Bound-state problem of the $N\Delta$ and $N\Delta\Delta$ systems

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We used the $N\Delta$ and $\Delta\Delta$ potentials derived from the chiral quark cluster model to analyze the bound-state problem of the $N\Delta$ and $N\Delta\Delta$ systems when the two-body subsystems can be in S-wave states. We found that the $N\Delta$ system has only one bound state right at the $N\Delta$ threshold while for the $N\Delta\Delta$ system there are no bound-state solutions in any of the allowed channels, although a couple of them are nearly bound. Our model predicts the $NN\ {}^1D_2$ resonance as being a bound state of the $N\Delta$ system at the $N\Delta$ threshold.
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I. INTRODUCTION

The study of the bound-state problem for the system of a nucleon and one or two deltas will provide information about the possible existence of dibaryon and tribaryon resonances that decay mainly into two nucleons and one pion or into three nucleons and two pions, respectively. Some of these resonances may even decay into two or three nucleons and no pions if the quantum numbers of the $N\Delta$ or $N\Delta\Delta$ bound states can be reached by the corresponding NN or NNN systems.

$N\Delta$ and $\Delta\Delta$ interactions have been derived in the past in the framework of meson-exchange models or phenomenological potentials [1,2]. These models have been used over the years to fit the NN data very accurately. However, in the $N\Delta$ and $\Delta\Delta$ sectors experimental data are so scarce that it is not possible to obtain reliable values of the parameters involved in the interaction. The situation is different in the case of chiral quark cluster models [3,4]. In these models the basic interaction is at the level of quarks involving only a quark-quark-field (pion or gluon) vertex. Therefore its parameters (coupling constants, cutoff masses, etc.) are independent of the baryon to which the quarks are coupled, the difference among them being generated by SU(2) scaling, as explained in Ref. [5]. Note that in the meson-exchange model of Peña *et al.* [1], the SU(2) scaling is used only in those cases where there is no experimental information about the $N\Delta$ coupling constants. Moreover, quark models provide a definite framework to treat the short-range part of the interaction. The Pauli principle between quarks determines the short-range behavior of the different channels without additional phenomenological assumptions. In this way, even in the absence of experimental data, one has a complete scheme which starting from the NN sector allows us to make predictions in the $N\Delta$ and $\Delta\Delta$ sectors. This fact is even more important if one takes into account that the short-range dynamics of the $N\Delta$ and $\Delta\Delta$ systems is to a large extent driven by quark Pauli blocking effects, which do not appear in the NN sector. Pauli blocking acts in a selective way in those channels where the spin-isospin-color degrees of freedom are not enough to accommodate all the quarks of the system [6,7]. Therefore, meson-exchange models cannot fully include the effect of quark Pauli blocking through its purely phenomenological short-range channel-independent part.

This is the second part of a project in which we have embarked to study the bound-state problem of the two- and three-body systems composed of nucleons and deltas. In a previous work [8] we presented the formalism and results for the cases of the $\Delta\Delta$ and $\Delta\Delta\Delta$ systems. Thus, we now go one step away from the systems of identical-particle by replacing one of the deltas by a nucleon. In a future work [9] we will present the results of our model for the NN and $NN\Delta$ systems which correspond to moving away from the identical-particle case by replacing two deltas by two nucleons.

In order to perform the $N\Delta\Delta$ calculations we follow the same procedure that we used with the $\Delta\Delta\Delta$ case [8] which has been taken from the experience gained in the NNN bound-state problem [10,12]. The three-body calculations are performed using a truncated T-matrix approximation where the inputs of the three-body equations are the two-body T matrices truncated such that the orbital angular momentum in the initial and final states is equal to zero. These two-body T matrices, however, have been constructed taking into account the coupling to the $l=2$ states due to the tensor force. This approximation in the case of the NNN system with the NN interaction taken as the Reid soft-core potential leads to a triton binding energy which differs less than 1 MeV from the exact value [10].

We describe in Secs. II and III our formalism. In the case of the $N\Delta$ system an important difference with the identical-particle case is the absence of the Pauli principle at the baryon level. In the case of the $N\Delta\Delta$ system the formalism differs considerably from the identical-particle case since now one has two types of particles. In Sec. IV we give our results and we present the conclusions in Sec. V.

II. TWO-BODY SYSTEM

The interaction between two baryons ($N\Delta$, $\Delta\Delta$) was obtained from the chiral quark cluster model developed elsewhere [4]. In this model baryons are described as clusters of three interacting massive (constituent) quarks, the mass coming from the breaking of the chiral symmetry. The ingredients of the quark-quark interaction are confinement, one-gluon-exchange (OGE), one-pion-exchange (OPE), and one-sigma-exchange (OSE) terms, and whose parameters are

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fixed from the NN data. Explicitly, the quark-quark (qq) interaction is

$$V_{qq}(\vec{r}_{ij})=V_{\rm con}(\vec{r}_{ij})+V_{\rm OGE}(\vec{r}_{ij})+V_{\rm OPE}(\vec{r}_{ij})+V_{\rm OSE}(\vec{r}_{ij}), \tag{1}$$

where $\vec{r}_{ij}$ is the ij interquark distance and

$$V_{\rm con}(\vec{r}_{ij})=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_j r_{ij}^2, \tag{2}$$

$$V_{\rm OGE}(\vec{r}_{ij})=\frac{1}{4}\alpha_s\vec{\lambda}_i\cdot\vec{\lambda}_j\left\{\frac{1}{r_{ij}}-\frac{\pi}{m_q^2}\left[1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right]\times\delta(\vec{r}_{ij})-\frac{3}{4m_q^2r_{ij}^3}S_{ij}\right\}, \tag{3}$$

$$V_{\rm OPE}(\vec{r}_{ij})=\frac{1}{3}\alpha_{ch}\frac{\Lambda^2}{\Lambda^2-m_\pi^2}\times m_\pi\left\{\left[Y(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}Y(\Lambda r_{ij})\right]\vec{\sigma}_i\cdot\vec{\sigma}_j+\left[H(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}H(\Lambda r_{ij})\right]S_{ij}\right\}\vec{\tau}_i\cdot\vec{\tau}_j, \tag{4}$$

$$V_{\rm OSE}(\vec{r}_{ij})=-\alpha_{ch}\frac{4m_q^2}{m_\pi^2}\frac{\Lambda^2}{\Lambda^2-m_\sigma^2}\times m_\sigma\left[Y(m_\sigma r_{ij})-\frac{\Lambda}{m_\sigma}Y(\Lambda r_{ij})\right], \tag{5}$$

where

$$Y(x)=\frac{e^{-x}}{x},\quad H(x)=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x), \tag{6}$$

a_c is the confinement strength, the $\vec{\lambda}$'s are the SU(3) color matrices, the $\vec{\sigma}$'s ($\vec{\tau}$'s) are the spin (isospin) Pauli matrices, S_{ij} is the usual tensor operator, m_q (m_π,m_σ) is the quark (pion, sigma) mass, α_s is the qq-gluon coupling constant, α_{ch} is the qq-meson coupling constant, and Λ is a cutoff parameter.

The present model does not contain any massive vector meson exchange (ρ or ω). These are known to be very important in one-boson-exchange models. The inclusion of additional vector-meson-exchange potentials between quarks could lead to double counting. This problem has been studied by Yazaki [11], concluding that the inclusion of vector meson exchanges between quarks indeed leads to problems with double counting whereas there is no problem with exchanging scalar and pseudoscalar mesons between quarks. In fact, in conventional one-boson-exchange models the vector mesons provide the short-range repulsion of the NN interaction. In the present quark model, the OGE and/or OPE combined with quark antisymmetrization takes over this task. Besides, the ρ meson is known to reduce the strength of the tensor force of the pion. In the present quark model, the quark-exchange terms of the OPE produce a similar effect.

TABLE I. Quark model parameters.

m_q (MeV)	313
b (fm)	0.518
α_s	0.485
a_c (MeV fm^{-2})	46.938
α_{ch}	0.027
m_σ (fm^{-1})	3.421
m_π (fm^{-1})	0.70
Λ (fm^{-1})	4.2

Usually, two-body calculations for the NN system have been carried out using the resonating group method (RGM). However, as our aim is to treat in the same framework two- and three-body systems, for the last case the RGM formalism is technically very much involved. Therefore, in order to derive a $B\Delta$ potential ($B=N,\Delta$) from the basic qq interaction defined above we use a Born-Oppenheimer approximation [6]. The validity of this approximation for bound and scattering states of the NN and $N\Delta$ systems will be discussed in Sec. IV A. Explicitly, the potential is calculated as follows:

$$V_{B\Delta(LST)\rightarrow B\Delta(L'S'T)}(R)=\xi_{LST}^{L'S'T}(R)-\xi_{LST}^{L'S'T}(\infty), \tag{7}$$

where

$$\xi_{LST}^{L'S'T}(R)=\frac{\left\langle\Psi_{B\Delta}^{L'S'T}(\vec{R})\left|\sum_{i<j=1}^{6}V_{qq}(\vec{r}_{ij})\right|\Psi_{B\Delta}^{LST}(\vec{R})\right\rangle}{\sqrt{\langle\Psi_{B\Delta}^{L'S'T}(\vec{R})|\Psi_{B\Delta}^{L'S'T}(\vec{R})\rangle}\sqrt{\langle\Psi_{B\Delta}^{LST}(\vec{R})|\Psi_{B\Delta}^{LST}(\vec{R})\rangle}}. \tag{8}$$

In the last expression the quark coordinates are integrated out keeping R fixed, the resulting interaction being a function of the B-Δ distance. The wave function $\Psi_{B\Delta}^{LST}(\vec{R})$ for the two-baryon system is discussed in detail in Refs. [6,7]. The parameters of the model are summarized in Table I.

If we consider two baryons in a relative S state interacting through a potential that contains a tensor force, then there is a coupling to the $B\Delta$ D wave so that the Lippmann-Schwinger equation of the system is of the form

$$t_{i;j_ii_i}^{l_is_il_i''s_i''}(p_i,p_i'';E)=V_{i;j_ii_i}^{l_is_il_i''s_i''}(p_i,p_i'')+\sum_{l_i's_i'}\int_0^\infty p_i'^2dp_i'V_{i;j_ii_i}^{l_is_il_i's_i'}(p_i,p_i')\times\frac{1}{E-p_i'^2/2\eta_i+i\epsilon}t_{i;j_ii_i}^{l_i's_i'l_i''s_i''}(p_i',p_i'';E), \tag{9}$$

where j_i and i_i are the angular momentum and isospin of the system, while l_is_i, $l_i's_i'$, and $l_i''s_i''$ are the initial, intermediate, and final orbital angular momenta and spin of the sys-

TABLE II. Coupled channels (l_2,s_2) that contribute to a given $N\Delta$ state with total angular momentum j_2 and isospin i_2.

j_2	i_2	(l_2,s_2)
1	1	(0,1),(2,1),(2,2)
1	2	(0,1),(2,1),(2,2)
2	1	(0,2),(2,1),(2,2)
2	2	(0,2),(2,1),(2,2)

tem, respectively. p_i and η_i are, respectively, the relative momentum and reduced mass of the two-body system.

We give in Table II the two-body channels that are coupled together for the four possible values of j_i and i_i that allow a nucleon and a delta to be in a relative S wave. As one can see from this table each state contains three coupled channels as a result of the coupling to the D waves produced by the tensor force. In the case of the $\Delta\Delta$ system the Pauli principle requires that $(-)^{l_i+s_i+i_i}=-1$ so that one can have at most three coupled channels for the eight possible states where two deltas can be in a relative S wave. We have given these $\Delta\Delta$ channels and states in Table I of Ref. [8].

As mentioned before, for the solution of the three-body system we will use only the component of the T matrix obtained from the solution of Eq. (9) with $l_i=l_i''=0$, so that for that purpose we define the S-wave truncated amplitude

$$t_{i;s_ii_i}(p_i,p_i'';E)\equiv t_{i;s_ii_i}^{0s_i0s_i}(p_i,p_i'';E). \tag{10}$$

III. THREE-BODY SYSTEM

We will assume that the nucleon is particle 1 and the two deltas are particles 2 and 3. If we restrict ourselves to the configurations where all three particles are in S-wave states, the Faddeev equations for the bound-state problem in the case of three particles with total spin S and total isospin I are

$$T_{i;SI}^{s_ii_i}(p_iq_i)=\sum_{j\neq i}\sum_{s_ji_j}h_{ij;SI}^{s_ii_is_ji_j}\frac{1}{2}\int_0^\infty q_j^2dq_j \times\int_{-1}^{1}d\cos\theta\, t_{i;s_ii_i}(p_i,p_i';E-q_i^2/2\nu_i) \times\frac{1}{E-p_j^2/2\eta_j-q_j^2/2\nu_j}T_{j;SI}^{s_ji_j}(p_jq_j), \tag{11}$$

where p_i and q_i are the usual Jacobi coordinates and η_i and ν_i the corresponding reduced masses:

$$\eta_i=\frac{m_jm_k}{m_j+m_k}, \tag{12}$$

$$\nu_i=\frac{m_i(m_j+m_k)}{m_i+m_j+m_k}, \tag{13}$$

with ijk an even permutation of 123. The momenta p_i' and p_j in Eq. (11) are given by

$$p_i'^2=q_j^2+\frac{\eta_i^2}{m_k^2}q_i^2+2\frac{\eta_i}{m_k}q_iq_j\cos\theta, \tag{14}$$

$$p_j^2=q_i^2+\frac{\eta_j^2}{m_k^2}q_j^2+2\frac{\eta_j}{m_k}q_iq_j\cos\theta. \tag{15}$$

$h_{ij;SI}^{s_ii_is_ji_j}$ are the spin-isospin coefficients,

$$h_{ij;SI}^{s_ii_is_ji_j}=(-)^{s_j+\sigma_j-S}\sqrt{(2s_i+1)(2s_j+1)}W(\sigma_j\sigma_kS\sigma_i;s_is_j) \times(-)^{i_j+\tau_j-I}\sqrt{(2i_i+1)(2i_j+1)}W(\tau_j\tau_kI\tau_i;i_ii_j), \tag{16}$$

where W is the Racah coefficient, and σ_i, s_i, and $S(\tau_i,\ i_i$, and $I)$ are the spins (isospins) of particle i, of the pair jk, and of the three-body system.

Since the variables p_i in Eqs. (9) and (11) run from 0 to ∞, it is convenient to make the transformation

$$x_i=\frac{p_i-b}{p_i+b}, \tag{17}$$

where the new variable x_i runs from -1 to 1 and b is a scale parameter. With this transformation Eq. (11) takes the form

$$T_{i;SI}^{s_ii_i}(x_iq_i)=\sum_{j\neq i}\sum_{s_ji_j}h_{ij;SI}^{s_ii_is_ji_j}\frac{1}{2}\int_0^\infty q_j^2dq_j \times\int_{-1}^{1}d\cos\theta\, t_{i;s_ii_i}(x_i,x_i';E-q_i^2/2\nu_i) \times\frac{1}{E-p_j^2/2\eta_j-q_j^2/2\nu_j}T_{j;SI}^{s_ji_j}(x_jq_j). \tag{18}$$

Since in the amplitude $t_{i;s_ii_i}(x_i,x_i';e)$ the variables x_i and x_i' run from -1 to 1, one can expand this amplitude in terms of Legendre polynomials as

$$t_{i;s_ii_i}(x_i,x_i';e)=\sum_{nm}P_n(x_i)\tau_{i;s_ii_i}^{nm}(e)P_m(x_i'), \tag{19}$$

where the expansion coefficients are given by

$$\tau_{i;s_ii_i}^{nm}(e)=\frac{2n+1}{2}\frac{2m+1}{2} \times\int_{-1}^{1}dx_i\int_{-1}^{1}dx_i'P_n(x_i)t_{i;s_ii_i}(x_i,x_i';e)P_m(x_i'). \tag{20}$$

Applying expansion (19) in Eq. (18) one gets

$$T_{i;SI}^{s_ii_i}(x_iq_i)=\sum_n T_{i;SI}^{ns_ii_i}(q_i)P_n(x_i), \tag{21}$$

where $T_{i;SI}^{ns_ii_i}(q_i)$ satisfies the one-dimensional integral equation

$$T_{i;SI}^{ns_ii_i}(q_i)=\sum_{j\neq i}\sum_{ms_ji_j}\int_0^\infty dq_jA_{ij;SI}^{ns_ii_ims_ji_j}(q_i,q_j;E)T_{j;SI}^{ms_ji_j}(q_j), \tag{22}$$

with

$$A_{ij;SI}^{ns_ii_ims_ji_j}(q_i,q_j;E)=h_{ij;SI}^{s_ii_is_ji_j}\sum_l \tau_{is_ii_i}^{nl}(E-q_i^2/2\nu_i)\frac{q_j^2}{2}\times\int_{-1}^{1}d\cos\theta\frac{P_l(x_i)P_m(x_j)}{E-p_j^2/2\eta_j-q_j^2/2\nu_j}. \tag{23}$$

The three amplitudes $T_{1;SI}^{ls_1i_1}(q_1)$, $T_{2;SI}^{ms_2i_2}(q_2)$, and $T_{3;SI}^{ns_3i_3}(q_3)$ in Eq. (22) are coupled together. The number of coupled equations can be reduced, however, since two of the particles are identical. The reduction procedure for the case where one has two identical fermions has been described before [13,14] and will not be repeated here. With the assumption that particle 1 is the nucleon and particles 2 and 3 are the deltas, only the amplitudes $T_{1;SI}^{ns_1i_1}(q_1)$ and $T_{2;SI}^{ms_2i_2}(q_2)$ are independent from each other and they satisfy the coupled integral equations

$$T_{1;SI}^{ls_1i_1}(q_1)=2\sum_{ns_2i_2}\int_0^\infty dq_3A_{13;SI}^{ls_1i_1ns_2i_2}(q_1,q_3;E)T_{2;SI}^{ns_2i_2}(q_3), \tag{24}$$

$$T_{2;SI}^{ms_2i_2}(q_2)=\sum_{ns_3i_3}(-)^{\text{iden}}\int_0^\infty dq_3A_{23;SI}^{ms_2i_2ns_3i_3}(q_2,q_3;E)T_{2;SI}^{ns_3i_3}(q_3)+\sum_{ls_1i_1}\int_0^\infty dq_1A_{31;SI}^{ms_2i_2ls_1i_1}(q_2,q_1;E)T_{1;SI}^{ls_1i_1}(q_1), \tag{25}$$

with the identical-particle factor

$$\text{iden}=1+\sigma_1+\sigma_3-s_2+\tau_1+\tau_3-i_2. \tag{26}$$

Substitution of Eq. (24) into Eq. (25) yields an equation with only the amplitude T_2:

$$T_{2;SI}^{ms_2i_2}(q_2)=\sum_{ns_3i_3}\int_0^\infty dq_3K_{23;SI}^{ms_2i_2ns_3i_3}(q_2,q_3;E)T_{2;SI}^{ns_3i_3}(q_3), \tag{27}$$

where

$$K_{23;SI}^{ms_2i_2ns_3i_3}(q_2,q_3;E)=(-)^{\text{iden}}A_{23;SI}^{ms_2i_2ns_3i_3}(q_2,q_3;E)+2\sum_{ls_1i_1}\int_0^\infty dq_1A_{31;SI}^{ms_2i_2ls_1i_1}(q_2,q_1;E)\times A_{13;SI}^{ls_1i_1ns_3i_3}(q_1,q_3;E). \tag{28}$$

In order to find the solutions of Eq. (27) we replace the integral by a sum, applying a numerical integration quadrature [15]. In this way Eq. (27) becomes a set of homogeneous linear equations. This set of linear equations has solutions only if the determinant of the matrix of the coefficients (the Fredholm determinant) vanishes for certain energies. Thus, the procedure to find the bound states of the system consists simply in searching for the zeros of the Fredholm determinant as a function of energy. We give in Table III the 16 $N\Delta\Delta$ states characterized by total spin and isospin (S,I) that are possible as well as the two-body $N\Delta$ and $\Delta\Delta$ channels that contribute to each state.

TABLE III. Two-body $N\Delta$ channels (s_2,i_2) and two-body $\Delta\Delta$ channels (s_1,i_1) that contribute to a given $N\Delta\Delta$ state with total spin S and isospin I.

S	I	(s_2,i_2)	(s_1,i_1)
1/2	1/2	(1,1),(1,2),(2,1),(2,2)	(1,0),(0,1)
1/2	3/2	(1,1),(1,2),(2,1),(2,2)	(0,1),(1,2)
1/2	5/2	(1,1),(1,2),(2,1),(2,2)	(0,3),(1,2)
1/2	7/2	(1,2),(2,2)	(0,3)
3/2	1/2	(1,1),(1,2),(2,1),(2,2)	(1,0),(2,1)
3/2	3/2	(1,1),(1,2),(2,1),(2,2)	(1,2),(2,1)
3/2	5/2	(1,1),(1,2),(2,1),(2,2)	(1,2),(2,3)
3/2	7/2	(1,2),(2,2)	(2,3)
5/2	1/2	(1,1),(1,2),(2,1),(2,2)	(2,1),(3,0)
5/2	3/2	(1,1),(1,2),(2,1),(2,2)	(2,1),(3,2)
5/2	5/2	(1,1),(1,2),(2,1),(2,2)	(2,3),(3,2)
5/2	7/2	(1,2),(2,2)	(2,3)
7/2	1/2	(2,1),(2,2)	(3,0)
7/2	3/2	(2,1),(2,2)	(3,2)
7/2	5/2	(2,1),(2,2)	(3,2)
7/2	7/2	(2,2)	

Our method of solution of the three-body problem is based in the separable expansion (19) of the two-body T matrices. We tested the convergence of this expansion by considering the three-nucleon bound-state problem with the Reid soft-core potential in the truncated T-matrix approximation (two-channel calculation) [12]. We show in Table IV the triton binding energy obtained when the number of terms of the expansion changes. As one can see from this table convergence is reached with $N=10$ although a very reasonable result is obtained already with $N=5$. In the calculations of

TABLE IV. Binding energy of the three-nucleon system with the Reid soft-core potential (two-channel calculation) obtained using different number of Legendre polynomials in the separable expansion (19).

N	B (MeV)
1	unbound
2	3.39
3	4.94
4	4.98
5	6.59
6	6.73
7	6.68
8	6.67
9	6.56
10	6.58
11	6.58
12	6.58
13	6.58
14	6.58
15	6.58

TABLE V. Binding energies B of the $N\Delta$ states with total angular momentum j_2 and isospin i_2.

j_2	i_2	B (MeV)
1	1	unbound
1	2	unbound
2	1	0.0
2	2	unbound

this paper we used $N=10$. Notice that the convergence of the separable expansion (19) is much slower than that obtained with the Ernst-Shakin-Thaler (EST) method [16,17].

IV. RESULTS

We discuss now our results considering separately the two-body and three-body systems.

A. Two-body system

We give in Table V the results for the binding energies of the $N\Delta$ system. Out of the four possible $N\Delta$ states of Table II only one, the $(j_2,i_2)=(2,1)$, has a bound state which lies exactly at the $N\Delta$ threshold. The states $(j_2,i_2)=(1,1)$ and $(j_2,i_2)=(2,2)$ are unbound because they present quark Pauli blocking [6] and therefore they have a strong repulsive barrier at short distances in the S-wave central interaction. These two states play an important role in the three-body spectrum. The state $(j_2,i_2)=(2,1)$ can also exist in the NN system and there it corresponds to the $^1D_2$ partial wave which has a resonance at an invariant mass of 2.17 GeV [18–21]. This means that the $N\Delta$ bound state may decay into two nucleons and appear in the NN system as a resonance. Since the $N\Delta$ bound state of Table V lies exactly at the $N\Delta$ threshold, its invariant mass is precisely 2.17 GeV. Thus, our model predicts the NN $^1D_2$ resonance as being a $N\Delta$ bound state. As far as we know, ours is the first calculation based on the quark model where this result has been obtained.

It is important to have in mind that this result has been obtained under the use of some approximations which need to be examined closely. First, there is the question of the validity of the Born-Oppenheimer approximation (7) and (8); second, we have neglected the coupling to the NN state in the $^1D_2$ channel; and finally, we have not taken into account the effects of the unstable nature of the Δ. We examine these points next.

We have investigated the validity of the Born-Oppenheimer approximation (7) and (8) in the case of the bound-state problem by considering the case of the deuteron. Our exact model of the NN interaction [3,4] does not use the Born-Oppenheimer approximation and of course it gives a deuteron binding energy of 2.225 MeV. If we now apply the Born-Oppenheimer approximation, we obtain instead a deuteron binding energy of 3.13 MeV. Thus, there is an error in the binding energy of less than 1 MeV. Same accuracy is obtained in the case of the scattering states of the NN system as is shown in Fig. 1. The validity of the Born-Oppenheimer approximation for the $N\Delta$ case has been also tested in the past for the scattering states. In Ref. [6] the phase shifts of the $N\Delta$ system are given and in Ref. [22] other observables are calculated, justifying the validity of such an approximation. Therefore, we expect that the binding energy of the $N\Delta$ bound state in the (2,1) channel (which as seen in Table II is very similar to the deuteron) will not change appreciably as a consequence of the Born-Oppenheimer approximation.

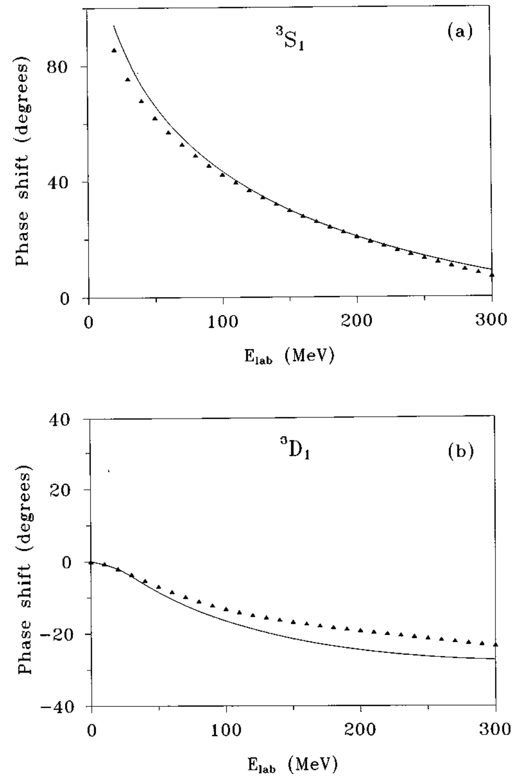

FIG. 1. $^3S_1$ and $^3D_1$ NN phase shifts using the Born-Oppenheimer approximation as a function of the laboratory energy. Experimental data are taken form Ref. [21].

We have neglected the coupling between the $N\Delta$ (2,1) state and the NN $^1D_2$ state. We do not expect that there will be a large effect in the binding energy of the $N\Delta$ state from this approximation due to the fact that the NN $^1D_2$ state is a D wave while the $N\Delta$ (2,1) state is predominantly an S wave. As is well known in the systems with coupled channels, the coupling between an $l=0$ and an $l=2$ state will have a big effect on the $l=2$ component while the $l=0$ component will not be much affected. In other words, the NN channel will be strongly modified (so much that a resonant structure will arise) while the $N\Delta$ channel will stay more or less the same. Of course, this coupling will generate a width in the $N\Delta$ bound state. The main contribution to the width of the $N\Delta$ state, however, will come from the unstable nature of the Δ (the decay $\Delta\rightarrow\pi N$) which we discuss next.

We have not taken into account the unstable nature of the Δ. In a previous investigation of $N\Delta$ bound states [23] we have shown by several numerical calculations of the energy eigenvalue that the effect of the unstable nature of the Δ (the decay $\Delta\rightarrow\pi N$) is to generate an imaginary part in the energy eigenvalue while having a very little effect in the real part of the eigenvalue (see Tables 1 and 2 of Ref. [23]).

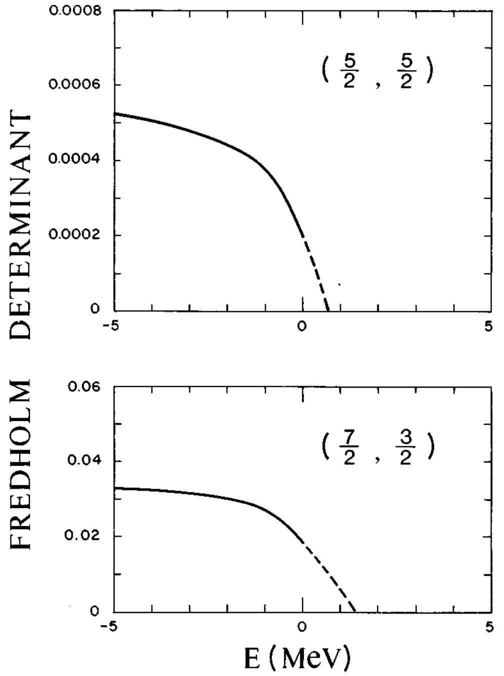

FIG. 2. Fredholm determinants of the $N\Delta\Delta$ system in the states $(S,I)=(\frac{5}{2},\frac{5}{2})$ and $(\frac{7}{2},\frac{3}{2})$ as a function of energy.

Thus, the effect of the decay $\Delta\rightarrow\pi N$ is mainly to broaden the $N\Delta$ bound state without changing its position noticeably.

Thus, we believe that our result for the $N\Delta(2,1)$ bound state rests in very solid ground. We should finally mention that the amplitudes of the reactions $\pi d\rightarrow\pi d$ [24] and $\pi d\rightarrow NN$ [25] exhibit also a resonant behavior in the partial waves corresponding to the $N\Delta$ (2,1) channel. The position of the resonance can be inferred from the amplitude of the elastic channel and again it corresponds to an invariant mass of 2.17 GeV [24]. Thus, since the three reactions $NN\rightarrow NN$, $\pi d\rightarrow\pi d$, and $\pi d\rightarrow NN$ couple very strongly to the $N\Delta$ (2,1) channel, they all feel the effect of the $N\Delta$ bound state.

B. Three-body system

We found in the case of the $N\Delta\Delta$ system that all of the 16 states of Table III are unbound. It is the structure of the interaction of the two-body system that is the one which largely determines the three-body spectrum. As mentioned before the $N\Delta$ states $(j_2,i_2)=(1,1)$ and $(j_2,i_2)=(2,2)$ present quark Pauli blocking. None of the three-body channels is absent of the repulsion generated by the quark Pauli blocking in the two-body states, and, as in the $\Delta\Delta\Delta$ case, the presence of this repulsive barrier completely destroys the bound state or allows just barely ones. There are two $N\Delta\Delta$ states that are almost bound. They are the $(S,I)=(\frac{5}{2},\frac{5}{2})$ and $(\frac{7}{2},\frac{3}{2})$ states. We show in Fig. 2 the Fredholm determinant of these two states as a function of energy. If the Fredholm determinant would pass through zero at a given negative energy, then there would be a bound state at that energy. As can be seen from Fig. 1, in the case of these two states the Fredholm determinant does not become zero for any negative energy, but if one extrapolates to positive energies, one can get an idea of how close the state is to being bound. From the results of Fig. 1 we get that the state $(S,I)=(\frac{5}{2},\frac{5}{2})$ is unbound by about 0.6 MeV and the state $(S,I)=(\frac{7}{2},\frac{3}{2})$ is unbound by about 1.4 MeV. Thus, the states $(S,I)=(\frac{5}{2},\frac{5}{2})$ and $(\frac{7}{2},\frac{3}{2})$ may be observable as tribaryon resonances which decay into three nucleons and two pions with masses close to the $N\Delta\Delta$ threshold.

There is a marked contrast between the spectra of the $N\Delta$ and $N\Delta\Delta$ systems studied here and those of the $\Delta\Delta$ and $\Delta\Delta\Delta$ system that were calculated in [8]. The $\Delta\Delta$ system has six bound states [8] while the $N\Delta$ system has only one. Similarly, the $\Delta\Delta\Delta$ system has seven bound states [8] while the $N\Delta\Delta$ system has none. However, the $N\Delta$ and $N\Delta\Delta$ systems are far more interesting than the $\Delta\Delta$ and $\Delta\Delta\Delta$ systems from the experimental point of view. For example, the six $\Delta\Delta$ bound states obtained in [8] correspond to dibaryon resonances of high mass ($\sim$2.4 GeV) which are hard to observe experimentally while the $N\Delta$ bound state of Table IV with a mass of $\sim$2.17 GeV corresponds to a dibaryon resonance that has already been observed. Similarly, the seven $\Delta\Delta\Delta$ bound states obtained in [8] correspond to tribaryon resonances of masses between 3.6 and 3.7 GeV while the two almost-bound states of the $N\Delta\Delta$ system correspond to tribaryon resonances with masses of $\sim$3.4 GeV so that the last ones should be much easier to observe experimentally.

V. CONCLUSIONS

We have examined the $N\Delta$ and $N\Delta\Delta$ bound-state problems using baryon-baryon interactions derived from the chiral quark cluster model for the case when all the two-body subsystems are in relative S-wave states. There is only one $N\Delta$ bound state which has the same quantum numbers and mass as the NN $^1D_2$ resonance. This suggests that the NN $^1D_2$ resonance is a true dibaryon. The $N\Delta\Delta$ bound-state problem comes largely determined by quark Pauli blocking effects in the two-body subsystems. The $N\Delta\Delta$ system has two almost-bound states at $(S,I)=(\frac{5}{2},\frac{5}{2})$ and $(\frac{7}{2},\frac{3}{2})$ which correspond to tribaryon resonances with masses close to the $N\Delta\Delta$ threshold.

Other systems that remain to be studied within our project of two- and three-body bound states of nucleons and deltas are the NN and $NN\Delta$ systems. These last two systems will be discussed in a future work [9]. Of particular interest is the $NN\Delta$ system which has the possibility of obtaining tribaryon resonances with even lower masses.

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