Radiative proton-antiproton annihilation and isospin mixing in protonium

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A detailed analysis of the radiative $p\bar{p}$ annihilation is made in the framework of a two-step formalism; the $p\bar{p}$ annihilates into meson channels containing a vector meson with a subsequent conversion into a photon via the vector dominance model (VDM). Both steps are derived from the underlying quark model. First, branching ratios for radiative protonium annihilation are calculated and compared with data. Then, details of the isospin interference are studied for different models of the initial protonium state and also for different kinematical form factors. The isospin interference is shown to be uniquely connected to the $p\bar{p}-n\bar{n}$ mixing in the protonium state. Values of the interference terms directly deduced from data are consistent with theoretical expectations, indicating a dominant $p\bar{p}$ component for the $^1S_0$ and a sizable $n\bar{n}$ component for the $^3S_1$ protonium state. The analysis is extended to the $p\bar{p}\rightarrow\gamma\Phi$ transition, where the large observed branching ratio remains unexplained in the VDM approach. [S0556-2813(99)00402-1]

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I. INTRODUCTION AND MOTIVATION

Nucleon-antinucleon ($N\bar{N}$) annihilation, due to the richness of possible final meson states, is considered one of the major testing grounds in the study of hadronic interactions. Both quark [1] and baryon exchange models [2–4] have been applied to $N\bar{N}$ annihilation data. However, the task of extracting information on the dynamics of the $N\bar{N}$ process is enormously complicated by the influence of initial- and final-state interactions. Some of the simplest annihilation channels, where the theoretical complexity of the $N\bar{N}$ annihilation process is partially reduced, are radiative two-body decay modes, where final-state interaction is negligible. Experimental branching ratios for radiative decay channels in annihilation from $p\bar{p}$ atoms were made available by recent measurements of the Crystal Barrel collaboration at CERN, performing a systematic study of the reactions $p\bar{p}\rightarrow\gamma X$ where $X=\gamma,\pi^0,\eta,\omega$ and η' [5]. Radiative decays of the $p\bar{p}$ atom where, in contrast to ordinary production of nonstrange mesonic final states, isospin is not conserved, are well suited for studying interference effects in the isospin transition amplitudes [5,6].

The simplest and most natural framework in studying radiative decay modes is the vector dominance model (VDM) [7]. In setting up the annihilation mechanism one adopts a two-step process where the $p\bar{p}$ system first annihilates into two mesons, with at least one of the mesons being a vector meson (ρ and ω), and where the produced vector meson converts into a real photon via the VDM [6]. In this case, production rates of radiative decay modes can be related to branching ratios of final states containing one or two vector mesons. A first analysis [5] in the framework of VDM was performed by Crystal Barrel, showing that the interference in the isospin amplitudes is sizable and almost maximally destructive for all channels considered. The phase structure of the interference term is determined by two contributions: (i) the relative signs of the generic strong transition amplitudes for $p\bar{p}\rightarrow X\omega$ or $X\rho$ acting in different isospin channels; (ii) the presence of the initial-state interaction in the $p\bar{p}$ atom, which mixes the $p\bar{p}$ and $n\bar{n}$ configurations. Similarly, analogous sources are responsible for the isospin interference effects in the strong annihilation reactions $p\bar{p}\rightarrow K\bar{K}$ [1,8,9]. Here, however, definite conclusions concerning the size and sign of the interference terms depend strongly on the model used for the annihilation dynamics.

In the present work we show how the determination of the interference terms in the analysis of the radiative decays can be uniquely connected to the isospin mixing effects in the $p\bar{p}$ atomic wave functions. The extraction of the magnitude and sign of the interference from the experimental data can in turn be used to investigate the isospin dependence, at least in an averaged fashion, of the S-wave $N\bar{N}$ interaction. We study this point for different $N\bar{N}$ interaction models.

This paper is organized as follows. In Sec. II we develop the formalism for radiative decays of protonium. As in Ref. [6] we adopt a two-step formalism, that is $p\bar{p}$ annihilation into two-meson channels containing a vector meson and its subsequent conversion into a photon via the VDM. Both steps are derived consistently from the underlying quark model in order to fix the phase structure of the isospin-dependent transition amplitudes. We also indicate the derivation of the branching ratios for radiative decays of S-wave protonium, where the initial-state interaction of the atomic $p\bar{p}$ system is included. Section III is devoted to the presentation of the results. We first perform a simple analysis to show that theoretically predicted branching ratios for radiative decays are consistent with the experimental data. We then show that the isospin interference terms present in the expression for the branching ratios can be uniquely connected to the $p\bar{p}$-$n\bar{n}$ mixing in the atomic wave function, induced by initial-state interaction. We quantify the details of

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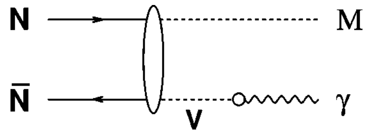

FIG. 1. Two-step process $N\bar{N}\to MV\to M\gamma$ with $V=\rho^0,\omega$ and $M=\pi^0,\eta,\rho^0,\omega,\eta',\gamma$ for radiative protonium annihilation.

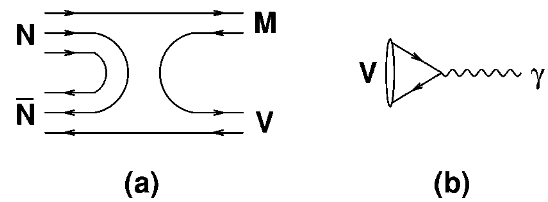

FIG. 2. Quark line diagrams corresponding to $N\bar{N}$ annihilation into two mesons (a) and vector meson-photon conversion (b).

this effect for different models of the $N\bar{N}$ interaction and apply the formalism developed in Sec. II to extract size and sign of the interference from data, which will be shown to be sensitively dependent on the kinematical form factors associated with the transition. Furthermore, we comment on the application of VDM on the transition $p\bar{p}\to\gamma\Phi$, where the corresponding large branching ratio plays a central role in the discussion on the apparent violations of the Okubo-Zweig-Iizuka (OZI) rule. A summary and conclusions are given in Sec. IV.

II. FORMALISM FOR RADIATIVE DECAYS OF PROTONIUM

In describing the radiative decays of protonium we apply the vector dominance model [6,7]. We consider the two-step process of Fig. 1, where the primary $p\bar{p}$ annihilation in a strong transition into a two-meson final state, containing the vector mesons ρ and ω, is followed by the conversion of the vector meson into a real photon. Here we restrict ourselves to orbital angular momentum $L=0$ for the initial $p\bar{p}$ state, corresponding to the dominant contribution in the liquid-hydrogen data of Crystal Barrel [5]. Furthermore, we consider the transition processes $p\bar{p}\to\gamma X$, where $X=\gamma,\pi^0,\eta,\rho,\omega$ and η', with $X=\phi$ presently excluded. The final state $\phi\gamma$ plays a special role in the discussion of the apparent violation of the Okubo-Zweig-Iizuka (OZI) rule, where a strong enhancement relative to $\omega\gamma$ was observed [10]. Within the current approach the description of the first-step process $p\bar{p}\to\omega(\rho)\phi$ and its phase structure cannot be accomodated due to the special nature of the ϕ, a dominant $s\bar{s}$ configuration. Later on we will comment on the possibility to explain the enhanced $\phi\gamma$ rate within the VDM, as suggested in the literature [11], and on the implications of the analysis presented here.

In the two-step process we have to introduce a consistent description for either transition in order to identify the source of the interference term. In particular, the relative phase structure of the strong transition-matrix elements $p\bar{p}\to\omega M$ versus $p\bar{p}\to\rho^0 M$ ($M=\pi^0,\eta,\rho,\omega$ and η') is a relevant input in determining the sign of the interference. Basic SU(3) flavor symmetry arguments [12] do not allow to uniquely fix the phase structure, hence further considerations concerning spin and orbital angular momentum dynamics in the $N\bar{N}$ annihilation process have to be introduced. Microscopic approaches to $N\bar{N}$ annihilation either resort to quark models (for an overview see Ref. [1]) or are based on baryon exchange models [2–4]. Here we choose the quark model approach, which allows us to describe both the strong transition of $p\bar{p}$ into two mesons and the vector meson conversion into a photon.

For the process $p\bar{p}\to VM$ where $V=\rho,\omega$ and $M=\pi^0$, η, ρ, ω and η' we apply the so-called A2 model [1], depicted in Fig. 2(a). In the discussion of annihilation models based on quark degrees of freedom this mechanism was shown to give the best phenomenological description in various meson branching ratios [1,13,14]. In a recent work [15] we showed that the A2 model combined with a corresponding annihilation mechanism into three mesons can describe $p\bar{p}$ cross sections in a quality expected from simple nonrelativistic quark models. The transition-matrix element of $p\bar{p}\to VM$ in the A2 model including initial-state interaction is given by

$$\begin{aligned} T_{N\bar{N}(IJ)\to VM} &= \langle V(j_1=1)M(j_2)l_f|\mathcal{O}_{A2}|N\bar{N}(IJ)\rangle \\ &= \sum_j \langle j_1 j_2 m_1 m_2|jm\rangle\langle j l_f m m_f|JM\rangle\cdot|\vec{k}|Y_{l_f m_f}(\hat{k}) \\ &\quad \times\langle VM||\mathcal{O}_{A2}||N\bar{N}(IJ)\rangle \end{aligned} \tag{1}$$

with the reduced matrix element defined as

$$\langle VM||\mathcal{O}_{A2}||N\bar{N}(IJ)\rangle = F(k)\langle IJ\to VM\rangle_{\rm SF}\mathcal{B}(I,J). \tag{2}$$

The atomic $p\bar{p}$ state is specified by isospin component I and total angular momentum $J=0,1$, the latter values corresponding to the $^1S_0$ and $^3SD_1$ states, respectively. The two-meson state VM is specified by the intrinsic spin $j_{1,2}$, the total spin coupling j, the relative orbital angular momentum $l_f=1$ and the relative momentum $\vec{k}$. Equation (2) includes a final-state form factor $F(k)$, the spin-flavor weight $\langle IJ\to VM\rangle_{\rm SF}$ and an initial-state interaction coefficient $\mathcal{B}(I,J)$, containing the distortion in the protonium state J with isospin component I. Detailed expressions for these factors are summarized in Appendix A.

For the process $V\to\gamma$ [Fig. 2(b)], where the outgoing photon with energy k^0 is on-mass shell, we obtain, with the details shown in Appendix B:

$$T_{V\to\gamma}=\vec{\epsilon}\cdot\vec{S}(m_1)\mathrm{Tr}(Q\varphi_V)\frac{e m_\rho^{3/2}}{(2k_0)^{1/2}f_\rho}, \tag{3}$$

where $\vec{\epsilon}$ and $\vec{S}(m_1)$, with projection m_1, are the polarization vectors of γ and V, respectively. The flavor dependence of the transition is contained in the factor $\mathrm{Tr}(Q\varphi_V)$, where Q is the quark charge matrix and φ_V the $Q\bar{Q}$ flavor wave function of vector meson V. In setting up the two-step process $N\bar{N}(IJ)\to VM\to\gamma M$ we use time-ordered perturbation theory with the resulting matrix element [16]

$$T_{N\bar{N}(IJ)\to VM\to\gamma M}=\sum_{m_1} T_{V\to\gamma}\frac{2m_V}{m_V^2-s}T_{N\bar{N}(IJ)\to VM}, \quad (4)$$

where the relativistic propagator for the intermediate vector meson in a zero width approximation is included. We resort to a relativistic prescription of the vector meson, since, with the kinematical constraint $\sqrt{s}=0$, V has to be treated as a virtual particle, which is severely off its mass shell. Accordingly, an additional factor $2m_V$, with the vector meson mass m_V, has to be included to obtain the proper normalization. From redefining

$$T_{V\to\gamma}\frac{2m_V}{m_V^2-s}\equiv\vec{\epsilon}\cdot\vec{S}(m_1)A_{V\gamma}, \quad (5)$$

we generate the standard VDM expression of

$$T_{N\bar{N}(IJ)\to VM\to\gamma M}=\sum_{m_1} T_{N\bar{N}(IJ)\to VM}\vec{\epsilon}\cdot\vec{S}(m_1)A_{V\gamma}. \quad (6)$$

The VDM amplitude $A_{V\gamma}$, derived in the quark model, is

$$A_{V\gamma}=\sqrt{2}\mathrm{Tr}(Q\varphi_V)\sqrt{\frac{m_V}{k^0}}\frac{e}{f_\rho}, \quad (7)$$

which in the limit $m_v\approx k^0$ reduces to the well-known results of [7]

$$A_{\rho\gamma}=e/f_\rho=0.055 \quad \text{and} \quad A_{\omega\gamma}=\frac{1}{3}A_{\rho\gamma}. \quad (8)$$

The phase structure of $A_{V\gamma}$, as determined by φ_V, is consistent with the corresponding definitions for the strong transition-matrix element.

In the radiative annihilation amplitude, the coherent sum of amplitudes for $V=\rho$ and ω, arising from different isospin channels, has to be taken. This gives

$$T_{N\bar{N}(J)\to\gamma M}=\sum_{V=\rho,\omega}\delta\cdot T_{N\bar{N}(IJ)\to VM\to\gamma M}, \quad (9)$$

where $\delta=1$ for $V\neq M$ and $\delta=\sqrt{2}$ for $V=M$. The additional δ accounts for the two possible contributions to the amplitude from an intermediate state with $V=M$, including a Bose-Einstein factor. For the decay width of $N\bar{N}\to\gamma X$ we write

$$\Gamma_{N\bar{N}(J)\to\gamma X}=2\pi\rho_f\sum_{M,\epsilon_T,m_2}\frac{1}{(2J+1)}|T_{N\bar{N}(J)\to\gamma M}|^2. \quad (10)$$

ρ_f is the final-state density and the sum is over the final-state magnetic quantum numbers of meson X (m_2) and of the photon (with transverse polarization ϵ_T). The corresponding branching ratio B is

$$B(\gamma X)=\frac{(2J+1)}{4\Gamma_{\mathrm{tot}}(J)}\Gamma_{N\bar{N}(J)\to\gamma X}, \quad (11)$$

where a statistical weight of the initial protonium state J with decay width $\Gamma_{\mathrm{tot}}(J)$ is taken. With the details of the evaluation indicated in Appendix C, we finally obtain for the branching ratios of $p\bar{p}\to\gamma X$ ($X=\pi^0,\eta,\eta'$):

$$B(\gamma\pi^0)=\frac{3}{4\Gamma_{\mathrm{tot}}(J=1)}f(\gamma,\pi^0)A_{\rho\gamma}^2\Big|\mathcal{B}(0,1)\langle{}^{13}SD_1\to\rho^0\pi^0\rangle_{SF}+\frac{1}{3}\mathcal{B}(1,1)\langle{}^{33}SD_1\to\omega\pi^0\rangle_{\mathrm{SF}}\Big|^2. \quad (12)$$

Alternatively, $B(\gamma\pi^0)$ can be expressed in terms of the branching ratios $B(V\pi^0)$ for the strong transitions $N\bar{N}\to V\pi^0$ [Eq. (A12) of Appendix A]:

$$B(\gamma\pi^0)=\frac{f(\gamma,\pi^0)}{f(V,\pi^0)}A_{\rho\gamma}^2\bigg(B(\rho^0\pi^0)+\frac{1}{9}B(\omega\pi^0)+\frac{2}{3}\cos\beta_{J=1}\sqrt{B(\rho^0\pi^0)B(\omega\pi^0)}\bigg) \quad (13)$$

with the interference phase $\beta_{J=1}$ determined by

$$\cos\beta_{J=1}=\frac{\mathrm{Re}\{\mathcal{B}(0,1)^*\mathcal{B}(1,1)\}}{|\mathcal{B}(0,1)\mathcal{B}(1,1)|}. \quad (14)$$

The same equations apply for $X=\eta$ and η' with π^0 being replaced by the respective meson. Here, a kinematical phase space factor f is introduced, which can be identified with those derived in specific models [Eqs. (A11) and (C10)] or taken from phenomenology. Values for the branching ratios on the right-hand side of Eq. (13) can either be taken directly from experiment or determined in the quark model considered in Appendix A. Magnitude and sign of the interference term, as determined by $\cos\beta_{J=1}$, solely depends on the initial-state interaction for the spin-triplet $N\bar{N}$ state ($J=1$), as expressed by the coefficients $\mathcal{B}(I,J=1)$.

Similarly, for the branching ratios of $p\bar{p}\to\gamma X$ ($X=\rho^0,\omega,\gamma$), now produced from the spin-singlet state ($J=0$) of protonium, we obtain

$$B(\gamma\rho^0)=\frac{f(\gamma,\rho^0)}{f(V,V)}A_{\rho\gamma}^2\bigg(\frac{1}{9}B(\rho^0\omega)+2B(\rho^0\rho^0)+\frac{2\sqrt{2}}{3}\cos\beta_{J=0}\sqrt{B(\rho^0\omega)B(\rho^0\rho^0)}\bigg), \quad (15)$$

$$B(\gamma\omega)=\frac{f(\gamma,\omega)}{f(V,V)}A_{\rho\gamma}^2\bigg(B(\rho^0\omega)+\frac{2}{9}B(\omega\omega)+\frac{2\sqrt{2}}{3}\cos\beta_{J=0}\sqrt{B(\rho^0\omega)B(\omega\omega)}\bigg), \quad (16)$$

and

$$B(\gamma\gamma)=\frac{f(\gamma,\gamma)}{f(V,V)}A_{\rho\gamma}^{4}\Bigg\{B(\rho^0\rho^0)+\frac{2}{9}B(\omega\rho^0)+\frac{1}{81}B(\omega\omega)+\frac{2}{9}\sqrt{B(\rho^0\rho^0)B(\omega\omega)}+\frac{2\sqrt{2}}{3}\cos\beta_{J=0}\sqrt{B(\rho^0\omega)}\times\left[\sqrt{B(\rho^0\rho^0)}+\frac{1}{9}\sqrt{B(\omega\omega)}\right]\Bigg\} \tag{17}$$

with the interference determined as

$$\cos\beta_{J=0}=\frac{\mathrm{Re}\{\mathcal{B}(0,0)^{*}\mathcal{B}(1,0)\}}{|\mathcal{B}(0,0)\mathcal{B}(1,0)|}. \tag{18}$$

Again, the sign and size of the interference $\cos\beta_{J=0}$ are fixed by the initial-state interaction, here for protonium states with $J=0$.

Equations (13), (15), and (16) are analogous to those of Ref. [6]; this is also true for Eq. (17) in the SU(3) flavor limit with $B(\rho^0\rho^0)=B(\omega\omega)$. However, the essential and new feature of the present derivation is that the interference term is completely traced to the distortion in the initial protonium state. The possibility to link the interference terms $\cos\beta_J$ to the initial-state interaction in protonium is based on the separability of the transition amplitude $T_{N\bar{N}(IJ)\to VM}$. The sign and size of $\cos\beta_J\,(J=0,1)$ will have a direct physical interpretation, which will be discussed in the following chapter.

We briefly comment on alternative model descriptions for the strong transition amplitudes $N\bar{N}\to VM$ and its consequences for the interference terms in radiative $p\bar{p}$ decays. Competing quark model approaches in the description of $N\bar{N}$ annihilation into two mesons concern rearrangement diagrams as opposed to the planar diagram of the A2 prescription of Fig. 2(a). In the rearrangement model a quark-antiquark pair of the initial $N\bar{N}$ state is annihilated and the remaining quarks rearrange into two mesons. The quantum numbers of the annihilated quark-antiquark pair are either that of the vacuum ($^3P_0$ vertex, R2 model [17]) or that of a gluon ($^3S_1$-vertex, $S2$ model [18,19]). In the $R2$ model, two ground-state mesons cannot be produced from an initial $N\bar{N}$ state in a relative S wave; hence R2 is not applicable to the annihilation process considered here. The S2 model generates transition-matrix elements for $p\bar{p}\to VM$, which are analogous to the ones of the A2 model of Eqs. (1) and (2), but with different absolute values for the spin-flavor weights $\langle IJ\to VM\rangle_{\mathrm{SF}}$ [18,19]. However, the relative signs of the matrix elements $\langle IJ\to\rho M\rangle$ and $\langle IJ\to\omega M\rangle$ are identical to the ones of the A2 model, except in the case $M=\eta$ where it is opposite. Therefore, results for branching ratios $B(\gamma M)$ of radiative decays expressed in terms of the branching ratios $B(VM)$ are identical both in the A2 and the S2 approach, except for $B(\gamma\eta)$ where $\cos\beta_{J=1}$ changes sign. But, as will be shown later, the sign structure of $\cos\beta_J$ deduced in the framework of the A2 model is consistent with the one deduced from experiment.

Possible deviations from the formalism presented here include contributions from virtual $N\bar{\Delta}\pm\Delta\bar{N}$ and $\Delta\bar{\Delta}$ states to the annihilation amplitudes as induced by the initial-state interaction. The role of the Δ state admixture and its effect on $p\bar{p}$ annihilation cross sections in the context of quark models was studied in Refs. [13,20]. Although contributions involving annihilation from $N\bar{\Delta}$ and $\Delta\bar{\Delta}$ states can be sizable [20], the overall effect on the annihilation cross section is strongly model dependent. In the case of the A2 model [13], these contributions are found to be strong for $N\bar{N}\,D$-wave coupling to channels with a virtual Δ in the S wave, hence dominantly for the $^{13}SD_1$ partial wave, where for isospin $I=0$ the tensor force induces strong mixing. However, for the radiative decay processes at rest considered here, the possible $N\bar{\Delta}\pm\Delta\bar{N}$ configurations only reside in the $^{33}SD_1$ state (here $^{33}SD_1\to\pi^0\omega$ and $^{33}SD_1\to\eta\rho^0$). Due to the weak D-wave coupling in the $I=1$ channel, $N\bar{\Delta}$ configurations play a minor role and are neglected.

Alternatively, the strong transition amplitudes $N\bar{N}\to VM$ can be derived in baryon exchange models [2–4]. Here however, the analysis is strongly influenced by the presence both of vector and tensor couplings of the vector mesons to the nucleon, by contributions of both N and Δ exchange (where the latter one contributes to the $\rho^0\rho^0$ and $\pi^0\rho^0$ channels) and by the addition of vertex form factors. The interplay between these additional model dependences complicates an equivalent analysis. Due to simplicity we restrict the current approach to a certain class of quark models, although deviations from the analysis given below when applying baryon exchange models cannot be excluded.

III. PRESENTATION OF RESULTS

In Sec. III A we discuss the direct application of the quark model approach to the radiative $N\bar{N}$ annihilation process. In Sec. III B we focus specifically on the isospin interference effects occuring in radiative transitions. We show that the interference term is solely determined by the isospin-dependent $N\bar{N}$ interaction, and give theoretical predictions for the phase $\cos\beta_J$ in various $N\bar{N}$ interaction models. Sign and size of $\cos\beta_J$ can be interpreted by the dominance of either the $p\bar{p}$ or the $n\bar{n}$ component of the protonium wave function in the annihilation region. Furthermore we show that extraction of the interference term from experimental data is greatly affected by the choice of the kinematical form factor. Finally we comment on the applicability of the vector dominance approach to the $p\bar{p}\to\gamma\phi$ transition.

A. Branching ratios of radiative protonium annihilation

In a first step we directly evaluate the expression for $B(\pi^0\gamma)$ and $B(X\gamma), X=\eta,\ \omega,\ \eta',\ \rho$, and γ, as given by Eqs. (13), (15)–(17), and (A12). To reduce the model dependences we choose a simplified phenomenological approach as advocated in studies for two-meson branching ratios in $N\bar{N}$ annihilation [21]. The initial-state interaction coefficients $\mathcal{B}(I,J)$ are related to the probability for a protonium state with spin J and isospin I, with the normalization condition $|\mathcal{B}(0,J)|^2+|\mathcal{B}(1,J)|^2=1$. The total decay width of state J is given by $\Gamma_{\mathrm{tot}}(J)$ with the separation into isospin contributions as $\Gamma_{\mathrm{tot}}(J)=\Gamma_0(J)+\Gamma_1(J)$. We identify the ratio of isospin probabilities $|\mathcal{B}(0,J)|^2/|\mathcal{B}(1,J)|^2$ with that of partial annihilation widths $\Gamma_0(J)/\Gamma_1(J)$. For our calculations

TABLE I. Isospin probabilities $|\mathcal{B}(I,J)|^2$. Values are deduced from calculation of partial annihilation widths of 1*s* protonium with Kohno-Weise potential [23].

State	$\lvert\mathcal{B}(0,J)\rvert^2$	$\lvert\mathcal{B}(1,J)\rvert^2$	$\Gamma_{tot}(J)$ (keV)
$^1S_0\,(J=0)$	0.60	0.40	1.26
$^3SD_1\,(J=1)$	0.53	0.47	0.98

we adopt the isospin probabilities deduced from protonium states obtained with the Kohno-Weise $N\bar{N}$ potential [22], where $p\bar{p}-n\bar{n}$ isospin mixing and tensor coupling in the the $^3SD_1$ state are fully included [23]. The resulting values for $\mathcal{B}(I,J)$ are shown in Table I. The kinematical form factor $f(\gamma,X)$ is taken of the form [24]

$$f(\gamma,X)=k\,\exp\{-A\sqrt{s-m_X^2}\}, \tag{19}$$

where k is the final state c.m. momentum with total energy $\sqrt{s}$. The constant $A=1.2\ \mathrm{GeV}^{-1}$ is obtained from a phenomenological fit to the momentum dependence of various multipion final states in $p\bar{p}$ annihilation [24]. Results for the branching ratios in this simple model ansatz are given in Table II. For the decay modes $\eta\gamma$ and $\eta'\gamma$ we use a pseudoscalar mixing angle of $\Theta_p=-17.3°$ [25]. The model contains a free strength parameter, corresponding to the strong annihilation into two mesons in the two-step process. Since we compare the relative strengths of the branching ratios, we choose to normalize the entry for $B(\gamma\pi^0)$ to the experimental number. The A2 quark model prediction for the hierarchy of branching ratios is consistent with experiment. In particular, the relative strength of transitions from the spin-singlet ($^1S_0$) and triplet ($^3SD_1$) $N\bar{N}$ states is well understood. The results of Table II give a first hint, that the VDM approach is a reliable tool in analyzing the radiative decays of protonium. Furthermore, all considered branching ratios are consistent with minimal kinematical and dynamical assumptions. We stress that the good quality of the theoretical fit to the experimental data of Table II should not be overemphasized given the simple phenomenological approach where initial-state interaction is introduced in an averaged fashion. Although the A2 model provides a reasonable account of $N\bar{N}$ annihilation data, discrepancies remain in certain two-meson channels [1,21]. In particular, observed two-meson annihilation branching ratios can show strong deviations from simple statistical or flavor symmetry estimates (dynamical selection rules), which in their full completeness cannot be described by existing models. Furthermore, theoretical predictions for two-meson branching ratios can be strongly influenced by initial-state $N\bar{N}$ interaction (see for example Ref. [26]), as in the case of radiative decays, but also by the possible presence of final-state meson-meson scattering [15,27]. Given these limitations in the understanding of two-meson annihilation phenomena we will in turn dominantly focus on the determination of the interference term present in radiative $p\bar{p}$ decays. Here $N\bar{N}$ annihilation model dependences are avoided by resorting to the experimentally measured two-meson branching ratios.

TABLE II. Results for branching ratios B for $p\bar{p}\to\gamma X$ with $X=\pi^0$, η, ρ^0, ω, η', and γ in the simple model estimate. The entry for $B(\pi^0\gamma)$ is normalized to the experimental value. Data are taken from Ref. [5].

Channel	$B\times10^6$ (model)	$B\times10^6$ (Expt.)
$^3SD_1\to\pi^0\gamma$	44	44 ± 4
$^3SD_1\to\eta\gamma$	14	9.3 ± 1.4
$^1S_0\to\omega\gamma$	68	68 ± 19
$^3SD_1\to\eta'\gamma$	8.3	$\leqslant 12$
$^1S_0\to\gamma\gamma$	0.14	$\leqslant 0.63$
$^1S_0\to\rho\gamma$	50	

B. Isospin interference and initial-state interaction

In a second step we focus on the determination and interpretation of the isospin interference terms $\cos\beta_J\,(J=0,1)$ given by Eqs. (14) and (18), which in turn are related to the $N\bar{N}$ initial-state interaction via the coefficients $\mathcal{B}(I,J)$ in Eq. (A8).

A full treatment of protonium states must include both Coulomb and the shorter ranged strong interaction, where the coupling of $p\bar{p}$ and $n\bar{n}$ configurations is included. The isospin content of the corresponding protonium wave function Ψ depends on r; for large distances Ψ approaches a pure $p\bar{p}$ configuration, i.e., $\Psi(I=0)=\Psi(I=1)$. As r decreases below 2 fm, Ψ starts to rotate towards an isospin eigenstate, i.e., Ψ takes the isospin of the most attractive potential in the short distance region. The $N\bar{N}$ annihilation process under consideration here is most sensitive to the behavior of Ψ for $r\leqslant1$ fm, where the strong spin- and isospin dependence of the $N\bar{N}$ interaction may cause either the $I=0$ or the $I=1$ component to dominate. The consequences of the spin-isospin structure for energy shifts and widths of low-lying protonium states have been discussed in Refs. [23,28,29]. The sensitivity of $p\bar{p}-n\bar{n}$ mixing in protonium states to changes in the meson-exchange contributions to the $N\bar{N}$ interaction was explored in [21].

Let us first discuss the physical interpretation of the interference terms $\cos\beta_J$. For a protonium state described by a pure $p\bar{p}$ wave function, the isospin-dependent initial-state interaction coefficients are related, $\mathcal{B}(I=0,J)=\mathcal{B}(I=1,J)$. Similarly, for a protonium state given by a pure $n\bar{n}$ wave function in the annihilation region, that is $\Psi(I=0)=-\Psi(I=1),\mathcal{B}(I=0,J)=-\mathcal{B}(I=1,J)$. Together with Eqs. (14) and (18), we obtain for the interference terms

$$\cos\beta_J=\begin{cases}+1 & \text{for pure } p\bar{p}\\ -1 & \text{for pure } n\bar{n}.\end{cases} \tag{20}$$

Therefore, a dominant $p\bar{p}$ component in the protonium wave function leads to constructive interference in radiative annihilation, with $\cos\beta_J=1$. Destructive interference reflects the dominance of the $n\bar{n}$ component in the annihilation region of the protonium state. Given this direct physical interpretation

TABLE III. Isospin interference terms $\cos\beta_J$ as calculated with the 1s protonium wave functions of the KW, DR1, and DR2 potential models. Values in brackets denote the results where the $^3D_1$ component of the atomic $^3SD_1$ state is included with the admixture fixed by the A2 model.

	KW	DR1	DR2
$\cos\beta_0$	$+1.00$	$+0.83$	$+0.63$
$\cos\beta_1$	$-0.90(-0.76)$	$+0.10$ $(+0.36)$	-0.41 $(+0.53)$

of the interference terms, radiative annihilation serves as a indicator for the isospin dependence of the $N\bar{N}$ protonium wave functions.

For quantitative predictions of the interference terms and for comparison, we resort to protonium wave functions calculated [23] with three different potential models of the $N\bar{N}$ interaction, that by Kohno and Weise [22] (KW) and the two versions of the Dover-Richard [30,29] (DR1 and DR2) potentials. The calculation of Ref. [23] takes fully into account the neutron-proton mass difference, tensor coupling, and isospin mixing induced by the Coulomb interaction.

Results for the interference terms $\cos\beta_J$ as deduced from the three different potential models are given in Table III. The value of the range parameter d_{A2} in the initial-state form factor entering in Eq. (A8) is adjusted to the range of the annihilation potential of the respective models. With the choice of $d_{A2}=0.12$ fm^2 (KW and DR2) and $d_{A2}=0.03$ fm^2 (DR1) the calculated ratios of isospin probabilities $|\mathcal{B}(0,J)|^2/|\mathcal{B}(1,J)|^2$ are close to those of partial annihilation widths $\Gamma_0(J)/\Gamma_1(J)$ calculated in Ref. [23]. All three potential models consistently predict constructive interference for radiative annihilation from the atomic $^1S_0$ state, indicating a dominant $p\bar{p}$ component. For radiative annihilation from the spin-triplet state $^3S_1$ predictions range from nearly vanishing (DR1) to a sizable destructive interference, where latter effect can be traced to a dominant short-ranged $n\bar{n}$ component in the protonium state. In Table III we also indicate predictions for the interference term $\cos\beta_1$, where the D-wave admixture in the $^3SD_1$ state has been included. The results are obtained for the specified values of d_{A2} with an additional choice of hadron size parameters (that is $R_N^2/R_M^2=0.6$ or $\langle r^2\rangle_N^{1/2}/\langle r^2\rangle_M^{1/2}=1.2$) entering in the expression of Eq. (A8). The inclusion of D-wave admixture in the initial-state interaction coefficients $\mathcal{B}(I,J=1)$ as outlined in Appendix A is a particular feature of the A2 quark model. Hence, predictions for $\cos\beta_1$ with the $^3D_1$ component of the atomic $^3SD_1$ state included are strongly model-dependent and should not be overestimated. Generally, inclusion of the D-wave component in the form dictated by the quark model tends to increase the values of the interference terms.

We also investigated the sensitivity of the interference term $\cos\beta_J$ on the range of the initial-state form factor, expressed by the coefficient d_{A2}. Although the absolute values for the initial-state interaction coefficients $\mathcal{B}(I,J)$ sensitively depend on the specific value for d_{A2}, variation of d_{A2} by up to 50% has little influence on sign and also on size of $\cos\beta_J$. Thus, predictions for the interference terms $\cos\beta_J$ in all three potential models considered, are fairly independent on the specific annihilation range of the $N\bar{N}$ initial state.

The models used for describing the $N\bar{N}$ initial-state interaction in protonium are characterized by a state independent, complex optical potential due to annihilation. Potentials of this type reproduce the low-energy $p\bar{p}$ cross sections and protonium observables, such as energy shifts and widths, fairly well. A more advanced fit [31] to $N\bar{N}$ scattering data, in particular to the analyzing powers for elastic and charge-exchange scattering, requires the introduction of an explicit state and energy dependence in the phenomenological short-range part of the $N\bar{N}$ interaction. At present, latter $N\bar{N}$ potential [31] was not applied to the protonium system; hence the model predictions of Table III should be regarded as a first estimate for the $p\bar{p}-n\bar{n}$ mixing mechanism in the $N\bar{N}$ annihilation region.

C. Isospin interference from data

The VDM approach allows us to relate the branching ratios of radiative annihilation modes to branching ratios with final states containing one or two vector mesons. Using these measured branching ratios in Eqs. (13) and (15)–(17) we can extract the interference terms $\cos\beta_J$ directly from experiment. However, conclusions on the sign and size of the interference terms strongly depend on the choice of the kinematical form factor $f(X_1,X_2)$, X_1 and $X_2=\gamma$ or meson, entering in the different expressions. A first analysis [5] for determining the interference terms from data was performed by the Crystal Barrel Group, assuming a form factor of [32]

$$f(X_1,X_2)=k\left(\frac{(kR)^2}{1+(kR)^2}\right), \tag{21}$$

where k is the final-state c.m. momentum and the interaction range is chosen as $R=1.0$ fm. This form factor is appropriate for small momenta k, taking into account the centrifugal barrier effects near threshold. However, for radiative decays, with high relative momenta in the final state, the choice of Eq. (19) is more appropriate, it contains an exponential which restricts the importance of each decay channel to the energy region near threshold. This can be regarded as a manifestation of multichannel unitarity, that is the contribution of a given decay channel cannot grow linearly with k [as in the form of Eq. (21)], since other channels open up and compete for the available flux, subject to the unitarity limit. Also, the latter form factor is given a sound phenomenological basis in $N\bar{N}$ annihilation analyses, for a more detailed discussion see, for example, Ref. [33]. Extracted values for the interference terms $\cos\beta_J$ with different J and different prescriptions for the kinematical form factor are given in Table IV. We also include there a third choice for the kinematical form factor [Eq. (A11)], as deduced from the A2 quark model description of the $N\bar{N}$ annihilation process. Although finite-size effects of the hadrons are included here, through the harmonic oscillator ansatz for the hadron wave functions the form factor is again useful for low relative momenta k. For the results of Table IV we use the measured branching ratios of $p\bar{p}\to\pi^0\rho^0$ [34], $\pi^0\omega$, $\eta\omega$, $\omega\omega$, $\eta'\omega$ [35], $\eta\rho$ [36] or [37], $\rho\omega$ [38], and $\eta'\rho$ [33]. Values for $\cos\beta_J$ using the phase-space factor of Eq. (21) are directly taken from the original analysis of Ref. [5]. Error estimates

TABLE IV. Isospin interference terms $\cos\beta_J$ as deduced from data. The label HQ refers to the kinematical form factor of von Hippel and Quigg as defined in Eq. (21). Similarly, labels VAN and A2 refer to the form-factor prescription of Vandermeulen and of the A2 quark model, as defined in Eqs. (19) and (A11), respectively. The analysis for $\cos\beta_J$ (HQ) is taken directly from Ref. [5]. The first line of the analysis for $\eta\gamma$ is done for $B(\eta\rho^0)=(0.53\pm0.14)\times10^{-2}$ [36], the second line for $B(\eta\rho^0)=(0.33\pm0.09)\times10^{-2}$ [37].

Channel	$\cos\beta_J$ (HQ)	$\cos\beta_J$ (VAN)	$\cos\beta_J$ (A2)
$^3SD_1\to\pi^0\gamma\,(J=1)$	-0.75 ± 0.11	-0.10 ± 0.28	$+1.00\pm0.38$
$^3SD_1\to\eta\gamma\,(J=1)$	-0.78 ± 0.25	-0.47 ± 0.72	-0.43 ± 0.76
	-0.58 ± 0.48	-0.17 ± 0.90	-0.12 ± 0.96
$^1S_0\to\omega\gamma\,(J=0)$	-0.60 ± 0.18	$+0.15\pm0.38$	-0.21 ± 0.28
$^3SD_1\to\eta'\gamma\,(J=1)$	$\leqslant-0.26$	$\leqslant1.65$	$\leqslant-0.12$

for the other entries in Table IV assume statistical independence of the measured branching ratios. For the radiative decay channel $\eta'\gamma$ only an upper limit for $\cos\beta_1$ can be given.

For all three choices of the kinematical form factor, the extracted values of $\cos\beta_J$ are consistent with the VDM assumption as they correspond to physical values. However, as evident from Table IV, conclusions on sign and size of the interference strongly depend on the form of the kinematical phase-space factor. For the preferred choice, i.e., Eq. (19), we deduce destructive interference for radiative annihilation from the $^3SD_1$ state, while for the $^1S_0$ state the corresponding isospin amplitudes interfere constructively. This is in contrast to the original analysis of Ref. [5], where the interference term is determined to be almost maximally destructive for all channels considered. Given the large uncertainties for $\cos\beta_J$ using the preferred form factor, the values deduced from data are at least qualitatively consistent with the theoretical predictions of Table III, indicating a dominant $p\bar{p}$ component for the $^1S_0$ and a sizable $n\bar{n}$ component for the $^3SD_1$ protonium wave function. As discussed in Sec. III B, precise values for $\cos\beta_J$ are rather sensitive to the isospin decomposition of the protonium wave function in the annihilation region. However, the current uncertainties in the experimental data should be very much improved to allow a more quantitative analysis of the isospin dependence of the $N\bar{N}$ interaction.

D. Vector dominance model and the $p\bar{p}\to\gamma\phi$ transition

Measurements on nucleon-antinucleon annihilation reactions into channels containing ϕ mesons indicate apparent violations of the Okubo-Zweig-Iizuka (OZI) rule [33]. According to the OZI rule, ϕ can only be produced through its nonstrange quark-antiquark component, hence ϕ production should vanish for an ideally mixed vector meson nonet. Defining the deviation from the ideal mixing angle $\theta_0=35{,}3°$ by $\alpha=\theta-\theta_0$ and asssuming the validity of the OZI rule, one obtains the theoretically expected ratio of branching ratios [1]:

$$R(X)=B(N\bar{N}\to\phi X)/B(N\bar{N}\to\omega X)=\tan^2\alpha\approx0.001-0.003, \tag{22}$$

where X represents a nonstrange meson or a photon. Recent experiments [33] have provided data on the ϕ/ω ratios which are generally larger than the standard estimate of Eq. (22). The most notable case is the $\phi\gamma$ channel for $p\bar{p}$ annihilation in liquid hydrogen [10], where data show a dramatic violation of the OZI rule of up to two orders of magnitude, that is $R(X=\gamma)\approx0.3$. Substantial OZI rule violations in the reactions $p\bar{p}\to X\phi$ can possibly be linked to the presence of strange quark components in the nucleon [39,40]. However, apparent OZI rule violations can also be generated by conventional second-order processes, even if the first-order term corresponds to a disconnected quark graph [41,42].

In Refs. [42,11] the apparently large value for the branching ratio $B(\gamma\phi)$ for $p\bar{p}$ annihilation in liquid hydrogen is explained within the framework of the VDM. Using the experimental rates of $B(\rho\phi)=(3.4\pm1.0)\times10^{-4}$ and $B(\omega\phi)=(5.3\pm2.2)\times10^{-4}$ [43] as inputs, the branching ratio $B(\gamma\phi)$ is given in the VDM by

$$B(\gamma\phi)=\frac{f(\gamma,\phi)}{f(\omega V)}(12.0+\cos\beta_0 8.5)\times10^{-7}. \tag{23}$$

Since the $\phi\omega$ and $\phi\rho$ channels also violate the OZI rule estimate, $R(X=\rho)\approx R(X=\omega)\approx10^{-2}$ [33], the standard $\omega-\phi$ mixing cannot be the dominant mechanism for the production of the $\phi\omega$ and $\phi\rho$ channels and the formalism developed in Sec. II cannot be used to determine the phase structure of the interference term $\cos\beta_0$ for $B(\gamma\phi)$. Consequently, the interference term $\cos\beta_0$ extracted in the $\gamma\omega$ reaction is not necessarily consistent with that of the $\gamma\phi$ decay channel. For maximal constructive interference ($\cos\beta=1$) one obtains an upper limit for $B(\gamma\phi)$ in the VDM calculation of

$$B(\gamma\phi)=2.7\times10^{-5} \quad \text{for } f=k^3\ [11],$$
$$B(\gamma\phi)=1.5\times10^{-6} \quad \text{for } f \text{ given by Eq. (19)}. \tag{24}$$

This is to be compared with the experimental result $B(\gamma\phi)=(2.0\pm0.4)\times10^{-5}$ [10]. The possibility of explaining the experimental value of $B(\gamma\phi)$ in VDM depends again strongly on the choice of the kinematical form factor. In Ref. [11] the form $f=k^3$ is used, appropriate for relative momenta k near threshold, resulting in an upper limit which lies slightly above the observed rate for $B(\gamma\phi)$. With the choice of Eq. (19) the upper value underestimates the experimental number by an order of magnitude. When we extract the interference terms $\cos\beta_J$ from the conventional radiative decay modes with the choice $f=k^3$, we obtain: $\cos\beta_1=-1.32$ for $\pi\gamma$, $\cos\beta_1=-0.94$ for $\eta\gamma$, and $\cos\beta_0=-0.90$ for $\omega\gamma$.

Hence, a near threshold prescription for the kinematical form factor in the VDM leads to maximal destructive interference for all channels considered, exceeding even the physical limit in the case of $\pi\gamma$. This would indicate a nearly pure $n\bar{n}$ component in the annihilation range of the protonium wave functions for both the $J=0$ and 1 states. These results are in strong conflict with the theoretical expectations for $\cos\beta_J$ reported in Sec. III B, where at least qualitative consistency is achieved with the kinematical form factor of Eq. (19).

Recent experimental results [44] for the reaction cross section $p\bar{p}\rightarrow\phi\phi$ exceed the simple OZI rule estimate by about two orders of magnitude. Therefore, in the context of VDM an additional sizable contribution to the branching ratio $B(\gamma\phi)$ might arise, although off-shell, from the $\phi\phi$ intermediate state. With an estimated cross section of $p\bar{p}\rightarrow\omega\omega$ of about 5 mb in the energy range of the $\phi\phi$ production experiment, the ratio of cross sections is given as $\sigma_{\phi\phi}/\sigma_{\omega\omega}\approx 3.5\ \mu\text{b}/0.5\ \text{mb}$ [44]. Given the measured branching ratios of $\omega\omega$ [35] and $\omega\phi$ [43] we can simply estimate the ratio of strong transition-matrix elements for annihilation into $\phi\phi$ and $\omega\phi$ from protonium of $\sqrt{B(\phi\phi)/B(\omega\phi)}\approx 0.43$. For this simple order of magnitude estimate we assume that $\sigma_{\phi\phi}/\sigma_{\omega\omega}$ is partial wave independent and phase-space corrections are neglected. With the VDM amplitude $A_{\phi\gamma}=(\sqrt{2}/3)A_{\rho\gamma}$ we obtain an upper limit of $B(\gamma\phi)\approx 2.3\times 10^{-6}$ with f given by Eq. (19), where the contribution of the $\phi\phi$ intermediate state is now included. Excluding an even further dramatic enhancement of the $\phi\phi$ channel for $N\bar{N}$ S-wave annihilation, inclusion of the $\phi\phi$ intermediate state does not alter the conclusions drawn from the results of Eq. (24). Hence, the large observed branching ratio for $\gamma\phi$ remains unexplained in the framework of VDM.

IV. SUMMARY AND CONCLUSIONS

We have performed a detailed analysis of radiative $p\bar{p}$ annihilation in the framework of a two-step process, that is $p\bar{p}$ annihilates into two-meson channels containing a vector meson which is subsequently converted into a photon via the VDM. Both processes are consistently formulated in the quark model, which allows us to uniquely identify the source of the isospin interference present in radiative transitions. Based on the separability of the transition amplitude $N\bar{N}\rightarrow VM$, sign and size of the interference terms can be linked to the dominance of either the $p\bar{p}$ or the $n\bar{n}$ component of the 1s protonium wave function in the annihilation region, hence constitutes a direct test of the isospin dependence of the $N\bar{N}$ interaction.

In a first step we directly applied the quark model in a simplified phenomenological approach to the radiative $N\bar{N}$ annihilation process. Model predictions are consistent with data and confirm the usefulness of VDM in the analysis of radiative transitions. In a second step we discussed sign and size of the interference term as expressed by $\cos\beta_J(J=0,1)$. Direct predictions of $\cos\beta_J$, as calculated for different potential models of the $N\bar{N}$ interaction, are qualitatively consistent, in that a sizable constructive interference is deduced for radiative annihilation from the atomic $^1S_0$ state, while for the $^3S_1$ state the interference term is vanishing or destructive. These predictions should be tested with more realistic parametrizations of the $N\bar{N}$ interaction [31]. Extraction of the interference effect from data is greatly influenced by the choice of the kinematical form factor associated with the transition. Values of $\cos\beta_J$ determined for the preferred form of Eq. (19) are qualitatively consistent with our theoretical study; however, a more quantitative analysis is restricted by the present uncertainties in the experimental data. Within the consistent approach emerging from the analysis of nonstrange radiative decay modes of protonium, an explanation of the measured branching ratio for the OZI suppressed reaction $p\bar{p}\rightarrow\gamma\Phi$ cannot be achieved. New mechanisms, linked to the strangeness content in the nucleon, may possibly be responsible for the dramatic violation of the OZI rule in the $\gamma\Phi$ final state.

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APPENDIX A: NUCLEON-ANTINUCLEON ANNIHILATION INTO TWO MESONS IN THE QUARK MODEL

In describing the annihilation process of $N\bar{N}\rightarrow VM$ where $V=\rho,\omega$, and $M=\pi^0$, η, ρ, ω, and η' we use the A2 model of Fig. 2(a). Detailed definitions and derivation of this particular quark model are found in Refs. [1,13]. The initial state $N\bar{N}$ quantum numbers are defined by $i=ILSJM$ (I is the isospin, L is the orbital angular momentum, S is the spin, and J is the total angular momentum with projection M). For the final two meson state VM we specify the angular momentum quantum numbers, with $j_{1,2}$ indicating the spin of mesons 1 and 2, j the total spin coupling and l_f the relative orbital angular momentum. For the transitions of interest the quantum numbers are restricted to $L=0$ and 2, corresponding to $p\bar{p}$ annihilation at rest in liquid hydrogen, $j_1=1$, representing the vector meson, and $l_f=1$, given by parity conservation. Taking plane waves for the initial and final-state wave functions with relative momenta $\vec{p}$ and $\vec{k}$, respectively, the transition-matrix element is given in a partial wave basis as

$$\begin{aligned}T_{N\bar{N}(i)\rightarrow VM}&=\langle V(j_1)M(j_2)l_f|\mathcal{O}_{A2}|N\bar{N}(i)\rangle\\&=\sum_j\langle j_1j_2m_1m_2|jm\rangle\\&\quad\times\langle jl_f m m_f|JM\rangle|\vec{k}|Y_{l_fm_f}(\hat{k})Y_{LS}^{JM\dagger}(\hat{p})\\&\quad\times\langle VM(j_1,j_2,j,l_f)||\mathcal{O}_{A2}||N\bar{N}(i)\rangle.\end{aligned}\qquad\text{(A1)}$$

The reduced matrix element of the two-meson transition is given in the A2 model as

TABLE V. Spin-flavor matrix elements $\langle i\to VM\rangle_{SF}$ for the decay $N\bar{N}(L=0)\to VM$ in the A2 quark model. These are relative matrix elements obtained from Ref. [1]. Here, η_{ud} refers to the nonstrange flavor combination $\eta_{ud}=(u\bar{u}+d\bar{d})/\sqrt{2}$.

Decay channel	$\langle i\to VM\rangle_{SF}$
$^{11}S_0\to\omega\omega$	$-\sqrt{243}$
$^{11}S_0\to\rho^0\rho^0$	$-\sqrt{243}$
$^{31}S_0\to\omega\rho^0$	$-\sqrt{1350}$
$^{13}S_1\to\pi^0\rho^0$	$+\sqrt{450}$
$^{13}S_1\to\eta_{ud}\omega$	$+\sqrt{450}$
$^{33}S_1\to\pi^0\omega$	$+\sqrt{338}$
$^{33}S_1\to\eta_{ud}\rho^0$	$+\sqrt{338}$

$$\langle VM||\mathcal{O}_{A2}||N\bar{N}(i)\rangle$$
$$=F_{L,l_f}p^L\exp(-d_{A2}(3/4k^2+1/3p^2)\langle i\to VM\rangle_{SF}. \quad \text{(A2)}$$

The factor F_{L,l_f} is a positive geometrical constant depending on the size parameters of the hadrons for given orbital angular momenta L and l_f. The exponentials arise from the overlap of harmonic oscillator wave functions used for the hadrons with the coefficient d_{A2} depending on the size parameters R_N and R_M of the nucleon and meson:

$$d_{A2}=\frac{R_N^2R_M^2}{3R_N^2+2R_M^2}. \quad \text{(A3)}$$

The matrix elements $\langle i\to VM\rangle_{SF}$ are the spin-flavor weights of the different transitions listed in Table V. Note that with the flavor part of the vector mesons defined as

$$\rho^0=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}),\quad \omega=\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) \quad \text{(A4)}$$

the matrix elements $\langle i\to\rho M\rangle$ and $\langle i\to\omega M\rangle$ have same sign. For the tensor force coupled channel $^3SD_1$ the spin-flavor matrix elements are simply related by a proportionality factor, dependent on the isospin channel, but independent of the VM combination, that is

$$F_{L=2,l_f=1}\langle^{2I+1,3}D_1\to VM\rangle_{SF}$$
$$=C(I)F_{L=0,l_f=1}\langle^{2I+1,3}S_1\to VM\rangle_{SF},$$

$$C(I)=\begin{pmatrix} I=0, & -\dfrac{2\sqrt{2}}{5} \\ I=1, & \dfrac{2\sqrt{2}}{13}\end{pmatrix}\left(-\frac{1}{3}\frac{(R_N^2+R_M^2)R_N^2}{3/2R_N^2+R_M^2}\right). \quad \text{(A5)}$$

In coordinate space the protonium wave function, including tensor coupling and isospin mixing, is written as

$$\Psi_{p\bar{p}}(J,S)=\sum_{L,I}\psi_{ILSJ}(r)Y_{LS}^{JM}(\hat{r}). \quad \text{(A6)}$$

Inserting this wave function into the expression for the transition-matrix element results in

$$T_{N\bar{N}(IJ)\to VM}$$
$$=\sum_j\langle j_1j_2m_1m_2|jm\rangle\langle jl_fmm_f|JM\rangle|\vec{k}|Y_{l_fm_f}(\hat{k})$$
$$\times F(k)\langle i\to VM\rangle_{SF}\mathcal{B}(I,J),$$
$$F(k)\equiv\exp(-d_{A2}3/4k^2). \quad \text{(A7)}$$

The distortion due to the initial-state interaction is contained in the coefficient $\mathcal{B}(I,J)$, which is simply the overlap of the isospin decomposed protonium wave function with the effective initial form factor arising in the transition. By taking the Fourier transform of the initial-state form factor contained in Eq. (A2), these coefficients for the $1s$ atomic states of protonium are defined as

$$\mathcal{B}(I,J=0)=F_{L=0,l_f=1}(2d_{A2}/3)^{-3/2}\int_0^\infty drr^2$$
$$\times\exp(-3r^2/(4d_{A2}))\psi_{I000}(r)\quad\text{for }{}^1S_0,$$

$$\mathcal{B}(I,J=1)=F_{L=0,l_f=1}\Big\{(2d_{A2}/3)^{-3/2}$$
$$\times\int_0^\infty drr^2\exp(-3r^2/(4d_{A2}))\psi_{I011}(r)-C(I)$$
$$\times(2d_{A2}/3)^{-7/2}\int_0^\infty drr^4\exp(-3r^2/(4d_{A2}))$$
$$\times\psi_{I211}(r)\Big\}\quad\text{for }{}^3SD_1. \quad \text{(A8)}$$

The partial decay width for the annihilation of a protonium state with total angular momentum J into two mesons VM is given by

$$\Gamma_{p\bar{p}\to VM}(I,J)=2\pi\frac{E_VE_M}{E}k\int d\hat{k}\sum_{m_1m_2m_f}|T_{N\bar{N}(IJ)\to VM}|^2, \quad \text{(A9)}$$

where E is the total energy and $E_{V,M}=\sqrt{m_{V,M}^2+\vec{k}^2}$ is the energy of the respective outgoing meson with $|\vec{k}|$ fixed by energy conservation. With the explicit form of the transition amplitude of Eq. (A7), the partial decay width is written as

$$\Gamma_{p\bar{p}\to VM}(I,J)=f(V,M)\langle i\to VM\rangle_{SF}^2|\mathcal{B}(I,J)|^2 \quad \text{(A10)}$$

with the kinematical phase-space factor defined by

$$f(V,M)=2\pi\frac{E_VE_M}{E}k^3\exp(-3/2d_{A2}k^2). \quad \text{(A11)}$$

Taking an admixture of initial states given by their statistical weight, the branching ratio of S-wave $p\bar{p}$ annihilation into the two meson final state VM is given by

$$B(VM)=B(p\bar{p}\to VM)=\sum_{J=0,1}\frac{(2J+1)\Gamma_{p\bar{p}\to VM}(I,J)}{4\Gamma_{tot}(J)}. \quad \text{(A12)}$$

APPENDIX B: VECTOR MESON-PHOTON CONVERSION IN THE QUARK MODEL

The transition $V\rightarrow\gamma$ [Fig. 2(b)], where $V=\rho$ or ω, can be formulated in the quark model, and related to the physical process of $V\rightarrow e^+e^-$. An explicit derivation of the latter process can be found in Ref. [45]. We just quote the main results necessary for the discussion of the radiative decays of protonium.

The $Q\bar{Q}\gamma$ interaction is defined by the Hamiltonian

$$H_I=e\int d^3x j^{\mu}_{em}(\vec{x})A_{\mu}(\vec{x}) \tag{B1}$$

with the quark current

$$j^{\mu}_{em}(\vec{x})=\overline{q}(\vec{x})Q\gamma^{\mu}q(\vec{x}), \tag{B2}$$

where $q(\vec{x})$ is the quark field and $A_{\mu}(\vec{x})$ the electromagnetic field given in a free field expansion. For emission of a photon with momentum $\vec{k}$, energy k^0, and polarization ϵ_{μ} from a vector meson with momentum $\vec{p}_V$ we obtain

$$\langle\gamma(\vec{k},\epsilon_{\mu})|H_I|V(\vec{p}_V)\rangle=\delta(\vec{k}-\vec{p}_V)T_{V\rightarrow\gamma} \tag{B3}$$

with

$$T_{V\rightarrow\gamma}=\frac{e(2\pi)^{3/2}}{(2k^0)^{1/2}}\epsilon^*_{\mu}\langle 0|j^{\mu}_{em}(\vec{x}=\vec{0})|V\rangle. \tag{B4}$$

For the conversion of a vector meson V into a real photon only the spatial part of the current matrix element contributes. Using standard techniques for the evaluation of the current matrix element we obtain

$$T_{V\rightarrow\gamma}=\frac{e\sqrt{6}}{(2k^0)^{1/2}}\vec{\epsilon}\cdot\vec{S}\,\mathrm{Tr}(Q\varphi_V)\psi(\vec{r}=0) \tag{B5}$$

with the quark charge matrix Q and the polarization $\vec{S}$ of the vector meson. The $Q\bar{Q}$ flavor wave function φ_V is consistently defined as in Eq. (A4) of Appendix A and contributes to the transition amplitude

$$\mathrm{Tr}(Q\varphi_V)=\begin{cases}\dfrac{1}{\sqrt{2}} & \text{for}\quad \rho^0\\[2ex] \dfrac{1}{3\sqrt{2}} & \text{for}\quad \omega.\end{cases} \tag{B6}$$

The spatial part of the $Q\bar{Q}$ wave function at the origin $\psi(\vec{r}=0)$ is given within the harmonic-oscillator description as $|\psi(0)|^2=(\pi R_M^2)^{-3/2}$, where the oscillator parameter R_M is related to the rms radius as $\langle r^2\rangle^{1/2}=\sqrt{3/8}R_M$.

Extending the outlined formalism to the physical decay process $V\rightarrow e^+e^-$ the decay width is given as [45]

$$\Gamma_{V\rightarrow e^+e^-}=\frac{16\pi\alpha^2}{m_V^2}\{\mathrm{Tr}(Q\varphi_V)\}^2|\psi(0)|^2 \tag{B7}$$

with $\alpha=e^2/(4\pi)$ and mass m_V of the vector meson. The latter result can be compared to the one obtained in the vector dominance approach resulting for example in [7]

$$\Gamma_{\rho^0\rightarrow e^+e^-}=\frac{4\pi}{3}\frac{\alpha^2 m_{\rho}}{f_{\rho}^2} \tag{B8}$$

with the decay constant f_{ρ}. Hence, we can identify

$$|\psi(0)|^2=\frac{m_{\rho}^3}{6f_{\rho}^2}, \tag{B9}$$

which with the experimental result of $\Gamma_{\rho^0\rightarrow e^+e^-}=6.77$ yields $f_{\rho}=5.04$ or equivalently $R_M=3.9\ \mathrm{GeV}^{-1}$, very close to the preferred value obtained in the analysis of strong decays of mesons. Hence, the matrix element for the conversion of a vector meson into a real photon is alternatively written as

$$T_{V\rightarrow\gamma}=\vec{\epsilon}\cdot\vec{S}\,\mathrm{Tr}(Q\varphi_V)\frac{e m_{\rho}^{3/2}}{(2k^0)^{1/2}f_{\rho}}. \tag{B10}$$

APPENDIX C: MATRIX ELEMENTS AND DECAY WIDTH IN RADIATIVE ANNIHILATION

In the following we present details for the evaluation of the matrix element of Eq. (6), which is explicitly written as

$$\begin{aligned}T_{N\bar{N}(IJ)\rightarrow VM\rightarrow\gamma M}=&\sum_{m_1}\langle j_1 j_2 m_1 m_2|jm\rangle\\ &\times\langle j l_f m m_f|JM\rangle|\vec{k}|Y_{l_f m_f}(\hat{k})\\ &\times\langle VM||\mathcal{O}_{A2}||N\bar{N}(IJ)\rangle\vec{\epsilon}\cdot\vec{S}(m_1)A_{V\gamma},\end{aligned} \tag{C1}$$

where $l_f=1$ and $j=1$, for the processes considered. The relative final-state momentum $\vec{k}$ and the photon polarization $\vec{\epsilon}$ are written in a spherical basis as

$$|\vec{k}|Y_{1m_f}(\hat{k})=\sqrt{\frac{3}{4\pi}}k_{m_f}\quad\text{and}\quad\vec{\epsilon}\cdot\vec{S}(m_1)=\epsilon_{m_1}, \tag{C2}$$

which together with Eq. (C1) leads to the result

$$T_{N\bar{N}(IJ)\rightarrow VM\rightarrow\gamma M}=\sqrt{\frac{3}{4\pi}}A_{V\gamma}\langle VM||\mathcal{O}_{A2}||N\bar{N}(IJ)\rangle\frac{i}{\sqrt{2}}\begin{cases}(\epsilon\times\vec{k})_M & \text{for}\ \ j_2=0, J=1\ (M=\pi^0,\eta),\\ \dfrac{(-)^{m_2}}{\sqrt{3}}(\epsilon\times\vec{k})_{-m_2} & \text{for}\ \ j_2=1,\ J=0\ \ (M=\rho^0,\omega).\end{cases} \tag{C3}$$

Consequently, for the process $N\bar{N}\rightarrow V_1V_2\rightarrow\gamma_1\gamma_2$ the transition-matrix element is determined as

$$T(N\bar{N}(IJ)\rightarrow V_1V_2\rightarrow\gamma_1\gamma_2)$$

$$=\sum_{m_2}\epsilon_{m_2}(2)A_{V_2\gamma}T(N\bar{N}\rightarrow V_1V_2\rightarrow\gamma_1V_2)$$

$$=\frac{1}{\sqrt{4\pi}}A_{V_1\gamma}A_{V_1\gamma}\langle V_1V_2||\mathcal{O}_{A2}||N\bar{N}(IJ)\rangle$$

$$\times\frac{i}{\sqrt{2}}(\vec{\epsilon}(1)\times\vec{k})\cdot\vec{\epsilon}(2), \quad (C4)$$

where $\vec{\epsilon}(i)$ refer to the polarization of the photon i.

The derivation of the decay widths for the radiative transitions is examplified here for the process $N\bar{N}\rightarrow\gamma\pi^0$. The corresponding matrix element is obtained by a coherent sum of intermediate vector meson states ρ and ω as

$$T_{N\bar{N}(J)\rightarrow\gamma\pi^0}=T_{^{13}SD_1\rightarrow\rho^0\pi^0\rightarrow\gamma\pi^0}+T_{^{33}SD_1\rightarrow\omega\pi^0\rightarrow\gamma\pi^0}$$

$$=\sqrt{\frac{3}{4\pi}}\frac{i}{\sqrt{2}}(\vec{\epsilon}\times\vec{k})_M\{A_{\rho^0\gamma}\langle\rho^0\pi^0||\mathcal{O}_{A2}||^{13}SD_1\rangle$$

$$+A_{\omega\gamma}\langle\omega\pi^0||\mathcal{O}_{A2}||^{33}SD_1\rangle\}. \quad (C5)$$

The decay width for $N\bar{N}\rightarrow\gamma\pi^0$ is then

$$\Gamma_{N\bar{N}\rightarrow\gamma\pi^0}=2\pi\rho_f\sum_{\epsilon_T,M}\frac{1}{2J+1}|T(N\bar{N}(J)\rightarrow\gamma\pi^0)|^2 \quad (C6)$$

with the final-state density

$$\rho_f=\frac{E_{\pi^0}k^2}{E_{N\bar{N}}}\int d\hat{k}, \quad (C7)$$

$|\vec{k}|=k$, and the sum is over the two transverse photon polarizations ϵ_T and the total projection M of the $N\bar{N}$ protonium with total angular momentum J. Using

$$\sum_{\epsilon_T,M}\int d\hat{k}|(\vec{\epsilon}\times\vec{k})|^2=8\pi k^2 \quad (C8)$$

together with the expression for the reduced matrix element in Eq. (2), we finally obtain

$$\Gamma_{N\bar{N}\rightarrow\gamma\pi^0}=f(\gamma,\pi^0)A^2_{\rho\gamma}\left|\langle^{13}SD_1\rightarrow\rho^0\pi^0\rangle_{\rm SF}\mathcal{B}(0,1)\right.$$

$$\left.+\frac{1}{3}\langle^{33}SD_1\rightarrow\omega\pi^0\rangle_{\rm SF}\mathcal{B}(1,1)\right|^2 \quad (C9)$$

with the kinematical phase-space factor defined in analogy to Eq. (A11) as

$$f(\gamma,M)=2\pi\frac{E_Mk^4}{E}\exp(-3/2d_{A2}k^2). \quad (C10)$$

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