Influence of multiple scattering on particle decays in a medium

A. V. Koshelkin
Moscow Institute for Physics and Engineering, Kashirskoye shosse 31, Moscow 115409, Russia
(Received 16 July 1998)

Effect of scattering in the matter on particle decays at high energies is studied. The systematic method for the calculation of the decay probability of the particles undergoing multiple elastic collisions in the equilibrium medium is developed. The probability of the decay of such particles is obtained. The Dalitz decay of pion in the matter is studied. [S0556-2813(99)02702-8]

PACS number(s): 12.39.Fe, 12.38.Mh, 12.40.Vv

I. INTRODUCTION

The problem of the influence of scattering in a medium on particle decays arises in the study of the states of the nuclear matter generated in process of collisions of high energy heavy ions [1–10]. Photons and leptons produced in the medium are a very important source of the information about the state of the nuclear matter because they are electroweak interacting particles only and therefore carry information about the primordial state of the nuclear matter generated in heavy-ion collisions. In this way one of the most important channels of the production of such particles is decays in the matter.

The effect of a medium on damping rates is studied in Refs. [2,5,9,11–18]. On the basis of the resummation method of the infinite sets of diagrams which has been developed by Braaten and Pisarski [19,20], in-matter polarization effects (renormalization of quark and pion masses) in QCD [5,20] is studied. In Refs. [11–16] the method of the hard thermal loops [19,20] has been used and developed for the calculation of the self-energy of an electron due to the electron-photon interaction in QED. In Refs. [2,21] the impact of the occupancy of the final states in a photon bath on decoupling constants has been investigated. The influence of the individual pair collisions of particles on the width of the vector mesons in the matter generated as a result of the collisions of heavy ions is considered in Refs. [17,18]. Since the matter is the high temperature and high density the collective effects play an important role here. In this way, if the energy fluctuations of the decaying particles due to their multiple scattering in the matter is of the order of the medium temperature, multiple scattering in the matter has a strong effect on the particle decay. This means that in this case the in-medium effects of multiple scattering should be taken into account in the calculations of decay widths of such particles in the matter.

In this paper we study the decays of the particles undergoing multiple elastic collisions in an equilibrium medium. The model-independent method of the calculation of the decay of such particles is developed. The obtained decay probability of the particles strongly depends on both the matter temperature, and the parameters characterizing the particle scattering in the medium. On the basis of the developed method the Dalitz decay of pion in an equilibrium pion gas is considered.

II. DECAY PROBABILITY OF A PARTICLE IN A SCATTERING MEDIUM

Let us consider in-matter particles with spin s and the mass M. We suppose that the particles undergo multiple elastic collisions in the thermodynamic equilibrium medium. We consider the particle decay determined by the processes $A \to A_1 + \cdots + A_N$.

In the quasiclassical case, when the wave length of the particle is less than the mean distance between them, the decoupling constant $\gamma(x,p)$ is given by the well-known expression (see, for example [22])

$$\gamma(x,p) = -2\,\mathrm{Im}\{\mathrm{Tr}\{\Sigma^{\mathrm{ret}}\}\}, \tag{1}$$

where Σ^{ret} is the retarded self-energy in the Keldysh diagram technique [23], $x=(t,\vec{r})$ and $=(p^0,\vec{p})$ are the four-coordinate and the four-momentum of the particle.

Let us suppose that the influence of the particle scattering on the decay vertex and on the interaction of the decay products in the final state is negligible. Then, we can write

$$\Sigma^{\mathrm{ret}} = \pm i \Sigma^{\alpha\beta}_{\mathrm{vac}}(p) G^{-+}_{\alpha\beta}(x,p) \prod_{j=1}^{N} (1 \pm n_j), \tag{2}$$

where $\Sigma^{\alpha\beta}_{\mathrm{vac}}(p)$ is the self-energy of the decaying particle in vacuum; $G^{-+}_{\alpha\beta}(x,p)$ is the Green's function of the particle in the matter; n_j are the occupancy numbers of the decay products (plus and minus signs correspond to the Boson and Fermi statistics, respectively); α and β are the spin variables.

To obtain the decay width Γ, which would be observed, we should average expression (1) over the four-coordinates and the four-momenta. Using the relation between $G^{-+}_{\alpha\beta}(x,p)$ and particle density $n(t,\vec{r},\vec{p})$ at the thermodynamic equilibrium [22,24] and integrating formula (1) over d^4x and d^4p, we obtain the following expression for the decay width of the on-shell particle:

$$\Gamma = \frac{1}{t_0} \prod_{j=1}^{N} (1 \pm n_j) \times \mathrm{Im}\left\{ \int_0^{t_0} dt \int \frac{d^3\vec{p}}{\varepsilon} \mathrm{Tr}\{\Sigma^{\alpha\beta}_{\mathrm{vac}}(p)\varrho_{\alpha\beta}\} \int \frac{d^3\vec{r}}{(2\pi)^3} \cdot n(t,\vec{r},\vec{p}) \right\}, \tag{3}$$

where $\varrho_{\alpha\beta}$ is the polarization density matrix of the decaying particle; t_0 is the time during which the decay is observed (the observation time); $p^0 \equiv \varepsilon = \sqrt{\vec{p}^2 + M^2}$. We also set the normalization volume $V=1$.

Let us suppose that the relaxation times in the matter $\tau \ll \min\{t_0, \Gamma^{-1}\}$. This means that the static approximation [25] for the description of the decaying particle movement in the medium can be used. In this case the density $n(t,\vec{r},\vec{p})$ is the solution of the standard transport equation [26]. Note that the function $n(t,\vec{r},\vec{p})$ in Eq. (3) is integrated over the particle coordinates $\vec{r}$. The integrating of the transport equation [26] over $d^3\vec{r}$ gives the following equation for the integral $f(\varepsilon,\vec{p},t)$ of the density $n(t,\vec{r},\vec{p})$ over the particle coordinates:

$$\frac{\partial f(p;t)}{\partial t} = \int d^3\vec{p}_1 d\sigma(p-p';p;p_1) v f_{\rm eq}(p_1) \times \{f(p';t) - f(p;t)\}, \tag{4}$$

where $d\sigma(p-p';p;p_1)$ is the cross section of the individual pair collision of two particles in the medium; $f_{\rm eq}(p_1)$ is the equilibrium distribution function of the particles in the matter; $v = \sqrt{s(s-4m^2)}/2\varepsilon\varepsilon_1$; $p=(\varepsilon,\vec{p})$; $p_1=(\varepsilon_1,\vec{p}_1)$; $p'=(\varepsilon',\vec{p}\,')$; $s=(p+p_1)^2$.

In the case of the homogeneous and isotropic matter the expression $\mathrm{Tr}\{\Sigma^{\alpha\beta}_{\rm vac}(p)\varrho_{\alpha\beta}\}$ in Eq. (3) depends only on the energy but does not depend on the momentum of the decaying particle, and we can rewrite the formula (3) as follows:

$$\Gamma = \frac{1}{t_0}\int_0^{t_0} dt \int_0^{+\infty} p^2 dp \Gamma_{\rm vac}(\varepsilon) F(\varepsilon,t). \tag{5}$$

Here

$$\Gamma_{\rm vac}(p) = \frac{1}{\varepsilon}\prod_{j=1}^{N}(1 \pm n_j)\mathrm{Im}\{\mathrm{Tr}\{\Sigma^{\alpha\beta}_{\rm vac}(p)\varrho_{\alpha\beta}\}\}, \tag{6}$$

$$F(\varepsilon,t) = \int \frac{d\Omega_{\vec{p}}}{(2\pi)^3} f(\varepsilon,\vec{p}), \tag{7}$$

and $d\Omega_{\vec{p}}$ is a solid angle in the direction of the vector $\vec{p}$. Function $F(\varepsilon,t)$ satisfies Eq. (8) obtained after the integration of the expression (4) over all directions of the vector $\vec{p}$:

$$\frac{\partial F(\varepsilon;t)}{\partial t} = \int d^3\vec{p}_1 \int_{\varepsilon - E_{\rm max}}^{\varepsilon + E_{\rm max}} d\varepsilon' \int d\Omega_{\vec{p}} \times \frac{d\sigma(\varepsilon-\varepsilon';\varepsilon;p_1)}{d\varepsilon' d\Omega_{\vec{p}}} v f_{\rm eq}(p_1)\{F(\varepsilon';t) - F(\varepsilon;t)\}, \tag{8}$$

where $E_{\rm max}$ is the maximum energy transferred in the process of the individual elastic collision of two particles of the matter. The energy $E_{\rm max}$ depends on the energies ε and ε_1 of the colliding particles.

When the matter is in equilibrium and only the elastic collisions take place the transferred energy is small as compared with the matter temperature. Then, we can expand the collision integral in the right-hand side of the last expression over the small parameter $|\varepsilon-\varepsilon'|/\varepsilon \ll 1$. As a result we have

$$\frac{\partial F(\varepsilon;t)}{\partial t} = \alpha \frac{\partial^2 F(\varepsilon;t)}{\partial \varepsilon^2}, \tag{9}$$

where α is the energy imparted to the particle in a unit time. In the derivation of the last equation we use the fact that the cross section is the even function of the transferred energy. The parameter α is defined by the following expression:

$$\alpha = \int d^3\vec{p}_1 d\Omega_{\vec{p}} \int_0^{E_{\rm max}} d\varepsilon' \frac{d\sigma(\varepsilon';\varepsilon;p_1)}{d\varepsilon' d\Omega_{\vec{p}}} v f_{\rm eq}(p_1)\varepsilon'^2. \tag{10}$$

For Eq. (9) to be solved the initial condition for the function $F(\varepsilon;t)$ should be specified. We suppose that some equilibrium particle distribution is given at time $t=0$:

$$F(\varepsilon;t=0) = f_{\rm eq}(\varepsilon)\,\eta(\varepsilon - M), \tag{11}$$

where $\eta(\varepsilon)$ is the standard unit function [$\eta(\varepsilon)=1$, when $\varepsilon \geq 0$, but $\eta(\varepsilon)=0$ at $\varepsilon<0$]; $\varepsilon = \sqrt{M^2+\vec{p}^2}$.

The parameter α determined by Eq. (10) depends on the particle energy ε. Besides in the case of the equilibrium matter the average value of the particle energy is approximately constant, but the energy transferred as a result of an individual collision of particles is small as compared with the matter temperature. Therefore, we can set $\varepsilon=\varepsilon_1$ in the formula (10) and also consider α as a constant. Then, solving Eq. (9) with the initial condition (11), we get

$$F(\varepsilon;t) = \frac{\eta(\varepsilon-M)}{\sqrt{4\pi\alpha t}}\int_0^{+\infty} d\varepsilon' \left\{\exp\left\{-\frac{(\varepsilon-\varepsilon'-M)^2}{4\alpha t}\right\} + \exp\left\{-\frac{(\varepsilon+\varepsilon'-M)^2}{4\alpha t}\right\}\right\} f_{\rm eq}(\varepsilon'). \tag{12}$$

Substituting the function $F(\varepsilon;t)$ (12) into Eq. (5), we find probability W of the particle decay in the scattering matter

$$W = \Gamma \cdot t_0 = \int_0^{t_0} \frac{dt}{\sqrt{4\pi\alpha t}} \int_0^{+\infty} p^2 dp \Gamma_{\rm vac}(\varepsilon(p)) \int_0^{+\infty} d\varepsilon' \times \left\{\exp\left\{-\frac{(\varepsilon-\varepsilon'-M)^2}{4\alpha t}\right\} + \exp\left\{-\frac{(\varepsilon+\varepsilon'-M)^2}{4\alpha t}\right\}\right\} f_{\rm eq}(\varepsilon'). \tag{13}$$

The last formula determines the decay probability of the particle undergoing multiple elastic collisions in the equilibrium matter. The obtained expression for the decay probability does not depend at all on the interaction model of the particle in the scattering medium. The influence of the matter on the probability W is expressed only in terms of the observable parameters such as the matter temperature (via the equilibrium distribution function $f_{\rm eq}$), the mean square of the energy transferred by the particle in the individual collision α and the observation time t_0.

We should note that the density of the states of the particle in the equilibrium matter has a sharp maximum at the energy $\varepsilon \sim T$ where T is the matter temperature. On the other hand, it follows from the formula (13) that the width of this maximum is determined by the relation between the value of the energy fluctuations of the particle due to the multiple scattering and the matter temperature. In this way the energy fluctuation during the observation time t_0 is defined by the magnitude of the parameter αt_0. If the $\alpha t_0 \geqslant T$ the influence of the multiple scattering on the particle decay is strong. In the opposite case $\alpha t_0 \leqslant T$ the effect is negligible.

Let us next consider the application of the developed method for the calculation of the decay probability of particles in the real scattering medium.

III. DALITZ DECAY OF A PION IN A SCATTERING MEDIUM

The pion decay $\pi^0 \to e^+ e^- \gamma$ (Dalitz decay) presents the interest in the study of dilepton pair production in the nuclear matter generated in the process of collisions of high energy heavy ions [7,8,27]. The contribution of this decay into the rate of dilepton pair production in the absence of multiple scattering of decaying particle in the matter is studied in the paper [28]. Since this electromagnetic decay is also virtually strong we use the quark model for the calculation of its probability. In this way the calculation result depends on the constants of strong and electromagnetic interactions. We note that presently the same combination of these constants appears in the formula for the width $\Gamma_{\pi^0 \to 2\gamma}$ of the decay $\pi^0 \to 2\gamma$. Therefore we change this combination of the constant by the expression for the width $\Gamma_{\pi^0 \to 2\gamma}$ in the final equation for the Dalitz decay probability of the pion in matter.

Neglecting the differences between masses of u and d quarks in the pion quark model we write the following relativistic invariant expression for the matrix element of the decay $\pi^0 \to e^+ e^- \gamma$:

$$\mathcal{M} = \frac{4\pi g e^{3/2}}{q^2 m_q^2} \epsilon_{ijsr} (v^*)^i (k) k^j q^s \overline{u}(-p_+) \gamma^r u(p_-),$$

$$q = p_+ + p_-, \tag{14}$$

where $p_- = (E_-, \vec{p}_-), p_+ = (E_+, \vec{p}_+), k = (\omega, \vec{k})$ are the four-momenta of electron, positron and photon, respectively; $u(p)$ and γ^r are the Dirac spinors and the Dirac matrixes; $v^j(k)$ is the photon polarization vector; ϵ_{ijsr} is the antisymmetric tensor; e is the electron charge; m_q is the quark mass; g is the constant of strong interaction which corresponds to the vertex of the pion quark loop.

Calculating the decay rate according to the conventional rules, we find the following expression for $\Gamma_{\text{vac}}(p)$ appears in Eq. (5) for the decay probability in the matter:

$$\frac{d\Gamma_{\text{vac}}}{dM_l} = \frac{4e^2}{3\pi \varepsilon M_l^4 M^5} (M^2 - M_l^2)^3 (M_l^2 + 2m^2) \times (M_l^2 - 4m^2)^{1/2} \Gamma_{\pi^0 \to 2\gamma},$$

$$\Gamma_{\pi^0 \to 2\gamma} = \frac{27\pi g^2 e^2 M^3}{m_q^2}, \tag{15}$$

where $\Gamma_{\pi^0 \to 2\gamma} = 8$ eV is the width of the decay of the pion being at rest [29], M and m are the masses of the pion and the electron, respectively; ε is the pion energy. $M_l = |p_+ + p_-|$ is the invariant mass of the dilepton pair. The last expression is obtained in the assumption that the occupation numbers of the decay products are equal to zero. Note that in the case of nonrelativistic pions the last formula coincides with the expression for the Dalitz decay rate [28].

Let us consider the Dalitz pion decay in an equilibrium pion gas. In the experimental situation the temperature T of the pion gas generated in the process of collisions of high energy heavy ions can be of the order of $T \sim 200 - 300$ MeV [3,27]. This means that we can approximately take that pions are ultrarelativistic. In this case the energy transferred in an individual elastic collision of two particles is small as compared with the particle energy which is of the order of the matter temperature. Estimating the maximum transferred energy $E_{\max}$ from the conservation laws we find that it is of the order of $E_{\max} \sim T(M/T)^2 \leqslant T$.

Then, substituting the obtained expression for $d\Gamma_{\text{vac}}/dM_l$ into the general formula (5) for the decay probability in the matter and assuming that the distribution function $f_{\text{eq}}(\varepsilon)$ has the Boltzmann form, we obtain

$$\frac{dW}{dM_l} = \frac{4e^2}{3\pi^{3/2} T M_l^4 M^5} (M^2 - M_l^2)^3 (M_l^2 + 2m^2) \times (M_l^2 - 4m^2)^{1/2} \Gamma_{\pi^0 \to 2\gamma} \times \left\{ \frac{2\alpha^{1/2} t_0^{3/2}}{3T} + \frac{T^2}{\alpha} \left[\exp\left(\frac{\alpha t_0}{T^2} \right) \int_{\sqrt{\alpha t_0}/T}^{+\infty} dx \times \exp(-x^2) - \frac{\sqrt{\pi}}{2} + \frac{\sqrt{\alpha t_0}}{T} \right] \right\}, \tag{16}$$

where T is the matter temperature, t_0 is the observation time which can be estimated as the life time of the equilibrium state of the matter which is of the order of ~ 10 fm [10].

Let us study the obtained expression for the Dalitz decay of the pion in the equilibrium pion gas.

When the energy fluctuation due to the elastic scattering is small as compared with the matter temperature, the parameter $(\sqrt{\alpha t_0}/T) \leqslant 1$. Then, calculating the integral in Eq. (16) at small low limit, we find

$$\frac{dW}{dM_l} = \frac{2e^2 t_0}{3\pi T M_l^4 M^5} (M^2 - M_l^2)^3 (M_l^2 + 2m^2) \times (M_l^2 - 4m^2)^{1/2} \Gamma_{\pi^0 \to 2\gamma} \times \left\{ 1 + \frac{\alpha t_0}{2T^2} + O((\alpha t_0 / T^2)^{3/2}) \right\}. \tag{17}$$

It follows from the last formula that the small energy fluctuations lead to the insignificant increase of the decay probability as compared with the situation of the particle decay in the absence of scattering in the matter.

In the opposite limiting case $(\sqrt{\alpha t_0}/T)\geqslant 1)$, when the energy fluctuations are large, we obtain from Eq. (16)

$$\frac{dW}{dM_l}=\frac{4e^2t_0}{3\pi^{3/2}TM_l{}^4M^5}(M^2-M_l{}^2)^3(M_l{}^2+2m^2)\times(M_l{}^2-4m^2)^{1/2}\Gamma_{\pi^0\to2\gamma}\times\left\{\frac{2\sqrt{\alpha t_0}}{3T}+O(T/\sqrt{\alpha t_0})\right\}. \quad (18)$$

From Eq. (18) it follows that the strong energy fluctuation leads to the considerable increase of the decay probability by the factor $(\sqrt{\alpha t_0}/T)\gg 1$.

Let us discuss the possibility of the observation of the obtained increase of the Dalitz pion decay in the equilibrium matter due to multiple elastic scattering. According to Eq. (10) the parameter α being the mean square fluctuation of the particle energy, can be approximately presented as the product of the collision frequency ν and the square of the maximum energy $E_{\max}{}^2$ transferred in the individual collision of two particles. Since $E_{\max}\sim T(M/T)^2$, the parameter $(\sqrt{\alpha t_0}/T)\sim\sqrt{\nu t_0}(M/T)^2=(M/T)^2\cdot\sqrt{N}$, where $N\gg 1$ is the number of particle collisions in the matter during the life time of the equilibrium state of the medium. In other words, if the number of particle collisions is larger than the ratio $(T/M)^2\gg 1$ the considerable increase of the pion decay probability takes place. When $T\sim 200-300$ MeV; $\nu\sim 1.33-4$ fm^{-1}, and $t_0\sim 10$ fm [3,10,27]. This corresponds to $\sqrt{\nu t_0}(M/T)^2\sim 1.38-1.79$. Thus, the multiple elastic scattering has to influence strongly on the pion Dalitz decay in the matter.

The numerical calculations according to the exact formulas (13) and (15) give

$$\frac{W(\alpha)}{W(\alpha=0)}=\begin{cases}1.74 & \text{when } T=150 \text{ MeV}\\ 1.36 & \text{when } T=200 \text{ MeV}\\ 1.32 & \text{when } T=300 \text{ MeV},\end{cases}$$

where $W(\alpha=0)$ is the decay probability in the absence of the multiple scattering. It follows from the last equations that in the most realistic case $T=150$ MeV the multiple elastic collisions of particles in the matter leads to the broadening of the pion state more than 70 percent.

In conclusion we should note that the dielectron yield due to the Dalitz decay of pions are not suppressed by any other processes of the dielectron production at low invariant mass $M_l\leqslant 100$ MeV of the dielectron pairs [27]. This allows us to discuss the possibility to observe the increasing of the pion decay probability due to the multiple elastic scattering of the pions in the hadronic matter.

IV. CONCLUSION

The decay of particles undergoing multiple elastic collisions in an equilibrium scattering medium is studied in this paper. The decay probability of such particles is calculated. The developed method of the calculation of decay probability does not depend on the dynamic model of particle scattering in the medium. In this way the effect of the medium on the decay probability is determined by the energy fluctuation of the decaying particle due to their multiple elastic scattering and the matter temperature. It is shown that when the energy fluctuation is of the order of the medium temperature, the multiple scattering results in the significant changes of the decay probability as compared with the case of scattering lack.

We study the Dalitz decay of pions undergoing multiple elastic collisions in an equilibrium pion gas. We have shown that the multiple scattering always leads to the increase of the decay probability. In this way when the energy fluctuations are of the order of the magnitude of the matter temperature, the multiple scattering leads to the strong broadening of the in-matter pion in the Dalitz decay channel. The probability of the pion Dalitz decay in the situations corresponding to the real temperatures of the matter has been numerically calculated. It is found that the multiple elastic scattering leads to the increase of the decay probability no less than 32 percent. We discuss the possibility to observe the increase of the Dalitz decay probability due to the multiple elastic scattering of pions in the matter generated in the process of collisions of heavy ions of high energy.

[1] K. Kajantie, J. Kapusta, L. D. McLerran, and A. Mekjian, Phys. Rev. D **34**, 2746 (1986).

[2] J. Kapusta, P. Lichard, and D. Seibert, Phys. Rev. D **44**, 2774 (1991).

[3] J. Cleymans, V. V. Goloviznin, and K. Redlich, Phys. Rev. D **47**, 989 (1993).

[4] J. Cleymans, V. V. Goloviznin, and K. Redlich, Phys. Rev. D **47**, 173 (1993).

[5] R. Pisarski, Phys. Rev. D **47**, 5589 (1993).

[6] S. K. Ghosh, S. C. Phatak, and P. K. Sahn, Nucl. Phys. **A596**, 670 (1996).

[7] U. Faschingbauer *et al.*, Nucl. Instrum. Methods Phys. Res. A **377**, 362 (1996).

[8] P. Rehak *et al.*, Nucl. Instrum. Methods Phys. Res. A **377**, 367 (1996).

[9] R. Pisarski and M. Tyrgat, Phys. Rev. D **54**, 2989 (1996).

[10] A. B. Migdal, D. N. Voskresensky, E. E. Saperstein, and M. A. Troitsky, *Pion Degrees of Freedom in Nuclear Matter* (Nauka, Moscow, 1991).

[11] G. Peressuti and B.-S. Skagerstam, Phys. Lett. **110B**, 406 (1982).

[12] G. Peressuti and B.-S. Skagerstam, Phys. Lett. **117B**, 171 (1982).

[13] R. Baier, H. H. Hakkagava, and A. Niegava, Can. J. Phys. **71**, 205 (1992).

[14] R. Baier, G. Kunstatter, and D. Shiff, Can. J. Phys. **71**, 208 (1993).

[15] R. Baier and R. Kobes, Phys. Rev. D **50**, 5944 (1994).

[16] T. Altherr, E. Petitgirard, and T. de Rio Gaztelurrutia, Phys. Rev. D **47**, 703 (1993).

[17] K. Haglin, Nucl. Phys. **A584**, 719 (1995).
[18] V. L. Eletsky, B. L. Ioffe, and J. I. Kapusta, nucl-th/9807072.
[19] R. Braaten and R. D. Pisarski, Nucl. Phys. **B337**, 569 (1990).
[20] R. Braaten and R. D. Pisarski, Nucl. Phys. **B339**, 310 (1990).
[21] I. E. Nieves and P. B. Pal, Phys. Rev. D **56**, 365 (1997).
[22] J. Knoll and D. N. Voskresensky, Ann. Phys. (N.Y.) **249**, 532 (1996).
[23] E. M. Lifshiz and L. P. Pitaevskii, *Physical Kinetics* (Nauka, Moscow, 1979).
[24] R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957); C. Martin and J. Schwinger, Phys. Rev. **115**, 1342 (1959).
[25] M. Gyulassy, X.-N. Wang, and M. Plumer, Phys. Rev. D **51**, 3436 (1995).
[26] S. R. de Groot, W. A. van Leeuwen, and Ch. G. van Weert, *Relativistic Kinetic Theory* (*Principles and Applications*) (North-Holland, Amsterdam, New York, Oxford, 1980).
[27] J. Cleymans, V. V. Goloviznin, and K. Redlich, Proceedings of Workshop on Pre-Equilibrium Parton Dynamics in Heavy Ions Collisions, LBL, Berkeley, 1993 (hep-ph/9310284, UCT-TP 199/93).
[28] J. Cleymans, K. Redlich, and H. Satz, Z. Phys. C **52**, 517 (1991).
[29] N. P. Samios, Phys. Rev. **121**, 275 (1961).