

Chiral symmetry and three-nucleon forces

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After a brief review of the role three-nucleon forces play in the few-nucleon systems, the chiral-perturbation-theory approach to these forces is discussed. Construction of the (nominal) leading- and subleading-order Born terms and pion-rescattering graphs contributing to two-pion-exchange three-nucleon forces is reviewed, and comparisons are made of the types of such forces that are used today. It is demonstrated that the short-range c term of the Tucson-Melbourne force is unnatural in terms of power counting and should be dropped. The class of two-pion-exchange three-nucleon forces then becomes rather uniform. [S0556-2813(99)02401-2]

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INTRODUCTION

Three-nucleon ($3N$) forces have come under increasing scrutiny recently [1]. Although these forces are rather weak, they play an important role in the theory of few-nucleon systems, where computational advances permit the calculation of new observables that are challenged by experiment [2,3]. The most-recent (second-generation) nucleon-nucleon (NN) potentials [4,5] fit the entire NN data base rather well (rivaling phenomenological partial-wave analyses in the best cases) and lead to predictions for most $3N$ observables that are in good agreement with experiment. In a few cases, such as the A_y puzzle [1,2] and the binding energies of few-nucleon ground states [6], there are inadequacies with this methodology that have focused attention on three-nucleon forces.

All realistic NN forces underbind the triton [7], and small differences among them can be traced to nonlocalities. Three-nucleon forces are incorporated into the Hamiltonian and adjusted to achieve the correct triton binding. With this addition ${}^4\text{He}$ is properly bound [8], while the two ${}^5\text{He}$ p levels have a splitting roughly 30% too small [9]. Binding of $A=6-8$ ground and low-lying excited states is too low [6].

The best studied of these problems is the A_y puzzle. The calculated asymmetry (A_y) in neutron-deuteron and proton-deuteron scattering at low energies is 25–30% too small, which looks suspiciously similar to the ${}^5\text{He}$ problem, since A_y is most sensitive to spin-orbit forces. A recent analysis of the former problem concludes [1] that reasonable changes in the NN force will not resolve the puzzle and that one should implement refined $3N$ force models. Although credible examples of these models first began to appear 40 years ago [10], technical problems associated with nuclear-force construction hampered the effort, and general acceptance of such forces was delayed until it was demonstrated that good NN forces could not reproduce the triton binding energy.

Construction of potentials always involves theoretical choices, since a potential is a subamplitude (an off-shell part of an amplitude) that when iterated (in the Schrödinger equation, for example) produces observables (on-shell amplitudes or energies). The off-shell question has always been a murky one, since it is usually ill defined. Nevertheless, the same

Lagrangian (i.e., the same theory) can lead to different potentials, although they should individually produce identical observables. Coupled to this is the worse problem of unraveling the underlying strong-interaction physics (i.e., deciding on a Lagrangian or equivalent formalism to use). In the early days a frequently asked question [11] was, how does one account for the off-shell nature of (virtual) pions exchanged between nucleons? Faced with such daunting theoretical obstacles, all models were simplified. Nonlocality (nucleon-momentum dependence) was typically ignored, for example. The early history of the field is well reviewed in Refs. [12, 13].

Since these early beginnings a new formalism [14–17] has been developed for implementing strong-interaction physics in low-momentum (for nucleons) regimes: chiral perturbation theory (CPT). This technique implements (approximate) chiral symmetry (manifested by the quarks in QCD) in constructing the strong-interaction building blocks, which are then assembled in *all* possible ways in the most general Lagrangian consistent with the symmetry. At the same time, the entire framework is organized with a power-counting scheme. A successful perturbation theory must guarantee that succeeding orders diminish, and chiral symmetry provides the constraints mandating that more complex calculations (loops, etc.) should yield progressively smaller results, even though strong-interaction coupling constants are not small. This scheme also provides a testing mechanism for nuclear interactions: naturalness and naive dimensional power counting [18].

Chiral perturbation theory simplifies the old-fashioned nuclear-physics approach of incorporating into a field theory all known meson and baryon resonances with energies less than some large (arbitrary) cutoff. All such heavy resonances (with the possible exception of the low-lying Δ isobar, which is ignored here for simplicity) are subsumed in short-range (pointlike) vertices. In the usual $\text{SU}(2)$ approach this means that only pion and nucleon fields contribute explicitly, although the entire zoo of heavy elementary particles contributes implicitly to the phenomenological constants of the theory.

Two scales that set the strength of the Lagrangian building blocks are $f_\pi \sim 93$ MeV (the pion-decay constant) and

$\Lambda \sim 1$ GeV (the large-mass QCD scale). Overall powers of Λ must be negative (i.e., $\Lambda^{-\Delta}$, with $\Delta \geq 0$), since they arise from the frozen propagation of the heavy states, and interactions in the Lagrangian are organized by these powers: $\mathcal{L}^{(\Delta)}$. Dimensionful coupling constants in this scheme can be written as powers of f_π and Λ times dimensionless coupling constants $\sim \pm 1$. The latter requirement is called naturalness. ‘‘Unnatural’’ implies very small or very large (compared to 1) and of either sign. We will use this test later.

We wish to examine and compare the two-pion-exchange three-nucleon forces (3NF’s) that incorporate at least minimal phenomenology from π - N scattering. There are basically four types (plus variants of each that we will not treat): (1) Tucson-Melbourne force [11] (the first of this class), based on current-algebra arguments; (2) Brazilian force [19], based on a chiral Lagrangian and a supplemental current-algebra constraint; (3) Texas [20] force, based on chiral perturbation theory; and (4) Ruhr(Pot) force [21], based on nonchiral Lagrangians. Each contains a σ term (or functional equivalent) for s -wave, isospin-symmetric pions, as well as p -wave pions in both isospin-symmetric and -antisymmetric configurations, such as might arise from virtual Δ -isobar excitation. We note that the august Fujita-Miyazawa [10] 3NF contained equivalents of all these elements (although the s -wave part was dropped) and the

Urbana-Argonne [22] model contains a conventional Fujita-Miyazawa Δ -mediated force plus an intermediate-range isospin- and spin-independent component.

CHIRAL PERTURBATION THEORY

We will make our comparisons using the framework of CPT, which allows us to define the theory in a consistent and transparent way. The relevant parts of the leading-order Lagrangian (corresponding to $\Delta = 0$) $\mathcal{L}^{(0)}$ are given by [20,23]

$$\mathcal{L}^{(0)} = \frac{1}{2} [\dot{\boldsymbol{\pi}}^2 - (\vec{\nabla} \boldsymbol{\pi})^2 - m_\pi^2 \boldsymbol{\pi}^2] + N^\dagger \left[i \partial_0 - \frac{1}{4f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \dot{\boldsymbol{\pi}}) \right] N + \frac{g_A}{2f_\pi} N^\dagger \vec{\sigma} \cdot \vec{\nabla} (\boldsymbol{\tau} \cdot \boldsymbol{\pi}) N, \quad (1)$$

whose three terms correspond to free pions, the free-nucleon energy and Weinberg-Tomozawa two-pion interaction, and the usual pion-nucleon interaction. We have simplified the nonlinear realizations of the SO(4) symmetry [20] and dropped terms that would have added even numbers of pion fields to all terms with pion fields; we do not require such terms in what follows. In addition, the $\Delta = 1$ Lagrangian $\mathcal{L}^{(1)}$ is given by [20,23]

$$\begin{aligned} \mathcal{L}^{(1)} = & \frac{1}{2m_N} \left[N^\dagger \vec{\nabla}^2 N - \frac{1}{4f_\pi^2} N^\dagger \{ \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \vec{\nabla} \boldsymbol{\pi}), \cdot \vec{p} \} N + \frac{g_A}{2f_\pi} N^\dagger \{ \boldsymbol{\tau} \cdot \dot{\boldsymbol{\pi}}, \vec{\sigma} \cdot \vec{p} \} N \right] \\ & + \frac{1}{f_\pi^2} N^\dagger \left[\left(c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \dot{\boldsymbol{\pi}}^2 - c_3 (\vec{\nabla} \boldsymbol{\pi})^2 - 2c_1 m_\pi^2 \boldsymbol{\pi}^2 - \frac{1}{2} \left(c_4 + \frac{1}{4m_N} \right) \varepsilon_{ijk} \varepsilon_{abc} \sigma_k \tau_c \partial_i \pi_a \partial_j \pi_b \right] N \\ & - \frac{d_1}{f_\pi} N^\dagger \vec{\sigma} \cdot \vec{\nabla} (\boldsymbol{\tau} \cdot \boldsymbol{\pi}) N N^\dagger N - \frac{d_2}{2f_\pi} \varepsilon_{ijk} \varepsilon_{abc} \partial_i \pi_a N^\dagger \sigma_j \tau_b N N^\dagger \sigma_k \tau_c N + \dots, \end{aligned} \quad (2)$$

where terms with additional pion fields have been dropped, and we have not listed [20] three separate spin- and isospin-dependent short-range 3NF terms [$\sim (N^\dagger N)^3$] with coefficients e_i . We have also ignored isospin violation. Where appropriate we have adopted the notation of Ref. [17] and have explicitly incorporated higher-order terms resulting from a nonrelativistic reduction of the pseudovector-coupling Born term. The phenomenological coefficients c_i and d_i must be determined from experiment.

We have not written down explicit Δ -isobar contributions above. They are implicitly included in the phenomenological coefficients. This hides the fact that those coefficients that contain tree-level Δ contributions are expected to be larger than ones that do not by a $\Lambda/(m_\Delta - m_N)$ factor. The alternative is to include a Δ field and count it as a nucleon field [20]. This shifts the nominal order of the isobar effects, but of course not their numerical value, and it unnecessarily complicates the following discussion.

For later use we also list infinitesimal generators for the (approximate) axial symmetry present in this Lagrangian, where again we ignore terms with more than two pion fields:

$$\boldsymbol{\pi} \rightarrow \boldsymbol{\pi} - f_\pi \boldsymbol{\epsilon}, \quad (3a)$$

$$N \rightarrow N - i \frac{\boldsymbol{\epsilon} \cdot \boldsymbol{\tau} \times \boldsymbol{\pi}}{4f_\pi} N, \quad (3b)$$

where $\boldsymbol{\epsilon}$ is (a constant) infinitesimal. Under this transformation the three terms in Eq. (1) are separately invariant in the limit of vanishing pion mass, as are the first bracketed term and each remaining term in $\mathcal{L}^{(1)}$ (in the same limit). Thus the Lagrangian in Eqs. (1) and (2) is term-by-term (as we have written them) invariant, except for the pion mass and c_1 term (also conventionally known as the σ term): $-4m_\pi^2 c_1 = \sigma$.

It is important to note that the Lagrangians $\mathcal{L}^{(i)}$ are not unique. Redefinition of the (unphysical) fields leads to other forms. The form we have chosen satisfies chiral constraints in a term-by-term fashion, rather than relying on cancellations between sets of terms. It is only important that the chosen form have sufficient generality (i.e., enough linearly independent terms). Different forms will then be physically equivalent on shell, but will in general be different off shell. Off-shell differences do not affect physical processes. Note

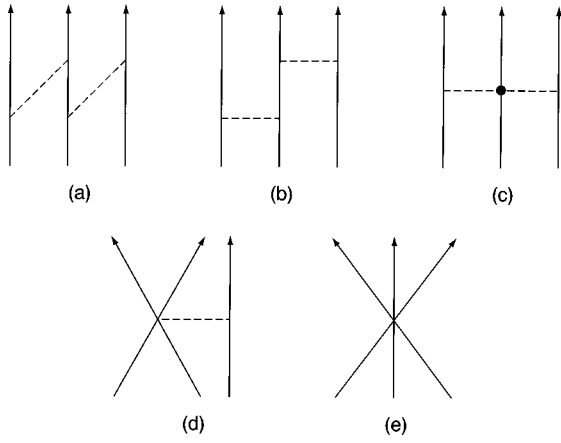


FIG. 1. Various three-nucleon-force components that arise in subleading order in chiral perturbation theory, as discussed in the text.

that the Lagrangian of Ref. [24], which is based on a nonrelativistic reduction of the relativistic pseudovector pion-nucleon coupling, used an off-shell extension specified by a continuous parameter μ . Only the choice $\mu=1$ corresponds to Eq. (2) and only that choice satisfies term-by-term chiral symmetry. Amplitudes calculated using various values of μ correspond to a unitary transformation of the Hamiltonian and therefore do not alter physical amplitudes (although they are different off shell). We note [24] that many of the older papers in the field have implicitly adopted different values of μ [viz., $-1, 0, 1$].

In order to determine the 3NF to (nominal) subleading order, we need to calculate the diagrams of Fig. 1. The two interaction terms in $\mathcal{L}^{(0)}$ together with the first two terms in $\mathcal{L}^{(1)}$ are usually called relativistic Born terms and are separately calculated using (the many orderings of) Figs. 1(a) and 1(c), and then subtracting the iteration of the one-pion-exchange potential (OPEP) given in Fig. 1(b). In the static (leading-order) limit ($m_N \rightarrow \infty$), they have long been known to vanish [25,24]. If one works to subleading order, one is faced with choices, because different off-shell choices for the subtracted OPEP lead to different forms for the 3NF. Thus the *choice* of form for OPEP [to order $(v/c)^2$] determines the form of this (Born-term) part of the 3NF. The reader is referred to Refs. [20] and [24], where different off-shell choices are made. The complete (μ, ν) off-shell ambiguity is discussed in the latter reference, and approximate Lorentz invariance is demonstrated. The former ambiguity arises from a nucleon-field transformation (a ‘‘chiral rotation’’) that breaks term-by-term chiral invariance, as we discussed below Eq. (2). Different values of μ have been implicitly assumed in the past by differing treatments of the Born terms (see the Appendix of Ref. [24]). The ν dependence arises through differing treatments of the difference between (four-vector) q^2 and \vec{q}^2 [see Eq. (4b) below], and is sometimes called the quasipotential parameter. Different quasipotential equations correspond (in part) to different values of ν , and the values $[0, 1/2, 1]$ have been commonly used [24]. Different values of ν correspond to different off-shell amplitudes, but unitarily equivalent on-shell values. Other calculations have ignored part or all of the subleading-order Born-term contributions. We will ignore the Born terms in what follows.

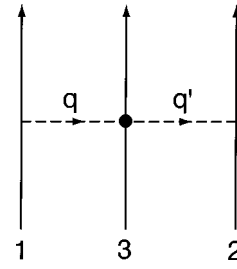


FIG. 2. Contribution to the three-nucleon force arising from a pion emitted by nucleon 1 and scattering from nucleon 3 before being absorbed by nucleon 2.

The remaining nine terms of $\mathcal{L}^{(1)}$ [labeled by c_i , d_i , e_i] generate 3NF’s of the type in Fig. 1(c) [c_1, c_3, c_4], Fig. 1(d) [d_1, d_2], and Fig. 1(e) [e_1, e_2, e_3]. The $\vec{\pi}^2$ term in $\mathcal{L}^{(1)}$ generates contributions of $\Delta=3$ size (each time derivative is the same as a nucleon-energy difference) and can be neglected. A wide range of physics is subsumed in each category. The c_3 and c_4 terms receive important contributions from Δ isobars at the solid circle of Fig. 1(c), while a heavy scalar-isoscalar meson would likewise contribute to c_1 . We note that all of the models we will compare contain this important physics, either through phenomenological input or via explicit heavy-particle intermediate states.

We summarize by noting that the Born term from $\mathcal{L}^{(0)}$, the c_i π -rescattering terms, the d_i one-pion-exchange terms, and the (purely) short-range e_i terms are all nominally the same size, although large Δ -isobar contributions can be expected to make some of the terms larger than others. We will not discuss the d_i and e_i terms further. This force was first derived in Ref. [20].

COMPARISONS

To facilitate comparisons we adopt the familiar framework of the Tucson-Melbourne (TM) collaboration [11] for the Born-subtracted amplitudes [26]:

$$S = 1 - iT, \quad (4a)$$

$$V_{3\text{NF}} = T = \left(\frac{g_A}{2f_\pi} \right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}'}{(\vec{q}^2 + m_\pi^2)(\vec{q}'^2 + m_\pi^2)} [-F^{\alpha\beta} \tau_1^\alpha \tau_2^\beta], \quad (4b)$$

$$t_{\pi N}^{\alpha\beta} = -F^{\alpha\beta} \cong \delta^{\alpha\beta} [a + b \vec{q} \cdot \vec{q}' + c(\vec{q}^2 + \vec{q}'^2)] - d(\tau_3^\gamma \epsilon^{\alpha\beta\gamma} \vec{\sigma}_3 \cdot \vec{q} \times \vec{q}'), \quad (4c)$$

where δ functions, phase-space factors, etc., have been ignored, and the invariant amplitudes of [11,26] have been expanded in $1/m_N$.

Equation (4b) is illustrated in Fig. 2, showing nucleon (3) scattering a pion emitted by nucleon (1) and absorbed by nucleon (2). The T matrix for π - N scattering (alone) is denoted $t_{\pi N}^{\alpha\beta}$ and is usually rewritten in terms of $F^{\alpha\beta}$, where α and β are the isospin labels of the initial and final pions. The final expression in Eq. (4c) holds for pions that have a low momentum ($\lesssim m_\pi$). Summing over the symmetric permutations of (1), (2), and (3) in Fig. 2 leads to the complete three-nucleon potential.

TABLE I. Low-energy CPT coefficients in GeV^{-1} from several recent fits.

Fit	c_1	c_2	c_3	c_4
$O(Q^2)$ [28]	-0.64	1.78	-3.90	2.25
$O(Q^3)$ [17]	-0.87	3.30	-5.25	4.12
$O(Q^3)$ [29]	-0.93	3.34	-5.29	3.63
$O(Q^3)$ [30]	-1.06	3.40	-5.54	3.25
$O(Q^3)$ [31]	-1.27	3.23	-5.93	3.44
$O(Q^3)$ [31]	-1.47	3.21	-6.00	3.52
$O(Q^3)$ [31]	-1.53	3.22	-6.19	3.51

One easily finds, from Eq. (2),

$$F^{\alpha\beta} = \frac{\delta^{\alpha\beta}}{f_\pi^2} [2\omega\omega'(c_2 + c_3) - 2c_3\vec{q}\cdot\vec{q}' - 4c_1m_\pi^2] - \frac{\tau_3^\gamma \epsilon^{\alpha\beta\gamma} \vec{\sigma}_3 \cdot \vec{q} \times \vec{q}'}{f_\pi^2} [c_4], \quad (5)$$

where ω and ω' are the initial and final pion energies. We have dropped Born term contributions to c_2 and c_4 in accordance with our earlier discussion. Equation (5) together with the dropped pieces generates the CPT π - N amplitude to $O(Q^2)$. Calculations including loops and new parameters at $O(Q^3)$ have also been performed. They have been used with different pieces of π - N scattering data to determine the coefficients c_i . In Table I we list some of these determinations. Earlier fits [28,17,29,30] were made to different sets of threshold and subthreshold parameters obtained from dispersion analyses of older data. Newer fits [31] were made to different phase-shift analyses (PSA's), the last two in Table I including the more modern meson-factory data. The $O(Q^3)$ determinations are consistent with each other when their error bars (not shown) are considered, except for c_1 , which reflects the higher value for the σ term in the newer PSA's. Note that the coefficients c_2 , c_3 , and c_4 , which receive contributions from the Δ at tree level, are larger than c_1 , as expected [32].

From the definition (4c) of the (a,b,c,d) coefficients, we obtain

$$a = \frac{4m_\pi^2 c_1}{f_\pi^2} = -\frac{\sigma}{f_\pi^2}, \quad (6a)$$

$$b = \frac{2c_3}{f_\pi^2}, \quad (6b)$$

$$c = 0, \quad (6c)$$

$$d = -\frac{c_4}{f_\pi^2}. \quad (6d)$$

Note that there is no c term and that the a term is opposite in sign to the TM result, although, with $c_3 < 0$ and $c_4 > 0$, b and d are negative and agree with the corresponding TM signs. A similar result was found in the first of the Brazil-force papers [19], where a field-theoretic calculation of isobar contributions was performed. The σ term was not calculated using

TABLE II. Low-energy pion-nucleon scattering parameters [with Z-graph (Born) terms removed] for a variety of 2π -exchange three-nucleon forces. We have also defined $a' = a - 2m_\pi^2 c$. The quantities a and a' are in units of m_π^{-1} , while b , c , and d are in units of m_π^{-3} .

Three-nucleon force	a'	b	c	d
Fujita-Miyazawa [10]	0.0	-1.15	0.0	-0.29
Tucson-Melbourne [11,27]	-1.03	-2.62	1.03	-0.60
Brazil [19,27]	-1.05	-2.29	1.05	-0.77
Urbana-Argonne [22,6]	0.0	-1.20	0.0	-0.30
Texas [20,31]	-1.87	-3.82	0.0	-1.12
Ruhr(Pot) [21]	-0.51	-1.82	0.0	-0.48

Feynman rules derived consistently from a Lagrangian, but inferred from a π - N amplitude derived elsewhere. In a later paper, a different off-shell amplitude (the one used in the TM calculation) was incorporated. Values of the a - d coefficients for popular three-nucleon force models are displayed in Table II. Note that $a = a' + 2m_\pi^2 c = 1.03/m_\pi$ for the TM force.

Given that CPT is a comprehensive approach to calculating strong-interaction physics based on chiral symmetry and subsumes current algebra [17,33], how can the CPT (corresponding to derivatively coupled pions) amplitude [Eq. (5)] and TM amplitude [Eq. (4c)] differ?

We answer that question by noticing that the difference resides only in terms that vanish when the pions are on shell [as we shall see in Eq. (11)]. We return to the earlier off-shell discussion and follow closely the approach of Ref. [33]. Off-shell amplitudes are not unique, and in a field-theoretic calculation, they depend on the fields chosen to represent pions and nucleons. Our form was chosen to satisfy chiral symmetry term by term, thereby attaining manifest power counting. Current-algebra constraints at certain off-mass-shell points [26] are not satisfied by our isospin-even π - N amplitude $F^{(+)}$ [$F^{\alpha\beta} = \delta^{\alpha\beta} F^{(+)} + \dots$]. These points all correspond to vanishing (four-vector) $q \cdot q'$, as well as ω and ω' (to the order we work). Consequently, we can ignore the c_2 and c_3 terms in Eq. (5) and concentrate on the remaining term, which can be written in the form

$$F_{\text{CPT}}^{(+)} = \frac{\sigma}{f_\pi^2}, \quad (7)$$

which holds everywhere.

Again following Ref. [33], we redefine the pion field as

$$\pi' = \pi \left(1 - \frac{\sigma}{m_\pi^2 f_\pi^2} N^\dagger N \right) \quad (8)$$

and work only to order $\Delta = 1$ (since $\sigma \sim 1/\Lambda$). Substituting Eq. (8) into $\mathcal{L}^{(0)}$, we generate the extra terms

$$\Delta \mathcal{L}^{(1)} = -\frac{\sigma}{m_\pi^2 f_\pi^2} \left[N^\dagger N (\pi' \cdot \square \pi' + m_\pi^2 \pi'^2) - \frac{g_A}{2f_\pi} N^\dagger \vec{\sigma} \cdot \vec{\nabla} [\tau \cdot \pi' N^\dagger N] N \right] + \dots \quad (9)$$

The last term involves four nucleon fields and is not immediately required. The three terms in $\mathcal{L}^{(1)}$ and $\Delta\mathcal{L}^{(1)}$ involving σ and two nucleon fields lead to

$$F_{\text{CA}}^{(+)} = \frac{\sigma}{m_\pi^2 f_\pi^2} (q^2 + q'^2 - m_\pi^2), \quad (10)$$

which agrees with Eq. (7) at any on-shell (e.g., Cheng-Dashen) point ($q^2 = q'^2 = m_\pi^2$), but vanishes at the Adler points ($q^2 = m_\pi^2$, $q'^2 = 0$, and $q'^2 = m_\pi^2$, $q^2 = 0$), and at the Weinberg point ($q^2 = q'^2 = 0$) has the value $F_{\text{CA}}^{(+)} = -\sigma/f_\pi^2$. Equation (10) therefore agrees with the usual current-algebra constraints [11,26], as does our entire amplitude $F^{(+)}$ in the new pion-field basis. Thus there is no conflict here between CPT (with derivatively coupled pions) and an approach based on current algebra. The only difference is in the *choice* of fields used to specify the chiral Lagrangian, and observables calculated for physical processes must be identical.

In the TM approach it was noted that rewriting Eq. (10) in terms of *inverse* pion propagators,

$$F_{\text{CA}}^{(+)} = \frac{\sigma}{f_\pi^2} + \frac{\sigma}{m_\pi^2 f_\pi^2} (q^2 - m_\pi^2 + q'^2 - m_\pi^2), \quad (11)$$

allows cancellation of the pion propagators in Fig. 2. The first (constant) term reproduces Eq. (7) ($F_{\text{CPT}}^{(+)}$). This rearrangement amounts to undoing the field transformation in Eq. (8) that led to Eq. (9) and leads to an effective a term (a') that has a common sign for all models: $a' = a - 2m_\pi^2 c$. Canceling the inverse propagators in the second term in Eq. (11) leads to a new short-range-plus-pion-range 3NF:

$$-\left(\frac{g_A}{2f_\pi}\right)^2 \frac{\sigma}{m_\pi^2 f_\pi^2} \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}' \left(\frac{1}{\vec{q}^2 + m_\pi^2} + \frac{1}{\vec{q}'^2 + m_\pi^2} \right) \tau_1 \cdot \tau_2. \quad (12)$$

However, a three-nucleon force of the same type is generated by the last term in Eq. (9), comprised of four nucleon fields and one pion field, together with the last term in Eq. (1): two graphs as in Fig. 1(d) give

$$\left(\frac{g_A}{2f_\pi}\right)^2 \frac{\sigma}{m_\pi^2 f_\pi^2} \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}' \left(\frac{1}{\vec{q}^2 + m_\pi^2} + \frac{1}{\vec{q}'^2 + m_\pi^2} \right) \tau_1 \cdot \tau_2. \quad (13)$$

This is *exactly equal in size and opposite in sign* to the new short-range contribution from the off-shell extrapolation of the π - N amplitude, Eq. (12). This cancellation is to be expected, since our original (chiral) Lagrangian produced no such terms to start with, and we have just been rearranging terms since then. In summary, the TM approach used a current-algebra representation of the amplitude, performed an implicit field redefinition to our (CPT) choice of fields, which resulted in an extra short-range term in their result. Why did they have an extra term and we do not?

The TM calculation was predicated upon current-algebra constraints on the off-shell π - N scattering amplitude, which

can be reproduced in the CPT approach, as well, as we have demonstrated. It is not enough, however, to worry about just that scattering amplitude, if one constructs a 3NF. To incorporate all of the chiral constraints into the three-nucleon force, current-algebra constraints on the pion-production amplitude from two nucleons would also be necessary [leading to the last term in Eq. (9)]: a daunting task in the current-algebra approach of TM, but one that is unnecessary in our approach. We emphasize that a detailed analysis of the off-shell region of π - N scattering (for example) is equivalent to a particular choice of fields and (while interesting) is not necessary for constructing a 3NF.

If one uses the pion-field redefinition in the symmetry generators, Eq. (3), one finds that the entire Lagrangian maintains its original symmetry, but that $\Delta\mathcal{L}^{(1)}$ generates new noninvariant terms that cancel against additional contributions from $\mathcal{L}^{(0)}$ (via the new term $\epsilon\sigma N^\dagger N/m_\pi^2 f_\pi$ in the pion generator). One might presume that since all of these terms violate chiral symmetry this poses no problem. Unfortunately, chiral-symmetry-breaking terms must vanish in the chiral limit. The additional terms in Eq. (9) (being just a redefinition of fields) exactly cancel each other in any on-shell amplitude. Individually, the two terms do not vanish in this limit because the presence of the $1/m_\pi^2$ in Eq. (9) removes the implicit m_π^2 in σ , and σ/m_π^2 does not vanish in the chiral limit ($m_\pi \rightarrow 0$). Reiterating, the structure of the additional terms in Eq. (9) means that they must individually vanish in that limit, or the entire set of terms must be kept to allow for exact cancellations between them to restore the proper limit. Because the TM approach (implicitly) kept only the first term in Eq. (9), that limit could not be guaranteed for the three-nucleon force. Another way of saying the same thing is that dimensional power counting (naturalness) is not satisfied for the *individual* terms in Eq. (9).

One can check this conclusion by dimensional power counting. An interaction of the form of the last term in Eq. (9) is chiral-symmetry breaking; if it alone is to be kept, it has to be implicitly proportional to m_π^2 and, hence, is nominally an $\mathcal{L}^{(3)}$ term. Such $\mathcal{L}^{(3)}$ coefficients have a generic size $x m_\pi^2 / f_\pi^3 \Lambda^3$, where the dimensionless coefficient x should be of order 1. If we equate this to $g_A \sigma / 2m_\pi^2 f_\pi^3$ [the coefficient of the last term in Eq. (9)], we obtain $x \sim g_A \sigma \Lambda^3 / (2m_\pi^4) \sim 100$, which is vastly unnatural. The unnatural coefficient [$g_A \sigma / 2m_\pi^2 f_\pi^3$] is entirely the result of σ/m_π^2 having a finite symmetry limit.

We recommend that the short-range c term in the TM force be dropped (but the full value of a' in Table II retained; note that the proper power counting has been maintained in a' by the factor of m_π^2 preceding c in the definition of $a' = a - 2m_\pi^2 c$, where now each term in this definition vanishes in the chiral limit). This had been previously advocated by the Brazil group for reasons having nothing to do with symmetry. We note that the d_1 and d_2 terms in the Texas force are also short range in one pair of nucleons and of pion range in the other. These parts of that force (and the corresponding terms in the Lagrangian) satisfy chiral constraints, as does a fully short-range force of the generic type contained in the UA 3NF and shown in Fig. 1(e).

In summary, we have briefly reviewed the class of “realistic” three-nucleon forces. We have demonstrated that the short-range c term of the TM approach is unnatural and should not be kept.

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