

# Mesons as $\bar{q}$ - $q$ bound states from Euclidean two-point correlators in the Bethe-Salpeter approach

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We investigate the two-point correlation function for the vector current. Gluons provide dressings for both the quark self-energy as well as the vector vertex function, which are described consistently by the rainbow Dyson-Schwinger equation and the inhomogeneous ladder Bethe-Salpeter equation. The form of the gluon propagator at low momenta is modeled by a two-parameter ansatz fitting the weak pion decay constant  $f_\pi$ . The quarks are confined in the sense that the quark propagator does not have a pole at timelike momenta. We determine the ground-state mass  $m_0$  in the vector channel from the Euclidean time Fourier transform of the correlator, which falls off as  $e^{-m_0 T}$  at large  $T$ .  $m_0$  lies around 590 MeV, and is almost independent of the model form of the gluon propagator. This method allows us to stay in Euclidean space and to avoid analytic continuation of the quark or gluon propagators into the timelike region. [S0556-2813(99)08302-8]

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## I. INTRODUCTION

The purpose of the present work is the determination of the ground-state mass of the vector  $\bar{q}q$  bound state within the Dyson-Schwinger (DS) and Bethe-Salpeter (BS) approach [1–3]. The starting point is the two-point correlation function

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq \cdot x} \langle \mathcal{T} J_\mu(x) J_\nu(0) \rangle \quad (1)$$

for the vector quark currents  $J_\mu(x) = \bar{q}(x) \gamma_\mu q(x)$ , with  $q(x)$  a quark field. Within the approximations used in the present paper, which will be described below,  $\Pi_{\mu\nu}$  is determined by the dressed quark propagator  $\langle \mathcal{T} \bar{q}(x) q(0) \rangle$  and the vertex three-point function  $\langle \mathcal{T} \bar{q}(x) J_\mu(y) q(0) \rangle$ . The objective of this work is to study the large-time behavior of the two-point correlator in Euclidean space in order to determine the mass of the lowest vector meson, as in lattice gauge calculations [4] and instanton models [5]. QCD sum rules also consider the Euclidean two-point correlator, but attempt to determine the ground-state mass by using the Borel transform to isolate the lowest mass pole [6]. Our method should be seen in contrast to an earlier approach, which solves the BS equation on shell [7].

We now briefly review the DS-BS approach used here. The nonperturbative nature of QCD at low and intermediate momentum transfers makes the use of effective models a natural and necessary tool. During the past few years, strong progress has been made in the framework of the DS approach [1–3], which is based on a coupled set of relations between dressed quark, gluon, and ghost propagators and vertex functions. In order to handle this system it is of course necessary to make certain simplifications and truncations. One approach, which is commonly referred to as *rainbow approximation*, uses an undressed quark-gluon vertex and

solves the Dyson-Schwinger equation (DSE) for the dressed quark propagator  $G(p)$ , with a given dressed gluon propagator  $D_{\mu\nu}^{ab}(p)$  as input (cf. Fig. 1):

$$\Sigma(p) = \int \frac{d^4q}{(2\pi)^4} g_s^2 D_{\mu\nu}^{ab}(p-q) \gamma_\mu \frac{\lambda_c^a}{2} G(q) \gamma_\nu \frac{\lambda_c^b}{2}. \quad (2)$$

The quark propagator  $G$  and the quark self-energy  $\Sigma$  are related by

$$G(p)^{-1} = i \gamma \cdot p + \Sigma(p). \quad (3)$$

The form of the gluon propagator  $D_{\mu\nu}^{ab}(p)$  for small momenta is basically unknown from QCD, and therefore has to be modeled while following certain requirements which will be discussed below. In case of this special truncation it is possible to formulate an effective field theory using functional integral techniques, which is known as the global color model (GCM) [2,3,8]. This has the striking advantage that in the chiral limit (zero current mass for the light quarks, i.e.,  $M_u = M_d = M_s = 0$ ) one is able to establish an easy connection between the dressed quark self-energy, leading to dynamical chiral symmetry breaking on one side and the existence of massless Goldstone bosons as pseudoscalar  $\bar{q}q$  bound states on the other side. As a consequence, it is possible to perform a systematic chiral low energy expansion [9] in terms of composite Goldstone boson field and therefore make a connection between the phenomenological chiral

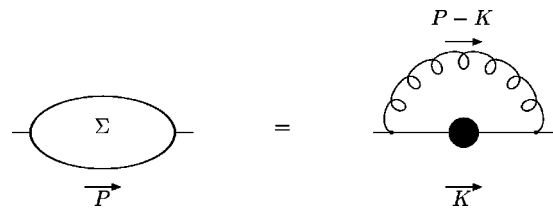


FIG. 1. Rainbow Dyson-Schwinger equation for the quark self-energy  $\Sigma$ .

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hadronic theory (chiral perturbation theory) [10,11] and an effective quark based theory of QCD, the GCM.

Furthermore one is also able to study the coupling of composite  $\bar{q}q$  systems to an external electromagnetic gauge field while guaranteeing local  $U(1)$  gauge invariance which is manifest in the corresponding Ward-Takahashi identity [12,7]. It turns out that the chiral invariance and electromagnetic current conservation, which are both essential for studying low-energy hadronic phenomena, are very difficult to maintain if one tries to go beyond the rainbow approximation [13], and are lost in general if model *Ansätze* for the quark-gluon vertex are used.

Though the rainbow approximation and the GCM naturally violate local color  $SU(3)_c$  gauge invariance and renormalizability, they provide a very successful description of

various nonperturbative aspects of strong interaction physics and the QCD vacuum as well as hadronic phenomena at low energies. These include, for instance, quark confinement [2], QCD vacuum condensates [8,14–16],  $U_A(1)$  breaking, and the  $\eta$ - $\eta'$  splitting [17], low-energy chiral dynamics of Goldstone bosons ( $\pi, K, \eta$ ) [18,9,19–21,16], meson form factors [22], heavy-light mesons [23], systems at finite temperature [24], or soliton [25] and Fadeev [26] descriptions of the nucleon.

The method is more involved when applied to higher mass states instead of Goldstone bosons, e.g.,  $\rho$ ,  $\omega$ ,  $\sigma$ , or  $A$  mesons. A mesonic  $\bar{q}q$  bound state with mass  $m$  in a channel with spin-flavor index  $\theta$  is determined in the ladder approximation as a solution of the *homogeneous* Bethe-Salpeter equation (BSE) [7]:

$$\Omega_\theta(P, q^2 = -m^2) = \int \frac{d^4K}{(2\pi)^4} g_s^2 D_{\mu\nu}^{ab}(P-K) \gamma_\mu \frac{\lambda_c^a}{2} G(K_+) \Omega_\theta(K, q^2 = -m^2) G(K_-) \gamma_\nu \frac{\lambda_c^b}{2}, \quad (4)$$

where we have used the notation  $K_\pm = K \pm q/2$ .  $\Omega_\theta(P, q)$  is related to the Bethe-Salpeter amplitude  $\chi_\theta(P, q)$  as

$$\chi_\theta(P, q^2 = -m^2) = G(P_+) \Omega_\theta(P, q^2 = -m^2) G(P_-). \quad (5)$$

As mentioned above, in the pseudoscalar channel the existence of a massless state, or, in other words a solution of Eq. (4) at  $q^2=0$ , is automatically guaranteed in the chiral limit if the rainbow DSE (2) is satisfied. For a small finite current quark mass  $M_0$ , a perturbative expansion in  $M_0$  and  $q^2$  can be performed. This is, however, not possible for larger meson masses, and therefore an explicit numerical solution of the corresponding homogeneous BSE (4) is required. The trouble hereby is that the meson momentum is defined in the timelike region,  $q^2 = -m^2$ , which in turn requires the dressed quark propagator  $G(p)$  in the timelike region. However,  $G(p)$  is *a priori* only determined at spacelike  $p$  from the DSE (2), because the model gluon propagator  $D(p)$  is defined only for spacelike Euclidean momenta. It is therefore necessary to perform an analytic continuation of the dressed quark propagator  $G(p)$  from Euclidean space into Minkowski space  $p_4 \rightarrow ip_0$  [7]. This is a very dangerous and unsafe procedure, because it requires a knowledge of the singularity structure of  $G(p)$  in the whole complex plane, which we do not have. In Ref. [27] an alternative method has been proposed which stays completely in Euclidean space, avoiding analytic continuations of any of the dressed propagators. It is the analog to what is used in instanton models [5] and lattice QCD calculations [4] where working in Euclidean space is essential.

The two-point correlation function

$$\Pi_{\theta_1\theta_2}(q) = \int d^4x e^{iq \cdot x} \langle T J_{\theta_1}(x) J_{\theta_2}(0) \rangle, \quad (6)$$

with quark currents  $J_\theta = \bar{q} T_\theta q$ , where  $\theta$  stands for the spin flavor of the operator  $T_\theta$ , is determined as a quark loop containing the dressed vertex function  $\Gamma_\theta(P, q)$  (cf. Fig. 2 for  $T_\theta = \gamma_\mu$ ).  $\Gamma_\theta(P, q)$  itself is given as the solution of the *inhomogeneous* ladder BSE (cf. Fig.3):

$$\Gamma_\theta(P, q) = -iT_\theta + \int \frac{d^4K}{(2\pi)^4} g_s^2 D_{\mu\nu}^{ab}(P-K) \times \gamma_\mu \frac{\lambda_c^a}{2} G(K_+) \Gamma_\theta(K, q) G(K_-) \gamma_\nu \frac{\lambda_c^b}{2}, \quad (7)$$

with the inhomogeneity  $-iT_\theta$ . This form can also be written as

$$\Gamma_\theta(P, q) = -iT_\theta + \Gamma_\theta(P, q)^{\text{NP}}, \quad (8)$$

where  $T_\theta$  is the bare vertex and  $\Gamma_\theta(P, q)^{\text{NP}}$  is the nonperturbative (NP) dressed vertex. The nonperturbative parts of the vertex functions can be constrained by the forms of the non-local condensates [28], which also has been used in our recent work [15]. Moreover, it has been shown that this formalism can be used for the nonperturbative part of the vector vertex function used at low momentum transfer for nuclear

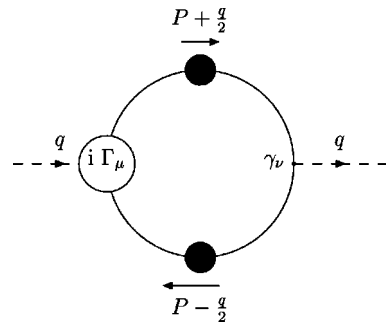


FIG. 2. Two-point correlator  $\Pi_{\nu\mu}$  in the vector channel.

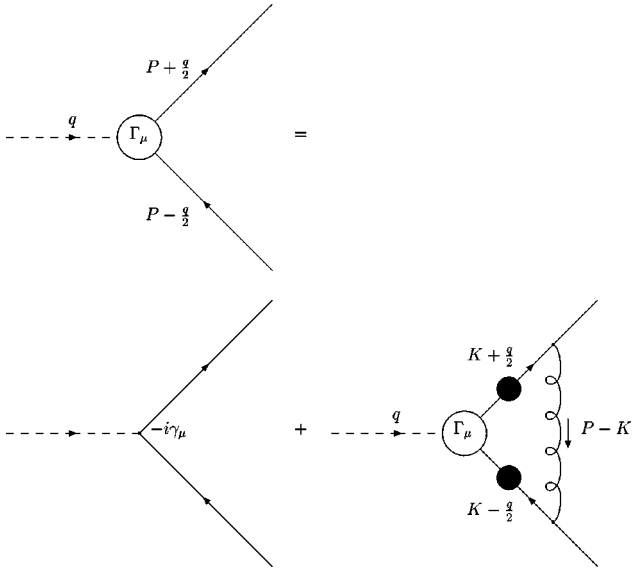


FIG. 3. Inhomogeneous ladder Bethe-Salpeter equation for the vector vertex function  $\Gamma_\mu$ .

magnetic dipole moments [29]. This gives possible constraints for the dressed vertex function treated in the present work, which we discuss below.

The momentum  $q$  is in general off shell and can be either space or timelike. Close to the mass shell  $q^2 \approx -m^2$ ,  $\Gamma_\theta(P, q)$  and the solution of the homogeneous BSE (4)  $\Omega_\theta(P, q^2 = -m^2)$  are related by

$$\Gamma_\theta(P, q^2 \approx -m^2) \approx \frac{\Omega_\theta(P, q^2 = -m^2)}{q^2 + m^2}. \quad (9)$$

In our study  $q^2$  will be always spacelike. Other than the dressed quark or gluon propagator the correlator  $\Pi_{\theta_1\theta_2}(q^2)$  has a known Källén-Lehmann spectral representation [30]: Its poles are at  $q^2 = -m_I^2$ ,  $I=0,1,\dots$ , where  $m_I$  are the meson resonance masses and the pole residua are basically given by the couplings  $f_I$  between the interpolating current  $J_\theta$  and the on-shell states  $|I\rangle$ . Otherwise  $\Pi_{\theta_1\theta_2}(q^2)$  is holomorphic in the complex plane, and falls off sufficiently fast for  $|q^2| \rightarrow \infty$ , so that Cauchy's theorem can be applied. A convenient method to filter out the lowest state with mass  $m_0$  is to take the one-dimensional Fourier transform of Eq. (6) with respect to the time component of the Euclidean momentum  $q_4$

$$\Pi_{\theta_1\theta_2}^*(T) = \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} \Pi_{\theta_1\theta_2}(q_4, \vec{q}=0), \quad (10)$$

which falls off as  $e^{-m_0 T}$  for large Euclidean times  $T$ .

It is obvious that the method as described is only able to extract properties of the ground state but not of excited states or the continuum. It presumably will also fail or run into trouble for states with a large width, bound states in the continuum, or if the interpolating quark current  $J_\theta$  couples predominantly not to the ground state but to an excited state. On the other hand, it is clear that we have indeed avoided making any assumption about the analyticity of the propagators. Instead we are using the analyticity structure of the

two-point correlator  $\Pi_{\theta_1\theta_2}(q^2)$  in the complex plane, which is well under control. The price which we have to pay is the fact that we have to solve the inhomogeneous BSE over the whole range of spacelike  $q^2$ , instead of the homogeneous BSE at only the on-shell point  $q^2 = -m_0^2$ .

It is the aim of our paper to perform an analysis of the two-point correlation function in the vector channel (i.e.,  $T_\theta = \gamma_\mu$ ) using this approach, and to extract the ground-state mass  $m_0$ . We will work in the chiral limit ( $M_u = M_d = M_s = 0$ ); therefore, isospin effects are not present. Following Ref. [9] we will use a model gluon propagator  $D(q^2)$  with an IR regularized Mandelstam  $1/q^4$  singularity [31,32] at  $q^2 \approx 0$ . For large  $q^2$  an asymptotic free UV tail is added. The parameters in  $D(q^2)$  are fixed in such a way that the weak pion decay constant in the chiral limit  $f_\pi = 88$  MeV is reproduced and, as shown in Refs. [9,14,15], a satisfactory description of low-energy chiral physics and vacuum condensates can be achieved. We demonstrate that the dressed quarks which are determined from the rainbow DSE (2) are confined in the sense that the quark propagator does *not* show an exponential falloff  $\sim e^{-MT}$  for large  $T$ . We then demonstrate the existence of a bound state in the vector channel by showing the exponential falloff of the correlator  $\Pi_{\mu\mu}^*(T)$  for large  $T$ , from which we can determine the ground-state mass  $m_0$ .

Our paper is organized as follows: In Sec. II we briefly review the definition of the dressed quark propagator  $G$  and the vector vertex  $\Gamma_\mu$  as well as the rainbow DSE for  $G$  and the inhomogeneous BSE for  $\Gamma_\mu$ , and demonstrate the validity of the vector Ward-Takahashi identity. We then show in Sec. III how to obtain the expression for the vector two-point correlator and its Fourier transform in Euclidean space. The numerical results for gluon and quark propagators, the vertex function, and the correlator are presented, analyzed, and discussed in Sec. IV. Finally, conclusions are offered in Sec. V.

## II. DRESSED QUARK PROPAGATOR AND VERTEX FUNCTION

### A. Rainbow Dyson-Schwinger and ladder Bethe-Salpeter equation

Following Ref. [12] the inverse of the dressed quark propagator  $G(p)$  has the form

$$G^{-1}(p) \equiv i\gamma p [A(p^2) - 1] + \Sigma(p) \equiv i\gamma p A(p^2) + B(p^2) \quad (11)$$

in momentum space. The quark self-energy dressing  $\Sigma(p)$  comprises the concept of *constituent* quarks by providing a running mass  $M(p^2) = B(p^2)/A(p^2)$ . It is determined as the solution of the rainbow DSE,

$$\Sigma(p) = \frac{4}{3} g_s^2 \frac{d^4 q}{(2\pi)^4} D(p-q) \gamma_\nu G(q) \gamma_\nu, \quad (12)$$

which is schematically shown in Fig. 1. For convenience the Feynman type gauge for the gluon propagator

$$D_{\mu\nu}^{ab}(q) = \delta_{\mu\nu} \delta^{ab} D(q^2) \quad (13)$$

has been used here, which defines the phenomenological function  $D(q^2)$  in Eq. (12). In the rainbow DS-BS approximation used in the present work, this function  $D(q^2)$  for the dressed gluon propagator is our main physical input.

In terms of the components  $A$  and  $B$ , Eq. (12) reads

$$[A(p^2)-1]p^2 = g_s^2 \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} D(p-q) \frac{A(q^2)q \cdot p}{q^2 A^2(q^2) + B^2(q^2)}, \quad (14a)$$

$$B(p^2) = g_s^2 \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} D(p-q) \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)}. \quad (14b)$$

In momentum space, the dressed vector vertex is determined by the *inhomogeneous ladder* BSE, which reads, in momentum space (7),

$$\begin{aligned} \Gamma_\mu(P, q) &= (-i) \gamma_\mu - \frac{4}{3} g_s^2 \\ &\times \int \frac{d^4K}{(2\pi)^4} D(P-K) \gamma_\nu G\left(K + \frac{q}{2}\right) \\ &\times \Gamma_\mu(K, q) G\left(K - \frac{q}{2}\right) \gamma_\nu, \end{aligned} \quad (15)$$

and is schematically displayed in Fig. 3.

As was shown in Ref. [12] both the rainbow DSE (12) and the ladder BSE (15) can be *consistently* derived from the action of the GCM in an external gauge field  $\mathcal{A}_\mu(z)$  using standard functional integration techniques. The main steps of this derivation are reviewed in Appendixes A and B.

A crucial consequence is that this formalism ensures by construction invariance under local U(1) gauge transformations. At the level of the dressed vertex  $\Gamma_\mu$  this invariance is reflected in the validity of the Ward-Takahashi identity (WTI)

$$q_\mu \Gamma_\mu(P, q) = G^{-1}\left(P - \frac{q}{2}\right) - G^{-1}\left(P + \frac{q}{2}\right), \quad (16)$$

which can be easily verified by substituting it into Eq. (15) and using the DSE (12) for the quark self-energy  $\Sigma$  as well as the definition (11).

Expanding the right-hand side of Eq. (16) in  $q_\mu$  and taking the limit  $q_\mu \rightarrow 0$  leads to the Ward identity

$$\Gamma_\mu(P, 0) = - \frac{\partial G^{-1}(P)}{\partial P_\mu}. \quad (17)$$

### B. General form of the dressed vertex $\Gamma_\mu(P, q)$

Following Ref. [7] in the Feynman-type gauge (13) for the model gluon propagator, the most general form for the vertex function  $\Gamma_\mu(P, q)$  which fulfills Eq. (15) reads

$$\begin{aligned} \Gamma_\mu(P, q) &= 1 \Lambda_\mu^{(1)}(P, q) + i \gamma_\nu \Lambda_{\nu\mu}^{(2)}(P, q) \\ &+ i \gamma_5 \gamma_\nu \epsilon_{\mu\nu\alpha\beta} P_\alpha q_\beta \eta^{-2} \Lambda^{(3)}(P, q), \end{aligned} \quad (18)$$

where for convenience an arbitrary mass scale  $\eta$  has been introduced in order to render all  $\Lambda^{(i)}$ 's dimensionless. Furthermore it turns out to be convenient to perform a decomposition into longitudinal and transversal components with respect to the external momentum  $q_\mu$ :

$$\Lambda_\mu^{(1)}(P, q) = \frac{q_\mu}{\eta} \frac{Pq}{q^2} \lambda_1^L + \frac{P_\mu^T}{\eta} \lambda_1^T, \quad (19a)$$

$$\begin{aligned} \Lambda_{\nu\mu}^{(2)}(P, q) &= \frac{P_\nu q_\mu}{\eta^2} \frac{Pq}{q^2} \lambda_2^L + \frac{P_\nu P_\mu^T}{\eta^2} \lambda_2^T \\ &- \frac{q_\nu q_\mu}{q^2} \lambda_3^L - \left( \delta_{\nu\mu} - \frac{q_\nu q_\mu}{q^2} \right) \lambda_3^T + \frac{q_\nu P_\mu^T Pq}{\eta^4} \lambda_4^T, \end{aligned} \quad (19b)$$

$$\Lambda^{(3)}(P, q) = \lambda_5^T. \quad (19c)$$

The eight scalar dimensionless coefficients  $\lambda_i^L$  ( $i = 1, \dots, 3$ ) and  $\lambda_i^T$  ( $i = 1, \dots, 5$ ) depend on  $P^2, q^2$  and  $C_{Pq}^2$ , where  $C_{Pq} = Pq/q^2$  is the direction cosine between  $P$  and  $q$ .  $P_\mu^T \equiv P_\mu - Pq q_\mu/q^2$  is the vector transverse to  $q_\mu$ , i.e.,  $q_\mu P_\mu^T = 0$ . The advantage of the decomposition (19) lies in the fact that the longitudinal components  $\lambda_i^L$  ( $i = 1, \dots, 3$ ) are determined automatically from the quark propagator  $G$  by means of the WTI (16),

$$\frac{Pq}{\eta} \lambda_1^L = B(P_-^2) - B(P_+^2), \quad (20a)$$

$$\frac{Pq}{\eta^2} \lambda_2^L = A(P_-^2) - A(P_+^2), \quad (20b)$$

$$2\lambda_3^L = A(P_-^2) + A(P_+^2), \quad (20c)$$

where  $P_\pm \equiv P \pm q/2$ .

This leaves only the five independent transversal components  $\lambda_i^T$  ( $i = 1, \dots, 5$ ) to be determined as solutions of the inhomogeneous BSE (15). In order to do this, one has to project out the single  $\lambda_i^T$  for each  $i$  which can be done by multiplying Eqs. (18) and (19) with appropriate Dirac matrices and taking traces. Performing the same operations on the right-hand side of Eq. (15) finally leads to a set of five coupled linear inhomogeneous integral equations for  $\lambda_i^T$  ( $i = 1, \dots, 5$ ) [7]:

$$\begin{aligned} \mathcal{X}_i &= \mathcal{N}_{ij} \lambda_j^T(P^2, q^2, C_{Pq}^2) \\ &+ \int_0^\infty dK K^3 \int_{-1}^{+1} dC_{Kq} \mathcal{M}_{ij}(q^2, P^2, C_{Pq}^2, K^2, C_{Kq}^2) \\ &\times \lambda_j^T(K^2, q^2, C_{Kq}^2). \end{aligned} \quad (21)$$

The indices  $i$  and  $j$  both run from 1 to 5. The vector  $\mathcal{X}_i$  and the nonzero components of the matrices  $\mathcal{N}_{ij}$  and  $\mathcal{M}_{ij}$  were derived in Ref. [7]. For completeness, and because we need them later for the determination of the correlator, they are also listed in Appendix C.

### C. Asymptotic behavior

For large values of the quark momentum  $P^2 \rightarrow \infty$  or for large values of the external momentum  $q^2 \rightarrow \infty$  the propagators reach their asymptotic free forms, and therefore the kernel in Eq. (15) vanishes, which implies that also  $\Gamma_\mu$  approaches the naked vertex. This means

$$\Gamma_\mu(P, q)|_{P^2 \rightarrow \infty} = (-i) \gamma_\mu \quad (22)$$

or

$$\lambda_i^L(P^2 \rightarrow \infty, q^2, C_{PQ}^2) = 0, \quad i = 1, 2, \quad (23a)$$

$$\lambda_i^L(P^2 \rightarrow \infty, q^2, C_{PQ}^2) = 1, \quad i = 3, \quad (23b)$$

$$\lambda_i^T(P^2 \rightarrow \infty, q^2, C_{PQ}^2) = 0, \quad i = 1, 2, 4, 5, \quad (23c)$$

$$\lambda_i^T(P^2 \rightarrow \infty, q^2, C_{PQ}^2) = 1, \quad i = 3 \quad (23d)$$

and

$$\Gamma_\mu(P, q)|_{q^2 \rightarrow \infty} = (-i) \gamma_\mu, \quad (24)$$

or

$$\lambda_i^L(P^2, q^2 \rightarrow \infty, C_{PQ}^2) = 0, \quad i = 1, 2, \quad (25a)$$

$$\lambda_i^L(P^2, q^2 \rightarrow \infty, C_{PQ}^2) = 1, \quad i = 3, \quad (25b)$$

$$\lambda_i^T(P^2, q^2 \rightarrow \infty, C_{PQ}^2) = 0, \quad i = 1, 2, 4, 5, \quad (25c)$$

$$\lambda_i^T(P^2, q^2 \rightarrow \infty, C_{PQ}^2) = 1, \quad i = 3, \quad (25d)$$

together with  $A(P^2 \rightarrow \infty) = 1$  and  $B(P^2 \rightarrow \infty) = 0$ .

### D. Constraints on vertex function

The nonperturbative part of the vector vertex function from Eqs. (7) and (8) can be constrained by the work using QCD sum rules with nonlocal condensates.  $\Gamma_\mu(y_1, y_2; z)^{\text{NP}}$ , defined as the second term in Eq. (B8), has also been shown to be given by the four-quark nonlocal condensate [28,29]

$$\Gamma_\mu(y_1, y_2; z)^{\text{NP}} = \langle 0 | : q(y_1) \bar{q}(z) \gamma_\mu q(z) \bar{q}(y_2) : | 0 \rangle. \quad (26)$$

Defining the nonlocal quark condensate by

$$\langle 0 | : \bar{q}(0) q(y) : | 0 \rangle \equiv g(y^2) \langle 0 | : \bar{q}(0) q(0) : | 0 \rangle, \quad (27)$$

the function  $g(y^2)$  gives the space-time structure of the nonlocal condensates. In particular, the vertex function  $\Gamma_\mu(0, 0; z)^{\text{NP}}$ , needed in the three-point determination of the vector vacuum susceptibility is given by Eqs. (26) and (27) with the approximation of vacuum factorization as

$$\Gamma_\mu(0, 0; z)^{\text{NP}} = (-i \gamma_\mu) \left( \frac{\langle \bar{q} q \rangle}{12} \right)^2 \int d^4 y g(y^2) g[(z-y)^2]. \quad (28)$$

The vacuum factorization approximation has been found to be satisfactory for deriving vacuum susceptibilities [28,29]. Using the forms of  $g(y^2)$  such as those studied in Refs.

[15,28,29], one obtains constraints on the dressed vertex functions and thus further constraints on the gluon propagator. This will be the subject of future work.

## III. VECTOR CORRELATOR

### A. Definition and properties

We are now ready to study the main object of our investigation which is the correlator  $\Pi_{\mu_1 \mu_2}$  of two vector currents  $j_\mu(x) = \bar{q}(x) \gamma_\mu q(x)$ . In momentum space, it is defined as

$$\Pi_{\mu_1 \mu_2}(q) \equiv \int d^4 x e^{iqx} \langle \mathcal{T} j_{\mu_1}(x) j_{\mu_2}(0) \rangle. \quad (29)$$

It can be expressed in terms of the dressed quark propagator  $G$  and the dressed vertex function  $\Gamma_\mu(P, q)$  as

$$\Pi_{\nu\mu}(q) = i \int \frac{d^4 P}{(2\pi)^4} \text{Tr}[\Gamma_\mu(P, q) G(P_+) \gamma_\nu G(P_-)], \quad (30)$$

which is schematically displayed in Fig. 2. The trace Tr goes over Dirac and color indices. A derivation of Eq. (30) from the GCM is given in Appendix D. It should be stressed that Eq. (30) is a mean-field result.

The WTI (16) for the dressed vertex  $\Gamma_\mu$  ensures that the correlator  $\Pi_{\nu\mu}(q)$  is *transversal*,

$$q_\mu \Pi_{\nu\mu}(q) = 0, \quad (31)$$

or

$$\Pi_{\nu\mu}(q) = \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Pi(q^2). \quad (32)$$

This can be easily verified by putting Eq. (16) into Eq. (30), and again reflects manifest invariance of our approach under U(1) gauge transformation and therefore conservation of the vector current  $j_\mu(x)$ . Because of Eq. (32), it is sufficient to consider the contraction  $\Pi_{\mu\mu}(q^2)$ .

### B. Spectral representation and bound states

The Källén-Lehmann spectral representation [30] for  $\Pi_{\mu\mu}(q^2)$  is obtained by saturating the correlator (29) with a complete set of on-shell mesonic bound states  $|I(m_I, k_I, \lambda_I)\rangle$  ( $I=0,1,2,\dots$ ) with mass  $m_I$ , 4-momentum  $P_I$  ( $P_I^2 = -m_I^2$  in Minkowski space), and polarization  $\lambda_I$ :

$$\Pi_{\mu\mu}(q^2) = \int_0^\infty ds \tilde{\rho}_{\mu\mu}(s) \frac{1}{s - q^2 + i\epsilon}, \quad (33)$$

where

$$\begin{aligned} \rho_{\mu\mu}(q) &= \tilde{\rho}_{\mu\mu}(q^2) \theta(q_0) = (2\pi)^3 \sum_I \delta^{(4)}(P_I - q) \\ &\times \langle 0 | j_\mu(0) | I(m_I, P_I, \lambda_I) \rangle \langle I(m_I, P_I, \lambda_I) | j_\mu(0) | 0 \rangle. \end{aligned} \quad (34)$$

The coupling between the on-shell state  $|I\rangle$  and the interpolating quark current  $j_\mu$  is denoted by  $f_I$ , and defined by

$$\langle 0|j_\mu(x)|I(m_I, P_I, \lambda_I)\rangle = f_I e^{ikx} \epsilon_\mu(P_I, \lambda_I) \frac{1}{\sqrt{(2\pi)^3 (2\omega_{P_I})}}, \quad (35)$$

where  $\epsilon_\mu(P_I, \lambda_I)$  denotes the polarization 4 vector and  $(\omega_{P_I})^2 = m_I^2 + \vec{k}_I^2$ . Inserting Eq. (35) into Eq. (34) leads in Euclidean space to

$$\Pi_{\mu\mu}(q^2) = 3 \sum_I \frac{f_I^2}{q_I^2 + m_I^2}. \quad (36)$$

In order to filter out the contribution from the ground state  $|I=0\rangle$ , we take the Fourier transform of (36) with respect to the Euclidean time  $T$ ,

$$\begin{aligned} \Pi_{\mu\mu}^*(T) &= \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} \Pi_{\mu\mu}(q_4, \vec{q}=0) \\ &= \frac{3}{2} \sum_I \frac{f_I^2}{m_I} e^{-m_I T}, \end{aligned} \quad (37)$$

---


$$\begin{aligned} \Pi_{\mu\mu}(q^2) &= (-N_c) \frac{1}{\eta^2 \pi^3} \int_0^\infty dP P^3 \int_{-1}^{+1} dC_{Pq} x \left\{ -\frac{P^2}{\eta^2} x^2 V \lambda_1^T(P^2, q^2, C_{Pq}^2) + \frac{P^2}{\eta^2} x^2 \left( F_1 - 2 \frac{P^2}{\eta^2} T \right) \lambda_2^T(P^2, q^2, C_{Pq}^2) \right. \\ &\quad \left. - \left( 3F_1 - 2 \frac{P^2}{\eta^2} x^2 T \right) \lambda_3^T(P^2, q^2, C_{Pq}^2) - 2 \frac{P^4 q^2}{\eta^6} x^2 C_{Pq}^2 T \lambda_4^T(P^2, q^2, C_{Pq}^2) - 2 \frac{P^2 q^2}{\eta^4} x^2 T \lambda_5^T(P^2, q^2, C_{Pq}^2) \right\}, \end{aligned} \quad (39)$$

where abbreviations from Appendix C have been used.

At this point we have to say how to handle the UV divergences in  $\Pi_{\mu\mu}(q^2)$ . For large quark loop momenta  $P$  the quark propagator and vertex functions reach their asymptotic forms specified in Sec. II C, and the loop in Fig. 2 is the free loop which is logarithmically divergent. In perturbation theory it can be renormalized in the standard way using dimensional regularization [33]. In our case it is actually not necessary to evaluate  $\Pi_{\mu\mu}(q^2)$ , because we need only its Fourier transform  $\Pi_{\mu\mu}^*(T)$ , which is UV finite, as any correlator in coordinate space is UV finite. The reason for that is that the quark propagator, which falls off like  $1/P$  in momentum space, will fall off like  $e^{-TP}$  after Fourier transform, and therefore the momentum integral  $\int_0^\infty dP P^3$  in Eq. (39) is UV finite. When evaluating  $\Pi_{\mu\mu}^*(T)$  from Eq. (39), it is therefore convenient to perform the Fourier transform  $\int_{-\infty}^{+\infty} (dq_4/2\pi) e^{iq_4 T}$  before the momentum integral  $\int_0^\infty dP P^3$ , which avoids a conceptually and technically intricate renormalization procedure.

#### IV. RESULTS AND DISCUSSION

Our model gluon propagator has the form

$$D(q^2) = D_{\text{IR}}(q^2) + D_{\text{UV}}(q^2), \quad (40)$$

and consider the limit  $T \rightarrow \infty$ ,

$$\Pi_{\mu\mu}^*(T \rightarrow \infty) \rightarrow \frac{3}{2} \frac{f_0^2}{m_0} e^{-m_0 T}. \quad (38)$$

It should be noted in this context that in case of the  $\rho$  meson there are in principal also  $2\pi$  intermediate states in sum (34), and therefore the asymptotic behavior of the correlator  $\Pi_{\mu\mu}$  for large Euclidean times is actually  $e^{-2m_\pi T}$ . However the diagrams with the two intermediate  $\pi$  states are suppressed by one order  $1/N_c$  with respect to the vector meson propagator, which is of leading order [11]. We are working at the mean-field level, which is compatible with the leading order in  $1/N_c$  as far as the quark degrees of freedom are concerned [9,17], and therefore these intermediate  $2\pi$  states do not appear in the present treatment.

#### C. Determination of $\Pi_{\mu\mu}$ in terms of $G$ and $\Gamma_\mu$

The final task which remains in order to evaluate  $\Pi_{\mu\mu}$  is to write it in terms of the quark self energy functions  $A$  and  $B$  [Eq. (11)] and vertex functions  $\lambda_i^T$  [Eqs. (18) and (19)]. This is done straightforwardly by inserting definitions (11), (18), and (19) into Eq. (30), and carrying out the matrix trace. The result is

where

$$g_s^2 D_{\text{IR}}(q^2) = (4\pi^2 d) \frac{\chi^2}{q^4 + \Delta}, \quad (41)$$

$$g_s^2 D_{\text{UV}}(q^2) = \frac{4\pi^2 d}{q^2 + \ln\left(\frac{q^2}{\Lambda_{\text{QCD}}^2} + \tau\right)}. \quad (42)$$

The first term  $D_{\text{IR}}$  [Eq. (41)], which dominates for small  $q^2$ , is a regularized Mandelstam *Ansatz* [31,32] with a strength  $\chi^2$  and an IR regulator  $\Delta$ , which models the IR strength of the quark-quark interaction. The second term  $D_{\text{UV}}$  [Eq. (42)], which dominates for large  $q^2$ , is an asymptotic UV tail and matches the known one-loop renormalization group result with  $d = [12/(33 - 2N_f)] = \frac{12}{27}$ ,  $\Lambda_{\text{QCD}} = 200$  MeV, and  $\tau = e$  [2]. The model parameters  $\chi$  and  $\Delta$  are adjusted to reproduce the weak pion decay constant in the chiral limit  $f_\pi = 88$  MeV. We are using three different parameter sets:

$$\text{Set 1: } \Delta = 1.0 \times 10^{-4} \text{ GeV}^4, \quad \chi = 1.02 \text{ GeV}, \quad (43a)$$

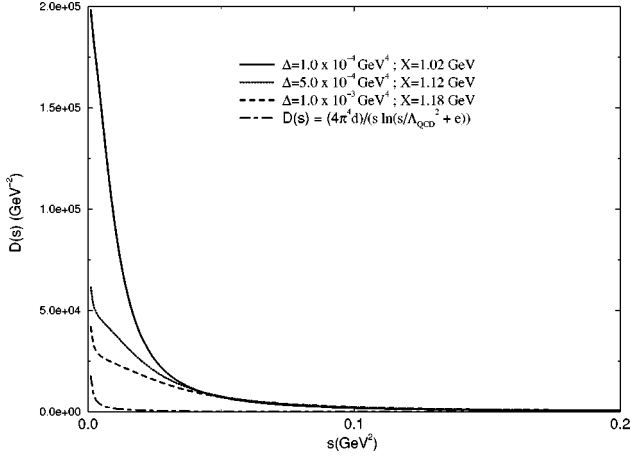


FIG. 4. Model gluon-2 point functions  $D(s)$  [Eq. (40)] for the three different parameter sets (43). In each case the value of the pion decay constant in the chiral limit  $f_\pi = 88$  MeV is reproduced. For comparison we also show the form of the UV tail  $D_{UV}(s)$  [Eq. (42)] (dash-dotted line).

$$\text{Set 2: } \Delta = 5.0 \times 10^{-4} \text{ GeV}^4, \quad \chi = 1.12 \text{ GeV}, \quad (43b)$$

$$\text{Set 3: } \Delta = 1.0 \times 10^{-3} \text{ GeV}^4, \quad \chi = 1.18 \text{ GeV}. \quad (43c)$$

The forms of  $D(q^2)$  are displayed in Fig. 4, which also shows for comparison the pure UV form  $D_{UV}(q^2)$ . As one can see in all three cases the asymptotic form is reached already for  $q^2 \approx 0.2$  GeV<sup>2</sup>. In Ref. [9] it has been shown that with those values a satisfactory description of all low-energy chiral observables can be achieved.

The second step is the determination of the dressed quark propagator (11) as a solution of the coupled set of integral equations (14a) and (14b). Figure 5 displays the running constituent mass  $M(p^2) = B(p^2)/A(p^2)$  at spacelike  $p^2 > 0$ . In order to demonstrate that the quarks are confined, one has to show that the quark propagator  $G(p^2)$  has no poles in the timelike region  $p^2 < 0$ . This can again be done by studying the Fourier transform with respect to the Euclidean time  $T$  [2]. It is sufficient to consider the scalar part  $G_s$ :

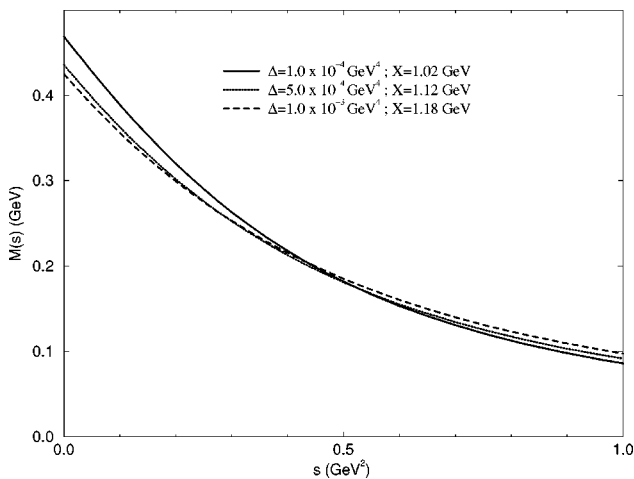


FIG. 5. The dynamical (constituent) quark masses  $M(s)$  for the three parameter sets (43).

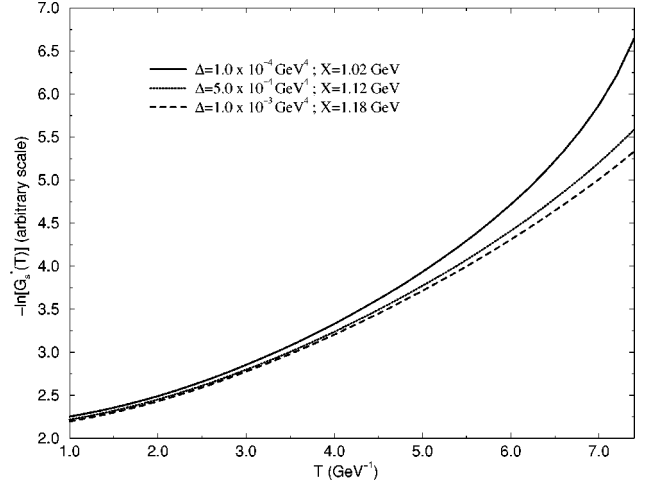


FIG. 6. The Euclidean time Fourier transform  $G_s^*(T)$  of the scalar part of the quark propagator  $G_s$  (arbitrary scale) for the three parameter sets (43).

$$\begin{aligned} G_s^*(T) &= \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} G_s(q^2) \\ &= \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)}. \end{aligned} \quad (44)$$

If  $G(p)$  had a pole at  $p^2 = -M^2$ , the Fourier transform  $G_s^*(T)$  would fall off as  $e^{-MT}$  for large  $T$  or

$$\ln[G_s^*(T \rightarrow \infty)] \sim -MT. \quad (45)$$

However, from Fig. 6 we can see that  $(-)\ln[G_s^*(T)]$  is far from rising linearly with  $T$  at large  $T$ ; it actually rises approximately quadratically with  $T$ . This strongly indicates that the parameter set (43) indeed produces confined quarks. We want to stress that this statement is based on a numerical argument, namely, Fig. 6. In the simple but somewhat unphysical case of a gluon propagator, which is a  $\delta$  function in momentum space,  $D(q^2) \propto \delta^4(q^2)$ , and which can be regarded as the limit of Eq. (41) for  $\Delta \rightarrow 0$ , the DSE for the quark propagator can be solved analytically, and it can be shown that the quark propagator indeed has no pole at timelike momenta [2,3]. This is equivalent to quark confinement in the sense mentioned above.

The third step is now to solve the inhomogeneous BSE, i.e., the coupled set of five inhomogeneous linear integral equations (21). We follow the numerical procedure from Ref. [7]. One applies a Gauss quadrature using about  $n_K \approx 30$  Gauss points for  $\int_{-\infty}^{+\infty} dK$  and  $n_{C_{Kq}} \approx n_{C_{KT}} \approx 10$  Gauss points for both  $\int_{-1}^{+1} dC_{Kq}$  and  $\int_{-1}^{+1} dC_{KT}$ . It should be noted that the angular integration over  $C_{KT}$  is running only over the gluon propagator  $D$  and not over the functions  $\lambda_i(K^2, q^2, C_{KQ}^2)$ . This transforms the integral equations into a high-dimensional set of ordinary inhomogeneous linear equations for the  $\lambda_i$  at the Gauss points, which is solved by inverting the  $(5n_K n_{C_{Kq}}) \times (5n_K n_{C_{Kq}}) \approx 1500 \times 1500$  coefficient matrix using standard packages [34].

Other than in Ref. [7], where the quark self-propagator  $G$  which appears in the kernel of the BSE (15) has been param-

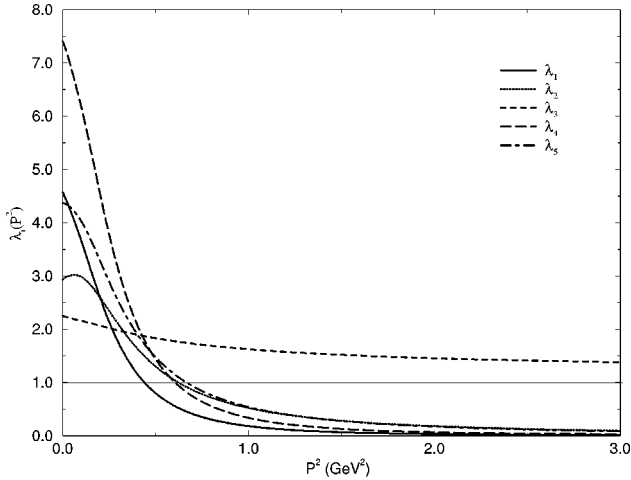


FIG. 7. The five components  $\lambda_i(P^2)$ ,  $i=1, \dots, 5$  of the vertex function  $\Gamma_\mu(P, q)$  as defined in Eq. (19) for a fixed spacelike value of the off shell momentum  $q^2 = +0.1$  GeV<sup>2</sup> and angle  $C_{Pq} = 0$  using the parameter set 1 [Eq. (43a)].

etrized by a simple analytic form without solving the DSE (14) explicitly, we are using the numerical solutions  $A$  and  $B$  of Eq. (14) in the kernel for the BSE (15). Our treatment is therefore a fully consistent solution of both the rainbow DSE for the quark propagator  $G$  and the inhomogeneous ladder BSE for the vector vertex function  $\Gamma_\mu$  using the same model gluon propagator  $D$  specified in Eq. (40) as input. As one can see from Eqs. (15) and (21), this involves a numerical interpolation of  $A(P^2)$  and  $B(P^2)$  to the points  $P_\pm$ . Figure 7 displays the coefficient functions  $\lambda_i(P^2)$  for the parameter set 1 [Eq. (43a)] for a fixed spacelike value of  $q^2 = 0.1$  GeV<sup>2</sup> and a fixed angle  $C_{Pq} = 0$ . As one can see, our result is qualitatively similar though quantitatively noticeably different than the one from Ref. [7]. Similar than Ref. [7], we also find that the  $\lambda_i(P^2, q^2, C_{PQ})$  are only very weakly dependent on the angle  $C_{PQ}$  if  $P^2$  and  $q^2$  are fixed.

Finally, Fig. 8 shows our main result, the Fourier-

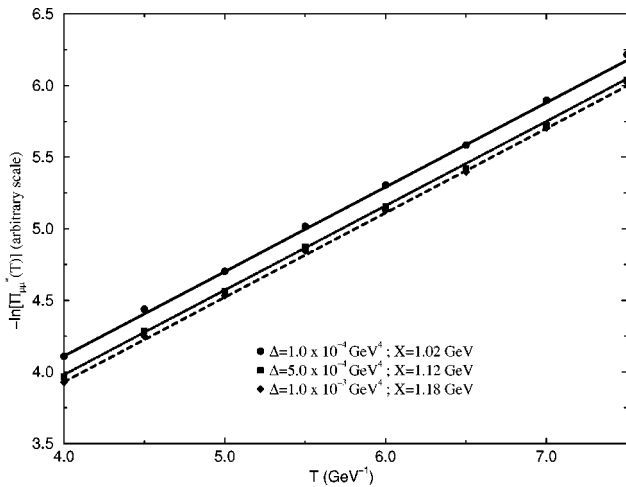


FIG. 8. The Euclidean time Fourier transform  $\Pi_{\mu\mu}^*(T)$  of the vector correlator (arbitrary scale) for the three parameter sets (43). Our numerical results are denoted by circles, squares, and diamonds, respectively. The straight full, dotted, and dashed straight lines are the best linear fits.

transformed correlator  $\Pi_{\mu\mu}^*(T)$  for large Euclidean times  $T$  evaluated with the three sets (43). As mentioned earlier it is essential to evaluate the Fourier integral  $\int_{-\infty}^{+\infty} (dq_4/2\pi) e^{iq_4 T}$  before doing the integral over the quark momentum  $\int_{-\infty}^{+\infty} dP P^3$  in order to avoid UV infinities. From Fig. 8 we see clearly that  $\Pi_{\mu\mu}^*(T)$  has an exponential falloff  $\sim e^{-m_0 T}$  for large  $T$ , and moreover that the ground-state mass  $m_0$  is practically the same for all three parameter sets, because the curves are parallel. The ground-state mass  $m_0$  can be extracted by carrying out a linear fit to the numerical result. One finds  $m_0 \approx 590$  MeV for all three sets. This has to be compared with the phenomenological values of 780 MeV for the ground-state vector meson mass. The instanton liquid model [5] finds  $m_0 = 950$  MeV, whereas quenched lattice QCD [4] obtains  $m_0 = 720$  MeV. Reference [7], which considers the same model as we do but solves the BSE in Minkowski space, finds a ground-state mass in the vector channel of about 880 MeV. However, it should be noted that the gluon propagator of Ref. [7] is not exactly the same as the one we use. We have taken care that we can indeed reproduce the weak pion decay constant  $f_\pi$ , which is not the case in Ref. [7]. Furthermore, as mentioned above, in Ref. [7] the SDE and BSE were not solved consistently in the sense that the quark propagator which enters the BSE is not the one which comes from the DSE. Instead, Ref. [7] employs an analytic form for the quark propagator, which only roughly fits the real solution of the DSE. A direct comparison between the numerical values of Ref. [7] and the value from our analysis is therefore not really conclusive.

## V. CONCLUSIONS

To summarize, we have studied a  $\bar{q}q$  bound state in the vector channel using a nonlocal and confining model quark-quark interaction, which respects all global symmetries of QCD. The model parameters have been chosen to give a good description of low-energy chiral physics for the Goldstone degrees of freedom ( $\pi, K, \eta$ ). We have demonstrated the existence of a higher mass state by evaluating the correlator of two interpolating vector currents in Euclidean space, and showing the exponential falloff of its Fourier transform at large Euclidean times. This employs a consistent treatment of the dressed quark propagator  $G$  and the dressed vector vertex  $\Gamma_\mu$ , which are both determined from the model quark-quark interaction by the rainbow Dyson-Schwinger equation for  $G$  and the inhomogeneous ladder Bethe-Salpeter equation for  $\Gamma_\mu$ . The method stays until the end in Euclidean space and all momenta are spacelike. This avoids any unsafe analytic continuations of the dressed propagators into the timelike region. We have found that the ground-state mass is practically independent of the parameters of the model interaction and lies at about 590 MeV.

For the creation of a bound state it is obviously necessary to have a highly nonlocal quark-quark interaction, which is reflected in its large nonperturbative strength in the IR. This leads not only to dynamical chiral symmetry breaking but also to a strong momentum dependence of the running dynamical quark mass  $M(p^2)$  and, in our case, even to confined quarks. This is an important advantage over with the Nambu–Jona-Lasinio (NJL) interaction [35], which can be



regarded as a special case of the GCM with a local quark-quark interaction  $D_{\mu\nu}^{ab}(x,y) = \delta^{ab} \delta_{\mu\nu} \delta^{(4)}(x-y)$ , and has been subject to extensive studies [36]. The NJL model has a momentum-independent constituent mass  $M$  and is therefore not confining. Though it gives a satisfactory description of the physics associated with the Goldstone degrees of freedom ( $\pi, K, \eta$ ) it has trouble dealing with higher  $\bar{q}q$  bound states, which lie around the  $2M$  threshold and are therefore unstable or barely stable against decay into a quark-antiquark pair.

As long as ground-state properties are considered, the method which we have used can also be applied to other channels, such as, e.g., the axial vector mesons, as well as to three-point functions, which are the basis for a study of meson form factors [27]. It is not adequate for extracting properties of excited states.

## ACKNOWLEDGMENTS

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## APPENDIX A: GLOBAL COLOR MODEL (GCM) IN AN EXTERNAL VECTOR FIELD

Following Ref. [12], we consider the Euclidean action of the GCM in an *external vector field*  $\mathcal{A}_\mu(x)$ :

$$S_{\text{GCM}}[\bar{q}, q; \mathcal{A}] = \int d^4x d^4y \left\{ \bar{q}(x) [\gamma \partial_x - i \gamma_\nu \mathcal{A}_\nu(x)] q(x) + \frac{g_s^2}{2} j_\mu^a(x) D_{\mu\nu}^{ab}(x-y) j_\mu^b(y) \right\}, \quad (\text{A1})$$

where  $j_\mu^a$  denotes the color octet vector current:

$$j_\mu^a(x) = \bar{q}(x) \gamma_\mu \frac{\lambda_c^a}{2} q(x). \quad (\text{A2})$$

For convenience we will employ the Feynman-type gauge (13) [2,3] for the model gluon propagator  $D$ . Applying the standard bosonization procedure [37,38] the generating functional

$$\mathcal{Z}[\mathcal{A}] \equiv e^{-\mathcal{W}[\mathcal{A}]} = \int \mathcal{D}\bar{q} \mathcal{D}q e^{-S_{\text{GCM}}[\bar{q}, q; \mathcal{A}]} \quad (\text{A3})$$

can be rewritten in terms of the bilocal auxiliary fields  $\mathcal{B}^\theta(x,y)$ ,

$$\mathcal{Z}[\mathcal{A}] = \int \mathcal{D}\mathcal{B}^\theta e^{-S_{\text{eff}}[\mathcal{B}^\theta; \mathcal{A}]}, \quad (\text{A4})$$

with the effective bosonic action

$$S_{\text{eff}}[\mathcal{B}^\theta; \mathcal{A}] = (-) \text{Tr} \ln \mathcal{G}^{-1}[\mathcal{B}^\theta; \mathcal{A}] + \int d^4x d^4y \frac{\mathcal{B}^\theta(x,y) \mathcal{B}^\theta(y,x)}{2g_s^2 D(x-y)} \quad (\text{A5})$$

and the quark operator

$$\mathcal{G}^{-1}[\mathcal{B}^\theta; \mathcal{A}] = [\gamma \partial_x - i \gamma_\nu \mathcal{A}_\nu(x)] \delta(x-y) + \Lambda^\theta \mathcal{B}^\theta(x,y). \quad (\text{A6})$$

The matrices  $\Lambda^\theta$  arise from Fierz reordering the current-current interaction in Eq. (A1), and are given by

$$\Lambda^\theta = \frac{1}{2} \left( \mathbf{1}_D, i \gamma_5, \frac{i}{\sqrt{2}} \gamma_\nu, \frac{i}{\sqrt{2}} \gamma_\nu \gamma_5 \right) \otimes \left( \frac{1}{\sqrt{3}} \mathbf{1}_F, \frac{1}{\sqrt{2}} \lambda_F^a \right) \otimes \left( \frac{4}{3} \mathbf{1}_C, \frac{i}{\sqrt{3}} \lambda_c^a \right). \quad (\text{A7})$$

In the mean-field approximation, which is the leading order in  $1/N_c$ , the fields  $\mathcal{B}^\theta(x,y)$  are substituted by their vacuum values  $\mathcal{B}_0^\theta(x,y)$ , which are given as the stationary points of the effective action (A5),

$$\left[ \frac{\delta S_{\text{eff}}}{\delta \mathcal{B}} \right]_{\mathcal{B}_0} = 0 \quad (\text{A8})$$

or

$$\mathcal{B}_0^\theta[\mathcal{A}](x,y) = g_s^2 D(x-y) \text{tr}[\Lambda^\theta \mathcal{G}_0[\mathcal{A}]^{-1}(x,y)], \quad (\text{A9})$$

where  $\mathcal{G}_0^{-1}(x,y)$  denotes the inverse propagator with the self-energy  $\Sigma(x,y) = \Lambda^\theta \mathcal{B}_0^\theta(x,y)$  in the external background field  $\mathcal{A}(x)$ :

$$\mathcal{G}_0[\mathcal{A}]^{-1}(x,y) = [\gamma \partial_x - i \gamma_\nu \mathcal{A}_\nu(x)] \delta(x-y) + \Lambda^\theta \mathcal{B}_0^\theta(x,y). \quad (\text{A10})$$

We want to stress that both  $\mathcal{B}_0^\theta(x,y)$  and  $\mathcal{G}_0^{-1}(x,y)$  have an implicit dependence on the external background field  $\mathcal{A}(x)$ . If the external field  $\mathcal{A}$  is switched off,  $\mathcal{G}_0$  goes into the dressed quark propagator  $G \equiv \mathcal{G}_0[\mathcal{A}=0]$ , which has the decomposition [Eq. (11)]

$$G^{-1}(p) \equiv i \gamma p [A(p^2) - 1] + \Sigma(p) \equiv i \gamma \cdot p A(p^2) + B(p^2) \quad (\text{A11})$$

in momentum space. The quark self-energy dressing  $\Sigma(p)$  is determined as the solution of the rainbow DSE [Eq. (A9) for  $\mathcal{A}=0$ ], rendering [Eq. (12)]

$$\Sigma(p) = \frac{4}{3} g_s^2 \frac{d^4 q}{(2\pi)^4} D(p-q) \gamma_\nu G(q) \gamma_\nu. \quad (\text{A12})$$

### APPENDIX B: VERTEX DRESSING AND INHOMOGENEOUS BETHE-SALPETER EQUATION

In coordinate space the dressed vector vertex  $\Gamma_\mu(y_1, y_2; z)$  is given as the functional derivative of the inverse quark propagator  $\mathcal{G}_0[\mathcal{A}]^{-1}$  (A10) with respect to the external field  $\mathcal{A}_\mu$ :

$$\Gamma_\mu(y_1, y_2; z) \equiv \left[ \frac{\delta \mathcal{G}_0[\mathcal{A}]^{-1}(y_1, y_2)}{\delta \mathcal{A}_\mu(z)} \right]_{\mathcal{A}=0}. \quad (\text{B1})$$

Taking the functional derivative in Eq. (A10) gives, for Eq. (B1),

$$\Gamma_\mu(y_1, y_2; z) = (-i) \gamma_\mu \delta(y_1 - y_2) \delta(y_1 - z) + \left[ \frac{\delta \Sigma[\mathcal{A}](y_1, y_2)}{\delta \mathcal{A}(z)} \right]_{\mathcal{A}=0}. \quad (\text{B2})$$

The second term on the right-hand side of Eq. (B2) can be determined by employing the stationary condition (A9), which, after Fierz reordering, can be cast into

$$\Sigma[\mathcal{A}](y_1, y_2) = \frac{4}{3} g_s^2 D(y_1 - y_2) \gamma_\nu \left[ \frac{\delta \mathcal{G}_0[\mathcal{A}](y_1, y_2)}{\delta \mathcal{A}_\mu(z)} \right]_{\mathcal{A}=0} \gamma_\nu. \quad (\text{B3})$$

In order to find an expression for  $[\delta \mathcal{G}_0[\mathcal{A}](y_1, y_2) / \delta \mathcal{A}_\mu(z)]_{\mathcal{A}=0}$  in terms of the quark propagator  $G$ , we write the definition (A10) schematically as

$$\mathcal{G}_0^{-1}[\mathcal{A}] = G^{-1} + \mathcal{A}_\mu \Gamma_\mu, \quad (\text{B4})$$

which leads to the formal expansion

$$\mathcal{G}_0[\mathcal{A}] = G - G \mathcal{A}_\mu \Gamma_\mu G \pm \dots \quad (\text{B5})$$

Putting Eq. (B5) into Eq. (B3) gives

$$\left[ \frac{\delta \Sigma[\mathcal{A}](y_1, y_2)}{\delta \mathcal{A}(z)} \right]_{\mathcal{A}=0} = -\frac{4}{3} g_s^2 D(y_1 - y_2) \int du_1 du_2 \gamma_\nu G(y_1, u_1) \Gamma_\mu(u_1, u_2; z) G(u_2, y_2) \gamma_\nu, \quad (\text{B6})$$

which, after substituting the result into Eq. (B2), renders the *inhomogeneous* BSE for  $\Gamma_\mu(y_1, y_2; z)$  in coordinate space:

$$\Gamma_\mu(y_1, y_2; z) = (-i) \gamma_\mu \delta(y_1 - y_2) \delta(y_1 - z) - \frac{4}{3} g_s^2 D(y_1 - y_2) \int du_1 du_2 \gamma_\nu G(y_1, u_1) \Gamma_\mu(u_1, u_2; z) G(u_2, y_2) \gamma_\nu. \quad (\text{B7})$$

Fourier transform leads then to the momentum space form (15).

### APPENDIX C: MATRIX ELEMENTS OF THE BS KERNEL

In this appendix we list the explicit forms for the quantities which appear as kernel of the five transversal inhomogeneous BS integral, and will be also needed in Eq. (39):

$$\mathcal{X} = (0, 3, 1, 0, 0), \quad (\text{C1})$$

$$\mathcal{N}_{11} = 1, \quad \mathcal{N}_{22} = -\frac{(P^T)^2}{\eta^2}, \quad \mathcal{N}_{23} = 3, \quad \mathcal{N}_{22} = -\frac{(P^T)^2}{\eta^2},$$

$$\mathcal{N}_{33} = 1, \quad \mathcal{N}_{42} = 1, \quad \mathcal{N}_{44} = \frac{q^2}{\eta^2}, \quad \mathcal{N}_{55} = 1, \quad (\text{C2})$$

and

$$\mathcal{M}_{11} = 2\pi \hat{D}_1 F_0 \frac{K}{P^T},$$

$$\mathcal{M}_{12} = 2\pi \hat{D}_1 \frac{K^3}{P^T \eta^2} \left( V + \frac{q^2}{\eta^2} C_{Kq}^2 W \right),$$

$$\mathcal{M}_{13} = -2\pi \hat{D}_1 \frac{K}{P^T} V,$$

$$\mathcal{M}_{14} = 2\pi \hat{D}_1 \frac{K^3 q r^2}{P^T \eta^4} C_{Kq}^2 \left( V + \frac{q^2}{\eta^2} W \right),$$

$$\mathcal{M}_{21} = -\pi \hat{D}_0 \frac{K^2}{\eta^2} x^2 V,$$

$$\mathcal{M}_{22} = \pi \hat{D}_0 \frac{K^2}{\eta^2} x^2 \left( F_1 - 2 \frac{K^2}{\eta^2} T \right),$$

$$\mathcal{M}_{23} = -\pi \hat{D}_0 \left( 3F_1 - 2 \frac{K^2}{\eta^2} T \right),$$

$$\begin{aligned}
\mathcal{M}_{24} &= -2\pi\hat{D}_0 \frac{K^4 q^2}{\eta^6} x^2 C_{Kq}^2 T, \\
\mathcal{M}_{25} &= 2\pi\hat{D}_0 \frac{K^2 q^2}{\eta^4} x^2 T, \\
\mathcal{M}_{31} &= -\pi\hat{D}_2 \frac{K^2}{\eta^2} V, \\
\mathcal{M}_{32} &= \pi\hat{D}_2 \frac{K^2}{\eta^2} \left( F_1 - 2\frac{K^2}{\eta^2} T \right), \\
\mathcal{M}_{33} &= -\pi \left( F_1 \hat{D}_0 - 2\frac{K^2}{\eta^2} T \hat{D}_2 \right), \\
\mathcal{M}_{34} &= -2\pi\hat{D}_2 \frac{K^4 q^2}{\eta^6} C_{Kq}^2 T, \\
\mathcal{M}_{35} &= \pi(\hat{D}_0 x^2 - \hat{D}_2) \frac{K^2 q^2}{\eta^4} T, \\
\mathcal{M}_{41} &= \pi\hat{D}_1 \frac{K^2}{P^L P^T} C_{Kq} \left( V + \frac{q^2}{\eta^2} W \right), \\
\mathcal{M}_{42} &= -\pi\hat{D}_1 \frac{K^2}{P^L P^T} C_{Kq} F_2, \\
\mathcal{M}_{43} &= -2\pi\hat{D}_1 \frac{K^2}{P^L P^T} C_{Kq} T, \\
\mathcal{M}_{44} &= -\pi\hat{D}_1 \frac{K^2}{P^L P^T} C_{Kq} \frac{q^2}{\eta^2} F_3, \\
\mathcal{M}_{53} &= -\pi\hat{D}_1 \frac{K}{P^T} T,
\end{aligned} \tag{C3}$$

We have used the abbreviations

$$\begin{aligned}
F_0 &\equiv \left( \frac{q^2}{4} - K^2 \right) \frac{1}{\eta^2} T + U, \\
F_1 &\equiv \left( K^2 - \frac{q^2}{4} \right) \frac{1}{\eta^2} T + U, \\
F_2 &\equiv F_1 + 2 \left( \frac{q^2}{4} - K^2 \right) \frac{1}{\eta^2}, \\
F_3 &\equiv F_1 + 2 \left( \frac{q^2}{4} - K^2 C_{Kq}^2 \right) \frac{1}{\eta^2}, \\
T &\equiv \eta^4 \alpha(K_+^2) \alpha(K_-^2), \\
U &\equiv \eta^2 \beta(K_+^2) \beta(K_-^2), \\
V &\equiv \eta^3 [\alpha(K_+) \beta(K_-) + \alpha(K_-) \beta(K_+)], \\
W &\equiv \eta^5 \left[ \frac{\alpha(K_+) \beta(K_-) - \alpha(K_-) \beta(K_+)}{2K \cdot q} \right],
\end{aligned} \tag{C4}$$

as well as

$$\begin{aligned}
\alpha(p^2) &= \frac{A(p^2)}{p^2 A^2(p^2) + B^2(p^2)}, \\
\beta(p^2) &= \frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)},
\end{aligned} \tag{C5}$$

and

$$\hat{D}_n(P^2, C_{Pq}, K^2, C_{Kq}) \equiv \int_{-x}^x dC_{KT} \frac{1}{3\eta^2 \pi^4} g_s^2 D(P^2 + K^2 - 2PK\sqrt{1 - C_{Pq}^2} C_{KT} - 2PKC_{Pq} C_{Kq}) (C_{KT})^n. \tag{C6}$$

$C_{KT}$  is the direction cosine between  $K_\mu$  and  $P_\mu^T$  and

$$x \equiv \sqrt{1 - C_{Kq}^2}, \quad P_L \equiv PC_{PQ}, \quad P_T = P\sqrt{1 - C_{Kq}^2}. \tag{C7}$$

#### APPENDIX D

In this appendix we give a derivation of Eq. (30). The time-ordered product of the two vector currents can be formally written as functional derivative of the generating func-

tional  $\mathcal{Z}[\mathcal{A}]$  [Eqs. (A3) and (A4)] with respect to the external vector field  $\mathcal{A}$ :

$$\langle T j_{\mu_1}(z_1) j_{\mu_2}(z_2) \rangle = \frac{1}{\mathcal{Z}[0]} (-) \left[ \frac{\delta^{(2)} \mathcal{Z}[\mathcal{A}]}{\delta \mathcal{A}_{\mu_1}(z_1) \delta \mathcal{A}_{\mu_2}(z_2)} \right]_{\mathcal{A}=0}. \tag{D1}$$

At the mean-field level the integration in over all possible

configurations of the bosonic auxiliary field  $\mathcal{B}$  is substituted by the stationary configuration  $\mathcal{B}_0$ , and we can therefore write

$$\mathcal{Z}[\mathcal{A}] = e^{-\mathcal{W}[\mathcal{A}]} \approx e^{-\mathcal{W}_0[\mathcal{A}]}, \quad (\text{D2})$$

where

$$\mathcal{W}_0[\mathcal{A}] = (-) \text{Tr} \ln \mathcal{G}_0[\mathcal{A}]^{-1} + \int d^4x d^4y \frac{\mathcal{B}_0^\theta(x,y) \mathcal{B}_0^\theta(y,x)}{2g_s^2 D(x-y)}. \quad (\text{D3})$$

This implies for the functional derivatives

$$\begin{aligned} \frac{\delta \mathcal{Z}[\mathcal{A}]}{\delta \mathcal{A}_\nu(z)} &= (-) \frac{\delta \mathcal{W}_0[\mathcal{A}]}{\delta \mathcal{A}_\nu(z)} e^{-\mathcal{W}_0[\mathcal{A}]}, \\ \left[ \frac{\delta \mathcal{W}_0[\mathcal{A}]}{\delta \mathcal{A}_\nu(z)} \right] &= \left[ \frac{\partial \mathcal{W}_0[\mathcal{A}]}{\partial \mathcal{A}_\nu(z)} \right] + \left[ \frac{\delta \mathcal{W}_0[\mathcal{A}]}{\delta \mathcal{B}_0[\mathcal{A}]} \right] \left[ \frac{\delta \mathcal{B}_0[\mathcal{A}]}{\delta \mathcal{A}_\nu(z)} \right] = \left[ \frac{\partial \mathcal{W}_0[\mathcal{A}]}{\partial \mathcal{A}_\nu(z)} \right] = (+i) \langle z | \text{tr} [\mathcal{G}_0[\mathcal{A}] \gamma_\nu] | z \rangle, \\ \left[ \frac{\delta \mathcal{W}_0[\mathcal{A}]}{\delta \mathcal{A}_\nu(z)} \right]_{\mathcal{A}=0} &= (+i) \text{tr} [G(z,z) \gamma_\nu] = 0, \\ \left[ \frac{\delta \mathcal{Z}[\mathcal{A}]}{\delta \mathcal{A}_\nu(z)} \right]_{\mathcal{A}=0} &= 0, \end{aligned} \quad (\text{D4})$$

$$\left[ \frac{\delta^{(2)} \mathcal{Z}[\mathcal{A}]}{\delta \mathcal{A}_{\mu_1}(z_1) \delta \mathcal{A}_{\mu_2}(z_2)} \right]_{\mathcal{A}=0} = (-) \mathcal{Z}[0] \left[ \frac{\delta^{(2)} \mathcal{W}_0[\mathcal{A}]}{\delta \mathcal{A}_{\mu_1}(z_1) \delta \mathcal{A}_{\mu_2}(z_2)} \right]_{\mathcal{A}=0}.$$

Therefore we find, for the correlator, using Eq. (B1),

$$\langle \mathcal{T} j_{\mu_1}(z_1) j_{\mu_2}(z_2) \rangle = iN_c \int d^4y_1 d^4y_2 \text{tr}_\gamma [\gamma_{\mu_1} G(z_1, y_1) \Gamma_{\mu_2}(y_1, y_2; z_2) G(y_2, z_1)], \quad (\text{D5})$$

which, after Fourier transform, gives Eq. (30).

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