)\{(E+V_0) + [(E+V_0)^2 + \tfrac{1}{4}\Gamma^2]^{1/2}\}\,, \tag{4}$$

$$k^2 = (\mu/\hbar^2)\{E + [E^2 + \tfrac{1}{4}\Gamma^2]^{1/2}\}\,, \tag{5}$$

$$KR = n\pi - \arctan(\Lambda/k)\,, \tag{6}$$

where the state number n is a positive integer ≥ 1;

$$K\Lambda = k\lambda = (\mu\Gamma/2\hbar^2)\,, \tag{7}$$

$$A = \frac{ik+\lambda+iK+\Lambda}{2(iK+\Lambda)}\,, \quad B = \frac{-ik-\lambda+iK+\Lambda}{2(iK+\Lambda)}\,. \tag{8}$$

In terms of the inner wave function, the transmission T is the ratio of outgoing part $Ae^{(iK+\Lambda)(r-R)}$ absolute squared, less the incoming part $Be^{-(iK+\Lambda)(r-R)}$ absolute squared, divided by the outgoing part absolute squared, all evaluated at $r=R$:

$$T \equiv \left.\frac{|\psi_{i\ \text{outg.}}|^2 - |\psi_{i\ \text{inc.}}|^2}{|\psi_{i\ \text{outg.}}|^2}\right|_{r=R} = \frac{|A|^2 - |B|^2}{|A|^2}\,. \tag{9}$$

Using (8) and then (7),

$$T = \frac{4kK + 4\lambda\Lambda}{(k+K)^2 + (\Lambda+\lambda)^2} = \frac{4kK}{(k+K)^2}\,, \tag{10}$$

which is the usual transmission of a plane wave in crossing a discontinuity from wave number K on one side to wave number k on the other. Formula (10) is also precisely the transmission coefficient $T = (1-|S|^2)$ for the inverse process of total capture (black nucleus) of an s-wave particle.[6]

Equation (1) may be rearranged to read

$$2\pi(\Gamma/D) = -\ln(1-T)\,, \tag{1'}$$

so that by substituting (10) we obtain

$$\pi(\Gamma/D) = -\ln\left|\frac{k-K}{k+K}\right|\,. \tag{11}$$

If we now compare (3) and (11), using (7), we find that Moldauer's equation reduces to

$$K = \mu R D/\pi\hbar^2\,, \tag{12}$$

which is to be tested.

III. LIMIT $E \to \infty$

The simplest test of a bound on Γ/D is to show that (12), and hence (1), is valid in the limit $E \to \infty$. In this limit $T \to 1$, and from (3)–(6); n, k, K, Γ, and Λ all tend to infinity, with $(k/K)\to 1$. From (4),

$$K\to k\left[1+\frac{V_0}{2(E^2+\frac{1}{4}\Gamma^2)^{1/2}}\right], \tag{13}$$

and substituting (13) into (3) shows that

$$\Lambda\to\left(\frac{1}{2R}\right)\ln\left|\frac{4(E^2+\frac{1}{4}\Gamma^2)^{1/2}}{V_0}\right|. \tag{14}$$

Now in the limit $E\to\infty$ either $\Gamma\gtrsim E$ or $\Gamma<E$. In the former case $K^2\propto\Gamma$ from (4), and using (7) in (14) we get $\sqrt{\Gamma}\propto\ln(2\Gamma/V_0)$ which is a contradiction; consequently $\Gamma<E$. It then follows that $k\propto\sqrt{E}$ and $K\propto\sqrt{E}$, whereas $\Lambda\propto\ln E$, thus $\Gamma^2/E^2\to 0$ and in (6) $\arctan(\Lambda/k)\to 0$. Defining $D\equiv\Delta E\equiv E_{n+1}-E_n$, we then find that

$$D\to\frac{\hbar^2}{2\mu}(K_{n+1}{}^2-K_n{}^2)\to\frac{\hbar^2}{2\mu}\left[\frac{(n+1)^2\pi^2}{R^2}-\frac{n^2\pi^2}{R^2}\right]\to\frac{\pi\hbar^2K_n}{\mu R}, \tag{15}$$

which proves (12), hence Moldauer's theorem (1) in in the limit $E\to\infty$, and we conclude that Γ/D is unbounded.

IV. ANALYTICAL PROOF

We next show that the Moldauer theorem (1) is true for all E. In its original form,[3] (1) was intended for average values of the parameters. In general, D is a function of E so that the type of average taken introduces a measure of ambiguity to (1). We argue that any meaningful average must be close to the actual spacing at the energy considered.

Another way of looking at (1) or (12) is to notice that K or Γ or T are evaluated *at* an energy E, whereas $D=\Delta E$ is the *difference* between two energies. So ambiguity in the choice of D or of E is implicit.

We argue that the most stringent control of this ambiguity in the proof of a relation like (12) is to show (12) for energy E, not only near the spacing D, but closely bounding such D.

In fact, we will show analytically that there exists an E with $E_{n-1}<E<E_{n+1}$ such that $D\equiv D_{n+1,n}\equiv E_{n+1}-E_n$ obeys

$$(R\mu D/\hbar^2\pi)=K(E), \tag{12$'$}$$

and we will later show numerically that E may be further restricted to lie within $E_n<E<E_{n+1}$ for all trial cases ranging from elementary particle to atomic dimensions.

Our proof is as follows: Let $n\pi=a$ in (6). Then the equations (3)–(6) give E as a function of a with the physical states existing only for $a=\text{integer}\times\pi$. Applying the mean-value theorem to $E(a)$ between $n\pi$ and $(n+1)\pi$, there exists some a, call it α, with $n\pi\leqslant\alpha\leqslant(n+1)\pi$ such that

$$E'(\alpha)=\frac{E[(n+1)\pi]-E(n\pi)}{\pi}\equiv\frac{D}{\pi}.$$

In what follows, quantities without subscripts are evaluated at a, whereas those subscripted α and $(n\pm1)$ are evaluated at α and $(n\pm1)\pi$, respectively. We prove in Appendix A that

$$E'=(\hbar^2K/\mu R)(1+G)^{-1}, \tag{16}$$

with

$$G\equiv\frac{k^2-(k\Lambda R/K)(K^2-k^2)}{R^2(k^4+K^2\Lambda^2)-(k\Lambda R/K)(K^2+k^2)}. \tag{17}$$

By the above mean-value theorem, (16) is equal to (D/π) at some E_α with $E_n\leqslant E_\alpha\leqslant E_{n+1}$.

We next show that

$$K_{n-1}<K_\alpha(1+G)^{-1}<K_\alpha<K_{n+1}, \tag{18a}$$

or that

$$K_{n-1}<(R\mu D/\pi\hbar^2)<K_{n+1}, \tag{18a$'$}$$

and since K is a monotonically increasing function of E, the number $(R\mu D/\hbar^2\pi)$ between K_{n-1} and K_{n+1} must equal some $K(E)$ with $E_{n-1}<E<E_{n+1}$, thus proving (12$'$). That K is a monotonically increasing function of E, as is k, is evident physically since they are wave numbers, but strictly speaking this remark must be proved for our equations, which we do in Appendix B [(B16) and (B17)], where we also derive inequalities useful in the proof of (18a) itself.

Since $\alpha<(n+1)\pi$ and $G>0$ it follows from (B14) that the right inequalities of (18a) hold:

$$K_\alpha(1+G)^{-1}<K_\alpha<K_{n+1}. \tag{18b}$$

To show the left inequality requires demonstration of

$$GK_{n-1}<?\ K_\alpha-K_{n-1}. \tag{19}$$

Using (B17), (B9), (B7), (B8), and (B3), (19) becomes

$$GK_{n-1}<\frac{(k^2/K^2)K_{n-1}}{(KR-1)^2}<\frac{K_{n-1}}{(KR-1)^2}<?\,[(n\pi-\tfrac{1}{4}\pi)-(n-1)\pi]\times\frac{1}{R}\leqslant K_n-K_{n-1}\leqslant K_\alpha-K_{n-1},$$

so we must show that [note that all state numbers n are positive integers, see (6), thus the state number $(n-1)\geqslant 1$]

$$\frac{K_{n-1}R}{(K_nR-1)^2}<\frac{(n-1)\pi}{(n\pi-\frac{1}{4}\pi-1)^2}<\frac{\pi}{(2\pi-\frac{1}{4}\pi-1)^2}=0.0494\pi<?\ \tfrac{3}{4}\pi,$$

which is true so the left inequality (18a) holds and

Moldauer's theorem [(1), (12′)] is proved for spherically symmetric square-well decay.

V. NUMERICAL TESTS

It turns out that because $\arctan(\Lambda/K)$ is limited in size to less than $\pi/4$, the equations in the order (6), (A2), (5), (3b), rapidly converge to a solution under numerical iteration. Such solutions always satisfy the tighter law

$$(R\mu D/\hbar^2\pi) = K(E)\,,$$

with (20)

$$E_n < E < E_{n+1}\,,$$

when expressed in the form

$$K_n < (R\mu D_{n+1,\,n}/\hbar^2\pi) < K_{n+1}\,, \tag{21}$$

so by (12′) again vindicating Moldauer's formula (1). The parameters tested are listed in Table I.

ACKNOWLEDGMENTS

It is a pleasure to express appreciation to Dr. W. Gibbs for his encouragement and perceptive comments. For critical readings of the manuscript, we are grateful to Dr. D. G. Foster, Jr., and Dr. W. Gibbs.

APPENDIX A. DERIVATION OF E'

Considering (3)–(6) to be functions of a, we take the derivatives from (3) to get:

$$\Lambda' R = \frac{Kk' - kK'}{K^2 - k^2}\,. \tag{A1}$$

We square (4) and use (7) to obtain

$$K^2 = \Lambda^2 + (2\mu/\hbar^2)(E + V_0)\,, \tag{A2}$$

whence

$$K^{2\prime} = \Lambda^{2\prime} + (2\mu/\hbar^2)E'\,. \tag{A3}$$

We square (5) and use (7) to obtain

$$k^4 - 2k^2(\mu/\hbar^2)E = K^2\Lambda^2\,, \tag{A4}$$

whence

$$2k^2k^{2\prime} - 2k^{2\prime}(\mu/\hbar^2)E - 2k^2(\mu/\hbar^2)E' = K^{2\prime}\Lambda^2 + K^2\Lambda^{2\prime}\,. \tag{A5}$$

From (6),

$$K'R = 1 - \frac{k\Lambda' - \Lambda k'}{\Lambda^2 + k^2}\,. \tag{A6}$$

(A1), (A3), (A5), and (A6) are four equations in the four derivatives Λ', k', K', and E'. Defining

$$\delta \equiv 2[(\Lambda/kKR)k^2 - (\Lambda^2 + k^2)] \tag{A7}$$

TABLE I. The following table is a partial list of parameters satisfying (3)–(6) and (20), thus verifying (1) in the form (20). As asserted, in every case the sixth column, D, lies between the seventh-column values, $(\pi\hbar^2K/\mu R)$.

μ (g)	V_0 (MeV)	R (fm)	n	E (MeV)	D (ergs)	$\frac{\pi\hbar^2}{\mu}\frac{K}{R}$ (ergs)
			Elementary-particle dimensions			
8.36×10^{-25}	300	0.8	1	13	2.9×10^{-3}	1.77×10^{-3}
			2	1828		3.9×10^{-3}
			10 000	6.4×10^{10}	20.5056	20.5044
			10 001			20.5065
			Nuclear dimensions			
1.67×10^{-24}	50	7.76	10	289	1.144×10^{-4}	1.091×10^{-4}
			11			1.200×10^{-4}
			296	298.7×10^{3}	32.39×10^{-4}	32.34×10^{-4}
			297	300.7×10^{3}		32.45×10^{-4}
			Atomic dimensions			
9.11×10^{-28}	0.03	100 000	30	3.35×10^{-3}	3.674×10^{-9}	3.614×10^{-9}
			31	5.65×10^{-3}		3.734×10^{-9}
			1000	37.6	1.2053×10^{-7}	1.2046×10^{-7}
			1001			1.2059×10^{-7}

and

$$\epsilon \equiv (\Lambda/kKR)(K^2+k^2) - 2[k^2-(\mu E/\hbar^2)], \tag{A8}$$

we obtain after some manipulation

$$\Lambda^{2\prime} = [2\mu E'/\hbar^2(K^2+\Lambda^2)] \times \{-(\Lambda^2+k^2)+(\delta/\epsilon)[k^2-(\mu E/\hbar^2)]\}, \tag{A9}$$

$$k^{2\prime} = (\mu\delta/\hbar^2\epsilon)E', \tag{A10}$$

$$K^{2\prime} = [2\mu E'/\hbar^2(K^2+\Lambda^2)] \times \{(K^2-k^2)+(\delta/\epsilon)[k^2-(\mu E/\hbar^2)]\}, \tag{A11}$$

$$(\mu/\hbar^2)E' = (K/R)(1+G)^{-1}, \tag{A12}$$

which is (16) with G given by (17).

APPENDIX B. INEQUALITIES

Restricting ourselves to positive E, all the variables of Eqs. (3)–(6) are real and positive. From (3),

$$\Lambda R = \text{arctanh}(k/K) = (k/K) + \tfrac{1}{3}(k/K)^3 + \tfrac{1}{5}(k/K)^5 \cdots, \tag{B1}$$

so that

$$\Lambda R > (k/K). \tag{B2}$$

From (4) and (5),

$$k<K. \tag{B3}$$

From (A2)

$$\Lambda < K \quad \text{and} \quad 2(\mu/\hbar^2)(E+V_0) < K^2. \tag{B4}$$

From (A4)

$$k^2 > K\Lambda \quad \text{and} \quad 2(\mu/\hbar^2)E < k^2, \tag{B5}$$

whence

$$\Lambda^2 < K\Lambda < k^2,$$

and applying (B3),

$$\Lambda < k < K. \tag{B6}$$

Therefore, $0 \leq \arctan(\Lambda/k) < \pi/4$, and using (6),

$$KR > n\pi - (\pi/4) \geq 3\pi/4 > 1, \tag{B7}$$

$$KR \leq n\pi \equiv a. \tag{B8}$$

Using (B2) and (B5) to eliminate Λ and using (B7) and (17),

$$G < \frac{(k^2/K^2)}{(kR-1)^2-(R/K)(K-k)^2} \leq \frac{(k^2/K^2)}{(KR-1)^2}. \tag{B9}$$

Using (A7), (B5), (B3), and (B7),

$$\delta < 0. \tag{B10}$$

Using (A8), (A4), (B2), (B6), and (B7),

$$\epsilon < 0. \tag{B11}$$

Using (B2), (B5), and (B3) in (17), we see that the denominator of G is greater than zero, and using (B1) the numerator of G is

$$k^2\{1-[1+3^{-1}(k/K)^2+5^{-1}(k/K)^4+\cdots - (k/K)^2 - 3^{-1}(k/K)^4 - \cdots]\}$$
$$= 2k^2[(1\times 3)^{-1}(k/K)^2 + (3\times 5)^{-1}(k/K)^4+\cdots]$$
$$>0 \text{ [all absolutely convergent by (B3)]},$$

so that

$$G>0. \tag{B12}$$

Using (A12),

$$E'>0. \tag{B13}$$

Using (B3), (B5), (B10), (B11), and (B13) in (A11),

$$K^{2\prime}>0, \quad \text{whence } K'>0. \tag{B14}$$

Finally, using (B10), (B11), and (B13) in (A10),

$$k^{2\prime}>0, \quad \text{whence } k'>0. \tag{B15}$$

From (B14) and (B13) it is clear that

$$\frac{dK}{dE} = \left(\frac{dK}{da}\right)\left(\frac{dE}{da}\right)^{-1} > 0. \tag{B16}$$

Consequently, for $E_{n-1} < E_n \leq E \leq E_{n+1}$ it follows that

$$K_{n-1} < K_n \leq K \leq K_{n+1}. \tag{B17}$$

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*Work performed under the auspices of the U.S. Atomic Energy Commission.

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PHYSICAL REVIEW C VOLUME 6, NUMBER 4 OCTOBER 1972

Mirror β Decays of ^{12}B and ^{12}N to $^{12}C_{4.44}$ †

David E. Alburger
Brookhaven National Laboratory, Upton, New York 11973
(Received 7 June 1972)

The β decays of ^{12}B and ^{12}N to the 4.44-MeV first excited state of ^{12}C have been studied by means of β-γ coincidence measurements using a 4π plastic scintillator and a NaI(Tl) detector. The ratio of $^{12}N/^{12}B$ β-ray branches to the 4.44-MeV state is found to be 1.52 ± 0.06 leading to $ft^+/ft^- = 1.06 \pm 0.04$ for this branch. This result removes the only severe exception to the systematic trend of positive $\delta = (ft^+/ft^-) - 1$ for mirror decays and is in agreement with a recent calculation of the binding-energy effect. In separate measurements the absolute β-ray branch of ^{12}B to the ^{12}C 4.44-MeV state is found to be $(1.27 \pm 0.06)\%$.

I. INTRODUCTION

During the past few years extensive studies[1] have been made of mirror β decays in order to investigate the questions of symmetry in β decay and possible second-class currents in the weak interaction. One of the first well-established cases of asymmetry was in the $A = 12$ system, where it had been found that the ft value for the β^+ decay of ^{12}N to the ^{12}C ground state is ~10% greater than the ft value for the corresponding β^- branch of ^{12}B. More exactly, the asymmetry $\delta = (ft^+/ft^-) - 1$ is $+0.115 \pm 0.009$[2] in this case. Accumulated evidence for almost all other mirror pairs of β^+ and β^- emitters has substantiated the conclusion that δ has a positive value. It is not yet clear whether δ increases linearly with the total decay energy, since the existing data could just as well be satisfied by the assumption of a constant value of δ averaging about +0.10.

Except for $A = 24$, where the measurements are exceedingly difficult and the slightly negative δ is in doubt, the only really severe exception to the systematics has been in the $A = 12$ system itself. Two measurements have been reported on the ratio of $^{12}N/^{12}B$ β-ray branches to the 4.44-MeV 2^+ first excited state of ^{12}C, namely 1.84 ± 0.1[3] and 1.72 ± 0.15.[4] The mean of these results led to $\delta = -0.117 \pm 0.041$[5] for this case which represented a large departure from the systematics.

The purpose of the present work was to remeasure the ratio of $^{12}N/^{12}B$ β decays to the $^{12}C_{4.44}$ state using a technique that should be less subject to systematic errors than previous methods, as well as providing greater accuracy in the result. This technique has also allowed a more accurate value to be obtained for the absolute β-ray branch of ^{12}B to the ^{12}C 4.44-MeV state.

II. EXPERIMENTAL PROCEDURES

In experiments on β-ray emitters using scintillation detectors in large solid-angle geometry one possible source of systematic error in the comparison of β^+ and β^- activities results from the fact that both detectors can respond to positron annihilation radiation. As far as the β-ray detector is concerned corrections for the effects of