Nuclei in a chiral SU(3) model

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Nuclei can be described satisfactorily in a nonlinear chiral SU(3) framework, even with standard potentials of the linear σ model. The condensate value of the strange scalar meson is found to be important for the properties of nuclei even without adding hyperons. By neglecting terms which couple the strange to the nonstrange condensate one can reduce the model to a Walecka model structure embedded in SU(3). We discuss inherent problems with chiral SU(3) models regarding hyperon optical potentials. [S0556-2813(98)07712-7]

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I. INTRODUCTION

Recently, the general principles of chiral symmetry and broken scale invariance in QCD have received renewed attention at finite baryon densities. There the underlying theory of strong interactions, QCD, is, however, not solvable in the nonperturbative low-energy regime. However, QCD constraints can be imposed on an effective ansatz for nuclear theory through symmetries that determine largely how the hadrons should interact with each other. In this spirit, models with $SU(2)_L \times SU(2)_R$ symmetry and scale invariance were applied to nuclear matter at zero and finite temperature and to finite nuclei [1–5]. As a new feature, a glueball field χ, the dilaton, was included which accounted for the broken scale invariance of QCD at tree level through a logarithmic potential [6]. The success of these models established the applicability of this approach to the relativistic description of the nuclear many-body problem.

Chiral SU(3) models have been quite successful in modeling hadron interactions. Meson-meson interactions can be described satisfactorily by using the linear SU(3) σ model [7]. Kaon-nucleon scattering data can be well reproduced using a chiral effective SU(3) Lagrangian [8] using a Lippmann-Schwinger approach [9]. The lowest order term is sufficient to describe the kaon-nucleon scattering data when including relativistic effects consistently and adding the η channel [10]. Especially, the in-medium properties of the kaon in nuclear matter are of considerable interest for recent measurements of kaon spectra at GSI, Darmstadt at subthreshold energies [11]. All of the above models lack the feature of including the nucleon-nucleon interaction on the same chiral SU(3) basis and therefore do not provide a consistent extrapolation to finite density.

Within SU(3) chiral models one can also take a different view at the properties of metastable exotic multihypernuclear objects [12]. The relativistic mean field model was extended to include the baryon octet and the vector nonet by using SU(6) symmetry for the coupling constants. The existence of strange hadronic matter and bound objects consisting purely of hyperons has been proposed [13,14]. The properties of strange hadronic matter are remarkably close to those of strangelets and can be negatively charged while carrying a positive baryon number. This has certain impacts for present heavy-ion searches looking for strangelets and other possible exotica [15,16].

We have recently extended the chiral effective model to $SU(3)_L \times SU(3)_R$ [17] including the baryon octet. This approach shall provide a basis to shed light on the properties of strange hadrons, as the in-medium properties of the kaon and the properties of strange hadronic matter, by pinning down the nuclear force in a chiral invariant way. This paper continues our previous work [17], which has applied a linear realization of chiral SU(3) symmetry and the concept of broken scale invariance to the description of hadronic matter in the vacuum and in the medium. It has been found that simultaneously both hadronic masses of the various SU(3) multiplets and the nuclear matter equation of state can be described reasonably well within a model respecting chiral symmetry.

However, it has been shown that the central potentials of the hyperons come out too large. They could not be corrected within a model with Yukawa-type baryon-meson interactions. The reason for this deficiency is threefold [17].

First, linear realizations of chiral symmetry restrict the coupling of the spin-0 mesons to the baryons to be symmetric (d type), while the spin-1 mesons are coupled to baryons antisymmetrically (f type). This destroys the balance between the repulsive contribution of the vector potential and the attraction due to the scalar potential. Therefore, the hyperon potentials attain too large values.

Secondly, the condensate of the strange scalar meson ζ changes considerably in the nuclear medium even for zero strangeness within this approach, because it couples to the nonstrange scalar field σ and therefore provides additional attraction. This is not counterbalanced by repulsive contributions from the strange vector field ϕ_μ, since ϕ_μ does not couple to either the nucleon or to the ω_μ field.

Thirdly, it is not possible to correct these values of the hyperon potentials through explicit symmetry-breaking terms, because they would destroy the relations for the partially conserved axial-vector currents (PCAC) of the pion and the kaon.

In order to deal with this general problem, nonlinear interaction terms of baryons with mesons were introduced in Ref. [17] in a chirally invariant way. However, although a cubic interaction of baryons with spin-0 mesons (with strong coupling of the strange condensate to the nucleons) produces

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reasonable hyperon potentials, such a form for this interaction seems quite artificial. Furthermore, the high mass of the strange meson (≈ 1 GeV) excludes a reasonable description of nuclei, since the oscillations in the charge density are too high for such an ansatz. Hence, although the cubic fit works satisfactorily for nuclear matter, it is not suitable to describe finite nuclei. The constraints imposed by the linear realization of chiral symmetry do not allow for a simultaneous description of both finite nuclei and hyperon potentials.

In this paper, we propose the adoption of the nonlinear realization of chiral symmetry as a solution to this problem. As was proven in Ref. [18], it is sufficient to have a local SU(3) invariance for the hadrons, with the pseudoscalar mesons appearing only in derivative couplings. Therefore, both d-type and f-type coupling is possible between baryons and scalar mesons. In addition, the pseudoscalar meson masses then depend only on the explicit symmetry breaking term. There is no reference to pseudoscalar mesons in the chirally invariant potential. Therefore, the potential only determines the masses of the scalar mesons. This allows us to decouple the strange condensate from the nonstrange one. Then both the results obtained with the SU(2) chirally symmetry models [1] and those of the nonlinear σ-ω model [19,20] can be reproduced as special realizations of the present general chiral SU(3) model. One can then systematically add terms of strange-to-nonstrange condensate coupling. Therefore, one can study the limiting case of a system consisting of nucleons only. Furthermore, explicit symmetry breaking terms (e.g., to correct the hyperon potentials) can be added without altering the PCAC relations of the pion and the kaon.

In this work it is demonstrated that one can simultaneously describe nuclei and the properties of strange hadrons within the framework of the nonlinear realization of chiral symmetry. Formally the sectors of scalar and pseudoscalar mesons are decoupled. However, the analysis [17] and the closeness of the coupling constant $g_{N\sigma}$ to m_N/f_π in the Boguta-Bodmer model seem to suggest to keep the constraints of the decay constants of the kaons and pions imposed on the vacuum expectation values (VEVs) of the nonstrange and strange scalar fields σ and ζ.

Our paper is structured as follows. The nonlinear realization and the connection between the linear and the nonlinear σ model of chiral symmetry are introduced in Sec. II. The chiral SU(3) Lagrangian is constructed and discussed in Sec. III. The equations of motion are solved in the mean field approximation which is described in Sec. IV. In Sec. V various parameter sets are presented which all account for a satisfactory description of finite nuclei: These include a Lagrangian with the potential of the linear SU(3) σ model as constructed in Ref. [17] with a modified baryon scalar-meson interaction. In the limit that the strange and nonstrange condensates are decoupled, the SU(2) chiral models of Refs. [1] and [20,21] are recovered, but embedded in the nonlinearly realized chiral SU(3) framework.

II. THE NONLINEAR REALIZATION OF CHIRAL SYMMETRY

In some neighborhood of the identity transformation, every group element $g'(x)$ of a compact, semisimple group G with a subgroup H can be decomposed uniquely into a product of the form [18]

$$g'(x)=\exp\left[i\sum \xi_a(x)S_a\right]\exp\left[i\sum \theta_b(x)T_b\right] \equiv u[\xi_a(x)]h[\theta_b(x)], \tag{1}$$

where $h(\theta_b)$ is an element of H. ξ_a and θ_b are parameters of the symmetry transformation which are generally space-time dependent. S_a and T_b represent the generators of the group G. For the case of $SU(3)_L\times SU(3)_R$ symmetry, the generators are the vectorial ($T_b=Q_b$) and axial ($S_a=Q_a^5$) charges, respectively, and the subgroup is $H=SU(3)_V$.

For our model, we assume invariance under *global* $SU(3)_L\times SU(3)_R$ transformations,

$$g=\exp\left[i\sum \alpha_L^a\lambda_{La}\right]\exp\left[i\sum \alpha_R^b\lambda_{Rb}\right]\equiv L(\alpha_L)R(\alpha_R). \tag{2}$$

Here, the representation of Gell-Mann matrices $\lambda_L=\lambda(1-\gamma_5)/2$ and $\lambda_R=\lambda(1+\gamma_5)/2$ with space-time-independent parameters α_L and α_R is used.

The product $gu[\xi_a(x)]$ is still an element of G and can be written as

$$g\ \exp\left[i\sum \xi_a S_a\right]=\exp\left[i\sum \xi_a'(g,\xi_a)S_a\right]\times\exp\left[i\sum \theta_b'(g,\xi_a)T_b\right], \tag{3}$$

where, in general, both ξ_a' and θ_b' depend on g and ξ_a. Let

$$\tilde{q}\to D(h)\tilde{q} \tag{4}$$

be a linear representation of the subgroup H of G. Then the transformation

$$g:\xi\to\xi',\quad \tilde{q}\to D\left(\exp\left[i\sum \theta_b'T_b\right]\right)\tilde{q} \tag{5}$$

constitutes a nonlinear realization of G.

The local parameters of the axial charges are identified with the fields of the pseudoscalar mesons [22]. In the representation of Gell-Mann matrices one has (also see the Appendix)

$$u[\pi_a(x)]=\exp\left[\frac{i}{2\sigma_0}\pi^a(x)\lambda_a\gamma_5\right]. \tag{6}$$

This assignment has the advantage that the pseudoscalar mesons are the parameters of the symmetry transformation. They will therefore only appear if the symmetry is explicitly broken or in terms with derivatives of the fields.

The composition of hadrons in terms of its constituents, the quarks, has to be determined in order to build models with hadronic degrees of freedom. This strategy has been followed, e.g., in Ref. [17] and is adopted also here. The transformation properties of the hadrons in the nonlinear representation can be derived if the ''old'' quarks q are related to the ''new'' quarks $\tilde{q}$ of the nonlinear representation.

The quarks of the nonlinear representation transform with the vectorial subgroup $SU(3)_V$ in accord with Eq. (1). Splitting the quarks in left- and right-handed parts, they can be written as

$$q_L = u\tilde{q}_L\,,\quad q_R = u^\dagger \tilde{q}_R\,. \tag{7}$$

These equations are connected by parity. The ambiguity in the choice of h is avoided by setting $h=1$. The transformation properties of the pions and the new quarks are found by considering how the old quarks transform:

$$q' = Lq_L + Rq_R = Lu\tilde{q}_L + Ru^\dagger \tilde{q}_R\,. \tag{8}$$

According to Eq. (3) (set $g=L$),

$$Lu = u'h, \quad Ru^\dagger = u^{\dagger\prime} h, \tag{9}$$

where the right equation is the parity transformed one of the left equation. Here and in the following, the abbreviations $u \equiv u[\pi_a(x)]$ and $u' \equiv u[\pi'_a(x)]$ are used. By inserting these relations into Eq. (8), one sees that $\tilde{q}$ transforms with $SU(3)_V$ as

$$\tilde{q}'_L = h\tilde{q}_L\,, \quad \tilde{q}'_R = h\tilde{q}_R\,. \tag{10}$$

According to Eq. (3), in general the vector transformation is a local, nonlinear function depending on pseudoscalar mesons $h = h[g,\pi_a(x)]$. Following Eq. (9), the pseudoscalar mesons transform nonlinearly as

$$u' = Luh^\dagger = huR^\dagger, \tag{11}$$

$$u'^\dagger = hu^\dagger L^\dagger = Ru^\dagger h^\dagger. \tag{12}$$

The second set of equalities are again due to parity. In contrast to the linear realization of chiral symmetry, there is no distinction between the left and right space. Therefore, only the representations **8** and **1** of the lowest-lying hadrons are possible. The various octets transform accordingly, e.g., for the scalar (X), vector ($V_\mu = l_\mu + r_\mu$), axial vector ($\mathcal{A}_\mu = l_\mu - r_\mu$), and baryon ($B$) matrices one has

$$X' = hXh^\dagger,\;\; V'_\mu = hV_\mu h^\dagger,\;\; \mathcal{A}'_\mu = h\mathcal{A}_\mu h^\dagger,\;\; B' = hBh^\dagger. \tag{13}$$

The present nonlinearly transforming hadronic fields can be obtained from the linearly transforming ones described in Ref. [17] by multiplying them by $u[\pi(x)]$ and its conjugate (see also Ref. [23]):

$$X = \tfrac{1}{2}(u^\dagger M u^\dagger + uM^\dagger u),\;\; Y = \tfrac{1}{2}(u^\dagger M u^\dagger - uM^\dagger u), \tag{14}$$

$$l_\mu = u^\dagger \tilde{l}_\mu u,\;\; r_\mu = u\tilde{r}_\mu u^\dagger, \tag{15}$$

$$B_L = u^\dagger \Psi_L u,\;\; B_R = u\Psi_R u^\dagger. \tag{16}$$

Here, $M = \Sigma + i\Pi$ and its conjugate contains the nonets of the linearly transforming scalar (Σ) and pseudoscalar (Π) mesons, whereas $\tilde{l}_\mu$, $\tilde{l}_\mu$, Ψ_L, and Ψ_R are the left and right-handed parts of the spin-1 mesons and baryons in the linear representation, respectively.

III. LAGRANGIAN

In this section, the various terms of the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \sum_{W=X,Y,V,\mathcal{A},u} \mathcal{L}_{\rm BW} + \mathcal{L}_{\rm VP} + \mathcal{L}_{\rm vec} + \mathcal{L}_0 + \mathcal{L}_{\rm SB} \tag{17}$$

are discussed in detail. $\mathcal{L}_{\rm kin}$ is the kinetic energy term, $\mathcal{L}_{\rm BW}$ includes the interaction terms of the different baryons with the various spin-0 and spin-1 mesons and with the photons. In $\mathcal{L}_{\rm VP}$, the interaction terms of vector mesons with pseudoscalar mesons and with photons is summarized. $\mathcal{L}_{\rm vec}$ generates the masses of the spin-1 mesons through interactions with spin-0 mesons, and $\mathcal{L}_0$ gives the meson-meson interaction terms which induce the spontaneous breaking of chiral symmetry. It also includes the scale breaking logarithmic potential. Finally, $\mathcal{L}_{\rm SB}$ introduces an explicit symmetry breaking of the $U(1)_A$, the $SU(3)_V$, and the chiral symmetry.

A. Kinetic energy terms

Since the vector transformation $h[\pi(x)]$ of the hadrons depends in general on the pseudoscalar mesons and thus is local, covariant derivatives have to be used for the kinetic terms in order to preserve chiral invariance. The covariant derivative, i.e., for the baryons, reads

$$D_\mu B = \partial_\mu B + i[\Gamma_\mu, B], \tag{18}$$

where

$$\Gamma_\mu = -\frac{i}{2}[u^\dagger \partial_\mu u + u\partial_\mu u^\dagger]. \tag{19}$$

This is a composite vector-type field, which transforms according to

$$\Gamma'_\mu = h\Gamma_\mu h^\dagger - ih\partial_\mu h^\dagger. \tag{20}$$

The spin-1 nonet of the strong interactions are here introduced as massive, homogeneously transforming fields, following the approach in Refs. [24, 25], in order to avoid complications arising from the mixing of the axial with the pseudoscalar mesons.

The kinetic energy term of the pseudoscalar mesons is introduced by defining [in analogy to Eq. (19)] the axial vector as

$$u_\mu = -\frac{i}{2}[u^\dagger \partial_\mu u - u\partial_\mu u^\dagger], \tag{21}$$

which transforms as $u'_\mu = hu_\mu h^\dagger$. The standard form for the kinetic energy of the pseudoscalar mesons is $\mathrm{Tr}(u_\mu u^\mu)$. However, the approximate validity of $g_{N\sigma} \approx m_N/f_\pi$, where $f_\pi = -\sigma_0$, in the Walecka-type models [20,21] and results obtained in Refs. [17,1] indicate that the constraints of the linear σ model on the scalar condensates in the vacuum, are also applicable to the description of hadronic matter and nuclei. To incorporate those constraints in the nonlinear realization we modify the standard kinetic energy term to include a coupling of the scalar and pseudoscalar mesons

$$\mathrm{Tr}(u_\mu X u^\mu X + X u_\mu u^\mu X). \tag{22}$$

This term contains, besides higher order self-interactions, the kinetic energy term for the pseudoscalar mesons if the pseudoscalar matrix in the exponential of Eq. (6) is defined as

$$\frac{1}{\sqrt{2}}\pi_a\lambda^a = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\pi^0 + \frac{\eta^8}{\sqrt{1+2w^2}}\right) & \pi^+ & 2\frac{K^+}{w+1} \\ \pi^- & \frac{1}{\sqrt{2}}\left(-\pi^0 + \frac{\eta^8}{\sqrt{1+2w^2}}\right) & 2\frac{K^0}{w+1} \\ 2\frac{K^-}{w+1} & 2\frac{\overline{K^0}}{w+1} & -\frac{\eta^8\sqrt{2}}{\sqrt{1+2w^2}} \end{pmatrix}. \tag{23}$$

The renormalization factors containing $w=\sqrt{2}\zeta_0/\sigma_0$ are included to obtain the canonical form of the kinetic energy terms for pseudoscalar mesons[1] from Eq. (22). For $w=1$, one has an $\mathrm{SU}(3)_V$ symmetric vacuum and the matrix (23) reduces to the matrix normally used, e.g., in chiral perturbation theory [26]. The advantage of Eq. (23) is that $\mathrm{SU}(3)_V$ breaking effects (such as $f_\pi \neq f_K$) are accounted for even at lowest order.

After computing the axial current for pions and kaons from Eq. (22), one obtains the same relations:

$$\sigma_0 = -f_\pi, \quad \zeta_0 = -\frac{1}{\sqrt{2}}(2f_K - f_\pi) \tag{25}$$

for the VEVs of the scalar condensates found in the linear σ model [17]. In order to construct a chirally invariant kinetic term for the spin-1 mesons, the ordinary derivatives must be replaced by the covariant derivatives as defined in Eq. (18),

$$V_{\mu\nu} = D_\mu V_\nu - D_\nu V_\mu, \tag{26}$$

and analogously for the axial vector mesons, where the symbol $\mathcal{A}_{\mu\nu}$ is used.

In summary, the kinetic energy terms read

$$\mathcal{L}_{\mathrm{kin}} = i\,\mathrm{Tr}\,\overline{B}\gamma_\mu D^\mu B + \tfrac{1}{2}\mathrm{Tr}\,D_\mu X D^\mu X + \mathrm{Tr}(u_\mu X u^\mu X + X u_\mu u^\mu X) + \tfrac{1}{2}\mathrm{Tr}\,D_\mu Y D^\mu Y \tag{27}$$

$$+\tfrac{1}{2}D_\mu\chi D^\mu\chi - \tfrac{1}{4}\mathrm{Tr}(V_{\mu\nu}V^{\mu\nu}) - \tfrac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \tfrac{1}{4}\mathrm{Tr}(\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu}), \tag{28}$$

where we included the usual field strength tensor of the photon $F_{\mu\nu}$ as we want to discuss electromagnetic form factors and nuclei later on. The pseudoscalar singlet is independent of the octet and has thus a kinetic term of its own. For the dilaton field χ (for its discussion see Sec. III D 3), which is also a chiral singlet, it makes no difference if the normal derivative is replaced with the covariant derivative because the additional commutator term vanishes.

B. Baryon-meson interaction

The various interaction terms of baryons with mesons are discussed in this section. The SU(3) structure of the the baryon-meson interaction terms are the same for all mesons, except for the difference in Lorentz space. For a general meson field W they read

$$\mathcal{L}_{\mathrm{BW}} = -\sqrt{2}g_8^W(\alpha_W[\overline{B}\mathcal{O}BW]_F + (1-\alpha_W)[\overline{B}\mathcal{O}BW]_D) - g_1^W\frac{1}{\sqrt{3}}\mathrm{Tr}(\overline{B}\mathcal{O}B)\mathrm{Tr}\,W, \tag{29}$$

with $[\overline{B}\mathcal{O}BW]_F := \mathrm{Tr}(\overline{B}\mathcal{O}WB - \overline{B}\mathcal{O}BW)$ and $[\overline{B}\mathcal{O}BW]_D := \mathrm{Tr}(\overline{B}\mathcal{O}WB + \overline{B}\mathcal{O}BW) - \frac{2}{3}\mathrm{Tr}(\overline{B}\mathcal{O}B)\mathrm{Tr}\,W$. The different terms to be considered are those for the interaction of baryons, with scalar mesons ($W=X$, $\mathcal{O}=1$), with vector mesons ($W=V_\mu$, $\mathcal{O}=\gamma_\mu$ for the vector and $W=V_{\mu\nu}$, $\mathcal{O}=\sigma^{\mu\nu}$ for the tensor interaction), with axial vector mesons ($W=\mathcal{A}_\mu$, $\mathcal{O}=\gamma_\mu\gamma_5$), and with pseudoscalar mesons ($W=u_\mu$, $\mathcal{O}=\gamma_\mu\gamma_5$), respectively. For u_μ, the singlet term is vanishing, because the matrix in the exponential of Eq. (6) involves only the pseudoscalar octet and is thus traceless. The interaction of the pseudoscalar chiral singlet Y with baryons has the structure $g_1^Y\mathrm{Tr}(\overline{B}\gamma_\mu\gamma_5 B)\mathrm{Tr}\,Y$.

Since the pseudoscalar mesons are solely contained in the exponential, the only possible form of their coupling with baryons is the pseudovector interaction. There, the coupling constant $g_A \equiv \sqrt{2}g_8^u$ is restricted by the Goldberger-Treiman relation. In contrast to the linear representation, the axial coupling constant g_A is not unity, but a value $g_A \simeq 1.26$ can be assigned to it. In this case the mixing angle between f-type and d-type coupling is $\alpha_u \simeq 0.4$ [27].

[1] The same normalization of the pseudoscalar matrix has to be taken if the kinetic energy term $\frac{1}{2}\mathrm{Tr}(\partial_\mu M^\dagger \partial^\mu M)$ is used with

$$M = u(X+iY)u, \quad M = u^\dagger(X-iY)u^\dagger \tag{24}$$

substituted.

In our approach, the quark model related SU(3) coupling scheme for the tensor coupling terms emerges naturally. This coupling scheme has been proven to be in good agreement with the observed small Lambda hypernuclear spin-orbit splitting [28–30]. For our present study, we will not discuss the tensor coupling terms as they have been found to be small for spherical nuclei [31]. We note in passing that this might be not the case for exotic nuclei and we leave this issue for forthcoming studies.

1. Scalar mesons

The baryons and the scalar mesons transform equally in the left and right subspace. Therefore, in contrast to the linear realization of chiral symmetry, a f-type coupling is allowed for the baryon-meson interaction. In addition, it is possible to construct mass terms for baryons and to couple them to chiral singlets. After insertion of the vacuum matrix $\langle X\rangle$ [Eq. (A7)], one obtains the baryon masses as generated by the VEVs of the two meson fields:

$$
\begin{aligned}
m_N &= m_0 - \tfrac{1}{3} g_8^S (4\alpha_S - 1)(\sqrt{2}\zeta - \sigma),\\
m_\Lambda &= m_0 - \tfrac{2}{3} g_8^S (\alpha_S - 1)(\sqrt{2}\zeta - \sigma),\\
m_\Sigma &= m_0 + \tfrac{2}{3} g_8^S (\alpha_S - 1)(\sqrt{2}\zeta - \sigma),\\
m_\Xi &= m_0 + \tfrac{1}{3} g_8^S (2\alpha_S + 1)(\sqrt{2}\zeta - \sigma)
\end{aligned}
\tag{30}
$$

with $m_0 = g_1^S(\sqrt{2}\sigma + \zeta)/\sqrt{3}$. The three parameters g_1^S, g_8^S, and α_S can be used to fit the baryon masses to their experimental values. Then, besides the current quark mass terms discussed in Sec. III E, no additional explicit symmetry breaking term is needed. Note that the nucleon mass depends on the *strange condensate* ζ. For $\zeta = \sigma/\sqrt{2}$ (i.e., $f_\pi = f_K$), the masses are degenerate, and the vacuum is $SU(3)_V$ invariant.

It is desirable to have an alternative way of baryon mass generation, where the nucleon mass depends only on σ. This can be accomplished by taking the limit $\alpha_S = 1$ and $g_1^S = \sqrt{6} g_8^S$. Then, the coupling constants between the baryons and the two scalar condensates are related to the additive quark model. This leaves only one coupling constant free to adjust for the correct nucleon mass. For a fine-tuning of the remaining masses, it is necessary to introduce an explicit symmetry breaking term, which breaks the SU(3) symmetry along the hypercharge direction. A possible term already discussed in Refs. [17,32], which respects the Gell-Mann-Okubo mass relation, is

$$
\mathcal{L}_{\Delta m} = -m_1 \mathrm{Tr}(\overline{B}B - \overline{B}BS) - m_2 \mathrm{Tr}(\overline{B}SB), \tag{31}
$$

where $S_b^a = -\frac{1}{3}[\sqrt{3}(\lambda_8)_b^a - \delta_b^a]$. As in the first case, the three coupling constants $g_{N\sigma} \equiv 3 g_8^S$, m_1 and m_2 are sufficient to reproduce the experimentally known baryon masses. Explicitly, the baryon masses have the values

$$
\begin{aligned}
m_N &= -g_{N\sigma}\sigma,\\
m_\Xi &= -\frac{1}{3} g_{N\sigma}\sigma - \frac{2}{3} g_{N\sigma}\sqrt{2}\zeta + m_1 + m_2,\\
m_\Lambda &= -\frac{2}{3} g_{N\sigma}\sigma - \frac{1}{3} g_{N\sigma}\sqrt{2}\zeta + \frac{m_1 + 2m_2}{3},\\
m_\Sigma &= -\tfrac{2}{3} g_{N\sigma}\sigma - \tfrac{1}{3} g_{N\sigma}\sqrt{2}\zeta + m_1 .
\end{aligned}
\tag{32}
$$

For both versions of baryon-meson interaction the parameters are fixed to yield the baryon masses $m_N = 939$ MeV, $m_\Lambda = 1115$ MeV, $m_\Sigma = 1196$ MeV, and $m_\Xi = 1331.5$ MeV.

2. Vector mesons

Two independent interaction terms of baryons with spin-1 mesons can be constructed in analogy with the baryon–spin-0-meson interaction. They correspond to the antisymmetric (f-type) and symmetric (d-type) couplings, respectively. From the universality principle [33] and the vector meson dominance model one may conclude that the d-type coupling should be small. For most of the fits $\alpha_V = 1$, i.e., f-type coupling, is used. However, a small admixture of d-type coupling allows for some fine-tuning of the single particle energy levels of nucleons in nuclei (see below).

As for the case with scalar mesons in Sec. III B 1, for $g_1^V = \sqrt{6} g_8^V$, the strange vector field $\phi_\mu \sim \bar{s}\gamma_\mu s$ does not couple to the nucleon implying that the strange vector form factor of the nucleon is very small. The remaining couplings to the strange baryons are then determined by symmetry relations

$$
\begin{aligned}
&g_{NN\omega} = (4\alpha_V - 1) g_8^V,\\
&g_{\Lambda\Lambda\omega} = \frac{2}{3}(5\alpha_V - 2) g_8^V, \quad g_{\Lambda\Lambda\phi} = -\frac{\sqrt{2}}{3}(2\alpha_V + 1) g_8^V,\\
&g_{\Sigma\Sigma\omega} = 2\alpha_V g_8^V, \quad g_{\Sigma\Sigma\phi} = -\sqrt{2}(2\alpha_V - 1) g_8^V,\\
&g_{\Xi\Xi\omega} = (2\alpha_V - 1) g_8^V, \quad g_{\Xi\Xi\phi} = -2\sqrt{2}\alpha_V g_8^V.
\end{aligned}
\tag{33}
$$

In the limit $\alpha_V = 1$, the relative values of the coupling constants are related to the additive quark model via

$$
\begin{aligned}
&g_{\Lambda\omega} = g_{\Sigma\omega} = 2 g_{\Xi\omega} = \frac{2}{3} g_{N\omega} = 2 g_8^V,\\
&g_{\Lambda\phi} = g_{\Sigma\phi} = \frac{g_{\Xi\phi}}{2} = \frac{\sqrt{2}}{3} g_{N\omega}.
\end{aligned}
\tag{34}
$$

Note that all coupling constants are fixed once, e.g., $g_{N\omega}$ is specified. Since the axial vector mesons have a vanishing expectation value at the mean-field level their coupling constants to the baryons will not be discussed here.

C. Electromagnetic structure of pseudoscalar mesons

The interaction Lagrangian of the vector mesons with pions and the photon takes the form

$$
\begin{aligned}
\mathcal{L}_{VP} = {}& e\ \mathrm{Tr}(A_\mu \Gamma^\mu) + g\ \mathrm{Tr}(V_\mu \Gamma^\mu)\\
&+ \frac{e}{4 g_\gamma} F_{\mu\nu} \mathrm{Tr}[(u^\dagger Q u + u Q u^\dagger) V^{\mu\nu}].
\end{aligned}
\tag{35}
$$

The first term originates from the kinetic energy term (22), if the photon A_μ is included in u_μ through the definition of

$$\tilde{u}_\mu = -\frac{i}{2}[u^\dagger(\partial_\mu + i g_v l_\mu)u - u(\partial_\mu + i g_v r_\mu)u^\dagger], \quad (36)$$

where $l_\mu = r_\mu = QA_\mu$, with $Q = T_3 + Y/2$, and the electrical charge, $g_v = e$. The remaining two terms can be motivated from a gauge and chiral invariant Lagrangian approach [34,35].

One obtains for the form factor of the pion ($Q^2 = -q^2$)

$$F_\pi(Q^2) = 1 - \frac{g}{g_\gamma}\frac{Q^2}{Q^2 + m_\rho^2}. \quad (37)$$

Note that the pion does not couple to the other vector mesons ω and ϕ.

With $g = 6.05$ from $\rho^0 \to \pi^+\pi^-$ [36] and $g_\gamma = 5.04$ from $\rho^0 \to e^+e^-$ the mean-square charge radius of the pion is

$$\langle r_\pi^2 \rangle \equiv -6\frac{dF(Q^2)}{dQ^2}\bigg|_{Q^2=0} = 0.48\ \text{fm}^2. \quad (38)$$

Experimentally, $\langle r_\pi^2 \rangle = 0.432 \pm 0.016$ fm^2 [37].

In analogy to the pion, one obtains for the form factor of the kaon

$$F_K^\pm(Q^2) = 1 - \frac{g_{KK\rho}}{g_{\gamma\rho}}\frac{Q^2}{Q^2 + m_\rho^2} - \frac{g_{KK\omega}}{g_{\gamma\omega}} \times \frac{Q^2}{Q^2 + m_\omega^2} - \frac{g_{KK\phi}}{g_{\gamma\phi}}\frac{Q^2}{Q^2 + m_\phi^2} \quad (39)$$

with the coupling constants

$$\frac{g_{KK\rho}}{g_{\gamma\rho}} = \frac{1}{2}\frac{g}{g_\gamma}, \quad \frac{g_{KK\omega}}{g_{\gamma\omega}} = \frac{1}{6}\frac{g}{g_\gamma}, \quad \frac{g_{KK\phi}}{g_{\gamma\phi}} = \frac{\sqrt{2}}{6}\frac{g}{g_\gamma}, \quad (40)$$

where $g/g_\gamma = 1.2$. Assuming equal masses for ρ and ω ($m_V \equiv m_\omega = m_\rho$), one gets

$$F_{K^\pm}(Q^2) = 1 - \frac{g}{g_\gamma}\left(\frac{2}{3}\frac{Q^2}{Q^2 + m_V^2} + \frac{\sqrt{2}}{6}\frac{Q^2}{Q^2 + m_\phi^2}\right). \quad (41)$$

Hence, for low Q^2, the momentum dependence of the kaon form factor differs from the pion form factor by a factor $2/3 + m_V^2/m_\phi^2\sqrt{2}/6 \approx 0.8$. The mean square charge radius of the kaon is given as

$$\langle r_{K^\pm}^2 \rangle = \frac{g}{g_{\gamma\rho}}\left(\frac{4}{m_V^2} + \frac{\sqrt{2}}{m_\phi^2}\right) = (0.32 + 0.06)\ \text{fm}^2 = 0.38\ \text{fm}^2 \quad (42)$$

as compared to $\langle r_{K^\pm}^2 \rangle = (0.34 \pm 0.05)$ fm^2, from experiment [38]. Hence, the contribution from the ϕ is small, but not negligible. If one takes into account only the ρ contribution, then $\langle r_K^2 \rangle = 1/2\langle r_\pi^2 \rangle = 0.24$ fm^2, which disagrees with the experimental value. For the form factor of the K^0, the contribution coming from the ρ meson changes its sign and one gets

$$\langle r_{K^0}^2 \rangle = \frac{g}{g_{\gamma\rho}}\left(-\frac{2}{m_V^2} + \frac{\sqrt{2}}{m_\phi^2}\right) = -0.10\ \text{fm}^2, \quad (43)$$

which is again in the range of the experimental value of $\langle r_{K^0}^2 \rangle = -0.054 \pm 0.101$ fm^2 [38]. The electromagnetic form factors of the baryons will be discussed in a forthcoming publication [39].

D. Meson-meson interaction

1. Vector meson masses

Here we discuss the mass terms of the vector mesons. The simplest scale-invariant form

$$\mathcal{L}_{\text{vec}}^{(1)} = \frac{1}{2}m_V^2\frac{\chi^2}{\chi_0^2}\ \text{Tr}\ V_\mu V^\mu + 2g_4^4\text{Tr}(V_\mu V^\mu)^2 \quad (44)$$

implies a mass degeneracy for the meson nonet. The first term of Eq. (44) is made scale invariant by multiplying it with an appropriate power of the glueball field χ (see Sec. III D 3 for details). To split the masses, one can add the chiral invariant [40,41]

$$\mathcal{L}_{\text{vec}}^{(2)} = \tfrac{1}{4}\mu\ \text{Tr}[V_{\mu\nu}V^{\mu\nu}X^2]. \quad (45)$$

Combining this with the kinetic energy term [Eq. (27)], one obtains the following terms for the different vector mesons:

$$-\frac{1}{4}\left[1 - \mu\frac{\sigma^2}{2}\right](V_\rho^{\mu\nu})^2 - \frac{1}{4}\left[1 - \frac{1}{2}\mu\left(\frac{\sigma^2}{2} + \zeta^2\right)\right](V_{K*}^{\mu\nu})^2,$$

$$-\frac{1}{4}\left[1 - \mu\frac{\sigma^2}{2}\right](V_\omega^{\mu\nu})^2 - \frac{1}{4}[1 - \mu\zeta^2](V_\phi^{\mu\nu})^2. \quad (46)$$

The coefficients are no longer unity, therefore the vector meson fields have to be renormalized, i.e., the new ω field reads $\omega_r = Z_\omega^{-1/2}\omega$. The renormalization constants are the coefficients in the square brackets in front of the kinetic energy terms of Eq. (46), i.e., $Z_\omega^{-1} = 1 - \mu\sigma^2/2$. The mass terms of the vector mesons deviate from the mean mass m_V by the renormalization factor,[2] i.e.,

$$m_\omega^2 = m_\rho^2 = Z_\omega m_V^2, \quad m_{K*}^2 = Z_{K*}m_V^2, \quad m_\phi^2 = Z_\phi m_V^2. \quad (47)$$

The constants m_V and μ are fixed to give the correct ω and ϕ masses. The other vector meson masses are given in Table I.

The axial vector meson masses can be described by adding terms analogous to Eq. (45). We refrain from discussing them further (see Refs. [40,42]).

[2] One could also split the ρ-ω mass degeneracy by adding a term of the form [40] (Tr $V_{\mu\nu}$)^2 to Eq. (46). Or, alternatively, one could break the SU(2) symmetry of the vacuum allowing for a nonvanishing vacuum expectation value of the scalar isovector field. However, the ρ-ω mass splitting is small ($\sim 2\%$), and, therefore, we will not consider these complications.

TABLE I. Parameters of the different potentials used (see text).

	k_0	k_1	k_2	k_3	k_{3m}	k_4	33δ
C_1	2.37	1.40	−5.55	−2.65	0	−0.23	2
C_2	2.36	1.40	−5.55	−2.64	0	−0.23	2
C_3	2.35	1.40	−5.55	−2.60	0	−0.23	2
M_1	1.28	0	0	0	0	0	6
M_2	1.29	0	0	0	0	0	6
W_1	−14.91	0	16.67	0	32.06	0	0
W_2	−12.96	0	16.67	0	32.06	0	0
W_3	10.44	7.32	−4.96	0	31.06	0	0

2. Scalar mesons

The nonlinear realization of chiral symmetry offers many more possibilities to form chiral invariants: the couplings of scalar mesons with each other are only governed by $SU(3)_V$ symmetry. However, only three kinds of independent invariants exist, namely,

$$I_1 = \mathrm{Tr}\, X, \quad I_2 = \mathrm{Tr}\, X^2, \quad I_3 = \det X. \tag{48}$$

All other invariants, Tr X^n, with $n \geqslant 3$, can be expressed as a function of the three invariants shown in Eq. (48). This can be shown from the characteristic equation of an arbitrary 3×3 matrix X

$$(X - x_1)(X - x_2)(X - x_3) = 0, \tag{49}$$

where x_i are the eigenvalues of X. By writing the coefficients of the powers of X in terms of invariants one obtains

$$X^3 - I_1 X^2 - \tfrac{1}{2}[I_2 - (I_1)^2]X - I_3 = 0. \tag{50}$$

Hence, one obtains the invariant Tr X^3 as a function of the base (48),

$$I_{3m} \equiv \mathrm{Tr}\, X^3 = I_1 I_2 + \tfrac{1}{2}[I_2 - (I_1)^2] I_1 + I_3. \tag{51}$$

By multiplying Eq. (50) with, e.g., X and taking the trace, the invariant for $n=4$ can be written in terms of Eq. (48):

$$I_4 \equiv \mathrm{Tr}\, X^4 = I_1 I_{3m} + \tfrac{1}{2}[I_2 - (I_1)^2] I_2 + I_3 I_1. \tag{52}$$

A similar expression can be found for all other n. Alternatively, instead of $I_3 = \det X$ the invariant $I_{3m} = \mathrm{Tr}\, X^3$, can be chosen as an element of the basis. Then, I_3 can be rewritten in terms of the new basis I_1, I_2, and I_{3m} as

$$I_3 = \tfrac{1}{3} I_{3m} - \tfrac{1}{2} I_1 I_2 + \tfrac{1}{6}(I_1)^3. \tag{53}$$

For our calculations, the invariants of Eq (48) are considered as building blocks, from which the different forms of the meson-meson interaction are constructed. They will be investigated including sets in which the models in Refs. [1] and [21] are embedded in a chiral SU(3) framework (see Sec. V).

3. Broken scale invariance

The concept of broken scale invariance leading to the trace anomaly in (massless) QCD, $\theta_\mu^\mu = (\beta_{\mathrm{QCD}}/2g)\mathcal{G}^a_{\mu\nu}\mathcal{G}_a^{\mu\nu}$ ($\mathcal{G}_{\mu\nu}$ is the gluon field strength tensor of QCD), can be mimicked in an effective Lagrangian at tree level [6] through the introduction of the potential

$$\mathcal{L}_{\mathrm{scale}} = -k_4 \chi^4 - \frac{1}{4}\chi^4 \ln \frac{\chi^4}{\chi_0^4} + \frac{\delta}{3}\chi^4 \ln \frac{I_3}{\det\langle X\rangle}. \tag{54}$$

The effect[3] of the logarithmic term $\sim \chi^4 \ln \chi$ is to break the scale invariance. This leads to the proportionality $\theta_\mu^\mu \sim \chi^4$, as can be seen from

$$\theta_\mu^\mu = 4\mathcal{L} - \chi \frac{\partial \mathcal{L}}{\partial \chi} - 2\partial_\mu \chi \frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} = \chi^4, \tag{55}$$

which is a consequence of the definition of the scale transformations [43]. This holds only if the meson-meson potential is scale invariant. This can be achieved by multiplying the invariants of scale dimension less then four with an appropriate power of the dilaton field χ.

The comparison of the trace anomaly of QCD with that of the effective theory allows for the identification of the χ-field with the gluon condensate:

$$\theta_\mu^\mu = \left\langle \frac{\beta_{\mathrm{QCD}}}{2g} \mathcal{G}^a_{\mu\nu}\mathcal{G}_a^{\mu\nu} \right\rangle \equiv (1-\delta)\chi^4. \tag{56}$$

The parameter δ originates from the second logarithmic term with the chiral invariant I_3 [see also Ref. [1] for the chiral SU(2) linear σ model]. An orientation for the value of δ may be taken from β_{QCD} at the one loop level, with N_c colors and N_f flavors,

$$\beta_{\mathrm{QCD}} = -\frac{11 N_c g^3}{48\pi^2}\left(1 - \frac{2N_f}{11 N_c}\right) + \mathcal{O}(g^5). \tag{57}$$

Here the first number in parentheses arises from the (antiscreening) self-interaction of the gluons and the second term, proportional to N_f, is the (screening) contribution of quark pairs. Equation (57) suggests the value $\delta = 6/33$ for three flavors and three colors. This value gives the order of magnitude about which the parameter δ will be varied.

For simplicity, we will also consider the case in which $\chi = \chi_0$, where the gluon condensate does not vary with density. We will refer to this case as the frozen glueball limit.

E. Explicitly broken chiral symmetry

In order to eliminate the Goldstone modes from a chiral effective theory, explicit symmetry breaking terms have to be introduced. Here, we use

$$\mathcal{L}_{\mathrm{SB}} = -\tfrac{1}{2} m_{\eta_0}^2 \mathrm{Tr}\, Y^2 - \tfrac{1}{2} \mathrm{Tr}\, A_p (uXu + u^\dagger X u^\dagger) - \mathrm{Tr}(A_s - A_p)X. \tag{58}$$

[3]According to Ref. [6], the argument of the logarithm has to be chirally and parity invariant. This is fulfilled by the dilaton χ which is both a chiral singlet as well as a scalar.

The first term, which breaks the $U(1)_A$ symmetry, gives a mass to the pseudoscalar singlet. The second term is motivated by the explicit symmetry breaking term of the linear σ model

$$\tfrac{1}{2}\mathrm{Tr}\, A_p(M+M^\dagger)=\mathrm{Tr}\, A_p[u(X+iY)u+u^\dagger(X-iY)u^\dagger], \quad (59)$$

with $A_p=1/\sqrt{2}\ \mathrm{diag}(m_\pi^2 f_\pi, m_\pi^2 f_\pi, 2m_K^2 f_K - m_\pi^2 f_\pi)$ and $m_\pi=139$ MeV, $m_K=498$ MeV. For simplicity, η_0/η_8 mixing is neglected by omitting Y from the second term of Eq. (58). If this term is included, we get a mixing angle of $\theta=16°$ for parameter set C_1 (see Sec. V A), which agrees well with experiment, $\theta^{\exp}\approx 20°$ from $\eta,\eta'\to\gamma\gamma$.

In the case of $SU(3)_V$ symmetry, the quadratic Gell-Mann-Okubo mass formula $3m_{\eta_8}^2+m_\pi^2-4m_K^2=0$ is satisfied. The third term breaks $SU(3)_V$ symmetry. $A_s=\mathrm{diag}(x,x,y)$ can be used to remove the vacuum constraints on the parameters of the meson-meson potential by adjusting x and y in such a way that the terms linear in σ and ζ vanish in the vacuum.

IV. MEAN-FIELD APPROXIMATION

The terms discussed so far involve the full quantum operator fields which cannot be treated exactly. To apply the model to the description of finite nuclei, we perform the mean-field approximation. This is a nonperturbative relativistic method to solve approximately the nuclear many body problem by replacing the quantum field operators by its classical expectation values (for a recent review see Ref. [44]).

In the following, we will consider the time-independent spherically symmetric case of finite nuclei with vanishing net strangeness, i.e., only nucleons and zero temperature. As usual, only the timelike component of the vector mesons $\omega\equiv\langle\omega_0\rangle$ and $\rho\equiv\langle\rho_0\rangle$ survive in the mean-field approximation. Additionally, due to parity conservation we have $\langle\pi_i\rangle=0$. The strange vector field ϕ does not couple to the nucleon. Therefore, for simplicity it is omitted in the mean-field version of the Lagrangian (17), which reads

$$\mathcal{L}_{\mathrm{kin}}=-i\bar{N}\gamma_i\nabla^i N-\frac{1}{2}\sum_{\varphi=\sigma,\zeta,\chi,\omega,\rho,A}\nabla_i\varphi\nabla^i\varphi,$$

$$\mathcal{L}_{\mathrm{BM}}+\mathcal{L}_{\mathrm{BV}}=-\bar{N}\gamma_0\left[g_{N\omega}\omega_0+g_{N\rho}\tau_3\rho_0+\frac{1}{2}e(1+\tau_3)A_0+m_N^*\gamma_0\right]N,$$

$$\mathcal{L}_{\mathrm{vec}}=\frac{1}{2}\frac{\chi^2}{\chi_0^2}(m_\omega^2\omega^2+m_\rho^2\rho^2)+g_4^4(\omega^4+6\omega^2\rho^2+\rho^4),$$

$$\mathcal{L}_0=-\frac{1}{2}k_0\chi^2(\sigma^2+\zeta^2)+k_1(\sigma^2+\zeta^2)^2+k_2\left(\frac{\sigma^4}{2}+\zeta^4\right)+k_3\chi\sigma^2\zeta+k_{3m}\chi\left(\frac{\sigma^3}{\sqrt{2}}+\zeta^3\right)-k_4\chi^4-\frac{1}{4}\chi^4\ln\frac{\chi^4}{\chi_0^4}+\frac{\delta}{3}\chi^4\ln\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0},$$

$$\mathcal{L}_{\mathrm{SB}}=-\left(\frac{\chi}{\chi_0}\right)^2[x\sigma+y\zeta]. \quad (60)$$

Equation (60) is the most general mean-field Lagrangian within our discussion of which different subsets of parameters and terms are discussed in Sec. V.

From the Lagrangian (17), the following equations of motion for the various fields are derived:

$$D\omega=-\left(\frac{\chi}{\chi_0}\right)^2 m_\omega^2\omega-4g_4^4(\omega^3+3\rho^2\omega)+g_{\omega N}\rho_B,$$

$$D\rho=-\left(\frac{\chi}{\chi_0}\right)^2 m_\rho^2\rho-4g_4^4(\rho^3+3\rho\omega^2)+g_{\rho N}\rho_3,$$

$$D\chi=-\frac{\chi}{\chi_0^2}(m_\omega^2\omega^2+m_\rho^2\rho^2)+k_0\chi(\sigma^2+\zeta^2)-k_3\sigma^2\zeta-k_{3m}\left(\frac{\sigma^3}{\sqrt{2}}+\zeta^3\right)+\left(4k_4+1+4\ln\frac{\chi}{\chi_0}-4\frac{\delta}{3}\ln\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0}\right)\chi^3+2\frac{\chi}{\chi_0^2}[x\sigma+y\zeta],$$

$$D\sigma=k_0\chi^2\sigma-4k_1(\sigma^2+\zeta^2)\sigma-2k_2\sigma^3-2k_3\chi\sigma\zeta-3k_{3m}\chi\frac{\sigma^2}{\sqrt{2}}-\frac{2\delta\chi^4}{3\sigma}+\left(\frac{\chi}{\chi_0}\right)^2 x+\frac{\partial m_N^*}{\partial\sigma}\rho_s,$$

$$D\zeta=k_0\chi^2\zeta-4k_1(\sigma^2+\zeta^2)\zeta-4k_2\zeta^3-k_3\chi\sigma^2-3k_{3m}\chi\zeta^2-\frac{\delta\chi^4}{3\zeta}+\left(\frac{\chi}{\chi_0}\right)^2 y+\frac{\partial m_N^*}{\partial\zeta}\rho_s. \quad (61)$$

TABLE II. Vacuum masses of the scalar mesons for different kinds of fits (explained in the text).

	$m_{a_0}(980)$	m_κ (900)	m_σ	m_{f_0}
C_1	953.54	995.70	473.32	1039.10
C_2	953.54	995.70	475.55	1039.10
C_3	953.54	995.70	478.56	824.17
M_1	482.41	448.96	422.79	482.41
M_2	488.55	441.41	408.79	488.55
W_1	500.83	457.49	401.54	500.83
W_2	519.95	478.34	425.14	519.95
W_3	1000.00	1255.32	480.50	1334.25

The Dirac equation for the nucleon and the equation for the photon field are of the form given, e.g., by Reinhard [45] and need not be repeated here. The densities $\rho_s = \langle \overline{N}N \rangle$, $\rho_B = \langle \overline{N}\gamma_0 N \rangle$, $\rho_3 = \langle \overline{N}\gamma_0 \tau_3 N \rangle$ can be expressed in terms of the components of the nucleon Dirac spinors in the usual way [44]. In Eqs. (61), the spatial derivatives are abbreviated by $D \equiv -\nabla^2 - (2/r)\nabla$.

The set of coupled equations are solved using an accelerated gradient iteration method following Ref. [31]. The Dirac equation for the nucleons can be cast in a modified Schrödinger equation with an effective mass. The meson field equations reduce to radial Laplace equations. In each iteration step, the coupled equations for the nuclear radial wave functions are solved for the given potentials, the corresponding densities are calculated, then the meson field equations are solved for the given densities, so that the new potentials are derived and the next iteration step can begin until convergence is achieved. The meson field equations are solved in the form

$$\left[-\frac{d^2}{dr^2} + m_{\varphi,0}^2 \right] (r\varphi^{(N+1)}) = -r f(\rho, \varphi^{(N)}), \qquad (62)$$

where $m_{\varphi,0}^2$ is the vacuum mass of the respective meson (or an arbitrary mass) which is subtracted on the right-hand side of the equation. The function $f(\rho, \varphi^N)$ stands for the interaction terms with other meson fields, the source terms coming from the nucleon density and the self-interaction terms as given above. This form achieves a five-point precision for the Laplacian by using only a three-point formula by solving for $(r\varphi)$. The scalar fields have to be solved by replacing, e.g., $\sigma \to (1 - \sigma/\sigma_0)$ to ensure the boundary condition that the field has to vanish for $r \to \infty$. The iteration is damped by taking into account only a fraction of the newly calculated density for the next iteration step.

The energy-momentum tensor can be used to obtain the total energy of the system in the standard way [44]. After eliminating the gradient terms on the fields by using the field equations, one obtains

$$E = \sum_{\alpha}^{\text{occ}} \epsilon_\alpha (2 j_\alpha + 1) - \frac{1}{2} \int d r r^2 (m_N^* \rho_s + g_{N\omega} \omega \rho_B + g_{N\rho} \rho \rho_3) + E_{\text{rearr}}. \qquad (63)$$

In the first term ϵ_α are the Dirac single particle energies and j_α is the total angular momentum of the single particle state. In nuclear matter this term becomes $4\Sigma_k (g_\omega \omega_0 + \sqrt{k^2 + m^{*2}})$. The rearrangement energy E_{rearr} is

$$E_{\text{rearr}} = \int d r r^2 \bigg[2 g_4^4 (\omega^4 + \rho^4 + 6\omega^2\rho^2 + 2\phi^4) - 2k_1(\sigma^2 + \zeta^2)^2 - 2k_2 \left(\frac{\sigma^4}{2} + \zeta^4 \right) - k_3 \chi_0 \sigma^2 \zeta - k_{3m} \chi_0 \left(\frac{\sigma^3}{\sqrt{2}} + \zeta^3 \right) + 2 \frac{\delta}{3} \chi_0^4 \ln \left(\frac{\sigma^2 \zeta}{\sigma_0^2 \zeta_0} \right) - x\sigma - y\zeta \bigg] - V_{\text{vac}}. \qquad (64)$$

The constant V_{vac} is the vacuum energy which is subtracted to yield zero energy in the vacuum. Equation (64) is the rearrangement energy for the frozen glueball model which is used for most of the fits discussed in the following. Let us now proceed to study the application to physical hadrons and hadronic matter fits.

V. CHIRAL MODELS THAT WORK

As was pointed out in Ref. [46], reproducing the nuclear matter equilibrium point is not sufficient to ensure a quantitative description of nuclear phenomenology. For this, one has to study the systematics of finite nuclei. This is done in the following for various potentials in a chiral SU(3) framework. Those include the potential of the SU(3) linear σ model, the potential of the Minnesota-group [1] and the Walecka model including nonlinear cubic and quartic self-interactions of the scalar field [19,20].

A. Potential of the linear σ model

The potential of the linear σ model is particularly interesting because the strange condensate couples to the non-strange condensate σ in such a way that it deviates from its VEVs even in the case of a system containing only nucleons. With the scale breaking logarithm included [$\mathcal{L}_{\text{scale}}$, see Eq. (54)], it reads

$$\mathcal{L}_0^C = -\tfrac{1}{2} k_0 \chi^2 I_2 + k_1 (I_2)^2 + k_2 I_4 + 2k_3 \chi I_3 + \mathcal{L}_{\text{scale}}. \qquad (65)$$

TABLE III. Condensates and nuclear matter properties at ρ_0.

	m_N^*/m_N	σ/σ_0	ζ/ζ_0	K
C_1	0.61	0.63	0.92	276.34
C_2	0.64	0.64	0.91	266.08
C_3	0.61	0.63	0.92	285.29
M_1	0.62	0.62	1	269.58
M_2	0.61	0.62	1.01	272.61
W_1	0.65	0.62	1	224.23
W_2	0.63	0.63	1	245.05
W_3	0.64	0.64	0.91	217.20

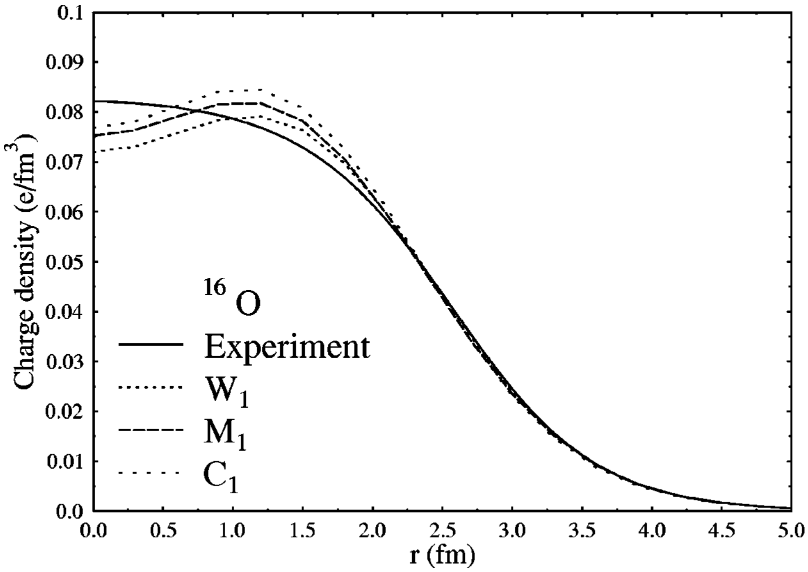

FIG. 1. Charge density for ^{16}O for the parameter sets indicated. The experimental charge density is fitted with a three-parameter Fermi model [48].

Here, the explicit symmetry breaking term of the linear σ model is used, i.e., $A_s = A_p$, which implies the same term to break the chiral symmetry in the scalar and pseudoscalar sector, respectively. In addition, the mass term of the pseudoscalar singlet is set to

$$m_{\eta_0}^2 = k_0\chi_0^2 - 4\left(\frac{k_2}{3}+k_1\right)(\sigma_0^2+\zeta_0^2)+\frac{4}{3}k_3\chi_0 \times(\zeta_0+\sqrt{2}\sigma_0) - \frac{4}{9}\delta\chi_0^4\left(\frac{1}{\sigma_0^2}+\frac{\sqrt{2}}{\sigma_0\zeta_0}\right). \quad (66)$$

This is equal to the pseudoscalar singlet mass which is obtained if M and $M^\dagger$ of the linear σ-model potential [17] are replaced by Eqs. (24).

The elements of the matrix A_p are fixed to fulfil the PCAC relations of the pion and the kaon, respectively. Therefore, the parameters of the chiral invariant potential k_0 and k_2 are used to ensure an extremum in the vacuum. As for the remaining constants, k_3 is constrained by the η' mass and k_1 is varied to give a σ mass of the order of $m_\sigma = 500$ MeV. The VEV of the gluon condensate χ_0 is fixed to fit the binding energy of nuclear matter $\epsilon_0/\rho - m_N = -16$ MeV at the saturation density $\rho_0 = 0.15$ fm^{-3}. The VEVs of the fields σ_0 and ζ_0 are constrained by the decay constants of the pion and the kaon, respectively [see Eq. (25)]. Throughout this work, the numerical values $f_\pi = 93.3$ MeV and $f_K = 122$ MeV are used

With the same potential, Eq. (65), fits with (C_1) and without (C_2) a dependence of the nucleon mass on the strange condensate ζ can be done. To see whether there is a significant effect from the gluon condensate χ at moderate densities, a nonfrozen fit is also studied (C_3) where we allow the condensate of the dilaton field to deviate from its vacuum value.

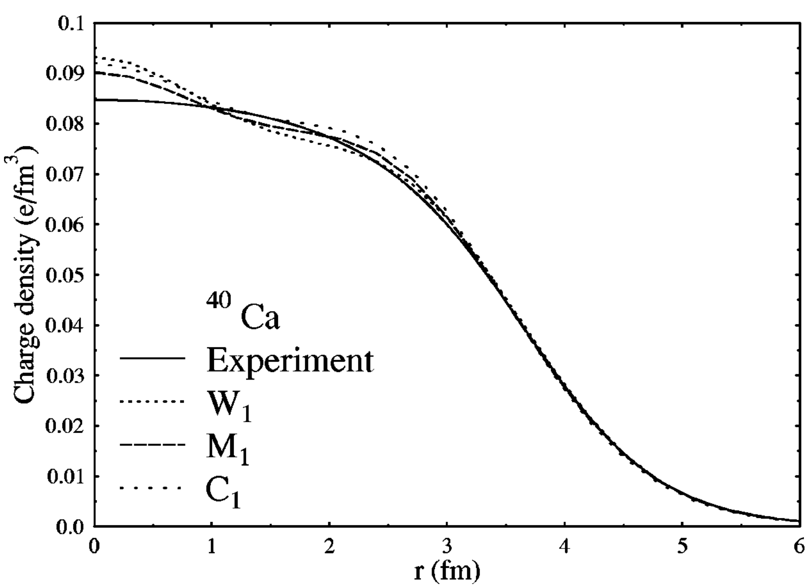

FIG. 2. As for Fig. 1, but for ^{40}C.

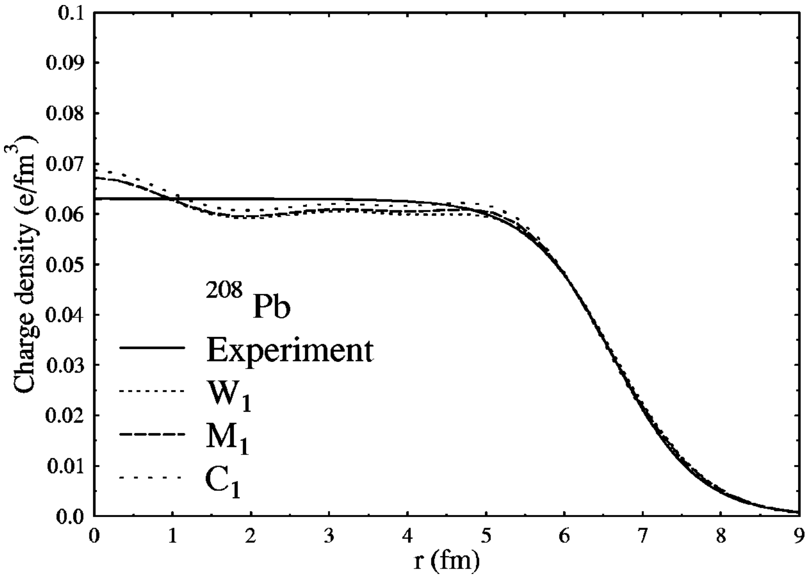

FIG. 3. As for Fig. 1, but for ^{208}Pb.

As can be seen from Table II, the hadronic masses in the vacuum have reasonable values. If the potential of Eq. (65) in combination with Eq. (66) is used, the mass of the η' meson depends on all constants k_i and on χ_0, which are also used to fit nuclear matter properties. In our fits, the pseudoscalar meson masses have the values $m_\eta = 574$ MeV and $m_{\eta'} = 969$ MeV.

According to Table III, the values of the effective nucleon mass and the compressibility in the medium (at ρ_0) are reasonable. For a fine-tuning of the single particle energy levels and a lowering of the effective nucleon mass, a quartic term for vector mesons [see Eq. (44)] has to be taken into account. Once the parameters have been fixed to nuclear matter at ρ_0 the condensates and hadron masses at high baryon densities can be investigated.

In Fig. 1 we display the scalar mean fields σ, ζ, and χ as a function of the baryon density for vanishing strangeness. One sees that the gluon condensate χ stays nearly constant when the density is raised, so that the approximation of a frozen glueball is reasonable. The strange condensate ζ is only reduced by about 10% from its vacuum expectation value. This is not surprising since there are only nucleons in the system and the nucleon-ζ coupling is fairly weak. The main effect occurs for the non-strange condensate σ. The field has dropped to 30% of its vacuum expectation value at 4 times normal nuclear density. If we extrapolate to even higher densities one observes that the σ field does not change significantly, so for all fields a kind of saturation takes place at higher densities.

From Eq. (30) one sees that the baryon masses are generated by the nonstrange condensate σ and the strange condensate ζ. So the change of these scalar fields causes the change of the baryon masses in medium.

The density dependence of the effective baryon masses m_i^* is shown in Fig. 2. When the density in the system is raised, the masses drop significantly up to 4 times normal nuclear density. This corresponds to the above-mentioned behavior of the condensates. Furthermore, one observes that

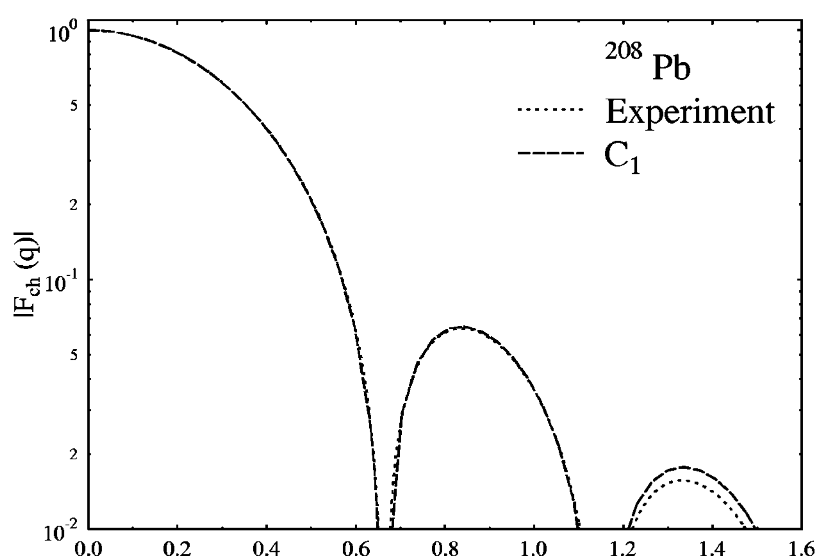

FIG. 4. The charge form factor of ^{208}Pb from the parameter set C_1 is compared to experiment [48].

the change of the baryon mass differs with the strange quark content of the baryon. This is caused by the different behavior of the nonstrange condensate σ which mainly couples to the nonstrange part of the baryons, and the strange condensate ζ which couples mainly to the strange part of the baryons. Without changing the parameters of the model, the properties of nuclei can be predicted readily.

The charge densities of ^{16}O, ^{40}Ca, and ^{208}Pb are found to have relatively small radial oscillations (Figs. 3, 4, and 5), though such oscillations cannot be found in the data.[4] The experimental charge densities are taken from Ref. [48], where a three-parameter Fermi model was used to fit the data.[5] The charge radii are close to the experimental observation (Table IV).

For the charge densities in coordinate space, it is difficult to assess the level of agreement with experiment. Therefore, we show exemplarily the charge form factor of ^{208}Pb for the parameter set C_1 in momentum space (Fig. 6). The low-momentum behavior of the charge form factor is well reproduced, although there is some departure from experiment [48] at higher momentum.

The binding energies of ^{16}O, ^{40}Ca, and ^{208}Pb are in reasonable agreement with the experimental data. Nevertheless they are low by approximately 0.5 MeV. To correct this, a direct fit to nuclear properties has to be done [39]. As can be seen from Table IV, models C_1 and C_2 exhibit a spin-orbit splitting that lies within the band of the experimental uncertainty given in Ref. [49]. The single-particle energies of ^{208}Pb are close to those of the Walecka model extended to include nonlinear σ^3 and σ^4 terms [20] or the model [1], both for neutrons (Fig. 7) and for protons (Fig. 8). This is encouraging since neither the nucleon-scalar meson nor the nucleon-ρ meson coupling constants can be adjusted to nuclear matter or nuclei properties, in contrast to the Walecka model [20].

B. Minnesota model

By incorporating the physics of broken scale invariance in the form of a dilaton field and a logarithmic potential, the Minnesota group succeeded in formulating a model with equally good results as those of Ref. [20] in the context of a linearly realized symmetry [1]. When switching to SU(3), it is necessary to use a nonlinear realization, because there is no freedom in the linear representation to correct for the unrealistic hyperon potentials [17] if one adopts a Yukawa-type baryon-meson interaction.

With a potential of the form

$$\mathcal{L}_0^M = -\tfrac{1}{2} k_0 \chi^2 I_2 + \mathcal{L}_{\rm scale} \tag{67}$$

the model [1] is embedded in SU(3). Those results can be reproduced exactly (M_1 fit). Here, the parameter for explicit symmetry breaking [see Eq. (58)] $A_s = \mathrm{diag}(0,0,y)$ is used, where y is adjusted as to eliminate the terms linear in ζ. For $y=0$ (or, generally, a matrix A_s proportional to the unit matrix) the vacuum is $SU(3)_V$ invariant. Even with the SU(3) constraint on the nucleon-ρ coupling, $g_{N\rho} = g_{N\omega}/3$ and with a coupling of the strange condensate to the nucleon according to Eq. 30 (fit M_2), the results are of the same quality as those obtained in Ref. [1].

Generally, the potential (67), in which the two condensates σ and ζ are decoupled from each other, leads to scalar masses which are all of the order of 500 MeV. To correct this failure, additional terms have to be included which lead to a σ/ζ mixing, as, e.g., the linear σ model potential (see Sec. V A).

C. Chiral Walecka model

As in the linear σ model, the coupling constant of the nucleon to the σ meson $g_{N\sigma}$ is constrained to yield the correct nucleon mass

$$g_{N\sigma} = \frac{m_N}{f_\pi}. \tag{68}$$

To reproduce exactly the results obtained in the nonlinear σ-ω model [20], it is necessary to keep this coupling as a free parameter. For that purpose, we introduce the additional term

$$-m_{\rm av} \mathrm{Tr}\, \overline{B} B, \tag{69}$$

which should be a small correction to the dynamically generated nucleon mass. In the nonlinear realization of chiral symmetry, this term is chirally invariant.

In order to obtain a chiral model which is capable of exactly reproducing the results of the nonlinear Walecka

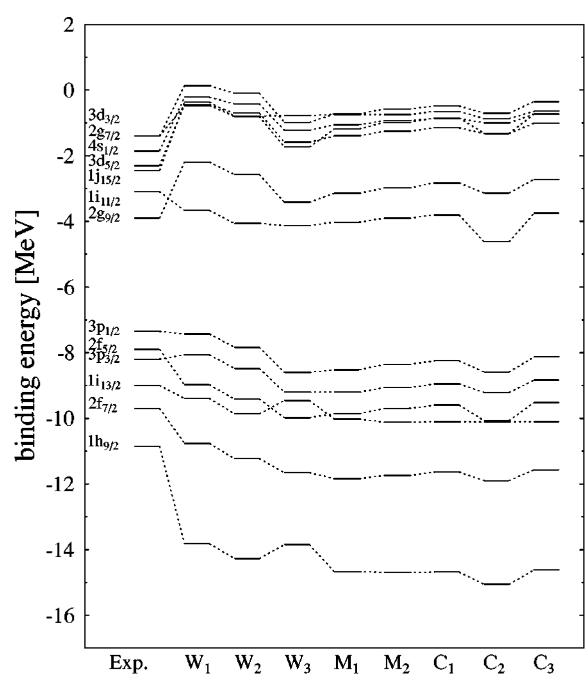

FIG. 5. Single particle energies of neutrons near the Fermi energy in ^{208}Pb. Experimentally measured levels are compared with predictions from various potentials used (see text).

[4] Similar problems exist also for nonchiral models, for a discussion see Refs. [46,47].

[5] A more sophisticated model-independent analysis by means of an expansion for the charge distribution as a sum of Gaussians would lead to an even closer correspondence between our results and the experimental data.

TABLE IV. Bulk properties of nuclei: Prediction (left) and experimental values (right) for binding energy E/A, charge radius r_{ch}, and spin-orbit splitting of oxygen (^{16}O with $\delta p \equiv p_{3/2} - p_{1/2}$), calcium (^{40}Ca with $\delta d \equiv d_{5/2} - d_{3/2}$), and lead (^{208}Pb with $\delta d \equiv 2d_{5/2} - 2d_{3/2}$).

	^{16}O			^{40}Ca			^{208}Pb		
	E/A	r_{ch}	δp	E/A	r_{ch}	δd	E/A	r_{ch}	δd
Exp.	−7.98	2.73	5.5–6.6	−8.55	3.48	5.4–8.0	−7.86	5.50	0.9–1.9
C_1	−7.30	2.65	6.05	−7.98	3.42	6.19	−7.56	5.49	1.59
C_2	−7.40	2.65	5.21	−8.07	3.42	5.39	−7.61	5.50	1.41
C_3	−7.29	2.65	6.06	−7.98	3.42	6.22	−7.54	5.49	1.61
M_1	−7.19	2.68	5.60	−7.93	3.45	5.83	−7.56	5.53	1.53
M_2	−7.34	2.67	5.90	−8.03	3.44	6.08	−7.61	5.52	1.58
W_1	−8.28	2.63	5.83	−8.63	3.42	5.91	−7.71	5.51	1.43
W_2	−8.23	2.63	5.84	−8.60	3.42	5.94	−7.75	5.51	1.45
W_3	−7.98	2.67	5.23	−8.47	3.44	5.45	−7.72	5.55	1.33

model [20], it is necessary to include only terms in the meson-meson potential, in which both condensates σ and ζ are decoupled from each other:

$$\mathcal{L}_0^W = -\tfrac{1}{2} k_0 \chi^2 I_2 + k_{3m} \chi I_{3m} + k_2 I_4 . \qquad (70)$$

Here, the scale breaking potential is neglected by taking the frozen glueball limit and setting $\delta = 0$. To allow for a free adjustment of the parameters k_0, k_{3m}, and k_4 to nuclear matter properties, A_s is set to

$$A_s = \mathrm{diag}(x,x,y). \qquad (71)$$

With x and y one then has two additional parameters to eliminate linear fluctuations in σ and ζ. The symmetry in the scalar sector is only broken explicitly if $y \neq x$.

The σ field used here has a nonvanishing vacuum expectation value as a result of the spontaneous symmetry breaking.[6] To compare this field σ with the field s used in the nonlinear Walecka model [20], one has to perform the transformation

$$\sigma = \sigma_0 + s. \qquad (72)$$

After inserting this transformation into the potential (60), one can identify the parameters used here with those of Ref. [20],

$$m_\sigma^2 = k_0 \chi_0^2 - 3 k_{3m} \chi_0 \sigma_0 \sqrt{2} - 6 k_2 \sigma_0^2, \qquad (73)$$

$$\kappa = -3 k_{3m} \chi_0 \sqrt{2} - 12 k_2 \sigma_0 , \qquad (74)$$

$$\lambda = -12 k_2 . \qquad (75)$$

Therefore, the results obtained in the framework of the Walecka model [20] can be reproduced exactly[7] within this ansatz (from now one denoted W_1) given a special choice of explicit symmetry breaking. However, in contrast to the Walecka model the hadron masses are generated spontaneously. The masses of the scalar multiplet as resulting from the parameterization of Ref. [20] are of the order of 500 MeV, as can be read off Table II. To correct for this, terms which mix the σ with the ζ have to be added (see below).

A problem, which is well known in the context of the Boguta-Bodmer model, exists here, too: For certain combinations of parameters the potential is not bound from below. To cure this problem, one can introduce additional terms, as was done in Ref. [51]. Another, more physical, way to circumvent this problem is to use the physics of broken scale invariance, as in Ref. [1] or the models used in Secs. V A and V B.

Beyond exactly reproducing an existing successful model, it is interesting to ask whether improvements in the phenom-

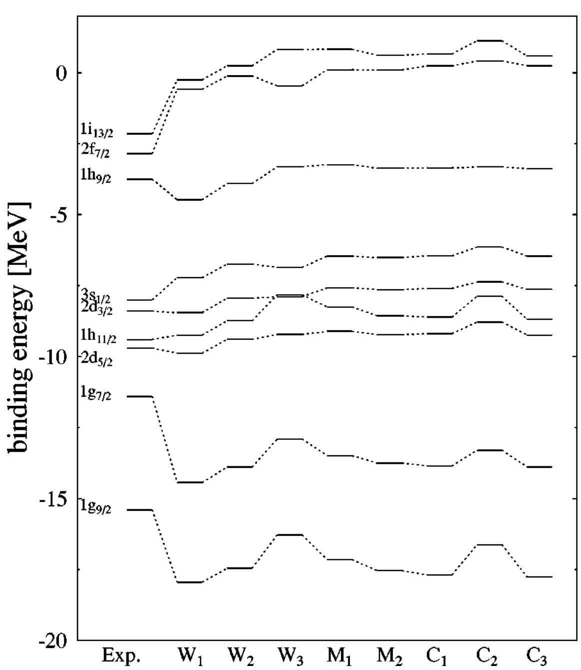

FIG. 6. As for Fig. 5, but for protons.

[6]In Ref. [50], the results of the Walecka model could also be reproduced in a nonlinear SU(2) chiral approach. There, however, the limit $m_\sigma \to \infty$ has been performed introducing in a second step a light scalar σ field mimicking correlated 2π exchange. In addition, the hadron masses were not generated dynamically.

[7]For this, a small d-type admixture of the baryon–vector-meson coupling is necessary, since the relation $g_{N\omega} = 3 g_{N\rho}$ is not fulfilled exactly in the Walecka model. (For the set W_1, $\alpha_v = 0.95$ is used.)

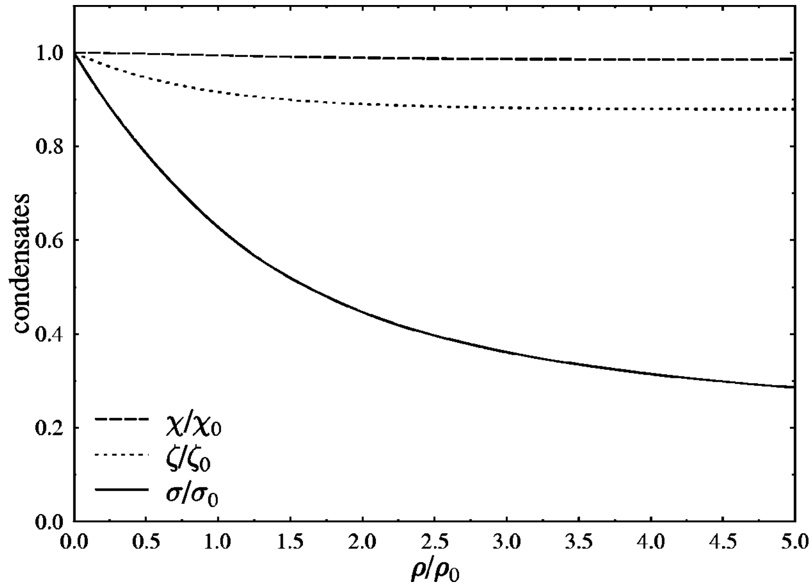

FIG. 7. Scalar condensates σ, ζ, and χ as a function of the baryon density for zero net strangeness.

enology can be made as compared to the Walecka model. This could mean either reducing the amount of parameters needed, or a significantly improved description of existing data, or the description of a broader range of physical phenomena.

Let us first consider the limit $m_{\rm av}=0$. Then, the relation

$$g_{N\sigma}=\frac{m_N}{f_\pi} \tag{76}$$

known from the linear σ model is valid. To reproduce exactly the results of Ref. [20] (fit W_1), $m_{\rm av}=32$ MeV, which is about 3% as compared to $g_{N\sigma}\sigma_0$ and which is roughly of the same order as the sum of the current quark masses in the baryon. Indeed, the model (fit W_1) does not give worse results than the model W_2 where the relation (76) and the SU(3)-symmetry constraint $g_{N\omega}=3g_{N\rho}$, corresponding to a value $a_v=1$ [Eq. (34)] is used.

Next, it is desirable to have masses for the scalar nonet which are (except for m_σ) on the order of 1 GeV. This can be achieved by admitting mixing between the σ and the ζ by including the term $k_1(I_2)^2$ to the scalar potential (70) (fit W_3). Therefore, in the SU(3) framework, even for a pure system of only nucleons it is *necessary* to take the strange condensate ζ into account.

D. Hyperon central potentials

As discussed in the Introduction, the reasons for unrealistic hyperon potentials in the linear σ model are the different types of coupling of the spin-0 and spin-1 mesons to baryons and a direct coupling of the σ with the strange condensate. The second reason produces too deep hyperon central potentials since the additional attraction stemming from the ζ cannot be compensated with an additional repulsion from the ϕ.

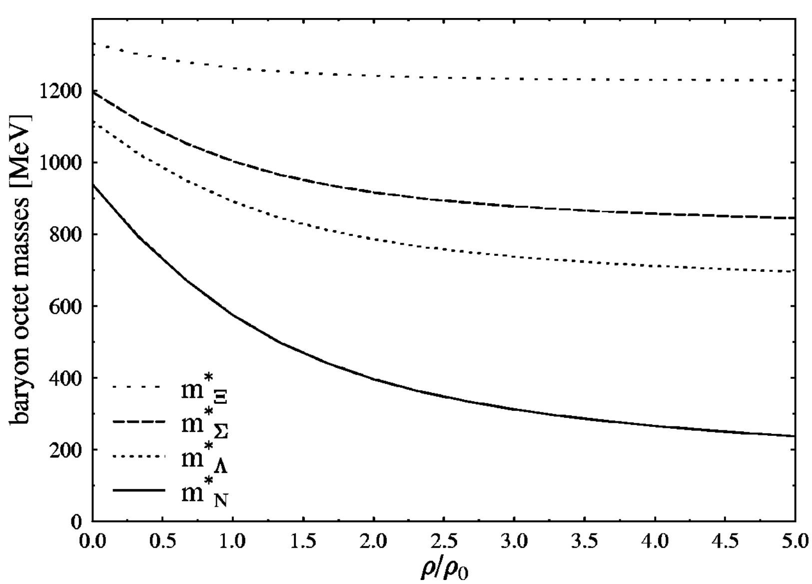

FIG. 8. Effective baryon masses as a function of the baryon density for zero net strangeness.

TABLE V. Baryon potentials and asymmetry energy at ρ_0. The hyperon potentials of the fits C_1, C_2, C_3, and W_3 are corrected with the explicit symmetry breaking term of Eq. (77).

	U_N	U_Λ	U_Σ	U_Ξ	a_4
C_1	−71.04	−28.23	3.17	30.3	40.41
C_2	−68.75	−30.50	−6.46	21.1	37.29
C_3	−71.06	−28.61	2.56	29.4	40.23
M_1	−70.18	−46.78	−46.78	−23.39	40.59
M_2	−70.67	−47.96	−28.71	−15.62	41.21
W_1	−68.84	−48.87	−42.92	−25.92	37.92
W_2	−69.02	−46.01	−46.01	−23.01	36.06
W_3	−68.21	−28.10	−28.10	12.0	35.22

This has a vanishing expectation value in nuclear matter at zero net strangeness since it does neither couple to the nucleon nor to the ω.

Both effects can be switched off in the nonlinear realization (fits W_1, W_2, M_1, and M_2). However, even in those fits, the experimentally extracted value for the Λ central potential of $U_\Lambda = -28 \pm 1$ MeV [52] cannot be reproduced. The nucleon central potential of $U_N \approx -70$ MeV is too deep: $\frac{2}{3} U_N \neq -28$ MeV. A shallower potential for the nucleon leads to a too small spin-orbit splitting of the energy levels of nucleons. Therefore, both the central potentials of the nucleon and of the Λ cannot be reproduced if the f-type quark-model motivated coupling constant is used for both baryon vector-meson and baryon scalar-meson interactions. The sensitive cancellation of large vector and scalar potentials amplifies and overemphasizes a (small) deviation from exact symmetry relations. Fortunately, explicit symmetry breaking can be introduced in the nonlinear realization without affecting, e.g., the PCAC relations. This allows for a parametrization of the hyperon potentials. Here, the term

$$\mathcal{L}_{\mathrm{hyp}} = m_3 \mathrm{Tr}(\overline{B}B + \overline{B}[B,S]) \mathrm{Tr}(X - X_0) \tag{77}$$

with the same $S^a_b = -\frac{1}{3}[\sqrt{3}(\lambda_8)^a_b - \delta^a_b]$ as in Sec. III B 1 is used. The explicit symmetry breaking term contributes only for hyperons at finite baryon densities along the hypercharge direction. With the parameter m_3 adjusted to the Λ potential of -28 MeV, the other hyperon potentials are determined. This leads to a repulsive Ξ potential ranging from 10–30 MeV (Table V). We do not take the numbers for the Ξ central potential too seriously because of the strongly varying values depending on the specific model and on the choice of the explicit symmetry breaking term.

VI. CONCLUSIONS

We studied a chiral SU(3) σ-ω-type model including the dilaton associated with broken scale invariance of QCD. Within such an approach it is possible to describe the multiplets of spin-0, spin-1, and spin-1/2 particles with reasonable values for their vacuum masses as well as the nuclear matter equilibrium point at $\rho_0 = 0.15$ fm^{-3} and the properties (e.g., binding energies, single particle energy spectra, charge radii) of nuclei. In contrast to other approaches to the nuclear many-body problem, all hadron masses are mainly generated through spontaneous symmetry breaking leading to a nonzero vacuum expectation value of a nonstrange (σ) and a strange (ζ) condensate. In the linear σ model, the vacuum expectation value of those two condensates is constrained by the decay constants of the pion (f_π) and of the kaon (f_K).

It was shown, however, that a SU(3) chiral model in the linear representation of chiral symmetry fails to simultaneously account for nuclei and hyperon central potentials (see also Ref. [17]). With that approach, it is either possible to describe nuclei with unrealistically low-high hyperon potentials *or* nuclear matter with reasonable hyperon potentials. This limitation does not exist if one switches to the nonlinear realization of chiral symmetry.[8] This is because of the following reasons.

First, an f-type baryon-scalar meson interaction can be constructed which does not destroy the balance between huge attractive and repulsive forces from the scalar and vector sector, respectively. This type of interaction improves the values for the hyperon potentials, though they remain too attractive.

Secondly, the nonstrange and strange condensates can be decoupled from each other, which reduces the level of attraction from the strange condensate. However, a decoupling of those condensates leads to masses for the whole scalar multiplet of the order of 500 MeV. A coupling of the condensates implying a mixing of the σ and ζ scalar masses is necessary for a correct description of the hadronic spectrum.

In contrast to the linear representation of chiral symmetry, it is possible to add an explicit symmetry breaking term which reduces the depth of the hyperon potentials without destroying basic theorems in the vacuum as the PCAC relations for the pseudoscalar mesons. However, in that direction further work has to be done to reduce the ambiguity of the explicit symmetry breaking term.

Within the nonlinear realization of chiral symmetry one also has the flexibility to construct some special potentials (in which the nonstrange and strange sectors are decoupled from each other) and to reproduce the results of SU(2) models, as, e.g., those obtained with the SU(2) model of the Minnesota group [1] and the nonlinear Boguta-Bodmer model [19].

However, to account for the scalar nonet masses, it is necessary to include terms which couple the nonstrange to the strange condensate. Particularly, it is possible to describe reasonably vacuum hadron masses, nuclear matter and nuclei within a single chiral SU(3) model in the nonlinear realization of chiral symmetry using the potential and some constraints of the linear σ model.

The results are similar, whether the strange condensate is allowed to couple to the nucleon, or not. However, only in the first case is it possible to reproduce the experimentally known baryon masses without an additional explicit symmetry breaking term except for the one which can be associated with the current quark masses and which produces finite

[8]However, we kept some constraints from the linear σ model such as, e.g., the dependence of the condensates on the decay constants in order to reduce the amount of the free parameter.

masses for the pseudoscalar bosons. If the nucleon mass is entirely generated by the nonstrange σ condensate, some additional explicit symmetry breaking is necessary to account for the correct baryon masses.

To improve our results, a direct fit to spherical nuclei, as was done in Ref. [31] has to be performed. This is currently under investigation [39]. Further studies are under way to investigate the effect of spin-3/2 resonances in hot and dense matter, the meson-baryon scattering and the chiral dynamics in transport models within *one single model* [53].

ACKNOWLEDGMENTS

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APPENDIX

The various hadron matrices used are (suppressing the Lorentz indices)

$$X=\frac{1}{\sqrt{2}}\sigma^a\lambda_a=\begin{pmatrix} (a_0^0+\sigma)/\sqrt{2} & a_0^+ & \kappa^+ \\ a_0^- & (-a_0^0+\sigma)/\sqrt{2} & \kappa^0 \\ \kappa^- & \overline{\kappa^0} & \zeta \end{pmatrix}, \tag{A1}$$

$$V=\frac{1}{\sqrt{2}}v^a\lambda_a=\begin{pmatrix} (\rho_0^0+\omega)/\sqrt{2} & \rho_0^+ & K^{*+} \\ \rho_0^- & (-\rho_0^0+\omega)/\sqrt{2} & K^{*0} \\ K^{*-} & \overline{K^{*0}} & \phi \end{pmatrix}, \tag{A2}$$

$$B=\frac{1}{\sqrt{2}}b^a\lambda_a=\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix} \tag{A3}$$

for the scalar (X), vector (V), baryon (B), and similarly for the axial vector meson fields. A pseudoscalar chiral singlet $Y=\sqrt{2/3}\,\eta_0\mathbb{1}$ can be added separately, since only an octet is allowed to enter the exponential 6.

The notation refers to the particles of the listed by the Particle Data Group (PDG) [36], though we are aware of the difficulties to directly identify the scalar mesons with the physical particles [54]. However, note that there is increasing evidence which supports the existence of a low-mass broad scalar resonance, the $\sigma(560)$ meson, as well as a light strange scalar meson, the $\kappa(900)$ (see Ref. [55], and references therein).

There is an experimental indication for a nearly ideal mixing between the octet and singlet states. Hence, the nine vector mesons are summarized in a single matrix. The relevant fields in the SU(2) invariant vacuum v_μ^0 and v_μ^8 (corresponding to λ_0 and λ_8, respectively) are assumed to have the ideal mixing angle $\sin\theta_v=1/\sqrt{3}$. This yields

$$\phi_\mu=v_\mu^8\cos\theta_v-v_\mu^0\sin\theta_v=\frac{1}{\sqrt{3}}(\sqrt{2}v_\mu^0+v_8^\mu),$$

$$\omega_\mu=v_\mu^8\sin\theta_v+v_\mu^0\cos\theta_v=\frac{1}{\sqrt{3}}(v_\mu^0-\sqrt{2}v_8^\mu). \tag{A4}$$

Similarly, for the scalar mesons

$$\sigma=\frac{1}{\sqrt{3}}(\sqrt{2}\sigma^0+\sigma^8), \tag{A5}$$

$$\zeta=\frac{1}{\sqrt{3}}(\sigma^0-\sqrt{2}\sigma^8) \tag{A6}$$

is used, where σ^0 and σ^8 belong to λ_0 and λ_8, respectively. However, there is no experimental indication for an ideal mixing of the scalar mesons σ and ζ. In general, depending on the interaction potential, mixing between σ and ζ occurs (see Sec. V A). This is also suggested by effective instanton-induced interactions of 't Hooft type [56].

The masses of the various hadrons are generated through their couplings to the scalar condensates, which are produced via spontaneous symmetry breaking in the sector of the scalar fields. There are nonvanishing vacuum expectation values (VEVs) of only two meson fields: of the nine scalar mesons in the matrix X only the VEVs of the components proportional to λ_0 and to the hypercharge $Y\sim\lambda_8$ are nonvanishing, and the vacuum expectation value $\langle X\rangle$ reduces to

$$\langle X\rangle=\frac{1}{\sqrt{2}}(\sigma^0\lambda_0+\sigma^8\lambda_8)\equiv\mathrm{diag}\left(\frac{\sigma}{\sqrt{2}},\frac{\sigma}{\sqrt{2}},\zeta\right), \tag{A7}$$

in order to preserve parity invariance and assuming, for simplicity, SU(2) symmetry[9] of the vacuum.

[9]This implies that isospin breaking effects will not occur, i.e., all hadrons of the same isospin multiplet will have identical masses. The electromagnetic mass breaking is neglected.

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