Proton particle-neutron hole states in ¹³²Sb with a realistic interaction

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The structure of the particle-hole nucleus ¹³²Sb provides a direct source of information on the effective neutron-proton interaction in the ¹³²Sn region. We have performed a shell-model calculation for this nucleus using a realistic effective interaction derived from the Bonn A nucleon-nucleon potential. The results are in very good agreement with the experimental data evidencing the reliability of our effective interaction matrix elements both with isospin T=1 and T=0. [S0556-2813(99)04402-7]

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I. INTRODUCTION

A fundamental problem in nuclear structure theory is to understand the properties of complex nuclei in terms of the nucleon-nucleon (NN) interaction. Since the initial work of Kuo and Brown [1], who derived an *s*-*d* shell effective interaction from the Hamada-Johnston potential [2], there has been much progress in this field. On the one hand, the manybody methods for calculating the matrix elements of the effective interaction have been largely improved [3]. On the other hand, more realistic NN potentials have been developed which are able to reproduce quite accurately all the known NN data. A comprehensive account of these developments through 1993 is given in the review paper by Machleidt and Li [4].

Motivated by these improvements, in recent years there has been a revival of interest in nuclear structure calculations with realistic effective interactions. As a result, calculations of this kind, which in earlier studies had been mainly confined to light nuclei, have been recently extended to mediumand heavy-mass nuclei. Until now, however, attention has been focused on identical particle systems, in particular on Sn isotopes and N=82 isotones [5–8]. In our own studies [6,7] (hereafter referred to as I and II) we considered the ¹⁰⁰Sn neighbors going from ¹⁰²Sn to ¹⁰⁵Sn, while for the N=82 isotones we were concerned with the ¹³²Sn neighbors with two and three valence protons. In both works we performed shell-model calculations using a realistic effective interaction derived from the meson-theoretic Bonn A potential [9].

The motivation for the theoretical study of nuclei around doubly magic ¹⁰⁰Sn and ¹³²Sn lies in the fact that they provide the best testing ground for the basic ingredients of shell-model calculations in the 100–150 mass region, especially as regards the matrix elements of the effective *NN* interaction. These nuclei, however, lie well away from the valley of stability and until recently experimental information on their spectroscopic properties was very scanty. The advent of large multidetector γ -ray arrays and radioactive ion-beam facilities is now making more and more accessible their study. In this context, it is worthwhile to mention that in I we could only predict the spectra of ¹⁰²Sn and ¹⁰³Sn since no experimental data for these two very neutron-deficient nuclei

were available at that time. After submission of this paper the identification of three excited states of 102 Sn in an inbeam γ -ray spectroscopic experiment was reported [10]. As is shown in Ref. [11], our predictions turned out to be in remarkably good agreement with the experimental results. Similarly, our study II of 134 Te and 135 I has largely benefitted from the results of very recent experimental work [12,13].

The very good agreement between theory and experiment achieved in I and II evidences the reliability of our T=1 effective interaction in the 100–150 mass region and makes apparent the motivation for the present study of the doubly odd nucleus ¹³²Sb. This nucleus, which has a single proton outside the Z=50 closed shell and a single neutron hole in the closed N=82 shell, offers the opportunity to test the T=0 matrix elements of the effective interaction. Experimental information on ¹³²Sb was earlier provided by the works of Refs. [14] and [15]. Recently, the detailed study of Ref. [16] has significantly improved our knowledge of its spectroscopic properties.

The outline of the paper is as follows. In Sec. II we give a brief description of our calculations. The results of our study are presented and compared with experiment in Sec. III. Section IV contains a summary of our conclusions.

II. CALCULATIONS

In our shell-model study of ¹³²Sb we consider the doubly magic ¹⁰⁰Sn as a closed core and treat the odd proton and the remaining 31 neutrons as valence particles. The model space consists of the five single-particle (sp) orbits $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$, and use is made of a two-body effective interaction derived from the Bonn A nucleonnucleon potential. This was obtained using a *G*-matrix formalism, including renormalizations from both core polarization and folded diagrams. We have chosen the Pauli exclusion operator Q_2 in the *G*-matrix equation,

$$G(\omega) = V + VQ_2 \frac{1}{\omega - Q_2 TQ_2} Q_2 G(\omega), \qquad (1)$$

as specified by $(n_1, n_2, n_3) = (11, 21, 45)$ [3]. Here *V* represents the *NN* potential, *T* denotes the two-nucleon kinetic

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energy, and ω is the so-called starting energy. We employ a matrix inversion method to calculate the above *G* matrix in an essentially exact way. The effective interaction, which is energy independent, can be schematically written in operator form as

$$V_{\rm eff} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots,$$
(2)

where \hat{Q} and \hat{Q}' represent the \hat{Q} box, composed of irreducible valence-linked diagrams, and the integral sign represents a generalized folding operation. We take the \hat{Q} box to be composed of *G*-matrix diagrams through second order in *G*. The shell-model oscillator parameter used by us is $\hbar\omega$ = 8 MeV. It should be noted that the *T*=1 matrix elements of the effective interaction used in the present calculation are just the same as those used in I. More details on our derivation of V_{eff} can be found in Ref. [3].

As regards the sp energies, we assume them to be the same for neutrons and protons. Our adopted values (in MeV) are: $\epsilon_{d_{5/2}} = 0.0$, $\epsilon_{g_{7/2}} = 0.20$, $\epsilon_{s_{1/2}} = 1.72$, $\epsilon_{d_{3/2}} = 1.88$, and $\epsilon_{h_{11/2}} = 2.70$. As compared to the set of sp energies used in I, only $\epsilon_{s_{1/2}}$ and $\epsilon_{d_{3/2}}$ have been modified. It should be noted that their position played a minor role in our earlier study I of the light Sn isotopes. Here we have kept constant the spacing between these two levels while shifting down their position by about 0.5 MeV. We have found that this change is essential to satisfactorily reproduce the experimental 1^+_2 state in ¹³²Sb and to place the calculated negative-parity states in the right energy range. In this context, it is worth mentioning that we have also calculated the spectra of ¹³³Sb and ¹³¹Sn making use of the above set of sp energies. These nuclei have a single proton outside the Z=50 closed shell and a single neutron hole in the N=82 closed shell, respectively. It turns out that the corresponding experimental spectra are reproduced within about 200 keV, a larger discrepancy (780 keV) occurring only for the $\frac{7}{2}$ + state in ¹³¹Sn. In this regard, it is interesting to note that our calculated position of the $\frac{3}{2}^+$ single proton level in ¹³³Sb (2.37 MeV) is in very good agreement with the new value (2.44 MeV) very recently provided by the high-sensitivity γ spectroscopic measurement of Ref. [17].

III. RESULTS

The experimental and theoretical spectra of 132 Sb are compared in Fig. 1, where all the observed levels are reported. In the calculated spectrum all levels up to 1.4 MeV excitation energy are included while in the higher energy region only the 1_2^+ and 3_2^- states are reported. It should be noted that the nature of the presently available experimental information is quite different for positive- and negativeparity levels. In fact, while the spin-parity assignments to the former have been clarified by the study of Ref. [16], this is not the case for the latter. More precisely, the excitation energy of the 8^- state is not known (the work of Ref. [15] places it between 150 and 250 keV) and the three other observed negative-parity states have not received firm spin assignments. We find that the first excited 8^- state lies at 126



FIG. 1. Experimental and calculated spectrum of ¹³²Sb.

keV while the 6_1^- , 5_1^- , 7_1^- , 3_1^- , and 4_1^- states are grouped in a very small energy interval (from 214 to 380 keV). As a consequence, any attempt to establish a one-to-one correspondence between the observed levels and those predicted by our calculation could be misleading. The relevant outcome of our calculation is that the above states, which all arise from the $\pi g_{7/2} \nu h_{11/2}^{-1}$ configuration, lie very close in energy and are well separated from the other two members of the multiplet, i.e., the 9⁻ and 2⁻ states, which are predicted at 1.01 and 1.42 MeV, respectively. A similar behavior is also predicted for the $\pi d_{5/2} \nu h_{11/2}^{-1}$ multiplet. In fact, the calculated energies of the 7⁻, 6⁻, 5⁻, and 4⁻ states belonging to this configuration go from 0.82 to 0.97 MeV while the highest- and lowest-spin members (8⁻ and 3⁻) lie at 1.12 and 1.56 MeV, respectively.

From Fig. 1 we see that the experimental energies of the seven excited positive-parity states are remarkably well reproduced by the theory, the largest discrepancy being 77 keV for the 5_1^+ state. The value of the rms deviation σ [18] is only 32 keV. In earlier works (Refs. [14,15], and references therein) the first four observed positive-parity states were attributed to the $\pi g_{7/2} \nu d_{3/2}^{-1}$ configuration and the $3^{(+)}$ level at 529 keV to the $\pi g_{7/2} \nu s_{1/2}^{-1}$ configuration. The 2_2^+ and the 1_1^+ states were interpreted as members of the $\pi g_{7/2} \nu d_{3/2}^{-1}$ multiplet, and the 1_2^+ state as a member of the $\pi g_{7/2} \nu d_{5/2}^{-1}$ multiplet. Our calculated wave functions, however, are not really pure. We find, in fact, that the contribution coming from configurations other than the dominant one is particularly significant for most of the calculated states. More pre-

| $J_i^{\pi} { ightarrow} J_f^{\pi}$ | $B(M1)_{\text{expt}} (10^{-3} \mu_N^2)$ | $\frac{B(M1)_{\text{calc}}}{(10^{-3}\mu_N^2)}$ | $\frac{B(E2)_{\text{expt}}}{(\text{e}^2\text{fm}^4)}$ | $\frac{B(E2)_{calc}}{(e^2 fm^4)}$ |
|------------------------------------|---|--|---|-----------------------------------|
| $3_{1}^{+} \rightarrow 4_{1}^{+}$ | 2.02 ± 0.03 | 12 | 36±11 | 42 |
| $2^{+}_{1} \rightarrow 3^{+}_{1}$ | 61 ± 7 | 35 | 76 ± 76 | 53 |
| 4_{1}^{+} | | | <26 | 5.8 |
| $3^+_2 \rightarrow 2^+_1$ | | 5.0 | | 6.1 |
| 3^{+}_{1} | >3.4 | 257 | | 3.4 |
| 4_{1}^{+} | >17.5 | 409 | | 18 |
| $2^+_2 \rightarrow 3^+_2$ | 4.8 ± 2.6 | 1.1 | $1.1^{+15.4}_{-1.1}$ | 0.85 |
| 2^{+}_{1} | 2.2 ± 1.7 | 0.0001 | 37 ± 30 | 4.4 |
| 3^{+}_{1} | 10 ± 5 | 0.90 | 36 ± 23 | 5.4 |
| 4_{1}^{+} | | | 8.4 ± 4.5 | 9.1 |
| $1_{1}^{+} \rightarrow 2_{2}^{+}$ | >29 | 1014 | >45 | 108 |
| 3^{+}_{2} | | | >0.15 | 0.04 |
| 2^{+}_{1} | >0.66 | 4.9 | >0.17 | 3.8 |
| 3^{+}_{1} | | | >0.51 | 16 |
| $1_2^+ \rightarrow 1_1^+$ | | 0.003 | | 0.85 |
| 2^{+}_{2} | | 19 | | 1.7 |
| 3^{+}_{2} | | | >0.09 | 11 |
| $2_{1}^{\tilde{+}}$ | >0.09 | 425 | | 7.1 |
| 3^{+}_{1} | | | | 2 |

TABLE I. Calculated and experimental transition rates in 132 Sb. The experimental data are from [16].

cisely, the percentage of these components ranges from 2 to 20%. Below 1.4 MeV we also predict the existence of the nonobserved members of the $\pi g_{7/2} \nu s_{1/2}^{-1}$ and $\pi d_{5/2} \nu d_{3/2}^{-1}$ multiplets. They are the 4_2^+ and the 3_3^+ and 4_3^+ states, respectively.

In Table I we compare the experimental electromagnetic transition probabilities in ¹³²Sb with the calculated ones. The E2 transitions have been calculated using an effective proton charge $e_n^{\text{eff}} = 1.55e$, which is the same as that adopted in II for the $\dot{N} = 82$ isotones. No effective charge has been attributed to the neutron hole. As regards the magnetic transitions, for the proton gyromagnetic factors we have adopted the values $g_s^{\text{eff}}(p) = 4.465$ and $g_l^{\text{eff}}(p) = 1.55$ while $g_s^{\text{free}}(n)$ and $g_{l}^{\text{free}}(n)$ have been used for the neutron hole. As we see from Table I, most of the experimental data are affected by large errors and for several transition rates only an upper or lower limit is available from experiment. In view of this, the agreement between theory and experiment for the B(E2)'s can be considered quite satisfactory. In fact, our calculated values lie all but two within the limits set by experiment. Concerning the B(M1)'s, the five transitions for which a definite value is available from experiment are not reproduced by our calculation within the error bars. In this connection, it is to be noted that, should one ignore configuration mixing, the three M1 transitions $2_2^+ \rightarrow 3_2^+$, $2_2^+ \rightarrow 2_1^+$ and $2_2^+ \rightarrow 3_1^+$ would be forbidden while the same value would be predicted for the $3_1^+ \rightarrow 4_1^+$ and $2_1^+ \rightarrow 3_1^+$ transitions. It therefore appears that our calculation produces a configuration mixing which goes in the right direction. It is clear, however, that for a stringent test of the wave functions additional and more accurate experimental information is needed.

At this point, it should be mentioned that several calculations have been previously performed to predict the structure of ¹³²Sb. A comprehensive account of the earlier works including references can be found in [15]. We only comment here on the most recent calculation of Ref. [19], which has been carried out by using the random-phase approximation with a phenomenological interaction. The results of this calculation are discussed in detail in Ref. [16].

As regards the positive-parity spectrum, the overall agreement with experiment obtained in Ref. [19] for the first five excited states is about the same as ours, while for the two higher-lying 1⁺ states our results are significantly better. As for the electromagnetic transition rates, most of the theoretical values reported in Ref. [16] are also close to ours. In the few cases where there are significant discrepancies no definite conclusion can be drawn owing to the large experimental uncertainties. One notable exception is the $B(E2;3_1^+ \rightarrow 4_1^+)$ for which a value of $36\pm11 \text{ e}^2\text{fm}^4$ has been measured [16]. Our calculated value, $42 \text{ e}^2\text{fm}^4$, falls within the error bar, while the calculation of Ref. [16] overstimates the experimental value by a factor of 2. This overestimate, however, can be mainly attributed to the large value of the neutron effective charge (0.9*e*) adopted in Ref. [19].

From the above comparison we see that our calculation seems to lead to an even better agreement with experiment than that produced by the sophisticated approach of Ref. [19]. We would like to emphasize that this has been achieved by using a realistic effective interaction with a sound mesontheoretic basis.

IV. SUMMARY

We have shown that our effective interaction derived from the Bonn A nucleon-nucleon potential leads to a very good description of the proton particle-neutron hole nucleus ¹³²Sb. To our knowledge, this is the first test of the T=0matrix elements of this interaction in the 100–150 mass region. A particular feature of the Bonn A potential relevant to the present study is its weak tensor force. In fact, in earlier works using different *NN* potentials it turned out that not enough attraction was provided by the calculated matrix elements of the T=0 effective interaction, which has a stronger dependence on the tensor force strength than the T=1interaction (a detailed discussion of this important point is given in Ref. [3]). Our results indicate that the Bonn A potential is quite suitable for use in shell-model studies of nuclei with both like and unlike nucleons in valence shells.

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