PHYSICAL REVIEW C

NUCLEAR PHYSICS

THIRD SERIES, VOL. 6, NO. 1 July 1972

Induced-Tensor Interaction in Weak Processes

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(Received 22 February 1972)

The nonrelativistic impulse approximation for the induced-tensor interaction has been analyzed in detail. ft values for mirror transitions have been calculated using electron Coulomb wave functions for the extended charge distribution and introducing higher-order corrections. 14 angular β-γ correlations, the β transition being allowed, have been analyzed, showing that this process is sensitive to the presence of the induced-tensor interaction. The amount of information about the induced tensor at present obtainable from circular γ polarization in nuclear transitions has been investigated. A limit on the induced-tensor coupling constant has been obtained which does not contradict the evidence based on ft values. β-γ angular correlations seem to yield evidence for the weak-magnetism term, while not excluding the existence of the induced tensor.

I. INTRODUCTION

Recent papers of Wilkinson[1] have initiated renewed interest in the search for the induced-tensor term in the weak axial-vector current matrix element.[2-8] Since this term was mentioned and investigated in our earlier papers,[9-14] we want to make our previous work up-to-date and fully relevant to the present level of knowledge. The impulse approximation was not quite correctly carried out in many of the previously published β-decay calculations.[9-16] In fact, the effective induced-tensor term as used in earlier papers[9-16] corresponds to a particular model[5] in which the effective off-mass-shell two-body exchange corrections are taken into account in a particular way. We conclude that the difference between the two induced-tensor forms [see Eqs. (1) and (2)] is due to exchange corrections estimated according to Ref. 5.

Forbidden β decays lead to rather uncertain conclusions about the value of the induced-tensor coupling constant f_T[11, 17] because of the many experimental and theoretical difficulties. $0^- \to 0^+$ transitions seem to favor $f_T < 0$, while unique forbidden transitions favor $f_T > 0$.

It has already been demonstrated[1] that ft values for mirror transitions seem to be rather sensitive to the presence of the induced tensor; they are also sensitive to the form of the effective induced-tensor interaction, i.e., our Eq. (22) or Eq. (23), as has already been noticed in a slightly different context.[5] In order to have this piece of evidence as a reference point, we have recalculated ft values including the contributions from various higher-order corrections to the spectrum-shape factor such as weak magnetism and the so-called "second-forbidden" contributions. The wave functions of the emitted electron (positron) were calculated using the Coulomb field for the finite-size nuclear charge distribution.[18] All these corrections do not significantly change previous conclusions.[1] The binding energy effects,[19-22] which lower the value of f_T or eliminate the necessity for the induced tensor in some cases, are obviously more important.

The study of various angular correlations has already been suggested[5,23] as a way of gaining further information about the sign of t_T, thus enabling us to differentiate various effective weak induced-tensor Hamiltonians. In this connection one should also include β-γ angular correlations. In the recent past, a few β-γ angular correlations have been measured in the case of allowed β decays,[24-35] the accuracy of measurements being unfortunately rather low. The theoretical expression given in Sec. IV shows that in allowed β decays the whole effect is proportional to the induced terms in higher-order corrections. The induced-tensor contribution, therefore, has to compete with relatively small terms. The sign of its contribution is different for the β^+ or the β^- decay, respectively, depending also on the γ transition which follows. This piece of evidence, combined with the analysis of *ft* values and experimental data of higher accuracy, might solve our problem completely. However, such a possibility is frustrated by experimental inaccuracies and theoretical difficulties in calculating nuclear-matrix-element ratios. The *ft* values and the β-γ correlations are actually rather sensitive to certain matrix-element ratios, as can be seen from Eqs. (B2) and (30). We analyzed β-γ correlations for 14 cases. Since experimental errors and uncertainties are rather large, we did not attempt any very elaborate calculations of nuclear-matrix-element ratios, but tried only to reach some general conclusions. It seems that in most of the cases the measured effect might be due to the induced weak-magnetism term. The inclusion of the induced tensor definitely favors $f_T > 0$, which together with the evidence based on the *ft* value decides in favor of our case A [Eq. (22)]. It is possible, however, that all effects, i.e., those in *ft* values as well as those in β-γ correlations, might be understandable with no tensor included.

Our next aim was to evaluate the amount of information on the induced tensor obtainable at present from the parity-violating processes in heavy nuclei. Limits on the induced-tensor effective coupling constant are deduced from circular γ polarization in nuclear transitions. A more elaborate evaluation of the effect leads to rather large limits on the induced-tensor coupling constant

$$-15 \leq f_T \leq 19\,,$$

in contrast to a preliminary estimate.[36] These limits do not seem to contradict the analysis of *ft* values.[1] A few rather large values required there (see Tables I, II, and IV; $A = 18$, 28, 30) cannot be taken too seriously, in view of experimental and theoretical uncertainties.

We hope that this paper might prompt some experimentalists to measure again β-γ angular correlations, the β transition being allowed.

II. IMPULSE APPROXIMATION

Since all questions raised recently[6] concerning the existence of the induced tensor and its connection with second-class currents have been cleared up,[7] we confine our consideration to the nonrelativistic approximation (NRA), often also called the impulse approximation. Although the off-mass-shell element of the axial-vector current can, in principle, be decomposed into 12 form factors,[5,37] numerical calculations involving the induced tensor have so far been limited to two forms, which are equal for nucleons on the mass shell:

$$(-)f_T(q^2)\frac{1}{2M}\,\bar{u}(p')\,\sigma_{\mu\nu}(p-p')_\nu\gamma_5 u(p)\,, \tag{1}$$

$$(-)f_T(q^2)\frac{1}{2M}\,\bar{u}(p')\,i(p_\mu+p'_\mu)\gamma_5 u(p)\,. \tag{2}$$

Other forms of the induced tensor are either equivalent to these or equal to zero on the mass shell.[5,38]

Questions have been raised regarding the relation between the expressions in Eqs. (1) and (2) off the mass shell.[5] Equation (1) was previously used in β-decay calculations[9-16] with the weak-interaction Hamiltonian given in the NRA by case B[39]:

$$H_{\text{int}} = \left(g_A \mp \frac{Y}{2M}E_0 - \frac{Y}{2M}2\xi\right)\vec{\sigma}\cdot\vec{\mathrm{L}}_4 \pm \frac{Y}{2M}\vec{\sigma}\cdot(-i\vec{\nabla})L_4\,. \tag{3}$$

Here only terms due to axial-vector currents are written. All notation has the usual meaning.[40] In deriving Eq. (3) the important off-mass-shell contributions were neglected. Equation (3) was derived from the following "relativistic"[41] expression for the induced tensor[42]:

$$\mathrm{IT} = -\frac{Y}{2M}\,\tfrac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\gamma_5\partial_\nu L_\mu\,, \tag{4}$$

$$L_\mu = \Psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\Psi_\nu\,. \tag{5}$$

In deriving the NRA of Eq. (3), the contribution coming from the combination of indices

$$\mu=j, \quad \nu=k, \quad j,k=1,2,3 \tag{6}$$

was neglected. In the strict nonrelativistic shell model for nucleons[43] this contribution, denoted by D, is not negligible in the NRA. It can be written in the form[44]

$$D=D_1+D_2, \tag{7}$$

$$D_1=-\frac{Y}{2M}\Psi_f^\dagger\gamma_4[(\vec{\sigma}\cdot\vec{p})(\vec{\alpha}\cdot\vec{L})+(\vec{\alpha}\cdot\vec{L})(\vec{\sigma}\cdot\vec{p})]\Psi_i, \tag{8}$$

$$D_2=\frac{Y}{2M}\Psi_f^\dagger\gamma_4\gamma_5[(\vec{p}\cdot\vec{L})+(\vec{L}\cdot\vec{p})]\Psi_i. \tag{9}$$

Here Ψ_i and Ψ_f refer to the initial and final "relativistic" nucleon wave functions, respectively, and the impulse p is taken to be an operator. The NRA follows from these functions by approximating[45]

$$\Psi_n\approx\begin{pmatrix}-\vec{\sigma}\cdot\vec{p}\,\chi_n/2M\\ \chi_n\end{pmatrix}. \tag{10}$$

Here χ_n are nuclear shell-model functions satisfying

$$p^2\chi_n=2M(E_n-V)\chi_n. \tag{11}$$

One then obtains[46]

$$\begin{aligned}D_1&=-\frac{Y}{(2M)^2}\chi_f^\dagger[p^2(\vec{\sigma}\cdot\vec{L})-(\vec{\sigma}\cdot\vec{L})p^2]\chi_i\\ &=\frac{Y}{2M}(E_f-E_i)\chi_f^\dagger\vec{\sigma}\cdot\vec{L}\chi_i.\end{aligned} \tag{12}$$

The importance of the nuclear shell-model equation (11) is obvious, since Eq. (12) would vanish for nucleons on the mass shell. The contribution (12) cancels the second and the third term in Eq. (3).[47]

We are left with the effective NRA of the interaction (4) for the case of allowed β transitions. In the case of forbidden β transitions, additional terms play an important role. As shown in Appendix A, one can extract from the term D_2 of Eq. (9) the term

$$D_{2b}=\frac{Y}{2M^2}(-i\vec{\sigma}\cdot\vec{\nabla})\vec{L}_4\cdot\vec{p}, \tag{13}$$

which is negligible in the case of allowed transitions, but does contribute to $0^\mp\to0^\pm$ and to $2^\pm\rightleftarrows0^\mp$ transitions.

An additional contribution, important for the case of $0^\mp\to0^\pm$ transitions, comes from the combination of indices in Eq. (4):

$$\mu=4, \quad \nu=j. \tag{14}$$

This contribution, denoted by C, is

$$C=\frac{Y}{2M}\Psi_f^\dagger\gamma_4[\vec{\sigma}\cdot\vec{p}, L_4]_-\Psi_i. \tag{15}$$

Previously, the relativistic nuclear wave functions Ψ_n were simply replaced by the nonrelativistic ones, thus giving the last term appearing in Eq. (3). As we know from the analogous induced-pseudoscalar case,[14] this replacement is not always justified. In Appendix A we show for β transitions in the NRA that

$$\begin{aligned}C&\propto\frac{Y}{2M}(-i\vec{\sigma}\cdot\vec{\nabla}L_4)+\frac{Y\Omega}{4M^2}L_4\vec{\sigma}\cdot\vec{p},\\ \Omega&=E_0+2\xi.\end{aligned} \tag{16}$$

In the case of $0^\mp\rightleftarrows0^\pm$ transitions the last term in Eq. (16) leads to the nuclear matrix element $\langle i\vec{\sigma}\cdot\vec{p}/M\rangle$, while the first term is proportional to $\langle\vec{\sigma}\cdot\vec{r}\rangle$. The ratio of the two matrix elements

$$f=\langle i\vec{\sigma}\cdot\vec{p}/M\rangle/\langle\vec{\sigma}\cdot\vec{r}\rangle \tag{17}$$

is rather large,[48] so the two terms in Eq. (16) contribute comparably.

The alternative form in Eq. (2) of the induced tensor leads to the effective β-decay interaction

$$\mathrm{IT}=-\frac{Y}{2M}\overline{\Psi}_f\gamma_5 i(p_\mu L_\mu+L_\mu p_\mu)\Psi_i, \tag{18}$$

where

$$\begin{aligned}p_\mu&=(\vec{p}, p_4),\\ \vec{p}&=-i\vec{\nabla}, \quad p_4=i(M+T_k),\\ T_k&=p^2/2M.\end{aligned} \tag{19}$$

In the NRA expression (18) is completely equivalent to expression (4). The selection of indices $\mu=j$ leads to the term D_2 of Eq. (9), while $\mu=4$ in the NRA (see Appendix A) leads to the term C of Eq. (16).[49]

Equations (1) and (2) lead to the same NRA in the usual nuclear shell-model impulse-approximation limit. In that sense the two forms are equivalent. They are, however, not equivalent when exchange effects, i.e., two-body contributions to the weak-interaction Hamiltonian are considered. This corresponds to a more elaborate form of the nuclear shell-model description, i.e., to a form which includes two-body residual interactions. It is in this sense, in our opinion, that one should understand the discussion given in Ref. 5.

Let us illustrate this by a simple one-pion-exchange diagram shown in Fig. 1. The nucleon propagator indicated in the diagram has the form

$$P=\sum_{s,\,E_s\lessgtr0}\frac{u^s(p_s)\overline{u}^s(p_s)}{E_s+\omega-E_1}. \tag{20}$$

Its $E_s>0$ part contributes to the nonrelativistic perturbation series, while its $E_s<0$ part leads to the two-body-exchange contribution

$$\sum_{s} \bar{u}(p_2)\gamma_5\tau_j \tfrac{1}{2}(1+\beta)O_T u(p_1)R\,. \tag{21}$$

Here the denominator was approximated by $\sim|E_s| + \omega - E_1 \approx 2M$, and the summation over the negative energy spinors u^s was carried out. O_T is the induced-tensor operator either in the form of Eq. (1) or in the form of Eq. (2). R symbolizes the left side of the Feynman diagram, Fig. 1. The end result is different for forms (1) and (2), the contribution from Eq. (1) being negligible in the NRA.[50] The summation over all possible $E_s > 0$ contributions, including higher-order terms, should somehow average to the shell-model potential appearing in Eq. (11). In this way higher-order terms are included in the impulse approximation.[51] We remind the reader that a completely analogous discussion for parity-nonconserving meson exchange, combined with γ emission, is given in detail in Ref. 52, especially in the appendixes.[53]

The contributions of the type (21) were estimated in Ref. 5 for a special case of the conserved second-class axial-vector current. In this special model, the induced tensor of the form (1) receives no significant contributions from the exchange effects. Two-body contributions to the induced tensor (2), when approximated through the effective single-body operators, add to Eq. (22) completing it back to Eq. (3),[54] i.e., case B [Eq. (23)] in the NRA. This explains how our results compare with those of Ref. 5. As mentioned in this reference, other models for exchange contributions could lead to significant complications, necessitating studies of other off-mass-shell induced terms.

In concluding this section, we want to summarize our results regarding the NRA:

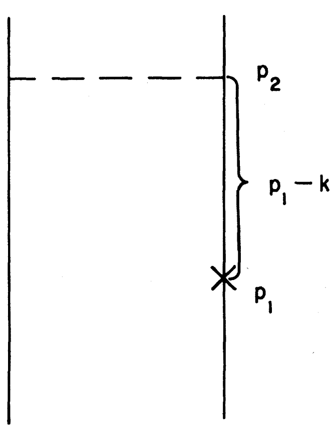

FIG. 1. The full lines are nucleons, the dashed line is the pion. The cross marks the induced-tensor vertex, while the curly bracket indicates the propagator corresponding to the energy denominator.

Case A,

$$H_{\rm int} = g_A\vec{\sigma}\cdot\vec{\rm L}_4 + \frac{Y}{2M}\vec{\sigma}\cdot(-i\vec{\nabla})L_4 + \frac{Y}{2M}\left[\tfrac{1}{2}(E_0+2\xi)L_4\frac{\vec{\sigma}\cdot\vec{\rm p}}{M} + (-i\vec{\sigma}\cdot\vec{\nabla})\frac{\vec{\rm L}_4\cdot\vec{\rm p}}{M}\right]. \tag{22}$$

Here all signs refer to β^- decay. The terms inside the square brackets are important only in forbidden decays. The vector-current contribution should be assumed. Case A follows from either Eq. (1) or Eq. (2) when two-body-exchange contributions are neglected. Equation (22) corresponds to Eq. (1) with the two-body-exchange contributions calculated as in Ref. 5.

Case B,

$$H_{\rm int} = H_{\rm int} \text{ of Eq. (22)}$$

with

$$g_A \to g_A - \frac{Y}{2M}(E_0+2\xi)\,. \tag{23}$$

This $H_{\rm int}$ corresponds to Eq. (2) with the two-body exchange contributions calculated as in Ref. 5.

III. *ft* VALUES AND MIRROR TRANSITIONS

The existence of the induced-pseudoscalar term is characterized by the ratio b of the effective coupling constant Y and the axial-vector coupling constant g_A,

$$b = Y g_A^{-1}\,. \tag{24}$$

Instead of merely quoting the values for b following from the analysis,[1] we recalculated them using Bhalla and Rose's wave functions[18] for electrons and taking into account vector–axial-vector interference corrections. The correction factors necessary to the spectrum shape were published a long time ago[9] and for the convenience of the reader are listed in Appendix B. We calculated the half-lives t by the formula

$$ft = t\int_0^{E_0{}^2-1} p^2q^2F(Z,E)\tilde{C}^B(E,b)dp = \frac{(2\pi)^3\ln 2}{g_A{}^2|\langle\sigma\rangle|^2}\,, \tag{25}$$

where E_0 is the maximal energy, E is the electron energy, and p and q are the electron and neutrino momenta, respectively. $F(Z,E)$ is the Fermi function, while $\tilde{C}^B(E,b)$ is the spectrum-shape correction factor depending on the induced tensor through the b of Eq. (24).

The *ft* values for electron and positron mirror transitions are expected to be approximately equal. Thus the magnitude of the deviation of δ from zero, when b is taken as zero and

$$\delta \equiv (ft)^+/(ft)^- - 1\,, \tag{26}$$

TABLE I. Deviations δ_0 of ft values and the induced-tensor coupling constant b for mirror nuclei.

A	E_0^+ / E_0^- (MeV) [a]	t^+ / t^- (sec) [b]	Case 1 δ_0	Case 1 b_A	Case 1 b_B	Case 2 δ_0	Case 2 b_A	Case 2 b_B
8	14.05 / 13.10	0.774 / 0.849	0.072 ± 0.005	6.6	−2.9	0.067 ± 0.005	6.2	−2.9
12	16.32 / 13.37	0.0116 / 0.0210	0.122 ± 0.006	9.6	−4.8	0.111 ± 0.006	9.2	−4.8
20	11.26 / 5.40	0.51 [c] / 11.03	0.046 ± 0.02 [c] / -0.064 [d]	7.3 / −9.2	−3.7 / 4.6	0.045 ± 0.02 / -0.061	7.3 / −9.9	−3.7 / 5.1
24	8.737 / 1.392	26.12 / 53 980	-0.018 ± 0.015 / 0.37 [e]	−4.8	2.2	-0.017 ± 0.015	−4.8	2.2
28	11.537 / 2.856	0.52 / 136.4	0.282 ± 0.005	45.9	−22.4	0.270 ± 0.005	44.8	−21.3

[a] Maximal energy of positrons and electrons.
[b] Partial half-lives.
[c] Partial half-life from D. H. Wilkinson and D. E. Alburger, Phys. Rev. Letters 24, 1134 (1970).
[d] For $t^+ = 0.453$ sec.
[e] For $t^+ = 36.25$ sec. All other data are taken from D. H. Wilkinson, Phys. Letters 31B, 447 (1970).

can be interpreted[1] as an indication of the existence of the induced tensor. With reference to Table I, we calculated δ_0 by putting $b = 0$ in Eq. (25) for $(ft)^+$ and $(ft)^-$. The actual value of b was selected in such a way as to give $\delta = 0$ when using the full expression for $\tilde{C}^\beta$, as given in Appendix B.

The results for case 1 in Table I were obtained by neglecting the cross terms with the vector coupling (i.e., weak magnetism). The values for b in case 2 in Table I were calculated taking into account all corrections. The calculations were performed both for case A [Eq. (22)] and for case B [Eq. (23)]. The ratios of matrix elements were estimated on the basis of the simple shell model as follows: $\alpha_1 = 2$, $\alpha_2 = -0.13$, $\beta_1 = 8\times 10^{-5}$, $\beta_2 = 3\times 10^{-4}$, $\beta_3 = 1$.

Inspection of Table I shows that the additional corrections are not of much importance. Newer measurements for $A = 20$ and $A = 24$ are also included in Table I. The signs of the induced-tensor coupling constants b for $A = 8$, 12, 20, 24, and 28 show complete mutual consistency and their absolute values for $A = 8$, 12, and 20 are rather close. The transitions with $A = 18$ and $A = 30$ (Table II) correspond to two successive β^+ decays, and the corresponding nuclear-matrix-element ratios in these cases are not necessarily correctly predicted by their single-particle values. Furthermore, as cases $A = 20$ and 24 show, slight changes in experimental values can reverse the sign of the theoretical b.

A detailed analysis confirms a rather obvious qualitative conclusion that, in contrast to the weak-magnetism term,[55] the induced tensor is not seen in the β spectrum shape.

Table III gives the slopes for the spectrum-shape factors for the decays of N^{12} and B^{12}, respectively, for various combinations of higher-order corrections as calculated using formulas from Appendix B.[9] Even for a rather large value of our parameter b ($b = \pm 2200Y$) the spectrum-shape-factor slope is practically unchanged.

In Table IV we recalculated the results appearing in Table I taking into account corrections to the matrix-element ratios.[20] The value of b was chosen in such a way as to obtain

$$(ft)^+(b)/(ft)^-(b) = \Lambda^-/\Lambda^+ . \tag{27}$$

The ratio Λ^-/Λ^+ was taken from case B in Ref. 20. As is to be expected, the values of the induced-tensor constant b are generally reduced. One case, $A = 20$, is explainable even with no induced tensor included. This becomes particularly suggestive when compared with the results on β-γ angular correlations presented in the next section.

TABLE II. Deviation δ_0 of the $(ft)^+/(ft)^+$ ratio from unity, and the induced-tensor coupling constant b.

A	E_0^{1+} / E_0^{2+} (MeV)	t_1^+ / t_2^+ (sec)	δ_0	b_A	b_B
18	3.425 / 0.633	1.58 / 6796	-0.097 ± 0.005 / 0.033 [a]	−110	51
30	5.085 / 3.22	5.9 / 150.6	-0.052 ± 0.003	−74	39

[a] $t_1^+ = 1.807$ sec, from D. W. Alburger and D. H. Wilkinson, Phys. Letters 32B, 190 (1970).

IV. β-γ ANGULAR CORRELATIONS

In the general search for the induced-tensor interaction it is worthwhile to turn our attention to β-γ angular-correlation measurements in the case of allowed β decays. In this case the whole effect is directly proportional to the induced terms and the so-called "second-order corrections." The induced tensor, therefore, has to compete with other terms of the same order of magnitude and its presence should be readily noticeable. The sign of its contribution changes going from β^- to β^+ decays, depending also on the γ transition. In general, the whole effect depends strongly on the sign of the induced-tensor term, as can be seen from the results presented in Table V. This should, in principle, enable us to distinguish case A from case B defined in Sec. II. In both cases, only the term

$$\frac{Y}{2M}\vec{\sigma}\cdot(-i\overleftrightarrow{\nabla})L_4 \tag{28}$$

can contribute. It therefore shows a certain similarity with other angular correlations, such as electron-neutrino or electron-nuclear spin.[5,23] We emphasize that the sign of Y should come out the same from the analysis of both the angular correlations and the ft values.

Starting with the standard interaction Hamiltonian written in Appendix B, and adding to it either Eq. (22) or Eq. (23), we arrive at the following expression for β-γ angular correlations:

$$W(\theta)=1+A_2(\beta)F_2(LLI_fI)P_2(\cos\theta)\,, \tag{29}$$

where

$$A_2(\beta)=-4F_2(11I_iI)\times\left\{2\beta_2N_{12}+\left[\frac{q}{9}(\beta_1+2\beta_2)+\frac{1}{2M}\pm\frac{g_V}{g_A}\frac{B_4}{2M}\pm\frac{b}{2M}-\frac{\alpha_2}{M}\pm\frac{1}{2M}\left(\frac{2}{3}\right)^{1/2}\frac{g_V}{g_A}\alpha_1\right]L_{12}\right\}. \tag{30}$$

The upper and lower sign refer to β^- and β^+ decay, respectively. We assumed a β transition $I_i\to I$ followed by a pure 2^L-pole γ transition $I\to I_f$. The nuclear-matrix-element ratios α_i and β_i are defined in Appendix B, while all other notation has the usual meaning.[28,56] Neglecting the so-called "second-order-correction" contributions, i.e., setting $\alpha_i=0$, $\beta_i=0$, and using plane waves for electrons we can approximate Eq. (30) by

$$A_2(\beta)=\frac{4}{3}\frac{p^2}{W}\left(\frac{1}{2M}\pm\frac{g_V}{g_A}\frac{B_4}{2M}\pm\frac{b}{2M}\right)F_2(11I_iI)\,, \tag{31}$$

where p and W are the momenta and the energy of the electron (positron), respectively. In order to indicate the order of magnitude of $A_2(\beta)$, we mention that the expression in the large brackets is 1.4×10^{-3} for β^- decay or $(-)0.9\times10^{-3}$ for β^+ decay, with no tensor included, i.e., when $b=0$.

Although several measurements of β-γ angular correlations have appeared in the literature,[24-35] most of them are still associated with large uncertainties and experimental errors. We decided to calculate $A_2(\beta)$ for all of them on the basis of the full formula of Eq. (30). The nuclear-matrix-element ratios α_i and β_i were estimated on the basis of the simple shell model, as in Sec. III. More elaborate nuclear-matrix-element calculations do not seem warranted by the present state of experiments.[57] Final results were obtained by averaging over the experimental range of electron energy. The errors quoted with the theoretical results correspond to the difference between β-γ correlation values for the average experimental energy and β-γ correlation values for the minimal or maximal experimental energies. Since it is difficult to decide which experiment should be given more weight in theoretical analysis, we want to look only for some general features at this stage of confrontation between theory and experiment.

General features emerging from Table V can be summarized as follows:

(i) Weak magnetism alone ($b=0$). Agreement between experimental and theoretical signs has been achieved in 10 cases out of 14. Absolute values

TABLE III. Slopes of spectra in $A=12$ mirror nuclei.

Decay	Energy interval (MeV)	Slope per MeV for $-0.6<x<0.6$ A[a]	B[b]	Experimental slope
^{12}N	10.08	−0.48	−0.54	−0.52 ± 0.06
^{12}B	7.48	0.60	0.53	0.55 ± 0.10

[a] All parameters from single-particle estimates.
[b] Only $C_{1\,\mathrm{GT}}$ and terms multiplied by x and B_4 are kept. See Appendix B. Here $x=b/2M$.

TABLE IV. Values of the induced-tensor coupling constant when binding-energy effects (Ref. 20) are taken into account.

A	δ_W[a]	$\delta_0-\delta_W$[b]	b_A^W	b_B^W
8	0.048	0.020	1.8	−0.9
12	0.098	0.013	1.0	−0.44
20	0.045	0.00	0	0
24	0.040	−0.058	−14.7	7.0
28	0.051	0.22	35.2	−16.9
18	0.007	0.026	23.9	−12.9
30	0.005	−0.057	−77	42

[a] δ_W from Ref. 20 is defined as Λ^-/Λ^+-1.
[b] δ_0 is the value from our case 2 in Table I.

TABLE V. Dependence of the angular-correlation coefficients on the induced coupling constant b.

Transition	W_0 $\overline{W}$ (units of mc^2)	$10^3 A_{2\,\mathrm{exp}}$	$10^3 A^{\beta}_{2\,\mathrm{th}}$ Case A $b=-0.002$	Case A $b=0$	Case A $b=0.002$	Case A $b=0.004$	Case B $b=-0.002$	Case B $b=0$	Case B $b=0.002$	Case B $b=0.004$
^{20}F $2^+(\beta^-)2^+$	11.6 >9	-10 ± 3 [a]	3.7 ± 0.4	-7.5 ± 1	-19 ± 2	-30 ± 4	8 ± 1	-3.2 ± 0.2	-14 ± 1	-25 ± 2
^{24}Na $4^+(\beta^-)4^+$	3.15 3.1	-0.4 ± 0.8 [b]	1.1 ± 0.1	-2.2 ± 0.1	-5.5 ± 0.2	-8.8 ± 0.3	1.7 ± 0.1	-1.64 ± 0.05	-4.9 ± 0.2	-8.2 ± 0.3
^{22}Na $3^+(\beta^+)2^+$	2.06 1.78	1.7 ± 2 [c] 3.0 ± 0.5 [b] 25 ± 5 [d] 0.5 ± 0.8 [e] 1.7 ± 8 [f]	0.30 ± 0.02	-0.19 ± 0.01	-0.69 ± 0.04	-1.18 ± 0.08	0.32 ± 0.01	-0.17 ± 0.01	-0.66 ± 0.04	-1.15 ± 0.08
^{56}Co $4^+(\beta^+)4^+$	3.935 3.348	-1.8 ± 4.1 [c] -7 ± 6 [g] -25 ± 4 [e] 2 ± 7 [f]	-2.2 ± 0.3	1.5 ± 0.15	5.0 ± 0.5	8.6 ± 0.9	-3.5 ± 0.4	0.19 ± 0.08	3.8 ± 0.5	7.6 ± 1
^{46}Sc $4^+(\beta^-)4^+$	1.4 1.35	-0.4 ± 0.8 [b]	0.38 ± 0.04	-0.78 ± 0.08	-1.9 ± 0.2	-3.1 ± 0.3	0.79 ± 0.09	-0.37 ± 0.03	-1.54 ± 0.16	-2.7 ± 0.3
^{58}Co $2^+(\beta^+)2^+$	1.95 1.68	0.5 ± 2.4 [c]	-0.93 ± 0.07	0.61 ± 0.05	2.16 ± 0.16	3.70 ± 0.28	-1.54 ± 0.10	0.03 ± 0.06	1.55 ± 0.13	3.09 ± 0.24
^{60}Co $5^+(\beta^-)4^+$	1.613 1.4	0.4 ± 0.3 [h] 0.7 ± 1.1 [c] 0.4 ± 0.7 [b] -0.7 ± 2.2 [i]	-0.15 ± 0.02	0.30 ± 0.04	0.76 ± 0.09	1.2 ± 0.2	-0.36 ± 0.05	0.09 ± 0.01	0.55 ± 0.07	1.0 ± 0.14
^{110}Ag $6^+(\beta^-)6^+$	2.035 1.78	0.6 ± 1 [c]	0.50 ± 0.03	-1.02 ± 0.07	-2.5 ± 0.1	-4.1 ± 0.3	1.7 ± 0.1	0.23 ± 0.02	-1.3 ± 0.1	-2.8 ± 0.2
^{134}Cs $4^+(\beta^-)4^+$	2.292 1.78	-1.5 ± 2.2 [c] 37 ± 7 [j] -2 ± 7 [f]	0.46 ± 0.03	-0.94 ± 0.07	-2.3 ± 0.1	-3.7 ± 0.2	1.7 ± 0.1	0.37 ± 0.03	-1.03 ± 0.07	-2.4 ± 0.2
^{154}Eu $3^-(\beta^-)2^+$	2.135 1.88	0.7 ± 2 [c] 7 ± 6 [h] 1.7 ± 3 [k]	-0.12 ± 0.01	0.25 ± 0.03	0.62 ± 0.08	1.0 ± 0.1	-0.52 ± 0.07	-0.14 ± 0.02	0.22 ± 0.02	0.60 ± 0.07
^{124}Sb $3^-(\beta^-)3^-$	2.215 1.88	-2.2 ± 2.8 [c] 6 ± 7 [f] 6 ± 8 [l]	0.51 ± 0.06	-1.0 ± 0.1	-2.6 ± 0.3	-4.2 ± 0.06	1.9 ± 0.3	0.32 ± 0.05	-1.2 ± 0.2	-2.8 ± 0.4
^{152}Eu $3^-(\beta^-)3^-$	2.37 >1.88	-17 ± 12 [h]	0.45 ± 0.12	-1.0 ± 0.2	-2.4 ± 0.4	-3.9 ± 0.6	2.0 ± 0.4	0.59 ± 0.11	-0.86 ± 0.13	-2.3 ± 0.4

TABLE V (*Continued*)

Transition	W_0 $\overline{W}$ (units of mc^2)	$10^3 A_{2\exp}$	$10^3 A^{\beta}_{2\,\mathrm{th}}$ Case A $b=-0.002$	$b=0$	$b=0.002$	$b=0.004$	Case B $b=-0.002$	$b=0$	$b=0.002$	$b=0.004$
^{152}Eu $3^-(\beta^-)2^-$	1.39 >1.31	16 ± 20 [h]	-0.08 ± 0.01	0.15 ± 0.01	0.39 ± 0.02	0.63 ± 0.02	1.4 ± 0.1	1.55 ± 0.11	1.8 ± 0.2	2.2 ± 0.3
^{160}Tb $3^-(\beta^-)3^-$	2.108 2.1	90 ± 30 [h] 40 ± 20 [h] 170 ± 30 [k]	0.44 ± 0.05	-0.89 ± 0.10	-2.2 ± 0.2	-3.5 ± 0.5	1.9 ± 0.2	0.57 ± 0.06	-0.76 ± 0.06	-2.1 ± 0.2

[a] Reference 34. [b] Reference 24. [c] Reference 25. [d] Reference 26. [e] Reference 27. [f] Reference 28. [g] Reference 35. [h] Reference 32. [i] Reference 29. [j] Reference 31. [k] Reference 33. [l] Reference 30.

agree within the errors only in two to three cases, though disagreement is generally not too bad.

(ii) Induced tensor $b>0$ included (case A). In nine or ten cases experimental signs seem to be in agreement with theoretical ones.[58] In three transitions there also exists excellent agreement in magnitude. In general, when the theoretical sign agrees with experiment the magnitude is closer to the experimental one than in the case of weak magnetism alone. One should mention the decay of F^{20}, where $b=7.3$, as fixed from ft values (see Table I), also fits the angular-correlation experiment. However, in that particular case (see Table IV), the binding-energy effects can probably completely[59] account for the discrepancy in ft values, thus leading to $b=0$.

(iii) Induced tensor $b<0$ included (case B). The sign seems to be predicted correctly only in three to four cases. In two of them, the predicted magnitude is almost correct.

At the present stage of experimental knowledge, our theoretical efforts displayed in Table V may serve mostly as an indication how important β-γ correlation analysis would be in our investigation of weak interactions. As experimental uncertainties are really large, it is difficult to estimate how much importance should be attached to our findings in favor of (i) and/or (ii).

V. PARITY VIOLATION IN HEAVY NUCLEI

It has been pointed out that the second-class-current contribution to the weak parity-violating (PV) internucleon potential should behave as an isovector.[60, 13] When applied to PV nuclear forces, the two cases of the NRA, Eq. (22) and Eq. (23), are equivalent. Roughly speaking, one has to introduce in those formulas the vector nuclear currents instead of the lepton bilinears $\vec{L}_4$ and L_4. In the very low-energy zero-momentum transfer limit and for unit or zero charged nucleons,[61] the quantities multiplied by E_0 and ξ are negligible, so both cases of the NRA are equivalent.

Neglecting many-body exchange corrections, one can use in the first approximation the potential already published in[60, 13]

$$V_{\mathrm{IT}}=\frac{YG}{4\sqrt{2}M}(\vec{\sigma}_1+\vec{\sigma}_2)[\vec{p}_{12}, f(r)]_- T^{(-)}, \qquad (32)$$

where

$$T^{(-)}=\tau_{1+}\tau_{2-}-\tau_{1-}\tau_{2+}\,. \qquad (33)$$

Here all notation has the usual meaning. The radial function appearing in Eq. (32) depends on the induced-tensor-producing mechanism, which is unknown. In order to have some numerical estimate, we make use of the function appearing with the normal (first-class-current) axial-vector vertex

$$f(r) \approx \frac{1}{4\pi} \frac{e^{-m_\rho r}}{r}, \tag{34}$$

where m_ρ is the ρ-meson mass. This approximation seems to be natural to some extent, if one imagines the corresponding PV nucleon-nucleon interaction being dominated by the particle having the necessary quantum numbers and the lowest possible mass. However, longer-range 2π exchange could, in principle, result in larger contributions. This 2π-exchange contribution is not as yet completely understood even for first-class currents. Preliminary estimates indicate that there it could be of the same order of magnitude as the contribution from ρ exchange. Unless something violent happens, this might continue to hold also for second-class currents. It means that our estimate based on Eqs. (32) and (34) can differ by a factor of 2, which does not change the conclusion in any significant way.

Some discussion on the value of Y, based on circular-polarization measurements, has already been presented.[36] As the approximation used there overestimates the effect,[52] some change in the conclusions is to be expected. The circular polarization of γ emission in the 482-keV decay of Ta^{181}, obtained by the method described in Ref. 52, is, for the conventional model of the weak Hamiltonian,

$$P = \left[\begin{pmatrix} 6.13 \\ 8.97 \end{pmatrix} - 7.19Y \right] \times 10^{-6}. \tag{35}$$

The two values in the first column correspond to two possible relative signs of the pion-exchange contribution, while the parameter Y, defined in Eq. (3) and Eq. (32), measures the strength of the induced-tensor contribution. When calculating short-range correlations among nucleons in a more exact way,[62] we obtain

$$P = \left[\begin{pmatrix} 3.07 \\ 5.75 \end{pmatrix} - 1.73Y \right] \times 10^{-6}. \tag{36}$$

One group of experimental results[63] leads to an average

$$P_{\text{exp}} = (-5.0 \pm 1.5) \times 10^{-6}. \tag{37}$$

An essentially different sign is given by Kuphal, Daum, and Kankeleit,[64] where

$$P_{\text{exp}} = (2.0 \pm 4.0) \times 10^{-6}. \tag{38}$$

In the wide spectrum of the published experimental results one can also find some giving larger P than those quoted above.[65,66] They would naturally set higher bounds on Y than the ones following from Eqs. (36) and (37), and therefore we are actually dealing with the most sensitive cases.

Looking for such combinations of numbers from Eq. (35) which would fit Eqs. (37) and (38), we obtain the limits

$$0 \leq Y \leq 2. \tag{39}$$

In the same way Eq. (36) leads to

$$-1.7 \leq Y \leq 7. \tag{40}$$

If additional residual nucleon-nucleon interactions are introduced in theoretical calculations, in the way outlined by Vinh-Mau,[67] the theoretical value for P is decreased; so we can estimate

$$-15 \leq Y \leq 19. \tag{41}$$

The estimated limits in Eqs. (40) and (41) are in agreement with the ft value estimated in Sec. III.

For other weak-Hamiltonian models, the situation is either approximately the same as for the conventional model, or the induced-tensor contribution is relatively insignificant in comparison with the strongly enhanced one-pion-exchange contribution. In that case, even larger limits on Y result.

APPENDIX A

The very form of expression D_2 in Eq. (9) shows that parts of it should be negligible in the NRA. It contains the derivative of the lepton combination L_4, which is equal to

$$-i\nabla L_4 = m_l \Psi_l^\dagger \beta\gamma_5 \Psi_\nu + (E_0 + 2\xi)\Psi_l^\dagger \gamma_5 \Psi_n = X^l. \tag{A1}$$

The corresponding part of D_2,

$$D_{2a} = \frac{Y}{2M} \overline{\Psi}_f \gamma_5 X^l \Psi_i, \tag{A2}$$

is, according to Eq. (2) of Ref. 14, equal to, in the NRA,

$$D_{2a} \approx \frac{Y}{(2M)^2} \left(-i\vec{\sigma} \cdot \vec{\nabla} X^l + \frac{\Omega}{2M} X^l \vec{\sigma} \cdot \vec{p} \right), \tag{A3}$$

and therefore is negligible.

The rest of D_2 in the NRA is of the form

$$D_{2b} \approx -i \frac{Y}{2M^2} \vec{\sigma} \cdot \vec{\nabla}(\vec{L}_4 \cdot \vec{p}). \tag{A4}$$

It gives no contribution to the allowed spectrum-shape factor in the lowest order of expansion of lepton covariants in powers of the nuclear radius r. The only vector one can build in that case is the one of wrong parity, $\vec{\sigma} \times \vec{p}$. One has to go to higher order in the expansion of the lepton combination, which is clearly negligible when multiplied by M^{-2}.

The term D_{2b} cannot be neglected in the case of $0^- \to 0^+$ transitions. One can construct the operator $\vec{\sigma} \cdot \vec{p}$ to the lowest order in r. There is a close analogy with the interaction $L_4 \vec{\sigma} \cdot \vec{p} M^{-1}$, which is very important in the case of $0^- \to 0^+$ transitions, while being a small "relativistic correction" in the case of allowed decays.[68]

It is easy to see that the expression in Eq. (2) and the corresponding expression in Eq. (18) lead straightforwardly to Eq. (22) in the NRA. The term D_2 in Eq. (9) is obtained when $\mu = j$ is selected. The $\mu = 4$ part

$$(\mathrm{IT})_4 = \frac{Y}{2M} \overline{\Psi}_f \gamma_5 L_4 \Psi_i (2M + T_f + T_i) \tag{A5}$$

is, in the NRA,[13] equal to

$$(\mathrm{IT})_4 \approx \frac{Y}{2M}(-i\vec{\sigma}\cdot\vec{\nabla} L_4)\left(1 + \frac{E_0}{4M} + \frac{T_f}{2M} + \frac{T_i}{2M}\right) + \frac{YE_0}{4M^2} L_4 \vec{\sigma}\cdot\vec{\mathrm{p}}. \tag{A6}$$

Equation (A6) is exactly equal to the NRA of the term C of Eq. (15). The proof closely follows Appendix A of Ref. 14. We have to find the NRA for the matrix element

$$\langle f | \gamma_4 [\vec{\sigma}\cdot\vec{\mathrm{p}}, L_4]_- | i\rangle_R . \tag{A7}$$

The index R in Eq. (A7) is to remind us of its "relativistic" character. We can approximate

$$\begin{aligned}
&\langle f | \gamma_4 [\vec{\sigma}\cdot\vec{\mathrm{p}}, L_4]_- | i\rangle_R \approx \langle f | [\vec{\sigma}\cdot\vec{\mathrm{p}}, L_4]_- | i\rangle + a ,\\
&a = -(2M)^{-2} \langle f [\vec{\sigma}\cdot\vec{\mathrm{p}}(\vec{\sigma}\cdot\vec{\mathrm{p}} L_4 - L_4 \vec{\sigma}\cdot\vec{\mathrm{p}})\vec{\sigma}\cdot\vec{\mathrm{p}}]\, i\rangle\\
&\quad = (-)\Big(\tfrac{1}{2}(p_f{}^2 - p_i{}^2)\langle f | \{\vec{\sigma}\cdot\vec{\mathrm{p}}, L_4\}_+ | i\rangle\\
&\qquad - \tfrac{1}{2}(p_f{}^2 + p_i{}^2)\langle f | [\vec{\sigma}\cdot\vec{\mathrm{p}}, L_4]_- | i\rangle\Big)(2M)^{-2} .
\end{aligned} \tag{A8}$$

The last line follows by successive partial integrations. For C in the NRA one encounters all terms that appeared in Eq. (A6). Neglecting small contributions and making the gauge-inspired replacement $E_0 \to \Omega$, one arrives at the expression of Eq. (16) from either Eq. (A6) or Eq. (A8).

APPENDIX B

Though detailed formulas for allowed transitions have already been published,[9] they might be relatively inaccessible. Therefore, we repeat some essentials.

A standard β-decay Hamiltonian is

$$H_{\mathrm{int}} = g_V L_4 + \frac{g_V}{2M}\vec{\mathrm{p}}_L\cdot\vec{\mathrm{L}}_4 + \frac{g_V}{M}\vec{\mathrm{L}}_4\cdot\vec{\mathrm{p}} + \frac{ig_V}{2M} B_4(\vec{\mathrm{p}}_L \times \vec{\mathrm{L}}_4) + g_A \vec{\sigma}\cdot\vec{\mathrm{L}}_4 + \frac{g_A}{2M}\vec{\sigma}\cdot\vec{\mathrm{p}}_L L_4 + \frac{g_A}{M} L_4 \vec{\sigma}\cdot\vec{\mathrm{p}}. \tag{B1}$$

The induced-pseudoscalar and -tensor terms are omitted here. To account for the induced-tensor terms, one should add to Eq. (B1) the H_{int} of Eq. (22) or (23), respectively. The notation $\vec{\mathrm{p}}_L$ indicates that $\vec{\mathrm{p}}$ affects only the lepton covariants $\vec{\mathrm{L}}_4$ and L_4.

The correction factor for Gamow-Teller transitions, including the induced-tensor contribution, is of the form

$$\begin{aligned}
\tilde{C}^\beta(E, b) = {}&(1 - 2x\{2\xi + E_0\})C_{1\mathrm{GT}} + \beta_1 C_{3\mathrm{GT}}\\
&\pm \frac{g_V}{g_A}(B_4 C_{4\mathrm{GT}} + B_4\beta_3 C_{5\mathrm{GT}} + \alpha_1 C_{6\mathrm{GT}})\\
&+ (C_{7\mathrm{GT}} + \beta_3 C_{8\mathrm{GT}}) + \alpha_2 C_{9\mathrm{GT}} \pm x(C_2 + \beta_3 C_3) .
\end{aligned} \tag{B2}$$

The upper and lower sign refer to β^- and β^+ decay, respectively. Here $x = (2M)^{-1}b$, $B_4 = \mu_p - \mu_n \approx 4.7$, and the nuclear-matrix-element ratios are defined as follows:

$$\beta_1 = \langle r^2 T_{10}\rangle / \langle T_{10}\rangle , \tag{B3}$$

$$\beta_2 = \sqrt{2}\langle r^2 T_{12}\rangle / \langle T_{10}\rangle , \tag{B4}$$

$$\beta_3 = \sqrt{2}\langle T_{12}\rangle / \langle T_{10}\rangle , \tag{B5}$$

$$\alpha_1 = \langle ir T_{11}(\vec{\mathrm{p}})\rangle / \langle T_{10}\rangle , \tag{B6}$$

$$\alpha_2 = \langle ir Y_1 \vec{\sigma}\cdot\vec{\mathrm{p}}\rangle / \langle T_{10}\rangle \sqrt{3} , \tag{B7}$$

where $T^M_{J\Lambda}$ are standard tensor operators

$$T^M_{J\Lambda} = \sum_{\nu, m} (1\nu\Lambda m | JM)\, \sigma_1^\nu Y^M_\Lambda . \tag{B8}$$

In α_1 the operator $\vec{\sigma}$ is replaced by the operator $\vec{\mathrm{p}}$.

Combinations of lepton wave functions are as follows:

$$C_{1\mathrm{GT}} = L_0 , \tag{B9}$$

$$C_{2\mathrm{GT}} = -\tfrac{1}{3} q^2 L_0 - \tfrac{2}{9} q N_0 , \tag{B10}$$

$$C_{3\mathrm{GT}} = -\tfrac{4}{9} q N_0 , \tag{B11}$$

$$C_{4\mathrm{GT}} = \frac{2}{3M}(P_0 - U L_0) , \tag{B12}$$

$$C_{5\mathrm{GT}} = \frac{1}{2M} C_3 = \frac{1}{3M}[P_0 - 3N_0 - (W - V)L_0] , \tag{B13}$$

$$C_{6\mathrm{GT}} = \frac{2}{M}(\tfrac{1}{3} q L_0 + N_0) , \tag{B14}$$

$$C_{7\mathrm{GT}} = \frac{1}{2M} C_2 = \frac{1}{3M}[(U + 2q)L_0 - P_0] , \tag{B15}$$

$$C_{8\mathrm{GT}} = C_{5\mathrm{GT}} , \tag{B16}$$

$$C_{9\mathrm{GT}} = \frac{2}{M}(\tfrac{1}{3} q L_0 - N_0) . \tag{B17}$$

All other notation has the usual meaning.

Equation (B2) as written corresponds to case B. In order to obtain case A, one should omit the term in the curly brackets, i.e., one should make the replacement

$$\{2\xi \pm E_0\} \to 0 .$$

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[39]This name is introduced for later reference. Some additional "relativistic corrections" to the axial-vector interaction term can be found in Appendix B.

[40]g_A is the axial-vector coupling constant; $Y=f_T(0)$; E_0 is the maximal lepton energy; $\xi=Z\alpha/2r_0$; $\vec{L}_4=\Psi_e^\dagger\vec{\sigma}\Psi_\nu$; and $L_4=\Psi_e^\dagger\gamma_5\Psi_\nu$.

[41]Apostrophes here indicate that we mean relativistic at the level of the Dirac equation and not at the level of field theory.

[42]The nucleon wave functions are not included explicitly.

[43]Nucleons interact only with the shell-model potential V, while possible residual two-body interactions are ignored at this level. This approximation is commonly assumed to underlie the impulse approximation.

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[47]Remember $E_0=E_i-E_f$. The term proportional to ξ appears automatically when the static Coulomb potential is introduced in Eq. (11). It appeared in Eq. (3) through the gauge-invariance arguments.

[48]It is of the order of magnitude 10; see Sec. VI in Ref. 17.

[49]Remember that the other terms appearing in Eq. (4) have been cancelled out.

[50]Note the difference between the vector and the axial-vector contributions exhibited through the identities

$$\bar{u}(p')\sigma_{\mu\nu}(p-p')_\nu\begin{pmatrix}1\\ \gamma_5\end{pmatrix}u(p) = \bar{u}(p')\left|\begin{pmatrix}2m\gamma_\mu\\ 0\end{pmatrix}+i(p'+p)_\mu\begin{pmatrix}1\\ \gamma_5\end{pmatrix}\right|u(p)\,.$$

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thus somewhat obscuring the whole situation.

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[68]See the terms multiplying α_2 in Eq. (B2).

PHYSICAL REVIEW C VOLUME 6, NUMBER 1 JULY 1972

Isospin Impurity of the 6.88-MeV State in ^{10}B

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(Received 25 June 1971)

The γ-ray yield for the reaction ^{9}Be$(p,\gamma)^{10}$B has been measured over the range of bombarding energies 0.20–0.85 MeV and the radiative decay of the 6.88-MeV resonance level in ^{10}B studied. A large nonresonant contribution to the reaction cross section has been identified and compared with calculated cross sections based on an extranuclear direct-capture process. The previously measured high isospin impurity of the 6.88-MeV state is consequently significantly reduced; the amplitudes for the $T=0$ and $T=1$ parts of the wave function for this state are determined to be 0.92 and 0.39, respectively.

I. INTRODUCTION

A substantial effort has been directed towards studying the level structure of the nuclei ^{10}Be and ^{10}B. The excitation region between 5.0 and 8.0 MeV in these nuclei has been of particular interest,[1-3] but the investigation of levels is made difficult by the high density of broad states in ^{10}B in this region. In addition, the 1^- level in ^{10}B at 6.88 MeV with nominal isospin zero has been measured to have a high $T=1$ admixture,[3,4] viz. more than 25%.

The present work was undertaken to help clarify the situation in ^{10}B and to search for another 1^- level lying close to the 6.88-MeV state but with $T=1$ which would give rise to the high isospin impurity.

Cooper *et al.*[5] and Roush *et al.*[1] have proposed that the 7.44-MeV level in ^{10}B (corresponding to a proton excitation energy of 948 keV) has $J^\pi=1^-$ and is the $T=1$ analog of the 1^- state at 5.96 MeV in ^{10}Be. It is further suggested that the $T=1$ strength of the 5.96-MeV state is divided between the two 1^- levels in ^{10}B; the states at 6.88 and 7.44 MeV would then be expected to be highly isospin mixed, a situation similar to that of the well-studied[6] 16.63-16.93-MeV doublet in ^{8}Be.

In the present investigation, a careful study of the γ decay of the 6.88-MeV level in ^{10}B was made with a high-resolution Ge(Li) spectrometer. Excitation curves for the various decay modes were measured and branching ratios obtained. No evidence was found for a 1^- level in ^{10}B lying closer to the 6.88-MeV state than the 7.44-MeV level.

II. SELECTION RULES

Self-conjugate nuclei have been used extensively for studying the isospin properties of nuclear levels, since several electromagnetic transition rules apply. In particular, $E1$ and $M1$ transitions between states of the same isospin in these nuclei are known to be strongly inhibited.[7-9] The energy level diagram of ^{10}B is given in Fig. 1; branching ratios and other level parameters are weighted means from previous work.[4]