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Strong and Weak 0^+ Excitations in the ^{142}Ce$(p, t)^{140}$Ce and ^{140}Ce$(p, t)^{138}$Ce Reactions*

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The 0^+ level structure of ^{140}Ce and ^{138}Ce as revealed by the ^{142}Ce(p,t) and ^{140}Ce(p,t) reactions is reported. Comparisons are made to a model which mixes the anharmonic pair vibration with "two multipole-phonon" 0^+ excitations and "one-phonon" 0^+ proton configurations. The 5.57-MeV 0^+ level in ^{140}Ce may be a pair analog to the weak 0^+ level in ^{138}Ce at 2.32 MeV.

I. INTRODUCTION

During the past few years much theoretical and experimental work has been directed at the pair-vibrational concept. In the region of closed neutron shells for $A \geq 90$, (p, t) experiments[1] have revealed frequent deviations from the uncoupled-harmonic (uh) pair-vibrational (pv) model.[2-4] These deviations are apparent in the pv excitation energy and its transition strength relative to the neighboring $N-2$ ground-state transition which it would equal in the uh model. However, recent theoretical calculations in the Zr ($N=50$) region[5] have shown that these deviations can, in large part, be understood by coupling the anharmonic pv to other nuclear states characterized by $J^\pi = 0^+$. These states included quadrupole pair vibrations,[3] two-multipole excitations such as $(2^+ \otimes 2^+)_{0^+}$, and admixtures of a 0^+ proton two-quasiparticle configuration with the neutron pv. The purpose of this work is to test the coupled-anharmonic (ca) pv description at $N=82$ by checking its consistency with the 0^+ structure observed in the ^{142,140}Ce(p,t) experiments.

II. EXPERIMENT

The reactions used a 30.3-MeV proton beam prepared by the Berkeley 88-in. cyclotron high-resolution beam line.[6] Self-supporting metallic foils of ^{142}Ce (0.400 mg/cm^2) and ^{140}Ce (0.790 mg/cm^2) were prepared. The ^{142}Ce target had 10% ^{140}Ce impurity; uncertainties in the target thicknesses are ±15%. Reaction particles were detected in Si detectors and identified in a Goulding-Landis ΔE-E particle identifying system.[7] Resulting ^{140}Ce and ^{138}Ce spectra are shown in Figs. 1(a) and 1(b). Energy resolutions (full width at half maximum) of 30 keV in ^{140}Ce and 55 keV in ^{138}Ce were obtained. It was possible to identify 46 discrete peaks in ^{140}Ce and 22 peaks in ^{138}Ce. Differential cross sections were extracted for most of these, but the remainder of this discussion will be concerned only with those states characterized by $J^\pi = 0^+$.

The 0^+ assignments are made on the basis of the familiar $L=0$ (p,t) angular distributions. These angular distributions are given in Figs 2 and 3 for ^{140}Ce and ^{138}Ce, respectively. Five such states are found in ^{140}Ce at $E_x < 6.36$ MeV. The ground-state and 3.23-MeV levels are most strongly populated. The latter state has been interpreted[8] as the anticipated pv state. The 1.90- and 3.03-MeV states are weak, but the differential cross sections are reliably extracted. Extraction of the 5.57-MeV differential cross section in ^{140}Ce was hampered by background and a ^{138}Ce contaminant peak. The shape does suggest a 0^+ assignment for this state. Two 0^+ states in ^{138}Ce have been found at $E_x < 3.62$ MeV. The ground state is strongly populated while the 2.32-MeV 0^+ level is comparatively weak. The solid lines in Figs. 2 and 3 are two-nucleon-transfer distorted-wave Born approximation (DWBA) calculations[9] which use structure amplitudes derived from wave functions that are to be described. Proton[10] and triton[11] optical-model parameters are taken from the literature. The DWBA calculations are normalized to the 35° point (lab angle) of the ^{142}Ce$(p,t)^{140}$Ce(3.23 MeV) transition. Generally, the DWBA fits are good.

III. THEORY

A coupling model is now described which attempts to understand the deviations of harmonic theory from experimental results. The calculations are quite similar to those performed in the Zr case,[5] and that work is summarized in relation to the Ce experiments. The collective boson wave functions used to describe the ground states

(0_+ or 0_- in the notation) of neighboring nuclei and the pv (0_-0_+) are calculated by a boson-expansion method[12, 13] using the Tamm-Dancoff basis. The neutron Hamiltonian is of the form

$$H_{\mathrm{sp}} + H_p = \sum_j \epsilon_j N_j - \sum_{jj'} G_{jj'} P_j^\dagger P_{j'}$$

with

$$P_j^\dagger = \tfrac{1}{2} \sum_m a_{j\bar{m}}^\dagger a_{jm}^\dagger$$

and

$$N_j = \sum_m a_{jm}^\dagger a_{jm} \,.$$

The $a_{jm}^\dagger$ and a_{jm} are the shell-model creation and annihilation operators. The residual pairing interaction is often represented by a constant matrix element $G_{jj'} = G$ that is inversely proportional to the nuclear mass. In order to extract pairing matrix elements that reproduce the experimental mass differences, which are corrected for effects other than pairing,[14] different pairing-force matrix elements are introduced for configurations below and above the closed shell.[5, 14] Three parameters were adjusted, G_{11} for pairs in configurations above $N = 82$, G_{22} for pairs below $N = 82$, and finally G_{12} for interactions between a pair below and a pair above $N = 82$. Anharmonicities then correspond to all terms of the pairing force which are not taken into account in the Tamm-Dancoff approximation. Neutron pairing strengths $G_{11} = 0.127$, $G_{22} = 0.197$, and $G_{12} = 0.155$ MeV are found to fit the ground-state mass differences of all stable Ce isotopes within experimental uncertainties. The method also yields predictions for noncollec-

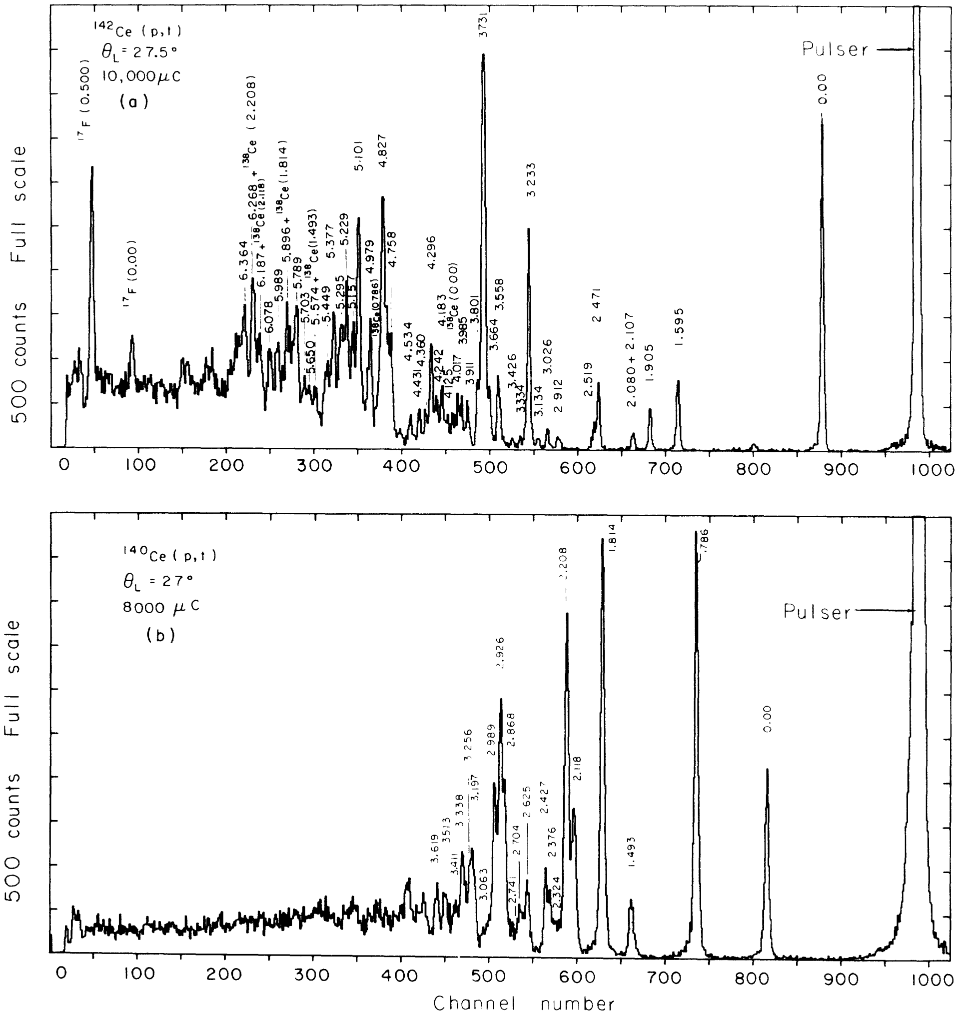

FIG. 1. (a) Triton spectrum from the ^{142}Ce(p,t) reaction taken at $E_p = 30.3$ MeV. (b) Triton spectrum from the ^{140}Ce(p,t) reaction taken at $E_p = 30.3$ MeV.

tive 0^+ states. These latter states are calculated in harmonic approximation only,[12] and these are represented in the notation as $(0'_{\pm})$.

The residual quadrupole interaction may have a dramatic effect on the pv energy in the ca calculation.[3] Spin-parity 2^+ assignments in ^{138}Ce at 0.796 and 1.510 MeV can be made from (p, t) angular distributions[8] and from a study of ^{138}Pr decay.[15] The ^{138}Pr decay study also revealed a 0^+ state at 1.477 MeV which if weakly populated in the (p, t) reaction would not have been resolved from the nearby 2^+ level in the present experiment. The two-neutron hole 2^+ configuration corresponding to the 0.79-MeV level (2_- in the notation) in ^{138}Ce is expected to combine in the uh approximation with the two-neutron particle configuration corresponding to the 0.64-MeV 2^+ state (2_+) in ^{142}Ce. The result would be a $J=0$ to 4 multiplet (2_-2_+) at $E_x(\text{uh}) = 5.33$ MeV in ^{140}Ce. In the ca theory a strong splitting between the (0_-0_+) and $(2_-2_+)_{0^+}$ states[16, 17] accounts for the energy depression of the uh pv.

The next step in the calculation involves the two multipole-phonon surface vibrations. One-phonon multipole vibrations, a 2^+ (1.59 MeV) and 3^- (2.46 MeV), have been seen in the ^{140}Ce(α, α')[18] and ^{140}Ce(p, p')[19] experiments. Thus one could expect 0^+ states in ^{140}Ce at 3.18 MeV with $(2^+ \otimes 2^+)_{0^+}$ configuration and at 4.92 MeV with $(3^- \otimes 3^-)_{0^+}$ configuration if the harmonic phonon model were accurate. A random-phase-approximation (RPA) calculation analogous to Ref. 5 is carried out in order to derive wave functions for the two-multipole phonon states. These wave functions do not have the coherence properties which could provide (p, t) transition strength of the observed magnitude, e.g., for the 3.03-MeV level in ^{140}Ce.

As a plausible explanation for the experimentally observed distribution of $L=0$ strength the admixture of the two-multipole surface vibrations mentioned above and the pv is considered. The mixing

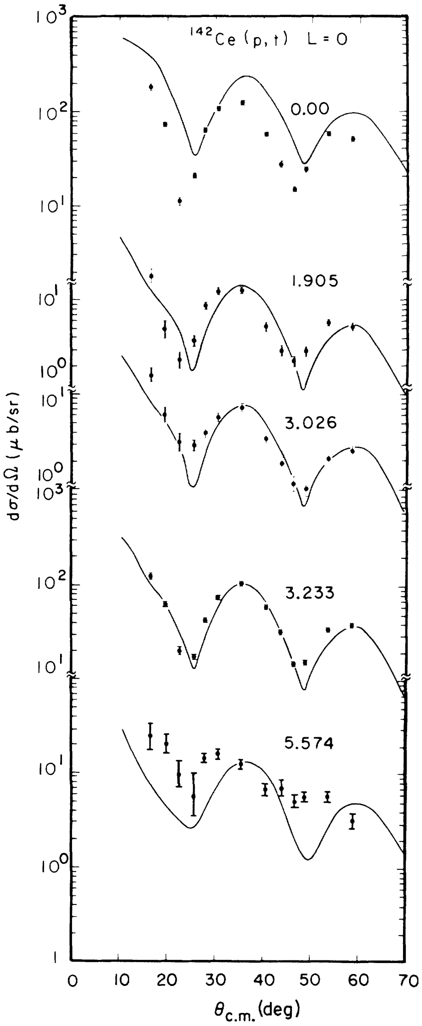

FIG. 2. Differential cross sections for ^{142}Ce(p,t) $L=0$ transitions and DWBA calculations using the coupled-anharmonic theory. A single normalization factor is used.

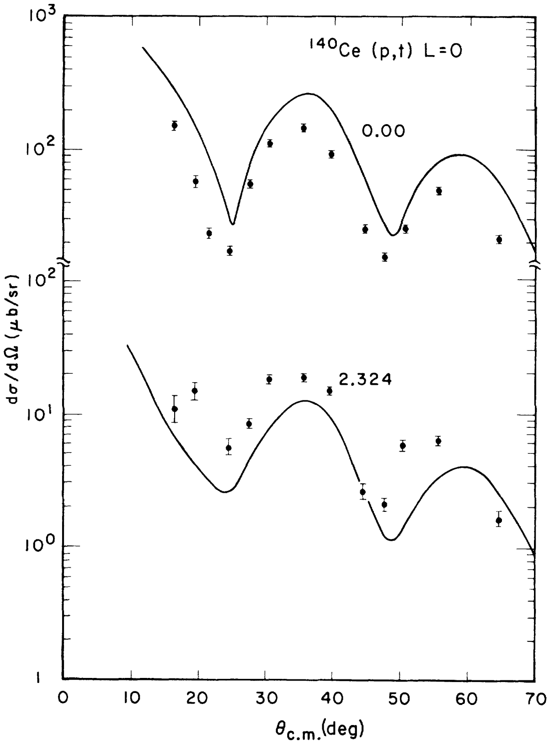

FIG. 3. Differential cross sections for ^{140}Ce(p,t) $L=0$ transitions and DWBA calculations using the coupled-anharmonic theory. A single normalization factor is used.

amplitudes are calculated[5] by diagonalizing the Hamiltonian

$$H = H_{sp} + H_p + H_2 + H_3$$

in an orthogonal basis constructed from the pv (0_-0_+), the two-multipole phonon states $(2^+)^2_{0^+}$ and $(3^-)^2_{0^+}$, the state $(2_-2_+)_{0^+}$, and four proton states described in the following paragraph. The single-particle part, H_{sp}, is derived from one-nucleon-transfer experiments.[20] The pairing force is H_p of which the neutron part was described above, and H_λ ($\lambda = 2, 3$) are the separable multipole forces.

Another 0^+ state in ^{140}Ce which is sharing two-nucleon-transfer strength is the 1.90-MeV state. This level may be analogous to the 1.76-MeV 0^+ state in ^{90}Zr, i.e., it appears to be a two-quasiparticle proton configuration orthogonal to the ^{140}Ce ground state. The 1.90-MeV state was observed with a weak spectroscopic factor in the ^{141}Pr(d, ^{3}He)^{140}Ce experiment and was not seen in the ^{139}La(^{3}He, d)^{140}Ce reaction.[21] Four proton states of two quasiparticle nature appear below 6 MeV in an RPA diagonalization[22] of a proton pairing force of strength $G_p = 0.201$ MeV (the same as was used to define the superfluid proton basis for the RPA diagonalization of the 2^+ and 3^- states). They couple weakly to the $(2^+)^2_{0^+}$ and $(3^-)^2_{0^+}$ states and only in second order with the pv. The proton quasiparticle states are all included in the admixture calculation outlined above. Reference 5 indicated that to be successful in describing the experimental transition strength to states of proton nature, one may have to add proton-neutron pairing forces. The proton-neutron parts of the multipole forces by themselves do in the present case explain the distribution of (p, t) strength among low-lying ^{140}Ce states. However, the energy of the proton "one-phonon" state is predicted too high (possibly due to inadequacies in the proton sp energies), so the (p, t) strength acquired by coupling may be overestimated.

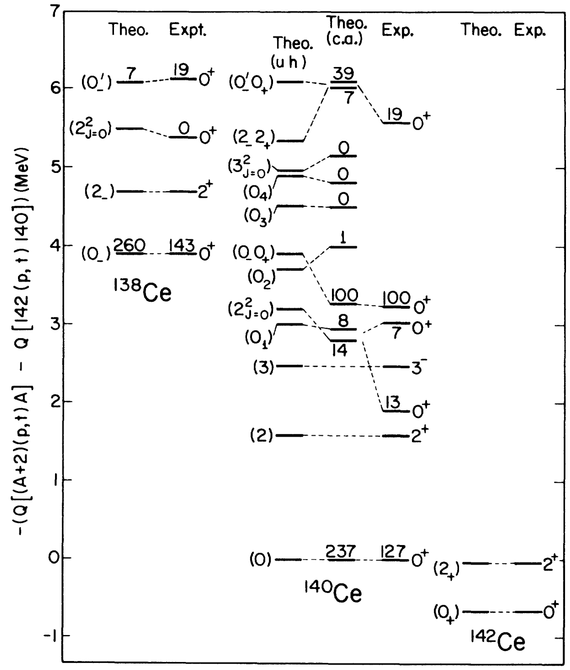

FIG. 4. Summary of calculated and experimental energies and (p,t) cross sections compared at the peak near 35° for the selected states considered.

The 5.57-MeV level in ^{140}Ce has about the same additional energy compared to the pv as the 2.32-MeV level in ^{138}Ce has above its ground state, and the (p, t) transition strengths are equal. It therefore appears natural to associate the 5.57-MeV level with the configuration $(0'_-0_+)$ corresponding to the noncollective state $(0'_-)$ at 2.32 MeV in ^{138}Ce. The $(0'_-0_+)$ and $(2_-2_+)_{0^+}$ states are predicted in the ca theory to be very close in energy, and they therefore mix although their coupling matrix element is small. Their calculated, summed (p, t) strength exceeds the 0^+ strength actually observed in this excitation region. However, unresolved moderately weak $L = 0$ (p, t) strength between 6- and 7-MeV excitation energy could be present.

Experimental and theoretical results are summarized in Fig. 4. The experimental (p, t) cross sections are normalized to the 3.23-MeV state which is associated with the pv in ^{140}Ce and the resulting relative strengths are given in large figures. This method of normalization is chosen in order to facilitate the comparison with the coupling calculation results which are normalized to the theoretical pv. The calculated ground-state cross sections are then overestimated. Using a different set of optical-model parameters still based on the average parameters of Refs. 10 and 11, the $A = 140$ and $A = 138$ ground-state cross sections reduced to 149 and 163 with a poorer fit to the angular distributions. In addition to optical-model-parameter ambiguities, other reasons for the overestimated ground-state cross sections could be slightly incorrect ratios of particle and hole (p, t) amplitudes or slightly incorrect amounts of pairing anharmonicities. Furthermore, the possible inadequacy of the Woods-Saxon form factors with fixed binding energy used in the DWBA calculations[9] should be kept in mind. A separate calculation using harmonic-oscillator form factors asymtotically matched to Hankel functions was performed.[23] In this case the ratio of the ^{140}Ce(g.s.) cross section to the ^{140}Ce(3.23 MeV) cross section increases by 20% relative to the Woods-

TABLE I. The table presents wave functions for the 0^+ states in ^{140}Ce derived from the coupled-anharmonic (ca) theory. The basis employed is orthogonal so the indicated components intend to give the main structure of each basis state.

E_x (MeV)	(0_-0_+)	$(2^2{}_{j=0})$	$(3^2{}_{j=0})$	$(2_-2_+)_{j=0}$	(0_1)	(0_2)	(0_3)	(0_4)	$(0'_-0_+)$
2.82	−0.2898	−0.8198	−0.0135	−0.1115	−0.1254	0.4500	−0.0028	−0.1141	0.0000
2.95	0.2254	0.0383	0.2037	0.0914	−0.9469	−0.0248	−0.0169	−0.0174	0.0000
3.27	0.7858	−0.2267	0.1703	0.4136	0.2493	0.2538	−0.0195	−0.0642	0.0002
3.99	−0.0613	0.4804	−0.0737	−0.0535	−0.0426	0.8510	0.0254	0.1729	0.0000
4.48	−0.0433	0.0512	0.2408	−0.0998	0.0519	0.0324	−0.9564	−0.0948	−0.0001
4.80	−0.0449	0.1696	0.5391	−0.2498	0.0950	0.0767	0.2533	−0.7329	−0.0002
5.17	−0.0335	0.1010	−0.6677	0.3300	−0.1024	0.0336	−0.1370	−0.6353	0.0004
6.02	0.0076	−0.0008	−0.0059	−0.0117	−0.0006	−0.0002	−0.0005	−0.0009	−0.9999
6.08	0.4888	−0.0520	−0.3599	−0.7895	−0.0375	−0.0108	−0.0302	−0.0540	0.0152

Saxon calculation. Similarly, the ratio of ^{138}Ce(g.s.) cross section to ^{140}Ce(3.23 MeV) cross section increases by 14%. Optical-model parameters were held constant in this analysis.

To the right of experimental levels in Fig. 4 known J^π values are given. To the left of the calculated states are the J values of the quanta of hole pairs (minus suffix), particle pairs (plus suffix), or "one phonons" on which numerical suffices indicate a proton two-quasiparticle state. As an energy scale the (p, t) Q value relative to that of the ^{142}Ce$(p, t)^{140}$Ce(g.s.) is used in order to emphasize the significant energy shift of the Q value for the (0_-0_+) pair vibration state away from ^{138}Ce(g.s.) = (0_-) and of the ^{142}Ce(g.s.) = (0_+) away from ^{140}Ce(g.s.) = (0). Both Q values are reproduced by theory. For ^{140}Ce both the uh and ca pictures are given. In the ca case the dashed line connecting a given state with a uh state intends to show the dominant configuration in the ca wave function. Table I gives the theoretical wave functions in the orthogonal basis[5] derived from the ca calculation.

IV. CONCLUSIONS

Summarizing, the (p, t) reactions on the ^{142}Ce and ^{140}Ce isotopes has revealed, as in most previous cases for $A \geq 90$,[1] a $J^\pi = 0^+$ level structure that deviates significantly from harmonic pv theory. However, the described model calculation seems able to explain the redistribution of (p, t) strength from the harmonic pv to a number of low-lying states, to give a plausible reason for the large energy shift of the pv, and for the occurrence of two 0^+ levels in ^{140}Ce(p, t) below the pv. The 5.57-MeV state is likely to represent the first finding of a pair analog of a noncollective state that was predicted in Ref. 12.

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Excitation Function for the $^{235}U(\alpha, 3n)^{236}Pu$ Reaction*

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Cross sections have been measured for the $^{235}U(\alpha,3n)^{236}Pu$, $^{235}U(\alpha,t)^{236}Np$, and $^{235}U(\alpha,f)$ reactions at several α-particle bombarding energies between 24.5 and 28.0 MeV.

I. INTRODUCTION

The determination of threshold energies for the production of fission isomers depends on the theoretical model employed.[1-3] A test of the statistical model used to analyze fission isomer excitation functions can be obtained by attempting to fit the excitation function for an evaporation reaction leading to an actinide nucleus in its ground state where the threshold is known. Previous results that are available on the population of actinide nuclei by (charged particle, xn) reactions are not of sufficient accuracy or detail to seriously test current statistical models.

In this paper we report the results of measurements of the excitation function for the $^{235}U(\alpha, 3n)$-^{236}Pu reaction. These results will be used in a refined analysis of fission isomer excitation functions which will be reported in a subsequent publication.[4]

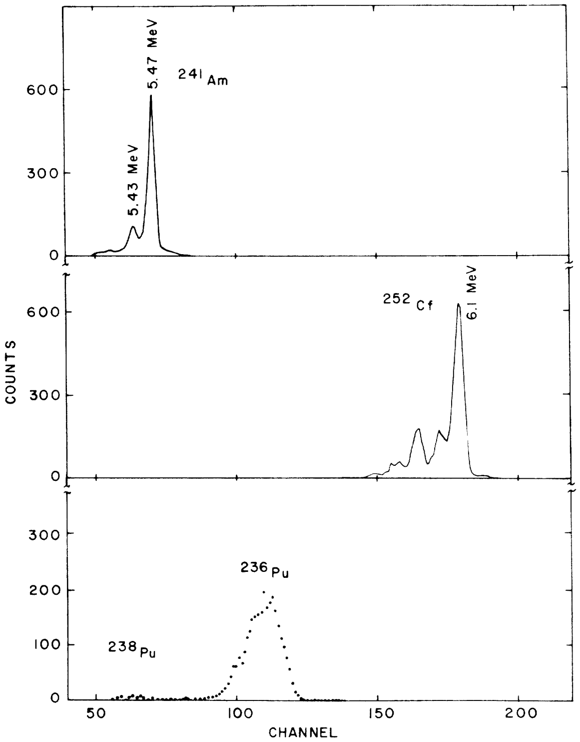

FIG. 1. α-particle spectra from ^{241}Am and ^{252}Cf calibration sources and a sample ^{235}U foil after bombardment.

II. EXPERIMENTAL PROCEDURE

Various energy beams of α particles between 24.5 and 28 MeV from the Los Alamos variable-energy cyclotron were used to irradiate ^{235}U targets. The beam energy was measured by placing a gold scattering foil in the beam and detecting the scattered particles at 90° after they passed through a series of aluminum foils. The aluminum