Relativistic calculations of induced polarization in $^{12}\mathrm{C}(e,e'\vec{p})$ reactions

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Relativistic calculations of the induced proton polarization in quasifree electron scattering on ^{12}C are presented. Good agreement with the experimental data of Woo *et al.* is obtained. The relativistic calculations yield a somewhat better description of the data than the nonrelativistic ones. Differences between the two approaches are more pronounced at larger missing momenta suggesting further experimental work in this region. [S0556-2813(99)05606-X]

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A measurement of induced proton polarization in the $^{12}\mathrm{C}(e,e'\vec{p})$ reaction has been reported recently by Woo *et al.* [1]. The data explore the low missing momentum region from 0 to 250 MeV/c in constant q-ω kinematics. The data were compared to nonrelativistic DWIA calculations using the effective momentum approximation. Momentum distributions used in these calculations were obtained by fitting the $^{12}\mathrm{C}(e,e'p)$ data of van der Steenhoven *et al.* [2]. The final state interactions of the outgoing proton were included using nonrelativistic optical potentials. Two such potentials were compared, one obtained by a reduction of the relativistic potentials of Cooper *et al.* [3], and the other based on an empirical effective interaction (EEI). Both models provide reasonable agreement with the data, with a slight preference for the EEI model when a proton is removed from the $1p_{3/2}$ shell. The results from both nonrelativistic models pass through the lowest missing momentum point for the knockout of a $1p_{3/2}$ proton. The change in polarization with increasing missing momentum is reproduced, but the calculations tend to fall below the data at higher missing momenta. The calculations predict that the polarization should rapidly become negative as the missing momentum is increased beyond 220 MeV/c.

For the set of data attributed to knockout of a $1s_{1/2}$ proton, the nonrelativistic calculations follow the trend of the data for small missing momenta, but become large and negative for missing momenta beyond 150 MeV/c. The data, on the other hand, seem to indicate a polarization becoming positive at these larger values of missing momenta.

There also exist relativistic calculations for the same reaction, which have primarily considered the cross section, or equivalently, the single-particle momentum distribution. These calculations have mainly been reported by Udias *et al.* [4,5], Jin and Onley [6], and Hedayatipoor *et al.* [7]. Relativistic calculations of proton polarization for the reaction $^{16}\mathrm{O}(e,e'\vec{p})$, have been reported previously by Johansson and Sherif [8].

In the present paper we compare our full relativistic calculations to the new data presented in Ref. [1]. We also point out differences between the results of the current model and those discussed in Ref. [1].

In the following text we outline the relativistic calculations for the quasifree electron scattering reaction, and discuss how the proton polarization is calculated. The results of our relativistic calculations are then presented along with discussion of the data and calculations of Woo *et al.*

The relativistic calculations of the amplitude, in the one photon exchange model for the $(e,e'p)$ process, are discussed in Refs. [7,9]. The main results are given briefly here in the notation of Johansson and Sherif [9]. (Note that we use the de Forest prescription $cc2$, as discussed in Ref. [7].) We do not include Coulomb distortion in the leptonic part of the amplitude as this is not expected to be important for the light nucleus considered here [10,11].

The relativistic expression for the differential cross section leading to a specific final state of the residual nucleus can be written as

$$\frac{d^3\sigma}{d\Omega_p d\Omega_f dE_f}=\frac{2}{(2\pi)^3}\frac{\alpha^2}{\hbar c}\times\left[\frac{(m_e c^2)^2 Mc^2|\boldsymbol{p}_p|c}{[(qc)^2]^2}\frac{|\boldsymbol{p}_f|c}{E_i}\right]\times\frac{c}{v_{\mathrm{rel}}}\frac{1}{R}\frac{\mathcal{S}_{J_iJ_f}(J_B)}{2J_B+1}\sum_{\mu M_B\nu_f\nu_i}|e^{\beta}_{\nu_f\nu_i}N^{\mu M_B}_{\beta}|^2, \tag{1}$$

where ν_i and ν_f are the spin projections of the incoming and outgoing electrons, respectively, while M_B and μ are the spin projections of the bound and continuum protons. The four-momenta of the initial and final electrons are p_i and p_f, respectively, while the final proton four-momentum is p_p. The four-momentum of the exchanged photon is q and is calculated as the difference between the initial and final electron four-momenta $q=p_i-p_f$. The four-momentum of the recoil nucleus is p_R, and the initial four-momentum of the struck proton, denoted p_m, is often referred to as the missing momentum. The recoil factor R is given in any frame by [12]

$$R=1-\frac{E_p}{E_R}\frac{1}{|\boldsymbol{p}_p|^2}\boldsymbol{p}_p\cdot\boldsymbol{p}_R. \tag{2}$$

The matrix element $N^{\mu M_B}_{\beta}$ which involves nuclear wave functions, can be written in the form

$$N_\beta^{\mu M_B} = \int d^3x \Psi_\mu^\dagger(p_p, \boldsymbol{x}) \Gamma_\beta \Psi_{J_B, M_B}(\boldsymbol{x}) \exp(i\boldsymbol{q}\cdot\boldsymbol{x}), \quad (3)$$

where the wave functions of the continuum and bound nucleons, denoted Ψ_μ and Ψ_{J_B,M_B}, respectively, are solutions of the Dirac equation containing appropriate potentials [13]. The 4×4 matrix Γ_β, operating on the nucleon spinors, and the four-vector which comes from the electron vertex, $e^\beta_{\nu_f\nu_i}$ in Eq. (1), are given in detail in Eqs. (2.8) and (2.9) of Ref. [7].

We define a matrix element which is a function of the spin projections of the initial and final particles by the relation

$$T^{\mu M_B}_{\nu_f\nu_i} = e^\beta_{\nu_f\nu_i} N^{\mu M_B}_\beta, \quad (4)$$

where summation over β is implied. The polarization of the proton along the y axis can then be written as

$$P_y = -2\frac{\mathrm{Im}\sum_{M_B\nu_f\nu_i} T^{1/2 M_B}_{\nu_f\nu_i}[T^{-1/2 M_B}_{\nu_f\nu_i}]^*}{\sum_{\mu M_B\nu_f\nu_i} |T^{\mu M_B}_{\nu_f\nu_i}|^2}. \quad (5)$$

We utilize kinematics in which the incident electron momentum defines the z axis, and the final electron momentum lies in the x-z plane with $\phi_e = 0°$, while the final proton momentum has $\phi_p = 180°$. This means that the polarization which we calculate is the negative of the polarization reported by Woo *et al.* We multiply our calculated polarization P_y by the factor (-1) to conform with the sign convention of Woo *et al.*

The present relativistic calculations use bound state wave functions generated using the Hartree potentials of Blunden and Iqbal [14]. We also use a bound state wave function generated using phenomenological Woods-Saxon potentials in order to test the sensitivity of the results to changes in the description of the bound state. The proton optical potentials are taken from Cooper *et al.* [3]. There are several sets of optical potentials available, some of which are energy dependent (E-dep) and constructed from a fit to data for a specific nucleus, such as ^{12}C, ^{16}O, and ^{40}Ca, in the proton kinetic energy range of $\sim$25 MeV to 1 GeV. Other potentials are parametrized in terms of target mass as well as proton energy ($E+A$-dep) and can be used to generate potentials for which no proton elastic scattering data exist. We shall perform calculations using both types of potentials.

Our results are compared in Fig. 1 with the nonrelativistic EEI calculations of Ref. [1]. The values of q and ω used in the nonrelativistic calculations were provided to us by Kelly [15]. Our values of q and ω are consistent with those used in the nonrelativistic calculations if we ignore the electron mass. For the $1p_{3/2}$ case, Fig. 1(a), the relativistic calculations provide a slight improvement over the nonrelativistic results. This is true for both types of optical potentials used in the relativistic calculations. We note that the differences between relativistic and nonrelativistic calculations are accentuated at higher missing momenta. There are also differences at very small p_m, with the relativistic model providing a 10% smaller polarization at missing momenta around 20 MeV/c.

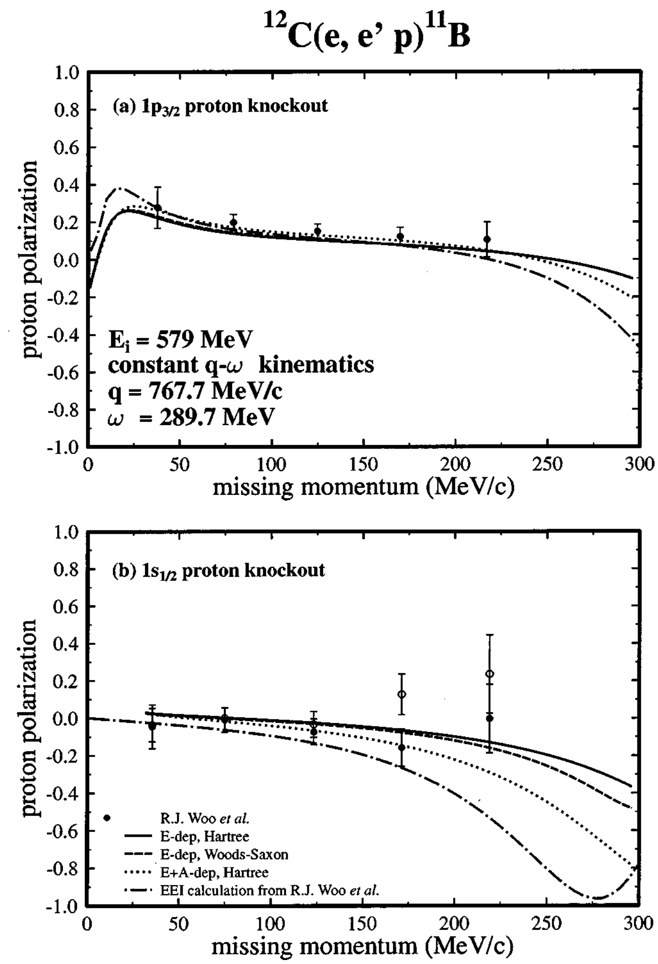

FIG. 1. Polarization of the knocked-out proton in the ^{12}C$(e,e'\vec{p})^{11}$B reaction. The energy of the incident electron is 579 MeV, with constant q-ω kinematics. The Hartree bound state wave functions are from Ref. [14] while the proton optical potentials are from Ref. [3]. (a) Knockout of a $1p_{3/2}$ proton. (b) Knockout of a $1s_{1/2}$ proton. Solid curves — Hartree binding potential and E-dep optical potential for ^{12}C. Dashed curves — Woods-Saxon binding potential and E-dep optical potential for ^{12}C. Dotted curves — Hartree binding potential and $E+A$-dep optical potential, fit 1. Dot-dashed curves — EEI calculations from Ref. [1]. The data are from Ref. [1]. Closed circles denote missing energy in the range $28 < E_m < 39$ MeV, and open circles denote missing energy in the range $39 < E_m < 50$ MeV.

The calculations for $1s_{1/2}$ proton knockout are shown in Fig. 1(b). It is more evident in this case that the relativistic calculations provide better agreement with the data, particularly at large missing momenta. (They are also better than the nonrelativistic calculations using the EDAIC potential reported in Ref. [1], but not shown here.) The most prominent feature here is that the nonrelativistic calculations with the EEI potential produce a large negative polarization in the region $p_m = 250$–300 MeV/c. The EDAIC potential produces a shallower minimum in the same region. By contrast the relativistic calculations indicate that the minimum would be at missing momenta larger than 300 MeV/c. Because of the large size of the error bars for the data points at the largest missing momenta, the behavior of the polarization is not well constrained in this region, but the relativistic calculations seem to be following the trend of the data somewhat better than the results of the nonrelativistic model.

Sensitivity to changes in the binding potentials has been examined by performing calculations using both Dirac Hartree and Woods-Saxon binding potentials. The Hartree potentials result in a binding energy that is slightly smaller than the experimental value. The Woods-Saxon potentials reproduce the experimental binding energy and also provide an rms radius for the bound state that is within one percent of that found from the Hartree potentials. We find little difference between the two binding potentials for the region of missing momenta considered in the present study. This occurs because the momentum-space wave functions for these bound states are very similar in the low momentum region explored in these kinematics.

We stress that the issue here is not one of final state interactions, since we are using the same potentials used by Woo *et al.* The issue is the difference between results arising in the relativistic and nonrelativistic treatments of these reactions. We have thus shown that the relativistic calculations appear capable of achieving better agreement with the nucleon polarization data than the nonrelativistic ones. This is consistent with observations made earlier in polarized nucleon scattering experiments. We also note a hint of large differences between the two calculations at large missing momenta. This suggests the advisability of pushing the measurements further into the high missing momentum region. It is expected that such measurements would strongly test both models and also clarify the role, if any, of two body currents in these reactions.

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