

## Strong $\rho NN$ coupling derived from QCD

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We study the two point correlation function of two nucleon currents sandwiched between the vacuum and the rho meson state. The light cone QCD sum rules are derived for the  $\rho NN$  vector and tensor couplings simultaneously. The contribution from the excited states and the continuum is subtracted cleanly through the double Borel transform with respect to the two external momenta,  $p_1^2$ ,  $p_2^2=(p-q)^2$ . Our results are  $g_\rho=2.5 \pm 0.2$ ,  $\kappa_\rho=(8.0 \pm 2.0)$ , in good agreement with the values used in the nuclear forces.

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### I. INTRODUCTION

Quantum chromodynamics (QCD) is asymptotically free and its high energy behavior has been tested to two-loop accuracy. On the other hand, the low-energy behavior has become a very active research field in the past years. Various hadronic resonances act as suitable labs for exploring the nonperturbative QCD. Among which, the inner structure of nucleon and mesons and their interactions is of central importance in nuclear and particle physics.

Internationally there are a number of experimental collaborations, like TJNAL (former CEBAF), COSY, ELSA (Bonn), MAMI (Mainz), and Spring8 (Japan), focusing on the nonperturbative QCD dynamics. Especially the Mainz research project MAMI (Mainzer Mikrotron) with its planned extension from a 855 MeV to 1.5 GeV electron c.w. accelerator and the Japan Hadron Facility (JHF) at Spring8 will extensively study the photo and electroproduction of vector mesons off nucleons.

Moreover, the strong  $\rho NN$  couplings are, like  $\pi NN$  and  $\pi N\Delta$  couplings, the basic inputs for the description of nuclear forces in terms of meson exchange between nucleons. So far the linkage between the underlying theory QCD and the phenomenological  $\rho NN$  couplings has not been made. Especially the commonly adopted tensor-vector ratio  $\kappa_\rho=6.8$  is much larger than the vector meson dominance model (VDM) result  $\kappa_\rho=3.7$ . One wonders whether it is feasible to calculate the  $\rho NN$  couplings directly with the fundamental theory QCD.

Although it is widely accepted that QCD is the underlying theory of the strong interaction, the self-interaction of the gluons causes the infrared behavior and the vacuum of QCD highly nontrivial. In the typical hadronic scale QCD is nonperturbative which makes the first principle calculation of these couplings unrealistic except the lattice QCD approach, which is very computer time consuming. Therefore, a quantitative calculation of the  $\rho NN$  couplings with a tractable and reliable theoretical approach proves valuable.

The method of QCD sum rules (QSR), as proposed originally by Shifman, Vainshtein, and Zakharov [1] and adopted, or extended, by many others [2,3,4], are very useful in ex-

tracting the low-lying hadron masses and couplings. In the QCD sum rule approach the nonperturbative QCD effects are taken into account through various condensates in the nontrivial QCD vacuum. A recent review of QSR is given by Shifman [5]. In this work we shall use the light cone QCD sum rules (LCQSR) to calculate the  $\rho NN$  couplings.

The LCQSR is quite different from the conventional QSR, which is based on the short-distance operator product expansion. The LCQSR is based on the OPE on the light cone, which is the expansion over the twists of the operators. The main contribution comes from the lowest twist operator. Matrix elements of nonlocal operators sandwiched between a hadronic state and the vacuum defines the hadron wave functions. When the LCQSR is used to calculate the coupling constant, the double Borel transformation is always invoked so that the excited states and the continuum contribution can be treated quite nicely. Moreover, the final sum rule depends only on the value of the hadron wave function at a specific point, which is much better known than the whole wave function [6]. In the present case our sum rules involve with the rho wave function (RWF)  $\varphi_\rho(u_0=\frac{1}{2})$ . Reviews of the method of LCQSR can be found in [7,8].

The LCQSR has been widely used to treat the couplings of pions with hadrons. Recently the couplings of pions with heavy mesons in full QCD [6], in the limit of  $m_Q \rightarrow \infty$  [9],  $1/m_Q$  corrections and mixing effects [10], the couplings of pions with heavy baryons [11], the  $\pi NN$  and  $\pi NN^*(1535)$  couplings [12], the  $\rho \rightarrow \pi\pi$  and  $K^* \rightarrow K\pi$  decays [13], and various semileptonic decays of heavy mesons [14] have been discussed.

The QCD sum rules were used to analyze the exclusive radiative  $B$ -decays with the help of the light-cone vector meson wave function in [15]. With the same formalism the off-shell  $g_{B^*B\rho}$  and  $g_{D^*D\rho}$  couplings in [16] and the  $\rho$  decay widths of excited heavy mesons [17] were calculated.

Our paper is organized as follows. Section I is an introduction. We introduce the two point function for the  $\rho NN$  vertex and saturate it with nucleon intermediate states in Sec. II. The definitions of the RWFs and the formalism of LCQSR are presented in Sec. III. In Sec. IV we present the LCQSR for the  $\rho NN$  coupling. In Sec. V we present some discussions of these RWFs and their values at the point  $u_0 = \frac{1}{2}$ . We make the numerical analysis and a short discussion in Sec. VI.

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## II. TWO POINT CORRELATION FUNCTION FOR THE $\rho NN$ COUPLING

Many authors have studied the strong  $\rho NN$  couplings. It was pointed out that the inclusion of an effective  $\rho$ -pole contribution leads to a large value for the tensor-vector coupling ratio  $\kappa_\rho = 6.6 \pm 1$  [18,19], in the dispersion-theoretical analysis of the nucleon electromagnetic form factors. The above value is consistent with other determination [20,21]. Brown and Machleidt have discussed the evidence for a strong  $\rho NN$  coupling from the measurement of  $\epsilon_1$  parameter in  $NN$  scattering [22]. Brown, Rho, and Weise suggested that  $\kappa_\rho = 2\kappa_v$  is consistent with a quark core radius of 0.5 fm for the nucleon and an equal factorization of the baryon charge between the quark and meson cloud in the two-phase Skyrme model [23]. Recently Wen and Hwang used the external field method in QCD sum rules to study the  $\rho NN$  couplings [24]. They obtained  $\kappa_\rho = 3.6$ , in agreement with VDM result  $\kappa_v = 3.7$  and  $\kappa_s = -0.12$ . In their work the authors introduced the vector-like  $\rho$ -quark interaction Lagrangian by hand and treated the vector meson quark coupling as free parameter. In other words, the  $\rho NN$  vector and tensor couplings cannot be determined simultaneously.

We shall calculate the  $\rho NN$  vector and tensor couplings simultaneously using vector meson light cone wave functions up to twist four, which will result in a reliable extraction of  $\kappa_\rho$ .

We start with the two point function

$$\Pi(p_1, p_2, q) = \int d^4x e^{ipx} \langle 0 | T \eta_n(x) \bar{\eta}_p(0) | \rho^+(q) \rangle \quad (1)$$

with  $p_1 = p$ ,  $p_2 = p - q$  and the Ioffe nucleon interpolating field [3]

$$\eta_p(x) = \epsilon_{abc} [u^a(x) C \gamma_\mu u^b(x)] \gamma_5 \gamma^\mu d^c(x), \quad (2)$$

$$\bar{\eta}_p(y) = \epsilon_{abc} [\bar{u}^b(y) \gamma_\nu C \bar{u}^{aT}(y)] \bar{d}^c(y) \gamma^\nu \gamma^5, \quad (3)$$

where  $a, b, c$  are the color indices and  $C = i\gamma_2\gamma_0$  is the charge conjugation matrix. For the neutron interpolating field,  $u \leftrightarrow d$ .

The rho nucleon couplings are defined by the  $\rho NN$  interaction:

$$\mathcal{L}_{\rho NN} = g_\rho \rho^\mu \bar{N} \left[ \gamma_\mu + \kappa_\rho \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} \right] N. \quad (4)$$

$\rho_\mu$  is an isovector in Eq. (4).  $g_\rho$  is the rho-nucleon vector coupling constant and  $\kappa_\rho$  is the tensor-vector ratio.

At the phenomenological level Eq. (1) can be expressed as

$$\begin{aligned} \Pi(p_1, p_2, q) &= i\lambda_N^2 g_\rho e^\mu(\lambda) \\ &\times \frac{(\hat{p}_1 + m_N) \left( \gamma_\mu + \kappa_\rho \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} \right) (\hat{p}_2 + m_N)}{(p_1^2 - m_N^2)(p_2^2 - m_N^2)} \\ &+ \dots, \end{aligned} \quad (5)$$

where  $e^\mu(\lambda)$  is the rho meson polarization vector. The ellipse denotes the continuum and the single pole excited states to nucleon transition contribution.  $\lambda_N$  is the overlapping amplitude of the interpolating current  $\eta_N(x)$  with the nucleon state

$$\langle 0 | \eta(0) | N(p) \rangle = \lambda_N u_N(p). \quad (6)$$

Expanding (5) with the independent variables  $P = (p_1 + p_2)/2, q$  and decomposing it into the chiral odd and chiral even part, we arrive at

$$\Pi = \Pi_o + \Pi_e, \quad (7)$$

where

$$\begin{aligned} \Pi_o(p_1, p_2, q) &= \frac{i\lambda_N^2 g_\rho}{(p_1^2 - m_N^2)(p_2^2 - m_N^2)} \\ &\times \left\{ (e \cdot P) \hat{P} + \frac{1 + \kappa_\rho}{2} q^2 \hat{e} - \frac{1 + 2\kappa_\rho}{4} (e \cdot q) \hat{q} \right. \\ &\left. + i(1 + \kappa_\rho) \epsilon_{\mu\alpha\beta\sigma} e^\mu P^\alpha q^\beta \gamma^\sigma \gamma_5 \right\} + \dots, \end{aligned} \quad (8)$$

and

$$\begin{aligned} \Pi_e(p_1, p_2, q) &= \frac{i\lambda_N^2 g_\rho}{(p_1^2 - m_N^2)(p_2^2 - m_N^2)} \left\{ \left( 2m_N + \frac{\kappa_\rho}{2m_N} q^2 \right) (e \cdot P) \right. \\ &\left. - \frac{1 + \kappa_\rho}{2} m_N (\hat{e} \hat{q} - \hat{q} \hat{e}) - \frac{\kappa_\rho}{2m_N} (e \cdot P) (\hat{q} \hat{p} - \hat{p} \hat{q}) \right\} \\ &+ \dots, \end{aligned} \quad (9)$$

with  $q^2 = m_\rho^2$ . We have not kept the single pole terms in Eqs. (8) and (9) since they are always eliminated after making double Borel transformation in deriving final LCQSRs.

It was well known that the sum rules derived from the chiral odd tensor structure yield better results than those from the chiral even ones in the QSR analysis of the nucleon mass and magnetic moment [3,25]. We shall consider chiral odd tensor structures only below.

## III. THE FORMALISM OF LCQSR AND RHO WAVE FUNCTIONS

Neglecting the four particle component of the rho wave function, the expression for  $\Pi(p_1^2, p_2^2, q^2)$  with the tensor structure at the quark level reads

$$\begin{aligned} &\int e^{ipx} dx \langle 0 | T \eta_n(x) \bar{\eta}_p(0) | \rho^+(q) \rangle \\ &= -4 \epsilon^{abc} \epsilon^{a'b'c'} \gamma_5 \gamma_\mu i S_d^{aa'}(x) \\ &\quad \times \gamma_\nu C \langle 0 | d^c(x) \bar{u}^{c'}(0) | \rho^+(q) \rangle^T \\ &\quad \times C \gamma_\mu i S_u^{Tbb'}(x) \gamma_\nu \gamma_5, \end{aligned} \quad (10)$$

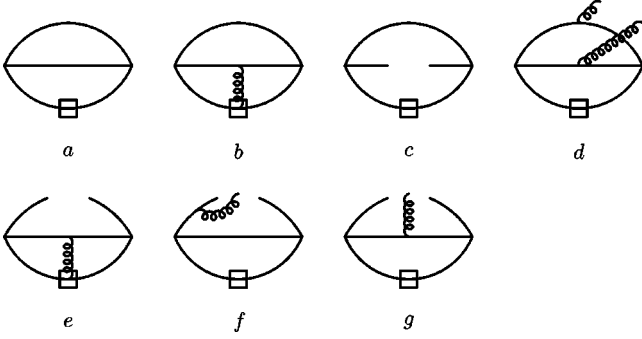


FIG. 1. The relevant Feynman diagrams for the derivation of the LCQSR for  $\rho NN$  coupling. The squares denote the rho wave function (RWF). The broken solid line, broken curly line, and a broken solid line with a curly line attached in the middle stands for the quark condensate, gluon condensate, and quark gluon mixed condensate, respectively.

where  $iS(x)$  is the full light quark propagator with both perturbative term and contribution from vacuum fields

$$\begin{aligned} iS(x) &= \langle 0 | T[q(x), \bar{q}(0)] | 0 \rangle \\ &= i \frac{\hat{x}}{2\pi^2 x^4} - \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2}{192} \langle \bar{q}g_s \sigma \cdot Gq \rangle \\ &\quad + \frac{g_s}{16\pi^2 x^2} \int_0^1 du \{ 2(1-2u)x_\mu \gamma_\nu \\ &\quad + i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\rho x^\sigma \} G^{\mu\nu}(ux) + \dots, \end{aligned} \quad (11)$$

where we have introduced  $\hat{x} \equiv x_\mu \gamma^\mu$ . In our calculation we take the tiny current quark mass to be zero.

The relevant Feynman diagrams are presented in Fig. 1. The squares denote the rho wave function (RWF). The broken solid line, broken curly line, and a broken solid line with a curly line attached in the middle stands for the quark condensate, gluon condensate, and quark gluon mixed condensate, respectively.

By the operator expansion on the light-cone the matrix element of the nonlocal operators between the vacuum and rho meson defines the two particle rho wave function. Up to twist four the Dirac components of this wave function can be written as follows [15,26,27]. For the longitudinally polarized rho mesons,

$$\begin{aligned} \langle 0 | \bar{u}(0) \gamma_\mu d(x) | \rho^-(q, \lambda) \rangle &= f_\rho m_\rho \left\{ q_\mu \frac{e^{(\lambda) \cdot x}}{q \cdot x} \int_0^1 du e^{-iuq \cdot x} \right. \\ &\quad \times \left[ \phi_\parallel(u, \mu^2) + \frac{m_\rho^2 x^2}{4} A(u, \mu) \right] \\ &\quad + \left( e_\mu^{(\lambda)} - q_\mu \frac{e^{(\lambda) \cdot x}}{qx} \right) \int_0^1 du e^{-iuq \cdot x} g^v(u, \mu^2) \\ &\quad \left. - \frac{1}{2} x_\mu \frac{e^{(\lambda) \cdot x}}{(q \cdot x)^2} m_\rho^2 \int_0^1 du e^{-iuq \cdot x} C(u, \mu^2) \right\}, \end{aligned} \quad (12)$$

and

$$\begin{aligned} \langle 0 | \bar{u}(0) \gamma_\mu \gamma_5 d(x) | \rho^-(q, \lambda) \rangle &= -\frac{1}{4} f_\rho m_\rho \epsilon_\mu^{\nu\alpha\beta} e_{\perp\nu}^{(\lambda)} q_\alpha x_\beta \int_0^1 du e^{-iuq \cdot x} g^a(u, \mu^2), \end{aligned} \quad (13)$$

where

$$C(u) = g_3(u) + \phi_\parallel(u) - 2g_\perp^{(v)}(u). \quad (14)$$

The link operators  $P \exp[ig \int_0^1 d\alpha x^\mu A_\mu(\alpha x)]$  are understood in between the quark fields. The distribution amplitudes describe the probability amplitudes to find the  $\rho$  in a state with quark and antiquark carrying momentum fractions  $u$  (quark) and  $1-u$  (antiquark), respectively, and have a small transverse separation of order  $1/\mu$  [27].

The vector decay constant  $f_\rho$  is defined as

$$\langle 0 | \bar{u}(0) \gamma_\mu d(0) | \rho^-(q, \lambda) \rangle = f_\rho m_\rho e_\mu^{(\lambda)}. \quad (15)$$

All distributions  $\phi = \{\phi_\parallel, g^v, g^a, A, C\}$  are normalized as

$$\int_0^1 du \phi(u) = 1. \quad (16)$$

The twist-three three-particle quark-antiquark-gluon distributions are [27]

$$\begin{aligned} \langle 0 | \bar{u}(0) \gamma_\alpha g_s G_{\mu\nu}(ux) d(x) | \rho^-(q, \lambda) \rangle &= iq_\alpha [q_\mu e_{\perp\nu}^{(\lambda)} - q_\nu e_{\perp\mu}^{(\lambda)}] f_{3\rho}^V \mathcal{V}(u, qz) + \dots, \end{aligned} \quad (17)$$

$$\begin{aligned} \langle 0 | \bar{u}(0) \gamma_\alpha \gamma_5 g_s \tilde{G}_{\mu\nu}(ux) d(x) | \rho^-(q, \lambda) \rangle &= q_\alpha [q_\nu e_{\perp\mu}^{(\lambda)} - q_\mu e_{\perp\nu}^{(\lambda)}] f_{3\rho}^A \mathcal{A}(u, qx) + \dots, \end{aligned} \quad (18)$$

where the operator  $\tilde{G}_{\alpha\beta}$  is the dual of  $G_{\alpha\beta}$ :  $\tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\delta\rho} G^{\delta\rho}$  and the ellipses denote higher twist contributions.

The following shorthand notation for the integrals defining three-particle distribution amplitudes is used:

$$\mathcal{F}(u, qx) \equiv \int \mathcal{D}\alpha e^{-iq \cdot x (\alpha_3 + u\alpha_g)} \mathcal{F}(\alpha_1, \alpha_g, \alpha_3). \quad (19)$$

Here  $\mathcal{F} = \{\mathcal{V}, \mathcal{A}\}$  refers to the vector and axial-vector distributions,  $\alpha$  is the set of three momentum fractions:  $\alpha_3$  ( $d$  quark),  $\alpha_1$  ( $u$  quark), and  $\alpha_g$  (gluon), and the integration measure is defined as

$$\int \mathcal{D}\alpha \equiv \int_0^1 d\alpha_1 \int_0^1 d\alpha_3 \int_0^1 d\alpha_g \delta\left(1 - \sum \alpha_i\right). \quad (20)$$

The normalization constants  $f_{3\rho}^V, f_{3\rho}^A, f_{3\rho}^T$  are defined in such a way that

$$\int \mathcal{D}\underline{\alpha}(\alpha_3 - \alpha_1) \mathcal{V}(\alpha_1, \alpha_g, \alpha_3) = 1, \quad (21)$$

$$\int \mathcal{D}\underline{\alpha} \mathcal{A}(\alpha_1, \alpha_g, \alpha_3) = 1. \quad (22)$$

The function  $\mathcal{A}$  is symmetric and the functions  $\mathcal{V}$  is antisymmetric under the interchange  $\alpha_1 \leftrightarrow \alpha_3$  in the SU(3) limit [28,27], which follows from the G-parity transformation property of the corresponding matrix elements.

In the infinite momentum frame the RWFs  $\phi_{\parallel}$  are associated with the leading twist two operator,  $A(u)$  correspond to twist four operators, and  $g^a(u), g^v(u)$  to twist three ones.

The three particle RWFs  $\mathcal{V}, \mathcal{A}$  are of twist three. Details can be found in [27].

#### IV. THE LCQSR FOR THE $\rho NN$ COUPLING

Expressing (10) with the longitudinal rho wave functions (LRWFs), we can obtain the expressions for the correlator in the coordinate space. Up to dimension six the gluon condensate is the only relevant condensate contributing to the chiral odd tensor structure. Yet the gluon condensate always appears with a large suppression factor, which arises from the two-loop internal momentum integration in the diagram (d) in Fig. 1. Its contribution is quite small, which is confirmed by our detailed calculation. For example, after double Borel transformation, diagram (d) is suppressed by a factor  $\langle g_s^2 G^2 \rangle / 24M_B^4$ , where  $\langle g_s^2 G^2 \rangle = 0.48 \text{ GeV}^4$  and  $M_B^2 \sim 1.5 \text{ GeV}^2$ .

Diagram (a) involves with two-particle LRWFs. After tedious but straightforward calculation, we get

$$\begin{aligned} \Pi_o^2(p_1, p_2, q) = & \int d^4x \int_0^1 du e^{i(p-uq)x} f_{\rho} m_{\rho} \left[ -\frac{g^a(u)}{\pi^4 x^6} \epsilon_{\alpha\beta\mu\sigma} e^{\beta} q^{\mu} x^{\sigma} \gamma^{\alpha} \gamma_5 - \frac{4}{\pi^4 x^6} \left( [\phi_{\parallel}(u) - g^v(u)] \hat{q} \frac{(e \cdot x)}{(q \cdot x)} + \hat{e} g^v(u) \right) \right. \\ & \left. + \frac{2}{\pi^4 x^6} m_{\rho}^2 g_3(u) \frac{(e \cdot x)}{(q \cdot x)^2} \hat{x} - \frac{1}{\pi^4 x^4} m_{\rho}^2 A(u) \hat{q} \frac{(e \cdot x)}{(q \cdot x)} - \frac{g^a(u)}{768 \pi^4 x^2} \langle g_s^2 G^2 \rangle \epsilon_{\alpha\beta\mu\sigma} e^{\beta} q^{\mu} x^{\sigma} \gamma^{\alpha} \gamma_5 + \dots \right]. \quad (23) \end{aligned}$$

The RWFs can be found in the previous section. Diagram (b) is associated with vacuum gluon fields. But its contribution vanishes due to isospin symmetry. Our explicit calculation confirms it.

We frequently use integration by parts to absorb the factors  $1/(q \cdot x)$  and  $1/(q \cdot x)^2$ , which leads to the integration of RWFs. For example,

$$\int_0^1 \frac{e^{-iuq \cdot x}}{q \cdot x} \psi(u) du = i \int e^{-iuq \cdot x} \Psi(u) du + \Psi(u) e^{-iuq \cdot x} \Big|_0^1, \quad (24)$$

where the function  $\Psi(u)$  is defined as

$$\Psi(u) = + \int_0^u \psi(u) du. \quad (25)$$

Note the second term in Eq. (24) vanishes after double Borel transformation or due to  $\phi_{\rho}(u_0) = \Psi(u_0) = 0$  at end points  $u_0 = 0, 1$ .

We first finish Fourier transformation. The formulas are

$$\int \frac{e^{ipx}}{(x^2)^n} d^D x \rightarrow i(-1)^{n+1} \frac{2^{D-2n} \pi^{D/2}}{(-p^2)^{D/2-n}} \frac{\Gamma(D/2-n)}{\Gamma(n)}, \quad (26)$$

$$\int \frac{\hat{x} e^{ipx}}{(x^2)^n} d^D x \rightarrow (-1)^{n+1} \frac{2^{D-2n+1} \pi^{D/2}}{(-p^2)^{D/2+1-n}} \frac{\Gamma(D/2+1-n)}{\Gamma(n)} \hat{p}, \quad (27)$$

$$\begin{aligned} & \int \frac{x_{\mu} x_{\nu} e^{ipx}}{(x^2)^n} d^D x \rightarrow i(-1)^n 2^{D-2n+1} \pi^{D/2} \\ & \times \left\{ \frac{g_{\mu\nu}}{(-p^2)^{D/2+1-n}} \frac{\Gamma(D/2+1-n)}{\Gamma(n)} \right. \\ & \left. + \frac{2p_{\mu} p_{\nu}}{(-p^2)^{D/2+2-n}} \frac{\Gamma(D/2+2-n)}{\Gamma(n)} \right\}. \quad (28) \end{aligned}$$

The next step is to make double Borel transformation with the variables  $p_1^2$  and  $p_2^2$  to Eq. (8) and Eqs. (30) and (31). The single-pole terms in Eq. (5) are eliminated. The formula reads

$$\begin{aligned} & \mathcal{B}_{1p_1^2}^{M_1^2} \mathcal{B}_{2p_2^2}^{M_2^2} \frac{\Gamma(n)}{[m^2 - (1-u)p_1^2 - up_2^2]^n} \\ & = (M^2)^{2-n} e^{-m^2/M^2} \delta(u-u_0), \quad (29) \end{aligned}$$

where  $u_0 = M_1^2 / (M_1^2 + M_2^2)$ ,  $M^2 \equiv M_1^2 M_2^2 / (M_1^2 + M_2^2)$ ,  $M_1^2, M_2^2$  are the Borel parameters.

Finally we identify the same tensor structures both at the hadronic level and the quark gluon level. Subtracting the

continuum contribution which is modeled by the dispersion integral in the region  $s_1, s_2 \geq s_0$ , we arrive at

$i\hat{e}$

$$\lambda_N^2 \sqrt{2} g_\rho \frac{1 + \kappa_\rho}{2} m_\rho^2 e^{-m_N^2/M^2} = \frac{f_\rho m_\rho}{2\pi^2} e^{-[u_0(1-u_0)m_\rho^2]/M^2} \times \left\{ g^v(u_0) M^6 f_2\left(\frac{s_0}{M^2}\right) - m_\rho^2 G_3(u_0) M^4 f_1\left(\frac{s_0}{M^2}\right) + \frac{g^v(u_0)}{24} \langle g_s^2 G^2 \rangle M^2 f_0\left(\frac{s_0}{M^2}\right) \right\}, \quad (30)$$

$i(e \cdot P)\hat{P}$

$$\lambda_N^2 \sqrt{2} g_\rho e^{-m_N^2/M^2} = -e^{-[u_0(1-u_0)m_\rho^2]/M^2} \frac{f_\rho m_\rho^3}{\pi^2} G_3(u_0) M^2 f_0\left(\frac{s_0}{M^2}\right),$$

where  $q^2 = m_\rho^2$ ,  $f_n(x) = 1 - e^{-x} \sum_{k=0}^n x^k/k!$  is the factor used to subtract the continuum,  $s_0$  is the continuum threshold. The sum rules are symmetric with the Borel parameters  $M_1^2$  and  $M_2^2$ . It is natural to adopt  $M_1^2 = M_2^2 = 2M^2$ ,  $u_0 = \frac{1}{2}$ . The functions  $G_i(u)$ ,  $i=0,1,2,3$ ,  $A(u)$  are defined as

$$G_0(u) = \int_0^u dt \phi_{||}(t), \quad (31)$$

$$G_1(u) = \int_0^u dt g^v(t), \quad (32)$$

$$G_2(u) = \int_0^u dt A(t), \quad (33)$$

$$G_3(u) = \int_0^u dt \int_0^t ds C(s). \quad (34)$$

## V. DISCUSSIONS OF RWFs AND PARAMETERS

The resulting sum rules depend on the RWFs, the integrals and derivatives of them at the point  $u_0 = \frac{1}{2}$ . The distri-

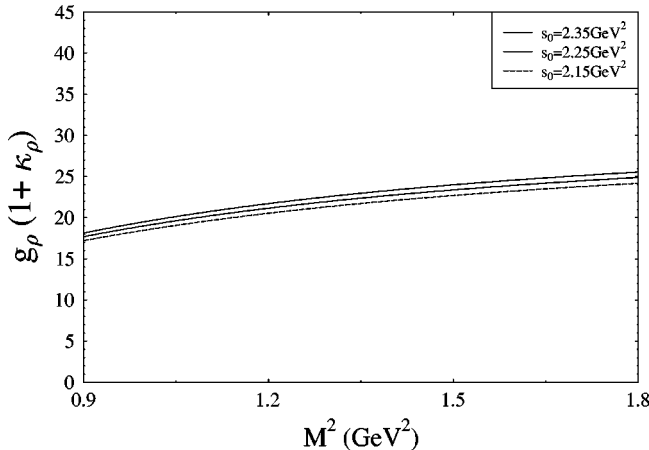


FIG. 2. The sum rule for  $g_\rho(1 + \kappa_\rho)$  as a function of the Borel parameter  $M^2$  for (30)/(35) with the model RWFs in [29]. From bottom to top the curves correspond to the continuum threshold  $s_0 = 2.35, 2.25, 2.15$  GeV $^2$ .

bution amplitudes of vector mesons have been studied in [28,26,27] using QCD sum rules. We adopt the model RWFs in Ref. [29].

The values of the two-particle RWFs, their derivatives and integrals at the point  $u_0 = \frac{1}{2}$  using the form in [29] are  $g^a = 1.15 \pm 0.23$ ,  $g^v = 0.64$ ,  $\phi_{||}(u_0) = 1.1$ ,  $G_0(u_0) = 0.5$ ,  $G_1(u_0) = 0.5$ ,  $G_2(u_0) = 0.58$ ,  $G_3(u_0) = -0.13$ ,  $A(u_0) = 2.18$ .

The experimental value for the rho meson mass  $m_\rho$  and the decay constant  $f_\rho$  is  $f_\rho = 198 \pm 7$  MeV and  $m_\rho = 770$  MeV [30].

The various parameters which we adopt are  $s_0 = 2.25$  GeV $^2$ ,  $m_N = 0.938$  GeV,  $\lambda_N = 0.026$  GeV $^3$  [3] at the scale  $\mu = 1$  GeV. The working interval for analyzing the QCD sum rules for nucleons is  $0.9$  GeV $^2 \leq M_B^2 \leq 1.8$  GeV $^2$ , a standard choice for analyzing the various QCD sum rules associated with the nucleon.

## VI. NUMERICAL ANALYSIS AND RESULTS

In order to diminish the uncertainty due to  $\lambda_N$ , we shall divide our sum rules by the famous Ioffe's mass sum rule for the nucleon:

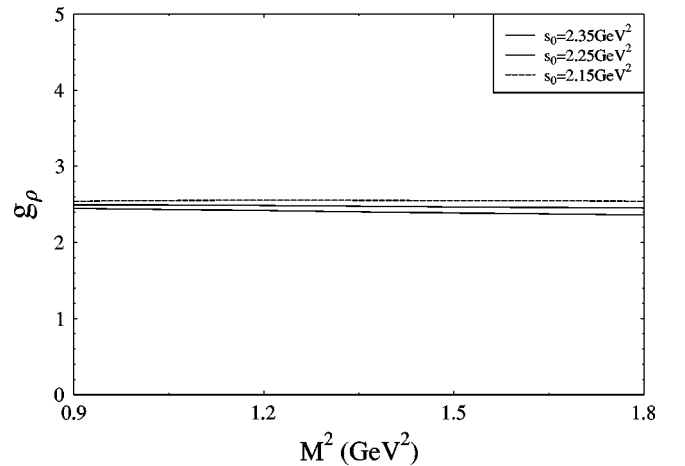


FIG. 3. The sum rule for  $g_\rho$  as a function of  $M^2$  and  $s_0$  for (31)/(35).

$$32\pi^4\lambda_N^2 e^{-M_N^2/M^2} = M^6 f_2\left(\frac{s_0}{M^2}\right) + \frac{b}{4} M^2 f_0\left(\frac{s_0}{M^2}\right) + \frac{4}{3} a^2 - \frac{a^2 m_0^2}{3M^2}. \quad (35)$$

Dividing Eqs. (30)–(38) by Eq. (35), we have two new sum rules for  $g_\rho(1+\kappa_\rho)$ ,  $g_\rho$ . The dependence on the continuum threshold  $s_0$  and Borel parameter  $M^2$  of these sum rules are presented in Figs. 2 and 3. From top to bottom the curves correspond to  $s_0 = 2.35, 2.25$ , and  $2.15$ , respectively.

$$1 + \kappa_\rho = - \frac{g^v(u_0)M^4 f_2\left(\frac{s_0}{M^2}\right) - m_\rho^2 G_3(u_0)M^2 f_1\left(\frac{s_0}{M^2}\right) + \frac{g^v(u_0)}{24} \langle g_s^2 G^2 \rangle f_0\left(\frac{s_0}{M^2}\right)}{m_\rho^4 G_3(u_0) f_0\left(\frac{s_0}{M^2}\right)}. \quad (38)$$

The result is presented in Fig. 4. Numerically,

$$1 + \kappa_\rho = (9.0 \pm 2.0), \quad (39)$$

which corresponds to

$$\kappa_\rho = (8.0 \pm 2.0). \quad (40)$$

Brown and Machleidt emphasized that the strong  $\rho NN$  coupling

$$\frac{g_\rho^2(1+\kappa_\rho)^2}{4\pi} = (37 \pm 13), \quad (41)$$

$$\kappa_\rho = (6.6 \pm 0.1) \quad (42)$$

should be adopted in order to reproduce experimental data [22]. The vector meson dominance (VMD) model yields

$$\frac{g_\rho^2(1+\kappa_\rho)^2}{4\pi} = 13.25. \quad (43)$$

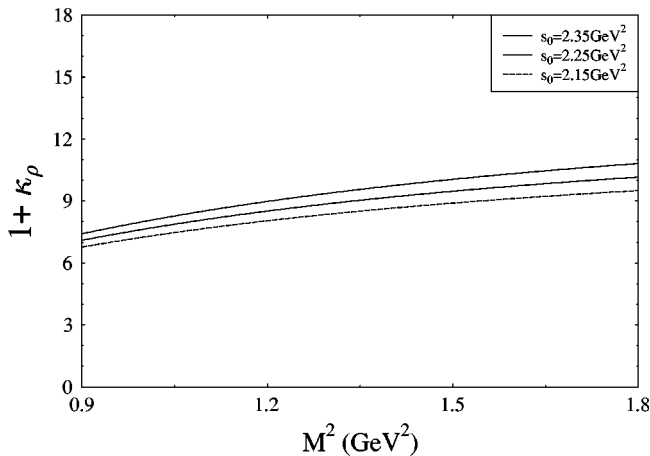


FIG. 4. The sum rule for  $1 + \kappa_\rho$  as a function of  $M^2$  and  $s_0$  from (30)/(31).

These sum rules are stable with reasonable variations of  $s_0$  and  $M^2$  as can be seen in Figs. 2 and 3. Numerically we have

$$g_\rho(1 + \kappa_\rho) = (22 \pm 3), \quad (36)$$

for (30)/(35) and

$$g_\rho = (2.5 \pm 0.2), \quad (37)$$

for (31)/(35).

We can also divide Eq. (30) by Eq. (31) to get a new sum rule for  $1 + \kappa_\rho$

Our result is

$$\frac{g_\rho^2(1 + \kappa_\rho)^2}{4\pi} = (39 \pm 10), \quad (44)$$

which agrees very well with Eq. (41) and deviates strongly from the VMD prediction (43).

We have included the uncertainty due to the variation of the continuum threshold and the Borel parameter  $M^2$  in our analysis. Other sources of uncertainty include (1) the truncation of OPE on the light cone and keeping only the few lowest twist operators; (2) the inherent uncertainty due to the model RWFs, etc. In the present case the major uncertainty comes from the RWFs since our final sum rules depend both on the value of RWFs and their integrals at  $u_0$ .

Our result  $\kappa_\rho = (8.0 \pm 2.0)$  is about two times larger than that derived in [24],  $\kappa_\rho = 3.6$  treating the rho meson as the external field. The reason is twofold. First, the vector-like  $\rho qq$  interaction is assumed in [24]. Moreover, for the anti-symmetric sum rules the susceptibilities  $\chi$ ,  $\kappa$ , and  $\xi$  take the same values as in the nucleon magnetic sum rules where the electromagnetic field is treated as the external field. In our opinion, such a treatment employs the vector meson dominance assumption inexplicitly, which may be the underlying reason for  $\kappa_\rho \approx \kappa_v = 3.7$ . Second, the vector meson mass corrections turn out to be large as pointed out in [29]. This effect is explicitly taken into account in our calculation.

In summary we have calculated the  $\rho NN$  couplings starting from QCD. The continuum and the excited states contribution is subtracted rather cleanly through the double Borel transformation in both cases. Our result strongly supports large value for the tensor-vector ratio  $\kappa_\rho$  in the nuclear force.

#### ACKNOWLEDGMENTS

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