Models for π^+-^{16}O Scattering at 270 MeV*

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Predictions of the Kisslinger and "local" optical models are compared to π^+-^{16}O elastic differential cross sections at 270-MeV lab kinetic energy. With no adjusted parameters, the local potential gives a remarkably good fit to the data.

Rohlin *et al.*[1] have recently measured π^+-^{16}O elastic scattering at a laboratory kinetic energy of 270 MeV. The oxygen ground-state wave function is simpler than that of carbon. Thus it offers a better test of various models of π-nucleus interactions in the N^* region than does the π^--^{12}C data reported earlier.[2] We present here π^+-^{16}O calculations based on the Kisslinger and "local" optical models, and discuss briefly the implications of the remarkable agreement obtained with the local model.

The Kisslinger model[3] assumes an s plus p wave π-N amplitude

$$\langle \vec{q}'|t|\vec{q}\rangle = a_0 + a_1 \vec{q}\cdot\vec{q}' . \tag{1}$$

This leads with suitable approximations to[4]

$$V_K(r) = -A[b_0 p_0^2 \rho(r) + b_1 \vec{p}\cdot\rho(r)\vec{p}]/2E. \tag{2}$$

Here $\vec{p} = -i\vec{\nabla}$; p_0 and $E \equiv T + m_\pi$ are the pion lab momentum and energy, respectively; $\rho(r)$ is the nuclear density. With free π-N amplitudes and electron scattering densities, Eq. (2) predicts the qualitative features of π-nucleus scattering at low energies ($T \lesssim 90$ MeV) for various nuclei[4] and also for ^{12}C from 120 to 280 MeV, the resonance region.[5]

With a different off-shell extrapolation,

$$\begin{aligned}\langle \vec{q}'|t|\vec{q}\rangle &= a_0 + \tfrac{1}{2} a_1 (q^2 + q'^2) - \tfrac{1}{2} a_1 (\vec{q} - \vec{q}')^2 \\ &\approx a_0' - \tfrac{1}{2} a_1 (\vec{q} - \vec{q}')^2 ,\end{aligned} \tag{3}$$

one obtains a local potential

$$V_L(r) = -A[(b_0 + b_1) p_0^2 \rho + \tfrac{1}{2} b_1 \nabla^2 \rho]/2E . \tag{4}$$

At low energies, i.e., $T \approx 80$ MeV, V_L gives somewhat poorer predictions[6] of the ^{12}C and ^{16}O data than does the Kisslinger model, V_K. The small-angle differential cross sections given by V_K are close to the experimental values, while those predicted by V_L are roughly 50% greater.

Lee and McManus[7] have recently applied WKB local potential calculations to the π^--^{12}C resonance region elastic and inelastic data. Solving the Klein-Gordon equation exactly, we have found that the difference between the small-angle π^--^{12}C cross sections of the two models diminishes as T increases, and is small for $T \gtrsim 200$ MeV. At angles beyond the first diffraction minimum, the predictions of V_L are about twice those of V_K, while the experimental points generally fall *between* the two curves.

In Fig. 1 we present results for π^+-^{16}O obtained with the two optical potentials, Eqs. (2) and (4). Coulomb forces are included in all cases. We assumed a density[8]

$$\rho(r) = \rho_0[1 + (Z-2)r^2/3a^2]\exp(-r^2/a^2) , \tag{5}$$

with $a = 1.7$ F; this is the electron scattering value corrected for the finite proton-charge radius. A Coulomb potential corresponding to (5) was also included.

The scattering parameters, b_0 and b_1, were inferred from the known free 270-MeV π-N amplitudes. Neglecting the mixing of partial waves when transforming to the laboratory frame[4,9] and correcting for Fermi motion, one obtains[10]

$$b_0 = -0.30 + 0.46i, \quad b_1 = -2.23 + 2.89i . \tag{6}$$

Using these parameters, both models predict the general features quite well. But it is remarkable how well the local model fits the data. For this curve, the mean square error, χ^2/N, is about 1.2 or just over one standard deviation per point.

Dedonder,[11] Fäldt,[12] and Wilkin[13] suggest a more exact way of transforming from the center of mass to the laboratory frame. Assuming again recoiless nucleons, they use the relation

$$f_{\text{lab}}(q^2) = \frac{k_{\text{lab}}}{k_{\text{cm}}}(1 + \cdots) f_{\text{cm}}(q^2) . \tag{7}$$

Here q is the 4-momentum transfer and k is the momentum of the incident pion. Using (7), one obtains

$$b_0' = 1.68 - 2.11i, \quad b_1' = -4.21 + 5.46i. \qquad (8)$$

Curves corresponding to these scattering parameters, Eq. (8), for both the local and Kisslinger model, are also shown in Fig. 1. There again the local model gives a good fit to the data with $\chi^2/N = 1.8$. The Kisslinger model with b_0', b_1' yields a poor fit with $\chi^2/N = 14$ and is much below the corresponding curve using b_0, b_1. For comparison, a Glauber model calculation[14] with similar inputs is also given. (Note that for all curves there are *no* adjusted or fitted parameters.)

It is surprising that the parameters obtained from a more careful transformation, Eq. (8), yield a somewhat poorer fit when used in the optical model. Perhaps it will turn out that correlation and local-field corrections[15] to the optical potential will lead to a set of parameters closer to Eq. (6). The fact that both curves for the local potential turn out to be very similar for the two sets of scattering parameters is consistent with Wilkin's observation that the local model is less sensitive to the choice of parameters than the Kisslinger model.[13]

A tentative conclusion one might draw is that the local model provides a good description of π-nucleus scattering above the resonance ($T \gtrsim 200$ MeV). This can only be tested when there are data at more energies and for other nuclei. We look forward to obtaining such data from the meson factories scheduled to soon begin operations.

If its validity is confirmed by further experiments, the local potential can be used to study differences in the distributions of neutrons and protons,[16] and in the analysis of complex reaction processes. The differences between the π^--^{12}C data and the V_L predictions are, in this view, a possible source of information about the deformed ^{12}C nucleus.

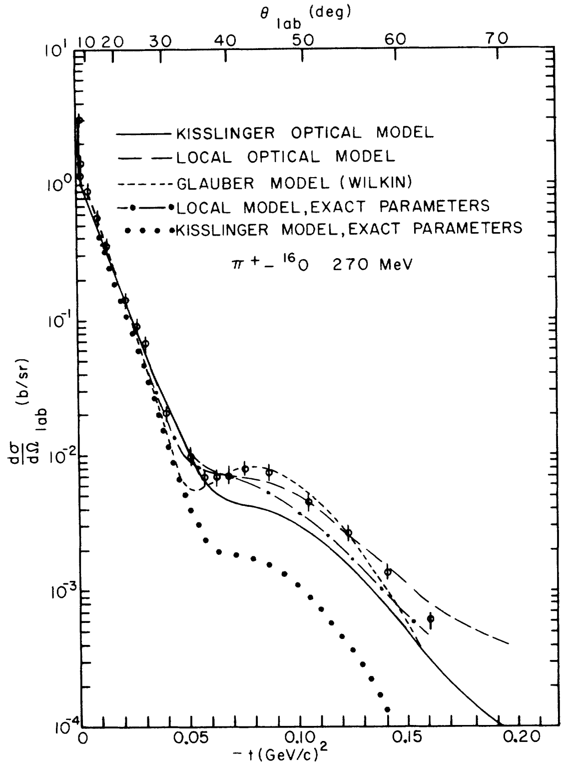

FIG. 1. Kisslinger optical model (—), local optical model (— —), and Glauber model (----) (Wilkin, Ref. 9) predictions. Two sets of averaged free π-N parameters and the electron scattering density are used, with Coulomb effects included. Data are from Ref. 1.

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