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Final conditions in high energy heavy ion collisions

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Motivated by recent experimental observations, we discuss the freeze-out properties of the fireball created in central heavy ion collisions. We find that the freeze-out parameters, temperature T, and velocity gradient near the center of the fireball are similar for different colliding systems and beam energies. This means that the transverse flow is stronger in collisions of heavy nuclei than that of light ones. [S0556-2813(99)06205-6]

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The system that is created in relativistic heavy ion collisions can have both longitudinal and transverse expansion (see, e.g., [1]). In order to study the hadronic experimental data [2,3], one needs a description of the final stage of collisions. At the region near midrapidity the boundary effect, due to the finite length of the hydrodynamic tube in the longitudinal direction, can be neglected [4]. Then it is possible to use Bjorken's model for longitudinal expansion: $v_L = z/t$. The same quasi-inertial flow is inherent to the Landau model at the freeze-out stage [5,6]. In this approach, the parameter $\tau = \sqrt{t^2 - z^2}$ was introduced to describe the proper time of the expanding system. For transverse expansion, we will use a rather general picture proposed in Ref. [7] where, due to the cylindrical symmetry in the transversal **r** direction, one has $v_T(r=0)=0$ while the derivative of the transverse velocity near the center of the fireball $v_T'(r=0) \neq 0$. The transverse velocity increases monotonously as a function of radius r.

The single particle spectra in a pure hydrodynamic picture without resonance decays are expressed by the integral of the Wigner function over the freeze-out surface:

$$p^0 \frac{d^3N}{d^3p} = \int d\sigma_\mu p^\mu f(x,p). \tag{1}$$

The Wigner function $f(x,p) = f_{\mathrm{th}}(x,p)\rho(x)$, where $f_{\mathrm{th}}(x,p)$ is the local thermal distribution function with a temperature parameter $T = 1/\beta$, chemical potential μ, and four-velocity field $u(x)$

$$f_{\mathrm{th}}(x,p) = \frac{(2j+1)}{(2\pi)^3} \frac{1}{\exp[\beta p^\mu u_\mu(x) - \beta\mu] \mp 1}, \tag{2}$$

and the function

$$\begin{aligned}\rho(x) &\propto \exp\left[\frac{\alpha}{2}[u(r,y_L) - u(0,y_L)]^2\right] \\ &= \exp\{-\alpha[\cosh y_T(r) - 1]\}\end{aligned} \tag{3}$$

is introduced to describe the finiteness of the relativistically expanding system in the transverse direction [7]. Note that the form of the transverse rapidity y_T-dependent density is similar to one used in Ref. [8]. In Eq. (3), y_L and y_T are the longitudinal and transverse rapidities, respectively, and $1/\sqrt{\alpha}$ is the intensity of the transverse flows:

$$\sqrt{\alpha} = \frac{[v_T'(0)]^{-1}}{\overline{R}_T} = \frac{\text{hydrodynamical length}}{\text{transverse radius}}. \tag{4}$$

For very intensive relativistic transverse flows, $\alpha \to 0$. On the other hand, for $\alpha \gg 1$ we have nonrelativistic transverse flows and Eq. (3) simplifies to

$$\rho(x) \propto \exp(-r^2/2\overline{R}_T^2), \tag{5}$$

provided that $y_T(r) \ll 1$.

Using the saddle point approximation, we have obtained from Eq. (1), for $m_T\beta \gg 1$, $m_T = \sqrt{m^2 + p_T^2}$ [7]:

$$\frac{d^2N}{m_T dm_T dy} \propto e^{-(\beta m_T + \alpha)(1-\overline{v}_T^2)^{1/2}}, \tag{6}$$

where transverse velocity at the saddle point $\overline{r}(p)$ is

$$\overline{v}_T \equiv \tanh y_T(\overline{r}(p)) = \frac{\beta p_T}{\beta m_T + \alpha}. \tag{7}$$

Such an approximation is correct in the region where the hypersurface $\tau(r)$ is smooth: $|d\tau/dr| \ll 1$. Actually, we will use our approximation in the region where the freeze-out hypersurface is a spacelike one: $|d\tau/dr| \leq 1$. In most models (see, e.g., [9]) the final transverse size of this region is larger than half of the radius of an initial hydrodynamical tube. The majority of particles are emitted just from this dense area. The problem of describing the freeze-out stage for other particles is server (see, e.g., [10]) because the correspondent part of the hypersurface near $r = \overline{R}_T$ (at an early stage of the evolution) is time like. We do not consider this problem here and will apply the expression in Eq. (6) for all particles. No big errors are expected in this approach since the experimentally observed transverse momentum distributions are all exponential in shape and the variation in local slope parameter as a function of p_T is small [11,12].

Within the model described in Eqs. (1)–(3), the transverse momentum distribution (6) is not very sensitive to the details of the velocity profile; see Ref. [7]. In the region where

$$m_T - m \ll \frac{(\beta m + \alpha)^2}{2\beta\alpha}, \tag{8}$$

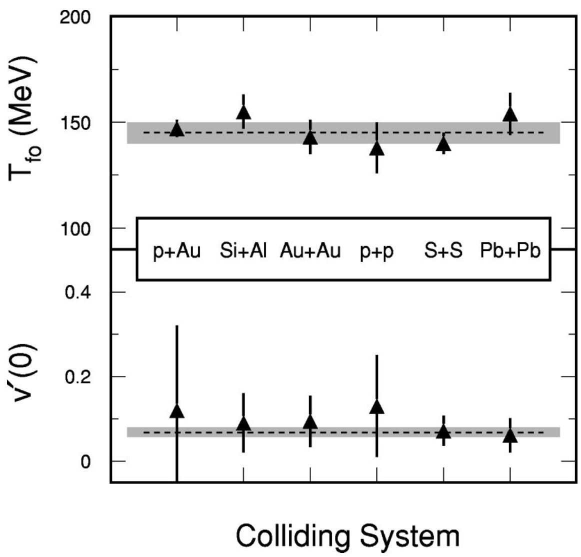

FIG. 1. The freeze-out temperature T (top) and flow velocity gradient (bottom) calculated at $r\approx 0$ for $T=145$ MeV. The set of data for this calculation is taken from the references listed in Table I.

one has the relationship

$$(\beta m_T+\alpha)(1-\overline{v_T^2})^{1/2}\approx(\beta m+\alpha)+\frac{\alpha\beta(m_T-m)}{(\beta m+\alpha)}. \quad (9)$$

Therefore we can simplify Eq. (6) as

$$\frac{d^2N}{m_T dm_T dy}\sim e^{-\beta_{\mathrm{eff}}(m_T-m)}, \quad (10)$$

with[1]

$$1/\beta_{\mathrm{eff}}=T_{\mathrm{eff}}=T+\frac{m}{\alpha}. \quad (11)$$

This equation connects the slope of the transverse mass spectra with the intensity of transverse flows. The intensity of transverse flows, $1/\sqrt{\alpha}$, is defined by the Eq. (4) and does not depend on m. However, it depends on the nucleus atomic number A.

The analysis of the experimentally measured slope parameters (T_{eff}) as a function of particle mass [11] with two parameters T and α shows that the freeze-out temperature T is approximately constant for different colliding systems at beam energy $\geqslant 10A$ GeV (see Fig. 1, top plot). Hence it is natural to conclude that the physical condition near the center is also the same for different colliding systems; i.e., $v'_T(0)$ is constant. From Eq. (4) we obtain

$$v'_T(0)=(\alpha\overline{R_T^2})^{-1/2}. \quad (12)$$

[1]The linear dependence of the slope parameter of the transverse momentum distributions on particle mass was obtained early for spherically symmetric nonrelativistic expanding systems in Ref. [13].

TABLE I. Experimental transverse velocity intensity and sideward radius parameters for different colliding systems. Error bars are statistical only. Note that for the AGS energies, the size parameters R_S is taken from two-dimensional fits assuming that $R_T=R_S$. All size parameters are obtained with a cut $p_T\geqslant 0.2$ GeV/c.

System	$1/\sqrt{\alpha}$	R_S [fm]
$p+p$ [11]	0.01 ± 0.11	0.8 ± 0.2
S+S [11]	0.26 ± 0.10	2.9 ± 0.2
Pb+Pb [11]	0.39 ± 0.12	5.1 ± 0.4
Si+Al [21]	0.29 ± 0.12	2.56 ± 0.17
Si+Au [21]	0.30 ± 0.12	2.4 ± 1.1
Au+Au [22]	0.42 ± 0.11	3.57 ± 0.52

In our approach $\overline{R_T}$ is the Gaussian-like radius of a decaying system. We assume that $\overline{R_T}$ depends on A only. It means that in the same collisions $\overline{R_T}$ is unique for different particle species. The value of $\overline{R_T}$ is connected to the so-called sideward interferometry radius R_s [14] which is obtained from the fit of the experimentally measured two-particle correlation functions in the sideward direction. When $\overline{v_T}=\beta p_T/(\beta m_T+\alpha)\ll 1$ (nonrelativistic approximation) it can be shown that only $v'_T(0)$ has an influence on the interferometry radii and the details of the transverse velocity profile $v_T(r)=\tanh y_T(r)$ are not important [7,15].[2] Within this approximation [7,15], the Gaussian transversal sideward radius is connected to $\overline{R_T}$ by

$$\overline{R_T}\sqrt{\frac{\alpha}{\beta m_T+\alpha}}=R_S(p_T). \quad (13)$$

Thus, with experimental data of $R_S(p_T)$, and β and α extracted from single particle spectra, the $\overline{R_T}$ can be readily extracted. To minimize the influence of the resonance decays, the interferometry radius for every analyzed particle species has to be measured at the point where p_T is large enough. On the other hand, in order to provide the validity of the nonrelativistic approximation and the correctness of the condition (8), the value of p_T should be limited to $\overline{v_T}\ll 1$.

Using the experimental data, listed in Table I, and fixing the optimal value of temperature $T=145$ MeV from the top plot of Fig. 1, we evaluate the velocity gradient at the center of the fireball. The results of the fit are shown in the bottom plot of Fig. 1. There are several interesting points needed to be stressed here. First, the value of the intrinsic temperature T seems to be a constant (top plot of Fig. 1). It does not depend on the size of the colliding system. Furthermore, for collisions at AGS energies (11–$15\,A$ GeV/c) and SPS energies (158–$200\,A$ GeV/c), this parameter is approximately

[2]Assuming the validity of this approach, we found that, even for the heaviest colliding system Pb+Pb [11], $\alpha=6.57$ and $\overline{v_T}=0.32$. These values were obtained at $m_T\approx 0.5$ GeV, the measured highest pair momentum region. For smaller p_T and lighter colliding systems, this assumption works better.

the same. Second, the value $v_T'(0)$ is also close to a constant, being calculated at the constant T (see the bottom plot of Fig. 1).

It has been noted that the temperature is limited for collisions at beam energies larger than 11A GeV (Fig. 2 of Ref. [11]). One should note that, although we do not observe a difference in $v_T'(0)$ between the collisions at AGS and SPS energies, it is quite possible that at lower beam energies one gets somewhat different values. The saturation of the freeze-out temperature might reflect the limiting temperature hypothesis pointed by Hagedorn some years ago [16,17]. On the other hand, the constant behavior of the velocity gradient near the center of the fireball is a new result yielded by the present study. It clearly shows that at the final stage, the freeze-out conditions are approximately the same for different sizes and different beam energies of the collisions. The constant transverse velocity gradient actually means that both the averaged transverse velocity and flow intensity are proportional to the size of the colliding nuclei. Indeed, the average transverse hydrodynamic velocity that can be calculated using the final distribution function (2) is

$$\langle v_T\rangle = \frac{\int (d^3p/p^0)\int d\sigma_\mu p^\mu v_T(x) f(x,p)}{\int (d^3p/p^0)\int d\sigma_\mu p^\mu f(x,p)} = \frac{\int d\sigma_\mu u^\mu(x)\rho(x) v_T(x)}{\int d\sigma_\mu u^\mu(x)\rho(x)}. \tag{14}$$

For $\tau(r)\cong$const at the freeze-out hypersurface we can get, from Eq. (14),

$$\langle v_T\rangle \approx \frac{\int \tanh y_T(r)\cosh y_T(r)\exp[-\alpha\cosh y_T(r)]r dr}{\int \cosh y_T(r)\exp[-\alpha\cosh y_T(r)]r dr}. \tag{15}$$

Using the saddle point approximation, in the limit of $\alpha \gg 1$, one can show that the average transverse hydrodynamic velocity is

$$\langle v_T\rangle \approx v_T'(0)\frac{\int \exp(-r^2/2\overline{R}_T^2) r^2 dr}{\int \exp(-r^2/2\overline{R}_T^2) r dr} \approx \sqrt{\frac{\pi}{2}} v_T'(0)\overline{R}_T = \sqrt{\frac{\pi}{2\alpha}}. \tag{16}$$

In our notation, $\overline{v}_T \neq \langle v_T\rangle$. The physical meaning is that, in the saddle point approximation, $\overline{v}_T$ is the transverse velocity of the fluid element which gives the main contribution to transverse momentum spectrum at a given p_T. On the other hand, $\langle v_T\rangle$ is the averaged transverse hydrodynamic velocity that characterizes whole system. From Eqs. (15) and (16), one can see that α is directly connected to $\langle v_T\rangle$. Using values for α from Table I we find that, in the approximation of Eq. (16), the averaged values of the transverse velocity $\langle v_T\rangle$ are 0.49c and 0.33c for Pb+Pb and S+S central collisions, respectively. Formally the averaged velocity $\langle v_T\rangle$ is consistent with zero for $p+p$ collisions although the application of the thermal model for such collisions remains an open question. At the freeze-out surface, the effective slopes of spectra for different colliding systems and different mass particle species can be presented by the nonrelativistic average transverse hydrodynamic velocity:

$$T_{\mathrm{eff}} \approx T + \frac{2}{\pi} m\langle v_T\rangle^2. \tag{17}$$

Note that factor of $2/\pi$ appears due to the cylindrical symmetry of the expanding systems.

The merit of this equation is that by fitting the measured slope parameter as a function of particle mass, one can separate the collective motion from the thermal motion. Therefore, the intrinsic freeze-out temperature T and averaged collective velocity $\langle v_T\rangle$ can be readily extracted from the data. The measured slope parameters of pions, kaons, and protons [11] seem to obey the linear mass dependence as given by Eq. (17). One new finding from this study is the linear tie of the averaged transverse velocity $\langle v_T\rangle$ with the radius $\overline{R}_T$; see Eq. (16). It is worth mentioning that hydrodynamic arguments for the behavior of the slope parameter as a function of a particle mass are valid as long as the considered particle species remain a part of the fireball and hence are participating in the frequent rescatterings. For those particles with a high probability of destruction, such a description is not valid anymore. It is possible that those particles freeze out earlier than the other hadrons which participate in the evolution of the system longer. They do not have enough time to acquire a common collective velocity and hence the corresponding slope parameters could be smaller than those predicted by Eq. (17). Indeed, the recent preliminary results reported by WA97 [18] and NA49 [19] show such deviations for hyperons, particularly for the Ω particle [20]. In order to understand the dynamics involved for those particles, one has to take the collision rate for the individual considered particle into account.

In summary, using a thermal model we analyzed the recent experimental data from heavy ion collisions. We found that the freeze-out temperature T and velocity gradient at the center of the fireball $v_T'(0)$ can be simultaneously constants for all colliding systems at $E_{\mathrm{beam}} > 10A$ GeV.

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