Perturbative Calculation of Spin Observables in Nucleon-Deuteron Elastic Scattering. II. Inclusion of a Tensor Force*

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A previous perturbative calculation of elastic nucleon-deuteron scattering using nucleon-nucleon S-, P-, and D-wave forces is extended to include the rank-1 $^3S_1$-$^3D_1$ tensor force of Yamaguchi and Yamaguchi. We find that both the tensor and the P-wave forces are important to the rank-2 polarizations of the deuteron. Using these forces we achieve excellent agreement with the data for these polarizations at 5.75 and 10.85 MeV. A comparison of calculations with just the tensor force to the exact calculations of Aarons and Sloan demonstrates the reliability of the perturbation technique.

I. INTRODUCTION

Recently an approximate calculation of the spin observables in nucleon-deuteron scattering was reported.[1,2] In the approximation, "exact" values of the half-off-shell nucleon-deuteron T matrices for the Aaron, Amado, and Yam model[3] (which consists of using spin-dependent separable potentials that act in just the nucleon-nucleon S-wave states) were obtained. These T matrices were then used to compute the first-order two-potential perturbation matrix elements (distorted-wave impulse approximation) of separable P- and D-wave N-N interactions. The resulting N-d amplitudes gave, at energies less than 20 MeV, quantitatively accurate nucleon polarizations and qualitative values of the deuteron vector polarization (it_{11}). At higher energies the nucleon polarizations were still in qualitative agreement with the data. The model failed, however, in computing the rank-2 polarizations of the deuteron.

Concurrently Aarons and Sloan[4] published an exact integration of the Faddeev-Lovelace equations[5] for the case of the $^3S_1$-$^3D_1$ separable tensor interaction of Yamaguchi and Yamaguchi.[6] This calculation gave qualitatively reasonable estimates of the rank-2 deuteron polarization. No tensor force had been used in I (although the potentials in I acted in both $^3S_1$ and $^3D_1$ states, there was no mixing of these states) and it was concluded that this was the reason that the rank-2 polarizations were not successfully represented.

In the present paper we extend the formalism[7,8] used in I to include the changes in the deuteron bound state caused by a tensor force. Calculations are made using separable rank-1 potentials in the N-N P waves and the $^3S_1$-$^3D_1$ channel. We find that the P waves make important contributions to the values of t_{20} and t_{21} computed by Aarons and Sloan[4] and the resulting tensor polarizations are in good agreement with the data. The moment t_{22} as computed with just the $^3S_1$-$^3D_1$ interaction had been quite satisfactory and is affected only slightly by the inclusion of the P waves. Calculations without the P waves are compared to the exact result of Aarons and Sloan and demonstrate that at these energies (up to 20 MeV) the perturbation theory is quite reliable.

II. TWO-POTENTIAL FORMALISM

We will describe the approximation to be used for the case of identical spinless particles. The generalization to the spin-$\frac{1}{2}$ case including the tensor force is then straightforward although somewhat tedious. Using the same notation as in Ref. 2, the N-N interaction is written as

$$t = t^{(1)} + t^{(2)} . \tag{1}$$

In I and Refs. 7 and 8 it was required that $t^{(2)}$ not effect the two-body bound state so that $\phi^{(1)}$, the bound state corresponding to $t^{(1)}$, was the same as the final bound state ϕ. We will now allow the final bound state to be different from $\phi^{(1)}$ and write

$$\phi = a\phi^{(1)} + \delta\phi , \tag{2}$$

where

$$a = \langle \phi^{(1)} | \phi \rangle , \tag{3}$$

so that

$$\langle \delta\phi | \phi \rangle = 0 . \tag{4}$$

The Alt, Grassberger, and Sandhas[9] equations for the interactions (1) are

$$T^{(1)}_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\gamma \bar{\delta}_{\beta\gamma} t^{(1)}_\gamma G_0 T^{(1)}_{\gamma\alpha} , \tag{5}$$

$$T_{\beta\alpha} = T^{(1)}_{\beta\alpha} + \sum_\gamma T^{(1)}_{\beta\gamma} G_0 t^{(2)}_\gamma G_0 T_{\gamma\alpha} . \tag{6}$$

Here $T_{\beta\alpha}$ is the three-body transition operation whose matrix elements, $\langle\phi_\beta|T_{\beta\alpha}|\phi_\alpha\rangle$, with respect to the asymptotic channel eigenstates are the physical transition amplitudes. The subscripts α, β label the channels and range from 1 to 3 for elastic and rearrangement scattering. The sign of the free Green's function, G_0, is defined in Eq. (12) and $\bar{\delta}_{\beta\alpha}=1-\delta_{\beta\alpha}$. As in I and Refs. 7 and 8 we assume that $t^{(1)}$ is separable and that Eq. (5) is solved exactly for $T^{(1)}_{\beta\alpha}$. We then use Eq. (6) in first order as

$$T_{\beta\alpha}=T^{(1)}_{\beta\alpha}+\sum_\gamma T^{(1)}_{\beta\gamma}G_0 t^{(2)}_\gamma G_0 T^{(1)}_{\gamma\alpha}, \tag{7}$$

and the physical transition matrix elements are

$$\begin{aligned}\langle\phi_\beta|T_{\beta\alpha}|\phi_\alpha\rangle &= a^2\langle\phi^{(1)}_\beta|T^{(1)}_{\beta\alpha}|\phi^{(1)}_\alpha\rangle + a\langle\phi^{(1)}_\beta|T^{(1)}_{\beta\alpha}|\delta\phi_\alpha\rangle\\ &\quad + a\langle\delta\phi_\beta|T^{(1)}_{\beta\alpha}|\phi^{(1)}_\alpha\rangle\\ &\quad + a^2\sum_\gamma\langle\phi^{(1)}_\beta|T^{(1)}_{\beta\gamma}G_0 t^{(2)}_\gamma G_0 T^{(1)}_{\gamma\alpha}|\phi^{(1)}_\alpha\rangle,\end{aligned} \tag{8}$$

where we have kept only the first-order terms in $t^{(2)}$ and $\delta\phi$.

We will assume that the final two-nucleon amplitude, t, consists of one separable term t_0 that is in the bound-state channel (i.e., once spin is included this will be the ${}^3S_1$-${}^3D_1$ separable potential of Yamaguchi) and a sum of separable terms t_i orthogonal to the bound state (the P-wave potentials). Thus $t^{(2)}$ is

$$t^{(2)}=(t_0-t^{(1)})+\sum_{i=1}^{n}t_i. \tag{9}$$

Since we will assume that each of these separable terms derives from a separable potential [this is not necessary and in general the $h_i(E)$ may be assumed to be arbitrary functions], they have the form

$$\langle\vec{q}'|t_i(E)|\vec{q}\rangle=v_i(\vec{q}')h_i(E)v_i(\vec{q}), \quad i=0,1,\ldots,n, \tag{10}$$

where

$$h_i(E)=\lambda_i\left[1-\lambda_i\int d^3q\,G_0(q,E)|v_i(\vec{q})|^2\right]^{-1}, \tag{11}$$

with

$$G_0(q,E)=(E+i\epsilon-q^2/2\mu)^{-1}. \tag{12}$$

For these potentials the bound states are

$$\phi^{(1)}(\vec{q})=N^{(1)}G_0(q,E_B)v^{(1)}(\vec{q}), \tag{13}$$

$$\phi(\vec{q})=NG_0(q,E_B)v_0(\vec{q}), \tag{14}$$

and

$$\delta\phi(\vec{q})=NG_0(q,E_B)\delta v, \tag{15}$$

where

$$\delta v=v-bv^{(1)}, \tag{16}$$

with

$$b=aN^{(1)}/N. \tag{17}$$

The symmetrized three-body scattering matrix element is

$$\begin{aligned}T(\vec{p}',\vec{p}) &= \tfrac{1}{3}\sum_{\beta,\alpha}T_{\beta\alpha}(\vec{p}'_\beta,\vec{p}_\alpha)\\ &= \tfrac{1}{3}\sum_{\beta,\alpha}\langle\phi_\beta(\vec{p}'_\beta)|T_{\beta\alpha}|\phi_\alpha(\vec{p}_\alpha)\rangle,\end{aligned} \tag{18}$$

and using Eqs. (8)–(15) with Eq. (5) this may be written as (see Refs. 7 and 8)

$$\begin{aligned}T(\vec{p}',\vec{p}) &= a^2T^{(1)}(\vec{p}',\vec{p})+2aN\Big\{N^{(1)}[B^{(1,2)}_\delta(\vec{p}',\vec{p})+B^{(2,1)}_\delta(\vec{p}',\vec{p})]\\ &\quad+\frac{1}{N^{(1)}}\Big[\int d^3p_1\,T^{(1)}(\vec{p}',\vec{p}_1)h^{(1)}\Big(W-\frac{p_1^2}{2n}\Big)B^{(1,2)}_0(\vec{p}_1,\vec{p})\\ &\quad+\int d^3p_1\,B^{(2,1)}_0(\vec{p}',\vec{p}_1)h^{(1)}\Big(W-\frac{p_1^2}{2n}\Big)T^{(1)}(\vec{p}_1,\vec{p})\Big]\Big\}\\ &\quad+4a^2\sum_{i=0}^{n}\int d^3p_2\Big[N^{(1)}B^{(1,2)}_i(\vec{p}',\vec{p}_2)+\frac{1}{N^{(1)}}\int d^3p_1\,T^{(1)}(\vec{p}',\vec{p}_1)h^{(1)}\Big(W-\frac{p_1^2}{2n}\Big)B^{(1,2)}_i(\vec{p}_1,\vec{p}_2)\Big]\\ &\quad\times h_i\Big(W-\frac{p_2^2}{2n}\Big)\Big[N^{(1)}B^{(2,1)}_i(\vec{p}_2,\vec{p})+\frac{1}{N^{(1)}}\int d^3p_1\,B^{(2,1)}_i(\vec{p}_2,\vec{p}_1)h^{(1)}\Big(W-\frac{p_1^2}{2n}\Big)T^{(1)}(\vec{p}_1,\vec{p})\Big]\\ &\quad-\Big(\frac{a}{N^{(1)}}\Big)^2\int d^3p_1\,T^{(1)}(\vec{p}',\vec{p}_1)h^{(1)}\Big(W-\frac{p_1^2}{2n}\Big)T^{(1)}(\vec{p}_1,\vec{p}),\end{aligned} \tag{19}$$

where, for example,

$$B^{(1,2)}_i(\vec{p}',\vec{p})=\langle\vec{p}_1=\vec{p}'\,|\,v^{(1)}(\vec{q}_1)G_0v_i(\vec{q}_2)\,|\,\vec{p}_2=\vec{p}\rangle, \tag{20}$$

and

$$v_\delta\equiv\delta v. \tag{21}$$

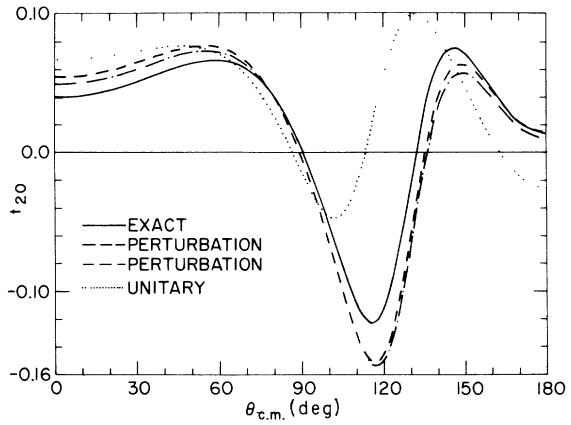

FIG. 1. Tensor polarization t_{20} of the deuteron at a nucleon laboratory energy of 10.85 MeV computed using the Aarons and Sloan potentials (Refs. 4 and 11). The two different curves labeled "perturbation" are explained in the text.

Equation (19) is the generalization of (2.1) of I to the case where the two-body bound state is changed to $t^{(2)}$. In a manner exactly analogous to that used in I it may be re-expressed in terms of the variational wave functions A, generalized to spin-$\frac{1}{2}$ particles and partial wave analyzed. Since the final result is somewhat lengthy we will not give it here.

In the actual calculations a small departure was made from Eq. (19): Namely the contributions due to the P-wave forces (the terms in the sum for $i \neq 0$) were not multiplied by a^2. Thus these contributions were the same as in I. Since for our potentials $a = 0.9636$, the factor a^2 is not very significant in either case.

III. NUMERICAL RESULTS

Calculations were made with the potential Sets

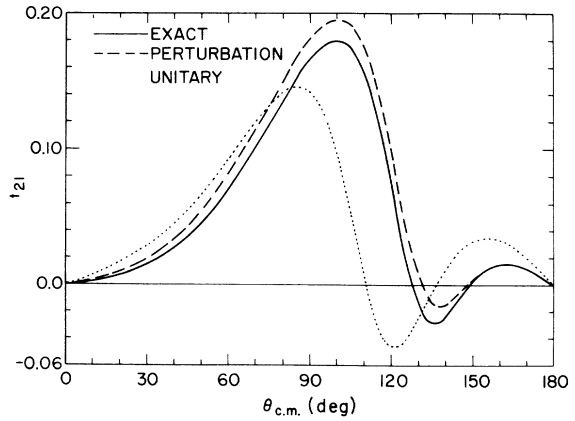

FIG. 2. Tensor polarization $t_{21}(=-T_{21})$ at 10.85 MeV for the Aarons and Sloan potentials.

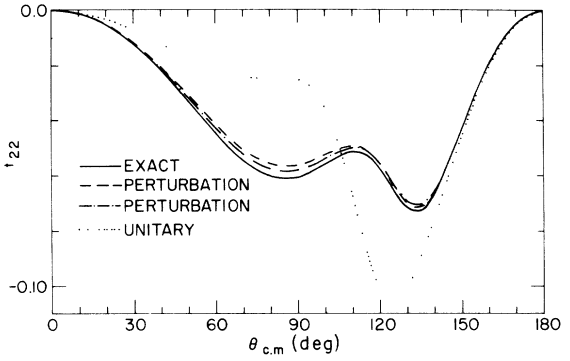

FIG. 3. Tensor polarization t_{22} at 10.85 MeV for the Aarons and Sloan potentials.

K, B, and C defined in PK[10] and with the potentials of Aarons and Sloan.[4,11] Table I contains a brief description of these potentials. The exact calculations that form the input to Eq. (19) were in general made with potential Set S (see I); however, when the final result was to be for the Aarons and Sloan potentials, the $^3S_1$ of Set S and a $^1S_0$ potential having $a = 20.4$ F and $r_e = 2.7$ F were used in the exact calculation.

As was the case in I, the $T^J_{l',s';l,s}$ were computed by Eq. (19) only for both l' and $l \leq 2$. The higher partial waves (up to $J = \frac{19}{2}$) were then computed using a unitary approximation due to Sloan.[10,12] In I a calculation with the cutoff at $l, l' \leq 4$ showed that the unitary amplitudes were accurate for $l > 2$.[12]

A. Comparison with an Exact Calculation

Figures 1–3 show the comparison of the present perturbation results with the exact calculations of Aarons and Sloan[4,13] for the rank-2 polarizations[14] of the deuteron. Also shown are the unitary model[10,12] values of these polarizations. It is clear that the perturbation technique provides an excellent approximation of the exact rank-2 polarizations. The two perturbation calculations shown in Figs. 1–3 were made using different $^3S_1$ potentials for the exact part of the calculation (in both cases the same final $^3S_1$-$^3D_1$ was used). For the dashed line the potential labeled 3*SC* in PK was used (this is fitted to the deuteron binding energy

TABLE I. Qualitative features of the potential sets used in the present paper.

Designation	Channels in which the potentials act	
K	$^1S_0$, $^3S_1$-$^3D_1$	
Aarons and Sloan	$^1S_0$, $^3S_1$-$^3D_1$	
B	$^1S_0$, $^3S_1$-$^3D_1$,	$^1P_0$, $^3P_0$, $^3P_1$, $^3P_2$
C	$^1S_0$, $^3S_1$,	$^1P_0$, $^3P_0$, $^3P_1$, $^3P_2$
S	$^1S_0$, $^3S_1$	

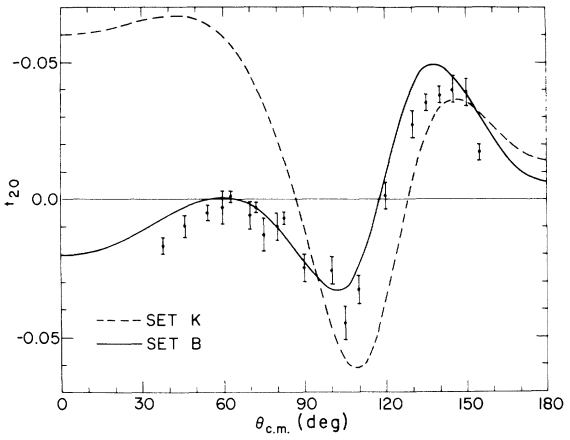

FIG. 4. Perturbation calculations of t_{20} at 5.5 MeV. The data are from White *et al.* (Ref. 16, 5.75 MeV, *p-d*).

and the ^3S scattering length), while for the dash-dot line a potential having the same S-wave inverse range (1.2390 F^{-1}) as the final $^3S_1$-$^3D_1$ potential and the proper binding energy was used. For this latter potential the S-wave part of δv is zero. The near equality of these two curves (for t_{21} in Fig. 2 they are the same curve) is an indication of the reliability of the perturbation technique. The Set K curves in Figs. 4–6 (5.5 MeV) and calculations at 22.7 MeV are also in very good agreement with Aarons and Sloan's results.

We have also been able to compare our partial wave amplitudes with those of Aarons and Sloan.[13] The errors in the diagonal amplitudes were typically 1–10% of their values while the off-diagonal amplitudes have errors of from 7–100%. If the errors in the elements are compared to the corresponding perturbation corrections one also finds large (10–100%) errors. In view of these substantial errors in the partial wave amplitudes, the accurate polarizations of Figs. 1–3 seem rather surprising and we do not have an explanation for the apparent insensitivity of the t_{2m} to these errors. It may be noteworthy that the $^4S_{3/2}$-$^4D_{3/2}$ mixing which is presumably important to the t_{2m}[15] (and in this case is considerably larger than when computed with just P-wave forces) has only a 10% error. A comparison of the partial wave amplitudes for the dash and dash-dot curves in Figs. 1–3 also shows differences comparable to those given above.

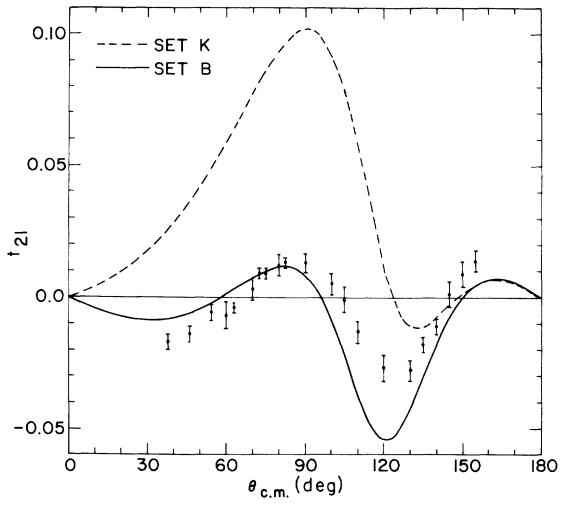

FIG. 5. Perturbation calculations of t_{21} at 5.5 MeV. The data are from White *et al.* (Ref. 16, 5.75 MeV, *p-d*).

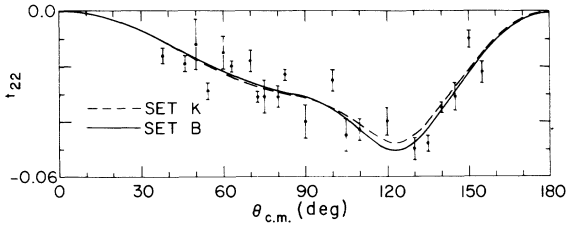

FIG. 6. Perturbation calculations of t_{22} at 5.5 MeV. The data are from White *et al.* (Ref. 16, 5.75 MeV, *p-d*).

The nucleon and deuteron vector polarizations computed by the perturbation technique disagree with the exact values[4, 11] by amounts (0.04) comparable with the errors in t_{2m}; however, since these quantities have unrealistically small values, these errors correspond to very large relative errors.

B. Calculation with Both *P*-Wave and Tensor Forces

In Figs. 4–8 [Refs. 16 and 17] we include P-wave forces in the calculation. We see that by using both the tensor force and the P-waves (Set B), excellent values of the t_{2m} can be predicted. The differences between the Set K curves (no P waves) and the Set B curves in Figs. 4, 5, and 7 show that the P-wave forces are as important as the tensor force in computing t_{20} and t_{21}, while in Figs. 6 and 8 we see that the P-wave forces make only a small correction to the already quite acceptable values for t_{22} computed with just the tensor force. The curves for Set C (P waves, no tensor force) in Figs. 7 and 8 show that the P waves by themselves are not adequate for any of the t_{2m}.[2]

We have also computed the elastic differential cross sections, nucleon polarizations, and vector polarizations of the deuteron using Set B. These are changed only slightly from the Set C (P-wave forces only) curves in I and hence will not be presented here. On the other hand the values of the

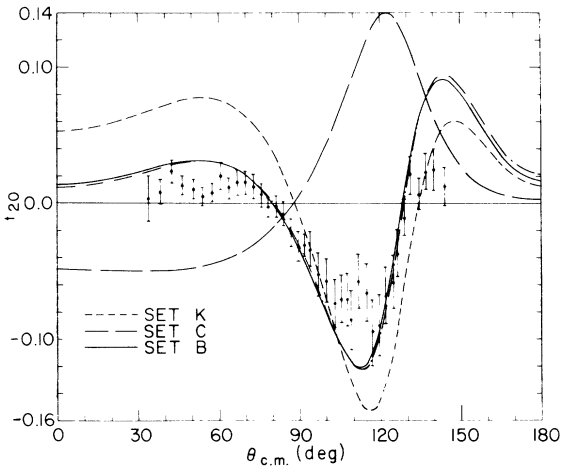

FIG. 7. Perturbation calculations of t_{20} at 10.85 MeV. The data are from Arvieux *et al.* (Ref. 17, 10.85 MeV, p-d). The dash-dot line is the same as the dash-dot line in Fig. 1 (see text) except potential Set B was used.

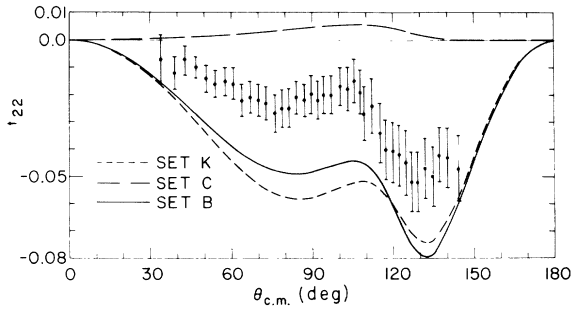

FIG. 8. Perturbation calculations of t_{22} at 10.85 MeV. The data are from Arvieux *et al.* (Ref. 17, 10.85 MeV, p-d).

Wolfenstein[18] parameters computed in I are strongly effected by the inclusion of the tensor force.

In PK we described some difficulties of using a rank-1 separable interaction for the $^3S_1$-$^3D_1$ channel and Figs. 3 and 9 of PK show that this potential is a very poor representation of the N-N data. For this reason we have not made an extensive set of calculations with potential Set B, nor have we studied the effects of varying the percent D state of the deuteron. Such a study does not seem warranted until one has a more realistic potential. We are at present trying to construct suitable rank-2 separable potentials for this purpose.

There have been a number of phase shift analyses of the low-energy N-d scattering.[15,19] For energies above the breakup threshold, the large number of phase parameters in N-d scattering have required that simplifying assumptions concerning the splitting or mixing of these phases or concerning the inelastic parts of the amplitudes be made. Sloan[20] has already commented on the inadequacy of the resulting inelastic parameters. We have not yet carried out a phase shift analysis of our amplitudes but some observations can be made from the amplitudes themselves (an example of the amplitudes for potential Set C is given in I). The doublet amplitudes all have small splitting and in the even angular momentum states the double-quartet mixing is generally small. The most important noncentral effects seem to be the ^2P_J-^4P_J and $^4S_{3/2}$-$^4D_{3/2}$ mixing amplitudes which are comparable in magnitude to the diagonal amplitudes (as was suggested in I, the latter mixing is important only when the tensor force is included). Successive amplitudes in the diagonal ^4P_J and ^4D_J series are split by 5–15% of their values. We conclude that although the doublet splitting may be neglected and some of the mixing parameters set equal to zero, neither l nor S is even approximately conserved in N-d scattering and a large number of complex phases and mixing parameters will have to be varied in a realistic phase shift analysis. Figure 1 of I shows that in such an analysis the amplitudes for $l>2$ (and perhaps $l=2$) could be reliably computed using the Sloan approximation.

Note added in proof: After this article was submitted, an article[21] by Doleschall was received in which the Faddeev-Lovelace equations are solved using rank-1 potentials for both the $^3S_1$-$^3D_1$ and P-wave channels. Although an exact comparison is not possible, since different potentials were used, the present approximate results appear to be in good agreement with Doleschall's exact calculation. Recently, Schmelzbach *et al.*,[22] have completed a very thorough phase shift analysis that answers the objections raised in Sec. III B against previous analyses.

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Test of Single-Channel Strength-Function Limit*

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Using the exact solution of a particle decaying from a square well, it is shown that the strength function, (Γ/D), is unbounded and has the form $(2\pi\Gamma/D) = -\ln(1-T)$ suggested by Moldauer (for average parameters), where T is the transmission.

I. INTRODUCTION

In the use of, especially the extrapolation of, average widths Γ_c and spacings D_c for a channel c, it is of critical interest whether the channel-strength function Γ_c/D_c is bounded. In the past it seemed evident that Γ_c/D_c was in fact bounded for one expected[1] that $2\pi\Gamma_c/D_c = T_c$, where T_c is the transmission and thus limited to a maximum of 1, so that $\Gamma_c/D_c \leq 1/2\pi$. A slightly different formula[2] has also been suggested leading to the bound $1/\pi$. Moldauer[3] has emphasized that the foregoing formulas are valid only in the limit of very small values of T_c, and has proposed rather that

$$T_c = 1 - e^{-2\pi\Gamma_c/D_c} . \quad (1)$$

In Eq. (1) the limit $T_c = 1$ does *not* imply any bound on the channel-strength function Γ_c/D_c. Moldauer demonstrates the validity of (1) for a number of simple analytic unitary models of the S matrix.[3] Ullah and Moldauer[4] have offered a proof of (1) assuming the plausible simple-pole expansion (Mittag-Leffler) of the statistical collision matrix. However, there remain some (hopefully minor) questions about the validity of the proofs,[4] and, more important, the generality of the assumed collision matrix has not been established by these authors.[5]

Accordingly, it seems useful to subject (1) to a test by means of a simple precise calculation. For such a trial we choose the spherically symmetric