Radiative capture of protons by deuterons

W. Schadow
Institute of Nuclear and Particle Physics, and Department of Physics, Ohio University, Athens, Ohio 45701

W. Sandhas
Physikalisches Institut der Universität Bonn, Endenicher Allee 11-13, D-53115 Bonn, Germany

(Received 17 August 1998)

The differential cross section for radiative capture of protons by deuterons is calculated using different realistic NN interactions. We compare our results with the available experimental data below $E_x = 20$ MeV. Excellent agreement is found when taking into account meson exchange currents, dipole and quadrupole contributions, and the full initial state interaction. There is only a small difference between the magnitudes of the cross sections for the different potentials considered. The angular distributions, however, are practically potential independent. [S0556-2813(99)07602-5]

PACS number(s): 21.45.+v, 25.40.Lw, 25.20.−x, 27.10.+h

The radiative capture of protons by deuterons and the inverse reaction, the photodisintegration of ^{3}He, have been investigated experimentally and theoretically over the past decades with quite some interest. Despite the various corresponding investigations, the theory is only in rough agreement with experiment, and there are inconsistencies between the data up to 30% in the magnitudes of the cross sections. The experimental results by Belt *et al.* [1] and King *et al.* [2,3] are in good agreement. Those by Matthews *et al.* [4] and Skopik *et al.* [5] agree in the angular distributions, but disagree in the magnitudes of the cross sections. This indicates a calibration problem of the measurements.

From the theoretical side several attempts have been made to describe the cross sections in this energy region. In the early calculations by Barbour *et al.* [6] phenomenological interactions were used. It was shown that the final state interaction is quite important, and that the $E2$ contributions in the electromagnetic interaction are needed in the differential cross section. King *et al.* [2] performed an effective two-body, direct capture calculation with the initial state being treated as a plane wave or as a scattering state generated from an optical potential. In the calculations by Gibson and Lehman [7], based on the Faddeev-type Alt-Grassberger-Sandhas (AGS) equations [8] adjusted to photoprocesses, a more realistic Yamaguchi interaction was used, but only the $E1$ components were employed. Fonseca and Lehman [9] obtained the polarization observables A_{yy} and T_{20} at different excitation energies with the same Faddeev-type formalism including only the $E1$ interaction. A calculation at $E_x = 15$ MeV based on realistic interactions and both $E1$ and $E2$ contributions has been done by Ishikawa and Sasakawa [10]. Another calculation of A_{yy} in this energy region is by Jourdan *et al.* [11]. It was found in all these investigations that T_{20} is independent of the deuteron and the helium D-state probability, whereas A_{yy} shows a weak dependence on these quantities.

Very-low-energy radiative capture processes are of considerable astrophysical relevance. The n-d radiative capture, which at such energies is almost entirely a magnetic dipole ($M1$) transition, was studied by several authors [12–14]. In Ref. [14] configuration-space Faddeev calculations of the triton wave function, with inclusion of three-body forces and meson exchange currents (MEC's) were employed. Various trends, e.g., the correlation between cross sections and triton binding energies, their potential dependence, and the role of different MEC's, were pointed out. More recently a rather detailed investigation of n-d and p-d radiative capture at low energies has been performed by Viviani *et al.* [15]. Their calculations employed the quite accurate three-nucleon bound and continuum states obtained in the variational pair-correlated hyperspherical method, developed, tested, and applied over years by this group.

In Refs. [16,17] we have treated the ^{3}He photodisintegration and the inverse radiative capture process within the integral equation approach discussed below. These calculations were based on the Paris, Bonn A, and Bonn B potentials in Ernst-Shakin-Thaler (EST) representation: PEST, BAEST, and BBEST [18,19]. We have demonstrated in particular the role of $E2$ contributions, meson exchange currents, and higher partial waves at $E_x = 12$ MeV and $E_x = 15$ MeV. The sensitivity against the underlying potentials, moreover, was pointed out. In the present paper we extend these investigations and compare our calculations with all sufficiently accurate data below $E_x = 20$ MeV.

The AGS equations are well known to go over into effective two-body Lippmann-Schwinger equations [8] when representing the input two-body T operators in separable form. The proton-deuteron scattering amplitude, thus, is determined by

$$\mathcal{T}(\mathbf{q},\mathbf{q}'') = \mathcal{V}(\mathbf{q},\mathbf{q}'') + \int d^3q' \, \mathcal{V}(\mathbf{q},\mathbf{q}') \mathcal{G}_0(\mathbf{q}') \mathcal{T}(\mathbf{q}',\mathbf{q}''). \tag{1}$$

Applying the same technique to the ^{3}He photodisintegration process, an integral equation of rather similar structure is obtained for the corresponding amplitude [7],

$$\mathcal{M}(\mathbf{q}) = \mathcal{B}(\mathbf{q}) + \int d^3q' \, \mathcal{V}(\mathbf{q},\mathbf{q}') \mathcal{G}_0(\mathbf{q}') \mathcal{M}(\mathbf{q}'). \tag{2}$$

In both equations the kernel is given by an effective proton-deuteron potential $\mathcal{V}$ and an effective free Green function $\mathcal{G}_0$. However, in Eq. (2) the inhomogeneity of Eq. (1) is replaced by an off-shell extension of the ^{3}He photodisintegration amplitude in plane-wave (Born) approximation,

$$B(\mathbf{q}) = \langle \mathbf{q} | \langle \psi_d | H_{\rm em} | \psi_{\rm He} \rangle. \tag{3}$$

Here, $|\psi_{\rm He}\rangle$ and $|\psi_d\rangle$ are the ^{3}He and deuteron states, $|\mathbf{q}\rangle$ is the relative momentum state of the proton, and $H_{\rm em}$ denotes the electromagnetic operator. In other words, with this replacement any working program for p-d scattering, based on separable representations or expansions of the two-body potential, can immediately be applied to calculating the full ^{3}He photodisintegration amplitude with inclusion of the final-state interaction.

The cross section for the p-d capture process is obtained from the corresponding photodisintegration expression by using the principle of detailed balance [20]:

$$\frac{d\sigma^{\rm dis}}{d\Omega} = \frac{3}{2} \frac{k^2}{Q^2} \frac{d\sigma^{\rm cap}}{d\Omega}. \tag{4}$$

Here, k and Q are the momenta of the proton and the photon, respectively. In the present treatment no Coulomb forces have been taken into account. The matrix element (3) for p-d capture differs from the corresponding n-d expression only in its isospin content.

The results presented in this paper are obtained by employing the PEST, BAEST, and BBEST potentials as input [18], however, with an improved parametrization by Haidenbauer [21]. The high quality of this input has been demonstrated in bound-state and scattering calculations [19,22,23].

The electromagnetic operator relevant in the total cross section is, at the low energies considered, essentially a dipole operator. In the differential cross section we have to include also the quadrupole operator. According to Siegert's theorem [24], these operators are given by

$$H_{\rm em}^{(1)} \sim -iE_\gamma \sum_{i=1}^{3} e_i r_i Y_{1\lambda}(\vartheta_i, \varphi_i) \tag{5}$$

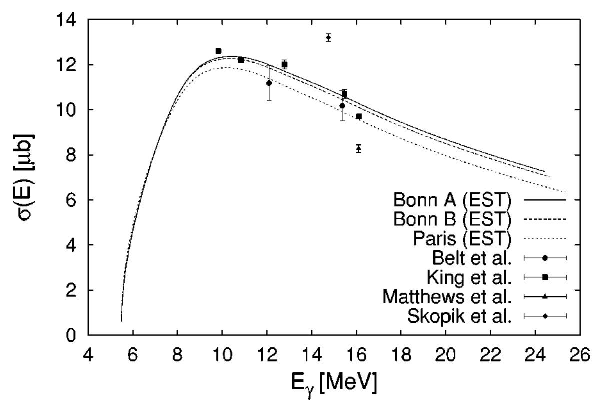

FIG. 1. Total cross section for the capture of protons by deuterons. The data are from [1–5].

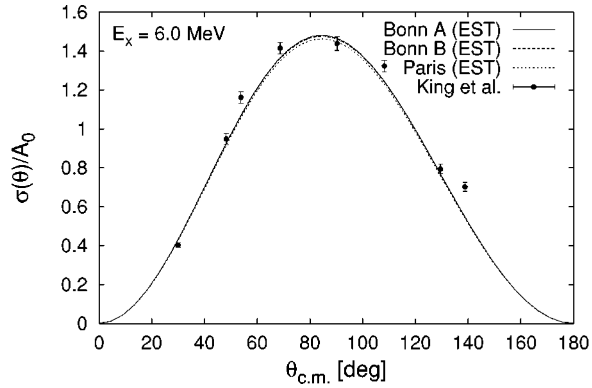

FIG. 2. Angular distribution of the cross section for p-d capture at $E_x = 6.0$ MeV. The data are from [2,3].

and

$$H_{\rm em}^{(2)} \sim \frac{E_\gamma^2}{\sqrt{20}} \sum_{i=1}^{3} e_i r_i^2 Y_{2\lambda}(\vartheta_i, \varphi_i), \tag{6}$$

where E_γ denotes the photon energy, r_i the nucleon coordinates, e_i the electric charges, and $\lambda = \pm 1$ the polarization of the photon.

Our method for determining the final state, i.e., the ^{3}He wave function, is described in Refs. [25,26]. In the calculation of the Faddeev components the total angular momentum j of the two-body potential was restricted to $j \leqslant 2$, while in the full state all partial waves with $j \leqslant 4$ (34 channels) have been taken into account. With this number of channels a converged calculation was achieved, incorporating 99.8% of the wave functions. Details concerning their high quality are given in [26]. For the initial state all partial waves with $j \leqslant 2$ have been included in order to get a converged calculation of the cross section [16,17].

Usually the differential cross section is expanded in terms of Legendre polynomials:

$$\sigma(\theta) = A_0 \left(1 + \sum_{k=1}^{4} a_k P_k(\cos\theta) \right). \tag{7}$$

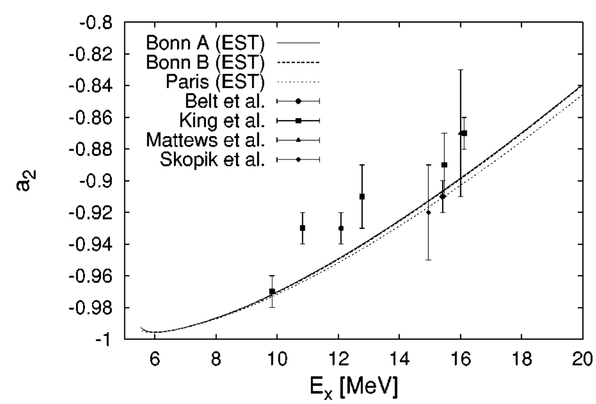

FIG. 3. The a_2 angular distribution coefficient as function of E_x. The data are from [1–5].

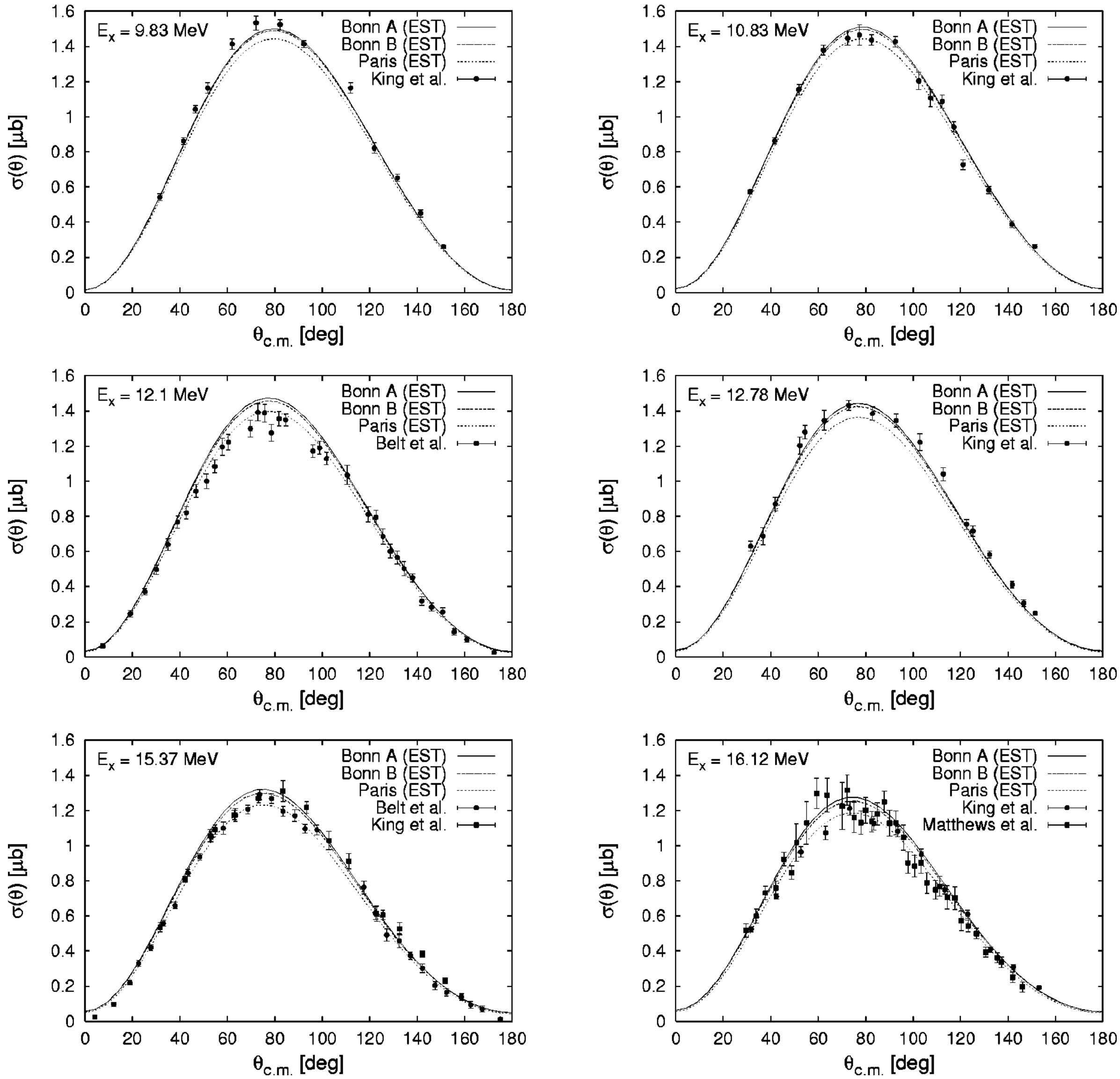

FIG. 4. Differential cross section for p-d capture for energies E_x from near threshold up to 16 MeV. The data are from [1–4]. The data set by Matthews *et al.* [4] has been renormalized with the A_0 from King *et al.* [2,3].

The total cross section is obtained by integrating over the angle θ between the incoming photon and the outgoing proton:

$$\sigma = 4\pi A_0 . \tag{8}$$

Figure 1 shows the total cross sections for the Paris, the Bonn *A*, and Bonn *B* potentials compared to the experimental data [1–5]. There is a small potential dependence of σ and, hence, of A_0 similar to the one observed in the corresponding photoprocess [16,17]. In view of the error bars, the experimental data by Belt *et al.* [1] and King *et al.* [2,3] are reproduced for all potentials with the same quality. Those by Matthews *et al.* [4] and by Skopik *et al.* [5] are not described by the theoretical curves.

Figure 2 shows the angular distribution of the differential cross section, i.e., the ratio of $\sigma(\theta)$ and the coefficient A_0 compared to the experimental data [3]. This distribution is evidently potential independent. In other words, its shape shows no correlation with the helium binding energy or the *D*-state probability of the ^{3}He wave function.

Figure 3 shows the angular distribution coefficient a_2 of the expansion (7) compared to the coefficients extracted from experiment [1–5]. In accordance with Fig. 2 there is almost no potential dependence, i.e., no dependence on the three-body binding energy and the *D*-state probability, although this probability varies for the three potentials considered between 6% and 8% [26].

Figure 4 shows the differential cross sections obtained for these potentials at various energies compared to the experimental data. As a result of the slight potential dependence of the total cross section and, thus, of A_0, the magnitudes of the curves differ correspondingly. In all cases there is good agreement between theory and experiment. As pointed out in [16,17] this agreement can only be achieved by taking into account $E1$ and $E2$ contributions of the electromagnetic interaction, meson exchange currents, and higher partial waves in the potential and in the three-body wave function. It should be mentioned that for increasing energies the peak is slightly shifted to the right-hand side, because of a smaller $E1$ and a somewhat higher $E2$ contribution. Note that, as a result of the missing $E1$-$E2$ interference term, the quadrupole contribution is practically negligible in the total cross section.

In [16,17] we have shown that for different potentials the low-energy peak heights of the ^{3}He photodisintegration cross sections are strictly correlated with the corresponding ^{3}He binding energies and with the number of partial waves included. The magnitude of the present radiative capture process, i.e., the constant A_0, appears to be similarly fixed by the three-body binding energy. In other words, at the energies discussed, the radiative capture cross section does not represent an additional observable for testing different potentials.

We acknowledge financial support from the Deutsche Forschungsgemeinschaft under Grant No. Sa 327/23-I. Part of this work was performed under the auspices of the U.S. Department of Energy under Contract No. DE-FG02-93ER40756 with Ohio University. The authors would like to thank H. R. Weller for providing additional information about the experimental data.

[1] B. D. Belt, C. R. Bingham, M. L. Halbert, and A. van der Woude, Phys. Rev. Lett. **24**, 1120 (1970).

[2] S. King, N. R. Roberson, H. R. Weller, and D. R. Tilley, Phys. Rev. C **30**, 21 (1984).

[3] S. King, N. R. Roberson, H. R. Weller, D. R. Tilley, H. P. Engelbert, H. Berg, E. Huttel, and G. Clausnitzer, Phys. Rev. C **30**, 1335 (1984).

[4] J. L. Matthews, T. Kruse, M. E. Williams, R. O. Owens, and W. Sawin, Nucl. Phys. **A223**, 221 (1974).

[5] D. M. Skopik, J. Asai, D. H. Beck, T. P. Dielschneider, R. E. Pywell, and G. A. Retzlaff, Phys. Rev. C **28**, 52 (1983).

[6] I. M. Barbour and A. C. Phillips, Phys. Rev. C **1**, 165 (1970); I. M. Barbour and J. A. Hendry, Phys. Lett. **38B**, 151 (1972).

[7] B. F. Gibson and D. R. Lehman, Phys. Rev. C **11**, 29 (1975).

[8] E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. **B2**, 167 (1967).

[9] A. C. Fonseca and D. R. Lehman, in *Few-Body Problems in Physics*, edited by Franz Gross, AIP Conf. Proc. No. 334 (AIP, New York, 1995); G. J. Schmid, R. M. Chasteler, H. R. Weller, D. R. Tilley, A. C. Fonseca, and D. R. Lehman, Phys. Rev. C **53**, 35 (1996).

[10] S. Ishikawa and T. Sasakawa, Phys. Rev. C **45**, R1428 (1992).

[11] J. Jourdan, M. Baumgartner, S. Burzynski, P. Egelhof, R. Henneck, A. Klein, M. A. Pickar, G. R. Plattner, W. D. Ramsay, H. W. Roser, I. Sick, and J. Torre, Nucl. Phys. **A453**, 220 (1986).

[12] E. Hadjimichael, Phys. Rev. Lett. **31**, 183 (1973).

[13] J. Torre and B. Goulard, Phys. Rev. C **28**, 529 (1983).

[14] J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Lett. B **251**, 11 (1990).

[15] M. Viviani, R. Schiavilla, and A. Kievsky, Phys. Rev. C **54**, 534 (1996).

[16] W. Schadow and W. Sandhas, Nucl. Phys. **A631**, 588c (1998).

[17] W. Sandhas, W. Schadow, G. Ellerkmann, L. L. Howell, and S. A. Sofianos, Nucl. Phys. **A631**, 210c (1998).

[18] J. Haidenbauer and W. Plessas, Phys. Rev. C **30**, 1822 (1984); J. Haidenbauer, Y. Koike, and W. Plessas, *ibid.* **33**, 439 (1986).

[19] J. Haidenbauer and Y. Koike, Phys. Rev. C **34**, 1187 (1986).

[20] B. A. Craver, Y. E. Kim, and A. Tubis, Nucl. Phys. **A276**, 237 (1977).

[21] J. Haidenbauer (private communication).

[22] T. Cornelius, W. Glöckle, J. Haidenbauer, Y. Koike, W. Plessas, and W. Witala, Phys. Rev. C **41**, 2538 (1990).

[23] W. C. Parke, Y. Koike, D. R. Lehman, and L. C. Maximon, Few-Body Syst. **11**, 89 (1991).

[24] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).

[25] L. Canton and W. Schadow, Phys. Rev. C **56**, 1231 (1997).

[26] W. Schadow, W. Sandhas, J. Haidenbauer, and A. Nogga, Phys. Rev. C (submitted).