

Bremsstrahlung in α decay

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A quantum mechanical analysis of the bremsstrahlung in α decay of ^{210}Po is performed in close reference to a semiclassical theory. We clarify the contribution from the tunneling, mixed, outside barrier regions, and from the wall of the inner potential well to the final spectral distribution, and discuss their interplay. We also comment on the validity of semiclassical calculations, and the possibility to eliminate the ambiguity in the nuclear potential between the alpha particle and daughter nucleus using the bremsstrahlung spectrum. [S0556-2813(99)50402-0]

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Bremsstrahlung during α decay and fission is one of the intriguing current topics of nuclear physics. Also, it will shed light on macroscopic quantum tunneling, which has been a very popular subject in the past decades in many fields of physics and chemistry. Kasagi *et al.* [1] measured the bremsstrahlung in the α decay from ^{210}Po , and showed that the emission of low energy photons is systematically hindered from what one expects from the pure Coulomb acceleration of the α particle in the outside region of the potential barrier. They speculated that this reduction is caused by a destructive interference between the radiations in the tunneling and classically allowed regions, and thus might offer a possibility to learn about the tunneling time. They also calculated the spectral distribution based on a semiclassical theory of Dyakonov and Gornyi [2]. Their calculations reproduce the experimental cross section well for low energy photons. In addition, an interesting thing is that their calculations give a hump in agreement with the measurement, which however has large error bars in that region. On the other hand, the intensity of the radiation observed by D'Arrigo *et al.* [3] for the alpha decay from ^{226}Ra and ^{214}Po is larger than the prediction of the pure Coulomb acceleration model. Also, their data show no structure in the spectral distribution as expected from a classical formula. In nuclear fission, a measurement by Luke *et al.* [4] gave an upper bound to the bremsstrahlung rate for the spontaneous fission of ^{252}Cf .

Recently, Papenbrock and Bertsch performed a quantum mechanical calculation for the bremsstrahlung in α decay in perturbation theory [5]. Their spectral distribution is monotonic, though it is within the error bars of the data in Ref. [1]. The authors claim that the contribution from the tunneling wave function under the barrier is small. However, the definition of either classical or tunneling is not unique. More studies from various points of view are clearly needed in order to clarify the situation. The aim of this paper is to resolve these puzzling problems relating to the spectral distribution, especially on the role of the tunneling region. To this end, we perform a quantum mechanical analysis in a way which has a close relation to semiclassical theory. As we show later, quantum mechanical calculations are required

to have quantitative accuracy, while a semiclassical theory provides a clear physical understanding of the phenomena by bridging the quantum mechanical and classical calculations of the bremsstrahlung. It also provides a more definitive understanding of the role of quantum tunneling by naturally dividing the whole region into the classical, mixed, and tunneling regions through classical turning points.

Fermi's golden rule gives the following expression for the photon emission per photon energy during α decay in the dipole approximation:

$$\frac{dP}{dE_\gamma} = \frac{4Z_{\text{eff}}^2 e^2}{3m^2 c^3} |\langle \Phi_f | \partial_r V | \Phi_i \rangle|^2 \frac{1}{E_\gamma}, \quad (1)$$

where $V(r)$ is the interaction between the α particle and the daughter nucleus as a function of their separation distance r , m is their reduced mass, and E_γ is the photon energy. The effective charge Z_{eff} is given by $Z_{\text{eff}} = (A_D Z_\alpha - A_\alpha Z_D) / A_P$, where A_D , Z_D , A_α , Z_α , and A_P are the mass and atomic numbers of the daughter nucleus and of α particle, and the mass number of the parent nucleus, respectively. $\Phi_i(r)$ and $\Phi_f(r)$ are the appropriately normalized radial wave functions of the initial and final states of the α particle, respectively [5].

As in Ref. [5], we take the following simple model for the interaction between the α particle and the daughter nucleus:

$$V(r) = (Z_\alpha Z_D e^2 / r) \Theta(r - r_0) - V_0 \Theta(r_0 - r). \quad (2)$$

The initial and final state wave functions are given by

$$\Phi_i = \begin{cases} (m/\hbar k_i)^{1/2} R_i(r)/r & (r > r_0), \\ N_i j_0(K_i r) & (r < r_0), \end{cases} \quad (3)$$

$$\Phi_f = \begin{cases} (2m/\pi\hbar^2 k_f)^{1/2} R_f(r)/r & (r > r_0), \\ N_f j_1(K_f r) & (r < r_0), \end{cases} \quad (4)$$

where $R_i(r) = G_0(\eta_i, k_i r) + iF_0(\eta_i, k_i r)$ and $R_f(r) = F_1(\eta_f, k_f r) \cos \alpha + G_1(\eta_f, k_f r) \sin \alpha$, α being the phase shift. The wave numbers in these equations are given by

$$K_i = \hbar^{-1} \sqrt{2m(E_i + V_0)}, \quad k_i = \hbar^{-1} \sqrt{2mE_i}, \quad (5)$$

$$K_f = \hbar^{-1} \sqrt{2m(E_f + V_0)}, \quad k_f = \hbar^{-1} \sqrt{2mE_f}, \quad (6)$$

where $E_i = E_\alpha$, E_α is the energy of the α decay, $E_f = E_\alpha - E_\gamma = E_\alpha - \hbar\omega$ and η the Sommerfeld parameter.

The transition matrix in Eq. (1) now reads

$$\langle \Phi_f | \partial_r V | \Phi_i \rangle = \sqrt{\frac{2m^2}{\pi \hbar^3 k_i k_f}} \left\{ I_W + \int_{r_0}^{\infty} dr A(r) R_f(r) R_i(r) \right\}, \quad (7)$$

where $A(r)$ stands for $\partial_r V = -Z_\alpha Z_D e^2 r^{-2}$, and the wall contribution I_W is given by

$$I_W = (Z_\alpha Z_D e^2 / r_0 + V_0) R_f(r_0) R_i(r_0). \quad (8)$$

If we denote the external turning points in the initial and final states by r_{ei} and r_{ef} , respectively, the integration in Eq. (7) can be naturally divided into three parts, i.e., the integrations between r_0 and r_{ei} , between r_{ei} and r_{ef} , and between r_{ef} and ∞ . We call them the tunneling, mixed, and outside regions, respectively. Straightforward calculations of each integral using the exact Coulomb wave functions will clarify the role of each term, especially the role of the tunneling region. Before we present the results of such quantum mechanical calculations, we wish to present semiclassical formulas for radiation.

They can be derived by replacing the quantum mechanical Coulomb wave functions by their semiclassical representations, which read [6]

$$F_l \sim \begin{cases} \left(\frac{k}{k_l(r)} \right)^{1/2} \sin \left[\int_{r_e}^r k_l(r) dr + \frac{\pi}{4} \right] & (r > r_e), \\ \frac{1}{2} \left(\frac{k}{\kappa_l(r)} \right)^{1/2} \exp \left[- \int_r^{r_e} \kappa_l(r) dr \right] & (r < r_e), \end{cases} \quad (9)$$

$$G_l \sim \begin{cases} \left(\frac{k}{k_l(r)} \right)^{1/2} \cos \left[\int_{r_e}^r k_l(r) dr + \frac{\pi}{4} \right] & (r > r_e), \\ \left(\frac{k}{\kappa_l(r)} \right)^{1/2} \exp \left[\int_r^{r_e} \kappa_l(r) dr \right] & (r < r_e), \end{cases} \quad (10)$$

where r_e is the external classical turning point, and k_l and κ_l are the wave numbers for the orbital angular momentum l in the classically allowed and forbidden regions, respectively. Notice that G_l and F_l increase and decrease exponentially, respectively, as r moves deeper inside the tunneling region. We remark that the phase shift α in Eq. (4) is proportional to the tunneling probability and is very small. This can be proved by matching the semiclassical wave function under the potential barrier to the wave function in the region of the potential well. We thus express $\tan \alpha$ as

$$\tan \alpha = C_\alpha P_{tf}, \quad P_{tf} = \exp \left[-2 \int_{r_0}^{r_{ef}} \kappa_{1f}(r) dr \right], \quad (11)$$

where P_{tf} is the tunneling probability in the final state in the WKB approximation. In Eq. (11) and in what follows, the

meaning of the lower indices will be obvious, e.g., κ_{1f} is the wave number with angular momentum 1 in the final state.

We now derive the semiclassical formulas for the contribution to the integral in Eq. (7) from the tunneling, mixed, and outside regions separately.

(1) *Tunneling region.* The main contribution from the tunneling region comes from the $F_1 \cdot G_0$ term, whose semiclassical representation is given by

$$\begin{aligned} & \int_{r_0}^{r_{ei}} A(r) F_1(\eta_f, k_f r) G_0(\eta_i, k_i r) dr \\ & \sim \frac{1}{2} \sqrt{k_i k_f} \exp \left[- \int_{r_{ei}}^{r_{ef}} \kappa_{1f}(r) dr \right] \\ & \quad \times \int_{r_0}^{r_{ei}} A(r) \left(\frac{1}{\kappa_{0i}(r) \kappa_{1f}(r)} \right)^{1/2} \\ & \quad \times \exp \left[- \int_r^{r_{ei}} [\kappa_{1f}(r') - \kappa_{0i}(r')] dr' \right] dr \\ & \sim \frac{\hbar \sqrt{k_i k_f}}{2m} \exp \left[- \int_{r_{ei}}^{r_{ef}} \kappa_{1f}(r) dr \right] \\ & \quad \times \int_0^{T_i} A[r(\tau_i)] \left(\frac{\kappa_{0i}[r(\tau_i)]}{\kappa_{1f}[r(\tau_i)]} \right)^{1/2} \\ & \quad \times \exp \left[- \frac{1}{\hbar} \int_{\tau_i}^{T_i} \left(\frac{\hbar^2 1(1+1)}{2m[r(\tau'_i)]^2} + \hbar\omega \right) d\tau'_i \right] d\tau_i \end{aligned} \quad (12)$$

In obtaining the last expression, we expanded the $\kappa_{1f}(r) - \kappa_{0i}(r)$ up to the leading order of $\hbar^2 1(1+1)/2m[r(\tau')^2] + \hbar\omega$ and introduced the time parameter along the (imaginary) time axis τ_i by

$$\tau_i(r) = \int_{r_0}^r \frac{dr}{v_i(r)}, \quad v_i(r) = \sqrt{\frac{2}{m} \left(\frac{Z_\alpha Z_D e^2}{r} - E_i \right)}. \quad (14)$$

$T_i = \tau_i(r_{ei})$ is the tunneling time in the initial channel. The lower index i of τ_i shows that the time is related to the distance r through the velocity in the initial state. Notice that the radiation amplitude in this region is given by the Laplace transform of the acceleration if one could ignore the centrifugal term. The $G_1 \cdot G_0$ term is the largest as long as the integral itself is concerned. However, the contribution of this term is reduced by the small value of $\tan \alpha$ in front of G_1 . We have a similar expression for the $G_1 \cdot F_0$ term, which is also expected to be small. The $F_1 \cdot F_0$ term is expected to be the smallest.

(2) *Mixed region.* The mixed region, which is classically allowed in the initial state but forbidden after radiation, is beyond the scope of classical calculations in Refs. [1] and [4]. Although quantum mechanical calculations are required to quantitatively estimate the contribution from this region as we show later, it is still interesting to see its semiclassical representation. The contribution of the F_1 term that dominates in this region reads,

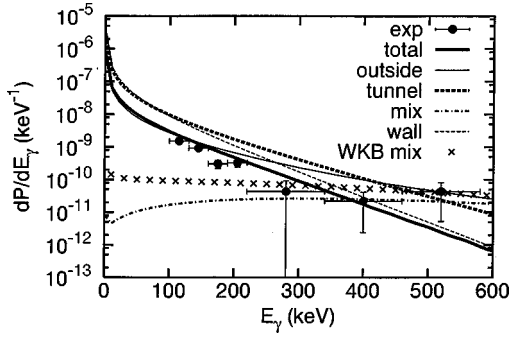


FIG. 1. Total photon emission probability and its decomposition. The potential parameters are $V_0=21.37$ MeV and $r_0=8.055$ fm. The crosses are the mixed region contribution calculated semiclassically. All the other lines have been obtained quantum mechanically. The data are taken from Ref. [1].

$$\begin{aligned} & \int_{r_{ei}}^{r_{ef}} A(r) F_1(\eta_f, k_f r) R_i(r) dr \\ & \sim \frac{1}{2} \int_{r_{ei}}^{r_{ef}} A(r) \left(\frac{k_i k_f}{k_{0i}(r) \kappa_{1f}(r)} \right)^{1/2} \\ & \quad \times \exp i \left[\int_{r_{ei}}^r k_{0i}(r') dr' + \frac{\pi}{4} \right] \\ & \quad \times \exp \left[- \int_r^{r_{ef}} \kappa_{1f}(r') dr' \right] dr. \quad (15) \end{aligned}$$

We show later that this region strongly influences the behavior of the spectral distribution at high energies.

(3) *Outside the barrier.* The main contribution in the classical region, i.e., outside the barrier, comes from the F_1 term. Its semiclassical expression reads

$$\begin{aligned} & \int_{r_{ef}}^{\infty} A(r) F_1(\eta_f, k_f r) R_i(r) dr \\ & \sim - \frac{\hbar \sqrt{k_i k_f}}{2im} \exp \left[i \int_{r_{ei}}^{r_{ef}} k_{0i}(r) dr \right] \\ & \quad \times \int_0^{\infty} A[r(t_f)] \left(\frac{k_{1f}[r(t_f)]}{k_{0i}[r(t_f)]} \right)^{1/2} \\ & \quad \times \exp \left[\frac{i}{\hbar} \int_0^{t_f} \left(\frac{\hbar^2 1(1+1)}{2m[r(t_f')]^2} + \hbar \omega \right) dt_f' \right] dt_f. \quad (16) \end{aligned}$$

This is nothing but the classical formula for the bremsstrahlung except for the existence of the centrifugal potential term. The lower index f of t_f means that the time t is related to the distance r through the velocity in the final state. We have ignored the term where the action integrals in the initial and final states enter with the same sign, because the sign of the integrand in that case rapidly changes as a function of r and the value of integration becomes negligibly small.

We next consider the contribution from the wall. This term has been overlooked in the calculations reported in Ref. [1]. As we clearly demonstrate later, this is one of the key issues to resolve the discrepancy between the calculations in

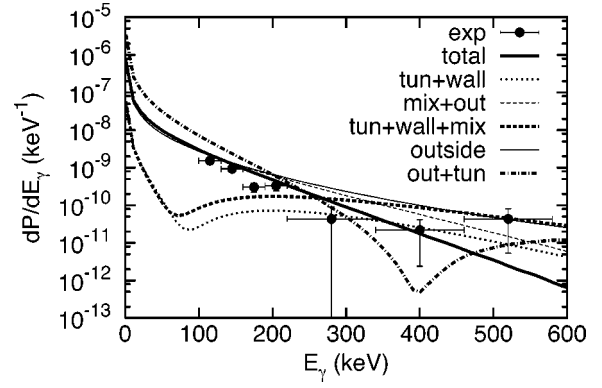


FIG. 2. Grouping of each component into classical and tunneling contributions. The potential parameters are the same as those for Fig. 1.

Refs. [1] and [5]. The contribution of the F_0 term can be safely ignored unless the energy of the α particle is very close to the top of the potential barrier. Using Eq. (11), the remaining terms can be expressed as

$$\begin{aligned} & R_f(r_0) G_0(\eta_i, k_i r_0) \\ & \sim \frac{1}{2} \left(\frac{k_f k_i}{\kappa_{1f}(r_0) \kappa_{0i}(r_0)} \right)^{1/2} \cos \alpha (1 + 2C_\alpha) \\ & \quad \times \exp \left[- \int_{r_{ei}}^{r_{ef}} \kappa_{1f}(r) dr \right] \\ & \quad \times \exp \left[- \frac{1}{\hbar} \int_0^{T_i} \left(\frac{\hbar^2 1(1+1)}{2m[r(\tau_i)]^2} + \hbar \omega \right) d\tau_i \right]. \quad (17) \end{aligned}$$

We now analyze the bremsstrahlung in the α decay from ^{210}Po , where $E_\alpha=5.3$ MeV. We determine the potential parameters in the same way as in Ref. [5], i.e., by matching the quantum mechanical wave functions at r_0 , and by calculating the decay width by normalizing the current at r_e to the probability inside. An alternative is to use the semiclassical quantization rule and the R -matrix theory for the decay rate [see Eqs. (3) and (4) in Ref. [7]]. We found, however, that the semiclassical quantization rule is not valid with significant influences on the bremsstrahlung spectrum (see later and Ref. [8]).

Figure 1 shows the spectral distribution calculated quantum mechanically with one of the potential parameter sets, $V_0=21.37$ MeV and $r_0=8.055$ fm, which gives 5 nodes to the wave function for the radial motion. The general behavior of the theoretical results do not depend so much on the particular choice of the potential parameters (see below, however). This potential parameter set corresponds to that in Refs. [5,9]. The thick solid line represents the total photon emission probability. It is a monotonically decreasing function and is consistent with the data except for the last data point. The figure also shows that various components have nearly the same order of magnitude, and that the final spectrum is the result of a complicated interference among them. In order to see the role of tunneling and classical regions and the interplay of each contribution more clearly, we show in Fig. 2 spectral distributions by dividing into the classical and tunneling contributions. We present two different combinations depending on whether we consider the mixed region

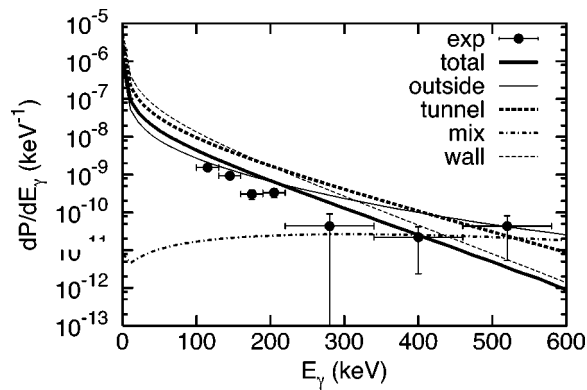


FIG. 3. The same as Fig. 1, but for $V_0=107.3$ MeV and $r_0=7.947$ fm.

classical or tunneling. We call the part containing the outside barrier component the classical. In either case, the soft photon emission is dominated by classical contributions. This follows from the strong cancellation between the wall contribution and the contribution from the integral in the tunneling region. If the potential has a finite slope at the inner turning point and if one could ignore the centrifugal term, the net contribution from the tunneling region should be zero in the soft photon limit. Our result matches with this general aspect. For high energy photons, the classical and the tunneling contributions have nearly the same magnitude and interfere destructively leading to a much smaller photon emission than the prediction of the classical theory. The dot-dashed line in Fig. 2 shows the photon emission probability obtained by summing the contributions from the outside barrier and tunneling regions. Interestingly, this has a hump at high photon energies, just like that discussed in Ref. [1], which overlooked the wall contribution and ignored the contribution from the mixed region. However, this interesting interference pattern is hidden by these contributions.

As is well known, the resonance position and the decay width do not uniquely determine the potential. Each potential gives a different value for the coefficient C_α in our formalism. An interesting question is then whether the bremsstrahlung can be used to give additional constraints to the potential. If one restricts to the simple square well potential in Eq. (2), various sets correspond to different number of nodes n in the radial wave function inside the potential well. We found that the spectral distribution is almost indistinguishable as long as $n \leq 7$. For larger values of n , one starts to notice a difference in that the soft photon emission probability is predicted to be recognizably larger than the prediction of the classical theory. As an example, we show in Fig. 3 the spectral distribution calculated by the potential set, $V_0=107.3$ MeV and $r_0=7.947$ fm, which corresponds to $n=11$ implied by the number of redundant states due to Pauli principle in a simple cluster model. Notice that the total photon spectrum (thick solid line) noticeably deviates from the classical spectrum (thin solid line) for photon energies smaller than 150 keV in contrast to Fig. 1. In this respect, it would be extremely interesting to extend the data towards lower photon energies to examine whether the shallow or the deep potential agrees better with the data of bremsstrahlung. C_α gradu-

ally decreases from 0.041 to -0.055 , and from -0.25 to -0.28 for the potentials in Figs. 1 and 3, respectively, as E_γ changes from 0 to 600 keV. Note that the emission probability of zero energy photons is not influenced by the potential parameters as discussed in Ref. [5]. We have confirmed that our numerical calculations reproduce the emission probability of zero energy photons given by Eq. (19) of Ref. [5] for both potential sets for Figs. 1 and 3. We found also that the finite spin of photons plays a negligible role in this respect.

We finally comment on the validity of semiclassical calculations. We already mentioned a problem with the semiclassical quantization rule in determining the potential parameters. Since C_α is sensitive to the slope of the wave function at the matching point, it causes a serious error in properly describing the bremsstrahlung spectrum. We found that the wall contribution itself gets very small because of the cancellation between the F_1 and G_1 terms in Eq. (17) if we adopt any potential set determined by using the semiclassical quantization rule. Consequently, the contribution from the tunneling region dominates the spectrum at soft photons giving larger cross section than the classical prediction. Another problem is that two turning points r_{ei} and r_{ef} lie too close to each other to use naive semiclassical wave functions. The crosses in Fig. 1 represent the contribution from the mixed region calculated semiclassically. The deviation from the dot-dashed line clearly shows the failure of the semiclassical calculations. Since the contribution from the mixed region strongly influences the high energy spectrum, this is a serious problem. The semiclassical calculations reproduce the qualitative behavior of the quantum mechanical results quite well for the contributions from the regions under and outside the barrier, but have some quantitative inaccuracy especially at high energies.

In summary, our analysis shows that the final bremsstrahlung spectrum results from a subtle interference of the contributions of the tunneling, mixed, and classical regions as well as the wall of the potential well, each of which has a comparable magnitude. Semiclassical as well as classical theories seem not to be reliable for describing these subtle interference effects, though they give some clear understanding of the phenomena. It would be very interesting if one could perform more exclusive experiments to pick up each contribution separately. The extension of data to more soft photon side is awaited to provide a stringent test of the potential between the alpha particle and the daughter nucleus. Our study is based on the assumption of the validity of the potential model. Though this assumption seems to work in our analysis, it is a very interesting question to examine in more detail whether the bremsstrahlung spectrum contains some information beyond the scope of the potential model such as the validity of the R-matrix theory and the preformation factor.

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