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Equation of state of finite nuclei and liquid-gas phase transition

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We construct the equation of state (EOS) of finite nuclei including surface and Coulomb effects in a Thomas-Fermi framework using a finite range, momentum and density dependent two-body interaction. We identify critical temperatures for nuclei below which the EOS so constructed shows clear signals for liquid-gas phase transition in these finite systems. Comparison with the EOS of infinite nuclear matter shows that the critical density and temperature of the phase transition in nuclei are influenced by the mentioned finite size effects. [S0556-2813(99)50301-4]

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The equation of state of infinite nuclear matter calculated in the mean-field approximation [1,2] shows a typical Van der Waals behavior. Below the critical temperature T_c ($\simeq 16$ MeV), the liquid and the gas phases are seen to coexist. A self-consistent determination of the EOS of a finite nuclear system is, however, still not available. The nature of the phase transition or even its occurrence may depend on the surface effects as well as on the Coulomb interaction between the protons. From the experimental side, the mass or charge distributions from energetic proton [3] or heavy ion induced reactions [4,5] show a power law behavior [6] indicating a possible liquid-gas phase transition [7] in finite nuclei. Recent experimental data on caloric curve also shows strong signals for liquid-gas phase transition. In the GSI data [8] for Au+Au at 600A MeV the temperature remains practically constant at $\simeq 5$ MeV in the excitation energy range of 3–10 MeV per nucleon beyond which the excitation energy increases linearly with temperature as in a classical gas which is suggestive of a sharp phase transition. Analysis of the data from the EOS Collaboration [9,10] for Au+C at 1A GeV also indicates a liquid-gas phase transition in finite nuclear systems. It would therefore be of utmost importance to investigate the EOS of finite nuclei in a self-consistent approach that would help in getting a clearer picture of the occurrence of phase transition in finite systems. An attempt in this direction is made in the Thomas-Fermi (TF) framework [11] in the present paper.

In the TF framework, the energy density of a nucleus of mass number A and proton number Z is constructed from a Seyler-Blanchard type [12] momentum and density dependent effective interaction [2,13] of finite range. The interaction is given by

$$v_{eff}(r,p,\rho)=C_{l,u}[v_1(r,p)+v_2(r,\rho)], \tag{1}$$

$$v_1=-\left(1-\frac{p^2}{b^2}\right)f(r), \tag{2}$$

$$v_2=d^2[\rho(r_1)+\rho(r_2)]^n \tag{3}$$

with

$$f(r)=\frac{e^{-r/a}}{r/a}. \tag{4}$$

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Here $r=|\mathbf{r}_1-\mathbf{r}_2|$ and $p=|\mathbf{p}_1-\mathbf{p}_2|$ are the separation of the two interacting nucleons in configuration and momentum space, and $\rho(r_1)$ and $\rho(r_2)$ are the densities at the sites of the two nucleons. The quantities C_l and C_u are the strengths of the interaction between like pair $(n-n)$ or $(p-p)$ and unlike pair $(n-p)$, respectively. The values of the potential parameters determined from a fit of the well-known bulk nuclear properties are given in Ref. [11]. The incompressibility of nuclear matter is then calculated to be 238 MeV. The Coulomb interaction energy density is given by the sum of the direct and exchange terms [11]. The energy density profile at a particular temperature T is then given by

$$\epsilon(r)=\sum_{\tau}\rho_\tau(r)\{TJ_{3/2}[\eta_\tau(r)]/J_{1/2}[\eta_\tau(r)] \times[(1-m^*_\tau)(r)V^1_\tau(r)]+\tfrac{1}{2}V^0_\tau(r)\}. \tag{5}$$

Here τ is the isospin index, the J's are the Fermi integrals, and V^0_τ is the single-particle potential which includes the Coulomb term for protons. The momentum dependence in the interaction gives rise to V^1_τ which determines the effective nucleon mass m^*_τ. The fugacity $\eta_\tau(r)$ is defined by

$$\eta_\tau(r)=[\mu_\tau-V^0_\tau(r)-V^2_\tau(r)]/T, \tag{6}$$

where μ_τ is the chemical potential and V^2_τ is the rearrangement energy term originating from the density dependence in the interaction. The total energy per particle is then given by

$$e(T)=\frac{1}{A}\int \epsilon(r)d\mathbf{r}. \tag{7}$$

The entropy per particle, from the Landau quasiparticle approximation, can be similarly calculated as

$$s(T)=\frac{1}{A}\int\sum_\tau \rho_\tau(r)\left[\frac{\frac{5}{3}J_{3/2}[\eta_\tau(r)]}{J_{1/2}[\eta_\tau(r)]}\right]d\mathbf{r}. \tag{8}$$

The free energy per particle is given by $f=e-Ts$. The density profiles of neutrons and protons at different confining volumes are determined self-consistently for a fixed temperature. The pressure is then determined from

$$P=-\left(\frac{\partial F}{\partial V}\right)_T, \tag{9}$$

where $F=Af$ is the total free energy and V stands for the confining volume of the nucleus. The isotherms at different temperatures can then be obtained. For the sake of comparison, the EOS of infinite nuclear matter is also calculated, expressions for which are simpler and are given in Ref. [2].

The EOS of infinite symmetric nuclear matter is displayed in the top panel of Fig. 1. It resembles closely that for the Van der Waals systems. The isotherms are shown for three temperatures, namely at $T=13.0$ MeV, 14.0 MeV, and at the critical temperature T_c which is found to be 14.5 MeV. In the isotherms the pressure is plotted as a function of $V/V_0=\rho_0/\rho$ where ρ_0 $(=0.153\,\mathrm{fm}^{-3})$ is the saturation density at zero temperature. The dotted line refers to the liquid-gas coexistence curve and the dashed line represents the spinodal line. We also display in the bottom panel the EOS of asymmetric nuclear matter with a representative asymmetry $X=(N-Z)/A=0.16$. The medium heavy nuclei, namely, ^{85}Kr and ^{150}Sm that we study in this paper have asymmetries close to 0.16. The critical temperature for this asymmetric nuclear matter decreases to $T_c=14.1$ MeV. The coexistence curve as well as the spinodal line are also presented.

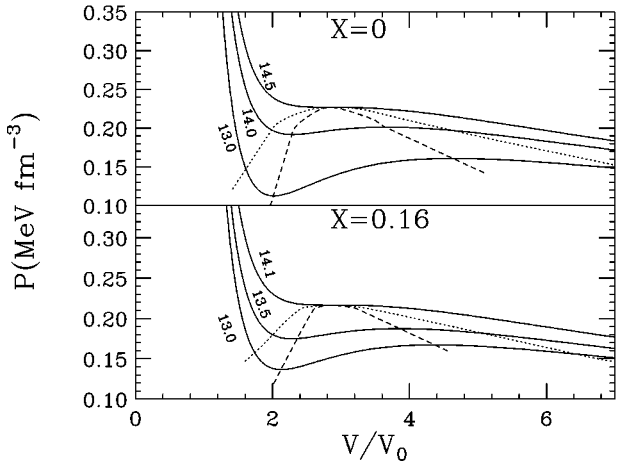

FIG. 1. The equation of state of symmetric nuclear matter (top panel) and of asymmetric nuclear matter with asymmetry $X=0.16$ (bottom panel). The temperatures (in MeV) for the isotherms are as marked in the figure. The dotted lines are the coexistence curves and the dashed lines are the spinodals.

The isotherms for the nucleus ^{85}Kr are displayed in Fig. 2 at four temperatures $T=9.5$, 10.5, 11.5, and 12.5 MeV, respectively. The critical temperature for this system is found to be $T_c=11.5$ MeV. Here the abscissa is V/V_0 where V_0 is the volume of the nucleus at zero temperature taken to be $V_0=\frac{4}{3}\pi r_0^3 A$ with $r_0=1.16$ fm and V is the confining volume in which the self-consistent density profiles are calculated. The isotherms for the finite nuclei are not much different in structure from those for infinite nuclear matter. The liquid-gas coexistence curve can be drawn following the Maxwell construction which is shown by the dotted line. The spinodals are represented by the dashed line. The notable difference between the spinodal line for infinite matter and that for the finite system lies in the rising part AO (Fig. 2) which is slanted backwards for the latter. In Fig. 3, the isotherms for

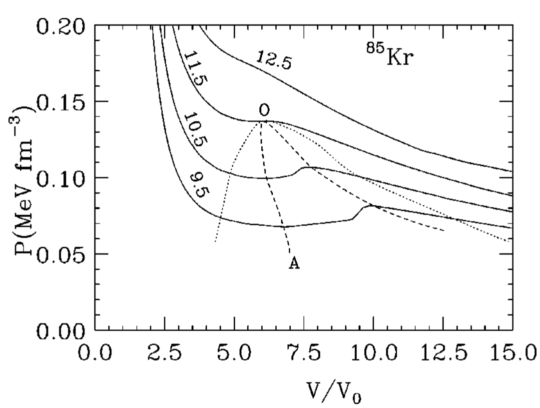

FIG. 2. The equation of state for the nucleus ^{85}Kr. The different notations used have the same meaning as in Fig. 1. For further details see the text.

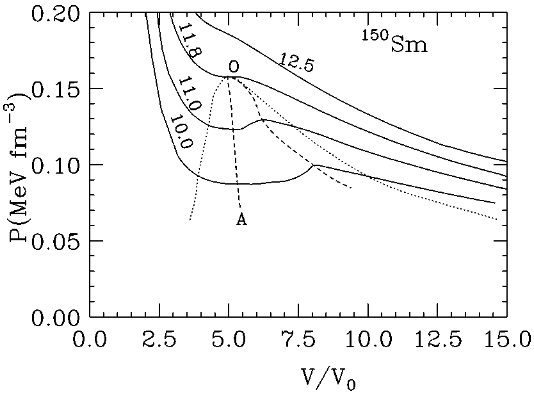

FIG. 3. Same as in Fig. 2, but for the nucleus ^{150}Sm.

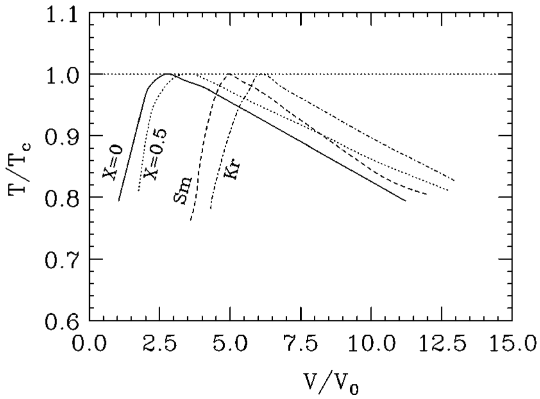

FIG. 4. The phase diagram for symmetric and asymmetric nuclear matter ($X=0$ and $X=0.5$) and for the nuclei ^{150}Sm and ^{85}Kr.

^{150}Sm are shown along with the coexistence curve and the spinodal line. The critical temperature for this nucleus is $T_c = 11.8$ MeV. The rising part AO of the spinodal line for this system is also a little backward slanted. The origin of this backward slant is traced back to the surface and Coulomb effects. We have checked that the slant changes from backward to forward with increasing size of the system.

The critical parameters T_c, P_c and the scaled critical volume V_c/V_0 for symmetric and asymmetric ($X=0.16$) nuclear matter and of the finite nuclei Kr and Sm are listed in Table I. To see the role of asymmetry more clearly on the critical parameters for infinite nuclear matter, the results with $X=0.5$ are also presented in Table I. With asymmetry, the critical temperature and pressure for the infinite system decreases, while the critical volume increases. For the finite systems, the critical temperature and pressure are considerably less than those for the infinite matter, while the scaled critical volume is significantly larger. The differences arise due to the surface and Coulomb effects. To delineate the surface effects, we repeated the calculations for Kr and Sm switching off the Coulomb interaction. From Table I, it is clear that both these effects act in the same direction. However, the mass region we consider shows the predominance of surface over the Coulomb effects.

In order to highlight the difference in the coexistence curves for finite nuclei and infinite nuclear matter, we display in Fig. 4 the phase diagram for infinite symmetric ($X=0$) and asymmetric ($X=0.5$) nuclear matter and for the nuclei ^{150}Sm and ^{85}Kr. The ordinate refers to the temperature scaled by T_c, the critical temperature corresponding to each of the four systems mentioned and the abscissa refers to the scaled volume V/V_0, obtained through Maxwell's construction from the respective isotherms. The full and the dotted lines refer to the coexistence curves for infinite matter with $X=0$ and 0.5, respectively; the dashed and dash-dot curves correspond to ^{150}Sm and ^{85}Kr. The increase in the critical volume with increasing asymmetry and decreasing mass is clear. It may be mentioned that in the phase diagrams we have displayed the scaled volume rather than the scaled density, as for a finite nucleus, the density is not constant in contrast to infinite nuclear matter. We may further mention that fluctuation effects due to finite number of particles may smear the phase transition, particularly near the critical temperature; blurring of the phase transition due to the finite size has been discussed in the literature earlier [14–16].

A discontinuity in the heat capacity C_v for an infinite system or a bump in C_v for a finite system [17] signals a phase transition. Our calculation also shows a peak structure in C_v (Fig. 5); the temperature at which peaking occurs depends on the choice of the confining volume. With increasing volume (beyond V_c) the transition temperature is smaller and the peak is sharper. We further see that for any specified confining volume larger than the critical volume, the transition temperature corresponds to that temperature the isotherm for which shows a maximum at the chosen volume. It is therefore obvious that the incompressibility at the transition temperature would vanish (or the compressibility would

TABLE I. Critical temperature, pressure, and volume for a few systems.

System	T_c (MeV)	P_c (MeV fm^{-3})	V_c/V_0
Symmetric nm	14.5	0.227	2.83
Asymmetric nm ($X=0.16$)	14.1	0.216	2.86
Asymmetric nm ($X=0.5$)	10.5	0.135	3.40
^{85}Kr (with Coulomb)	11.5	0.137	6.01
^{85}Kr (no Coulomb)	12.1	0.135	6.66
^{150}Sm (with Coulomb)	11.8	0.158	4.95
^{150}Sm (no Coulomb)	12.5	0.145	6.19

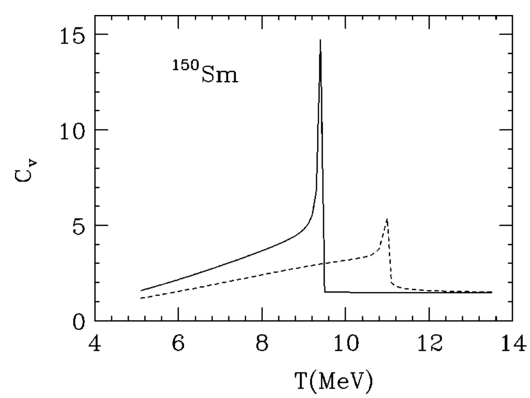

FIG. 5. The specific heat at constant volume C_v plotted as a function of temperature for the system ^{150}Sm. The full line and the dashed line correspond to the confining volumes $10V_0$ and $6V_0$, respectively.

show a singularity) which confirms further the occurrence of the phase transition. At this point on the isotherm, it is found that the density profile self-consistently evolves to a nearly uniform phase [18], the so-called low density gas phase that condenses out to the fragments of nuclei [6]. In an exact calculation, the transition should occur at the crossing of the coexistence curve [19] defined here by the Maxwell construction; our calculation done in the mean-field approach being an approximate one shows the transitions on the spinodal line at volumes larger than the critical volume. The dependence of the critical volume on the nuclear mass may have an important bearing in understanding the nature of phase transition deduced from the shape of the fragment mass or charge distributions obtained in statistical models by fixing a ''freeze-out'' volume where the system is assumed to be homogeneous. The critical volume possibly defines its lower bound.

To summarize, we have calculated the equation of state of finite nuclei in a self-consistent mean-field theory including surface and Coulomb effects. The critical parameters for finite nuclei, namely, the critical temperature, pressure and volume are found to be system dependent and differ significantly from those corresponding to nuclear matter. The nature of the isotherms, the peaked structure of the heat capacity and the singularity in the compressibility as obtained unambiguously point to a liquid-gas phase transition in finite nuclear systems.

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