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$\nu d \to \mu^- \Delta^{++} n$ reaction and axial vector N-Δ coupling

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The reaction $\nu d \to \mu^- \Delta^{++} n$ is studied in the region of low q^2 to investigate the effect of deuteron structure and width of the Δ resonance on the differential cross section. The results are used to extract the axial vector N-Δ coupling C_5^A from the experimental data on this reaction. The possibility to determine this coupling from electroweak interaction experiments with high intensity electron accelerators is discussed.
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I. INTRODUCTION

The study of electromagnetic and weak couplings in the N-Δ transition amplitude can provide valuable information about the hadron structure. For example, the electromagnetic couplings in the magnetic dipole ($M1$) and electric quadrupole ($E2$) transition amplitudes, determined from the experiments on photoproduction and electroproduction of the Δ resonance, are found to be about 30% larger than those computed in many theoretical models of hadron structure [1]. To explain this discrepancy is a challenging task for these models. A similar comparison between theoretical and experimental values of the various couplings in the weak transition amplitude has not been made, even though there exists considerable literature on the study of weak N-Δ transitions [2]. However, in a recent paper, Hemmert, Holstein, and Mukhopadhyay (HHM) [3], using the low q^2 data from the Argonne National Laboratory (ANL) experiment of Barish *et al.* [4] and the Brookhaven National Laboratory (BNL) experiment of Kitagaki *et al.* [5] on the reaction $\nu d \to \mu^- \Delta^{++} n$, have determined the value of the axial vector N-Δ coupling C_5^A. They find that, in the weak sector too, the experimental value of C_5^A is about 30% larger than the theoretical estimates obtained in most of the quark models. This value is, however, consistent with the value obtained in a calculation that uses the hypothesis of partial conservation of axial current (PCAC), when the experimental value is used for the $g_{\Delta N\pi}$ coupling. The underestimation of the electromagnetic and weak couplings in the N-Δ transitions may be a manifestation of the large violations of $SU(6)$ symmetry, while maintaining the chiral symmetry of the Lagrangian, and needs further investigation. On the experimental side, a better determination of these couplings might become available in near future, when the electromagnetic and weak interaction reactions planned to be studied at high intensity electron accelerators are performed [6,7].

In this paper, we undertake the determination of C_5^A using the data from the BNL experiment of Kitagaki *et al.* [5] on the ratio of the differential cross sections for the inelastic $\nu d \to \mu^- \Delta^{++} n$ and the quasielastic $\nu d \to \mu^- pp$ reactions. We also analyze the experimental results from the ANL experiment of Radecky *et al.* [8], which has about three times more events than the experiment of Barish *et al.* [4]. In the inelastic reaction, all the experimental analyses [4,5,8] exclude the region of very low $|q^2|$, i.e., $|q^2| \leq 0.1\ \mathrm{GeV}^2$. In this region, the nuclear corrections due to the deuteron target have not been calculated. We take into account the effect of deuteron structure in the present work. We also study the effect of the width of the Δ resonance on the differential cross section, and its influence on the determination of C_5^A using an energy dependent P-wave width for the Δ. In the earlier analyses of this reaction [4,5,8], an energy dependent S-wave width was used. These effects were not included in the analysis of HHM [3], which could influence the determination of C_5^A, especially when the low q^2 data is used for the ratio of the differential cross section of the inelastic reaction $\nu d \to \mu^- \Delta^{++} n$ and the quasielastic reaction $\nu d \to \mu^- pp$. The analysis presented here brings out in detail the various uncertainties involved in the extraction of C_5^A from the data, when extrapolated to $q^2 = 0$.

In Sec. II, we calculate the effects of deuteron structure and width of the Δ resonance on the differential cross sections. We determine the value of C_5^A in Sec. III, where the possibility of extracting it from electron scattering experiments is also discussed. Section IV provides a summary of the results presented in this paper.

II. DIFFERENTIAL CROSS SECTION

A. Differential cross section for $\nu p \to \mu^- \Delta^{++}$

The weak N-Δ transition is described in terms of eight form factors $C_i^{V,A}(i=3-6)$, where superscripts V and A refer to the vector and axial vector form factors, respectively. In the standard notation [9–11], the amplitude $\mathcal{M}$ is written as

$$\mathcal{M} = \frac{G}{\sqrt{2}} \cos\theta_c l_\alpha J^\alpha, \tag{1}$$

with

$$l_\alpha = \overline{u}(k')\gamma_\alpha(1-\gamma_5)u(k), \tag{2}$$

and

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$$\begin{aligned} J^\alpha = \sqrt{3}\bar{\psi}_\mu(p')\Bigg\{ \Bigg[\frac{C_3^V}{M}(g^{\mu\alpha}\not{q} - q^\mu\gamma^\alpha) \\ + \frac{C_4^V}{M^2}(g^{\mu\alpha}qp' - q^\mu p'^\alpha) + \frac{C_5^V}{M^2}(g^{\mu\alpha}qp - q^\mu p^\alpha)\Bigg]\gamma_5 \\ + \frac{C_3^A}{M}(g^{\mu\alpha}\not{q} - q^\mu\gamma^\alpha) + \frac{C_4^A}{M^2}(g^{\mu\alpha}qp' - q^\mu p'^\alpha) \\ + C_5^A g^{\mu\alpha} + \frac{C_6^A}{M^2}q^\mu q^\alpha\Bigg\} u(p), \end{aligned} \tag{3}$$

where M is the nucleon mass; $\psi_\mu(p')$ and $u(p)$ are the Rarita Schwinger and Dirac spinors for Δ and nucleon of momentum p' and p; $q = p' - p = k - k'$ is the momentum transfer. The weak form factors $C_i^V(i = 3-6)$ are obtained using the conserved vector current (CVC) hypothesis, which requires $C_6^V = 0$ and relates the remaining three form factors to the various amplitudes in the photoproduction and electroproduction of the Δ resonance. From the experimental data on these processes, the following values of the vector form factors are obtained, which are used in the analysis of the neutrino scattering experiments [4,5,8,11]:

$$C_5^V = 0, \quad C_4^V = -\frac{M}{M'}C_3^V, \tag{4}$$

with

$$C_3^V(q^2) = \frac{2.05}{(1 - q^2/0.54\ \mathrm{GeV}^2)^2}. \tag{5}$$

Here M' is the mass of Δ resonance. The weak axial form factors $C_i^A(i = 3-5)$ are determined by fitting the available data on the differential cross section $d\sigma/dq^2$ in neutrino scattering, mainly from the deuteron target, in order to minimize the nuclear corrections. However, these values of the form factors are also compatible with the data on neutrino scattering from nuclear targets [12]. It is to be noted that C_6^A is not determined from these experiments as it is proportional to the lepton mass, which is neglected in these analyses. Instead, this form factor is determined in terms of C_5^A using the hypothesis of PCAC. The values of the axial form factors most often used in the analysis of the neutrino experiments are [4–6,8,10–12]

$$C_{i=3,4,5}^A(q^2) = C_i^A(0)\left[1 - \frac{a_i q^2}{b_i - q^2}\right]\left(1 - \frac{q^2}{M_A^2}\right)^{-2} \tag{6}$$

and

$$C_6^A(q^2) = C_5^A\frac{M^2}{m_\pi^2 - q^2}, \tag{7}$$

with $C_3^A(0) = 0$, $C_4^A(0) = -0.3$, $C_5^A(0) = 1.2$, $a_4 = a_5 = -1.21$, $b_4 = b_5 = 2\ \mathrm{GeV}^2$, and M_A is treated as a free parameter. For our present purpose, we take $M_A = 1.28$ GeV [5]. Using the matrix element of Eqs. (1)–(3), the differential cross section is written as

$$\begin{aligned} \frac{d^2\sigma}{dq^2 dk'^0} = \frac{1}{128\pi^2}\frac{M}{M'}\frac{1}{(s-M^2)^2}G^2\cos^2\theta_c L_{\alpha\beta}J^{\alpha\beta} \\ \times \frac{\Gamma(W)}{(W-M')^2 + \Gamma^2(W)/4}, \end{aligned} \tag{8}$$

with

$$L_{\alpha\beta} = k_\alpha k'_\beta + k'_\alpha k_\beta - g_{\alpha\beta}kk' + i\epsilon_{\alpha\beta\gamma\delta}k^\gamma k'^\delta \tag{9}$$

and

$$J_{\alpha\beta} = \overline{\Sigma}\Sigma J_\alpha^\dagger J_\beta, \tag{10}$$

where the summation is performed over the hadronic spins, using a spin 3/2 projection operator $P_{\mu\nu}$ given by

$$\begin{aligned} P_{\mu\nu} = -\frac{\not{p}' + M'}{2M'}\Bigg(g_{\mu\nu} - \frac{2}{3}\frac{p'_\mu p'_\nu}{M'^2} \\ + \frac{1}{3}\frac{p'_\mu\gamma_\nu - p'_\nu\gamma_\mu}{M'} - \frac{1}{3}\gamma_\mu\gamma_\nu\Bigg). \end{aligned} \tag{11}$$

In Eq. (8), $s = (p+k)^2$, W is the Δ invariant mass $W^2 = p'^2$ and $\Gamma(W)$ its decay width given by [13]

$$\Gamma = \Gamma_0\frac{M'}{W}\frac{q_{\mathrm{c.m.}}^3(W)}{q_{\mathrm{c.m.}}^3(M')}, \tag{12}$$

with $\Gamma_0 = 120$ MeV [14] and $q_{\mathrm{c.m.}}(W)$ the modulus of the pion momentum in the rest frame of a Δ with invariant mass W; k'^0 is the muon energy in the laboratory frame.

B. Effect of deuteron structure

When the reaction takes place in deuteron, i.e., $\nu(k) + d(p) \rightarrow \mu^-(k') + \Delta^{++}(p'_1) + n(p'_2)$, the differential cross section in the impulse approximation is calculated to be

$$\begin{aligned} \frac{d^2\sigma}{dq^2 dk'^0} = \frac{1}{128\pi^2}\frac{M_d^2}{M'(s-M_d^2)^2}G^2\cos^2\theta_c L_{\alpha\beta}J^{\alpha\beta} \\ \times \int \frac{d\boldsymbol{p}'_2}{(2\pi)^3 p_2'^0}\frac{\Gamma(W)}{(W-M')^2 + \Gamma^2(W)/4}\phi^2(|\boldsymbol{p}'_2|), \end{aligned} \tag{13}$$

where M_d is the deuteron mass and $\phi(|\boldsymbol{p}'_2|)$ is the Fourier transform of the deuteron radial wave function. This expression is derived assuming the neutron to be spectator, and neglecting meson exchange currents and final state interactions. The contribution of these effects on the differential cross section $d\sigma/dq^2$ has been studied earlier for the case of the quasielastic reaction [15] and found to be small in the kinematical region considered here. Using Eq. (13), we calculate the differential cross section for the reaction $\nu d \rightarrow \mu^- \Delta^{++} n$ for various deuteron wave functions corresponding to Hulthen [16], Paris [17], and Bonn [18] poten-

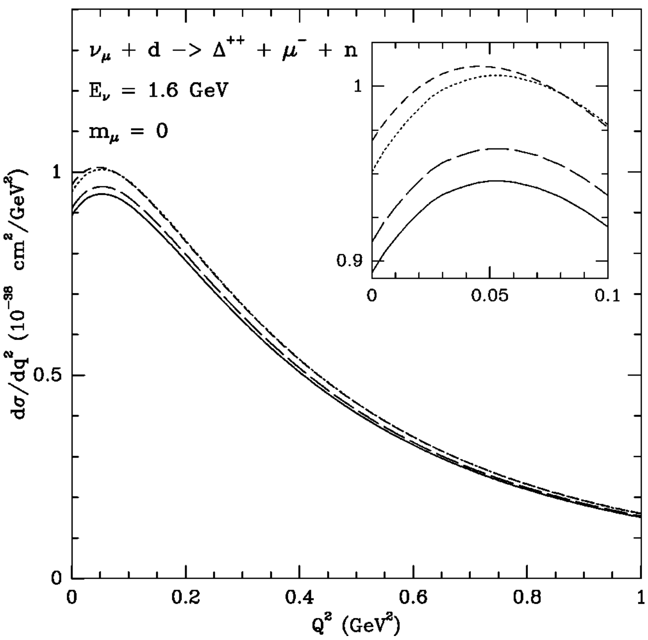

FIG. 1. Differential cross section for weak charged current neutrino production of Δ on deuteron. In the short-dashed line, deuteron effects are neglected while dotted, long-dashed, and solid lines include these effect using Hulthen, Bonn and Paris deuteron wave functions, respectively.

tials, and compare them with the differential cross section results for the free case, calculated from Eq. (8).

The results for $d\sigma/dq^2$ as a function of $Q^2=-q^2$ for the incident neutrino energy $E_\nu=1.6$ GeV are shown in Fig. 1. We see that the deuteron effects are small, not exceeding 8% even at low Q^2, i.e., $Q^2<0.1$ GeV^2. This is the region where they give a large reduction in the quasielastic reaction $\nu d\rightarrow\mu^- pp$ [15]. The different behavior of deuteron effects in these two reactions is due to the nature of the vector current contribution. In the inelastic reaction, the vector contribution vanishes for proton as well as for deuteron targets in the limit of $Q^2\rightarrow 0$, and the only contribution is from the axial vector piece, which is only slightly affected by the deuteron structure. On the other hand, in the quasielastic reaction, while both vector and axial vector currents contribute for the nucleon case, the vector contribution is completely suppressed in the deuteron. The only contribution left in the case of deuteron is from the axial vector current with an effective strength, which is strongly reduced due to symmetry considerations of the two nucleons in the final state [15]. In the range of $Q^2>0.1$ GeV^2 the deuteron effects are found to be quite small on the differential cross section $d\sigma/dq^2$ for the inelastic reaction. The situation is then similar to the case of quasielastic reaction [15], where the deuteron effects are almost negligible in this region.

We compare the deuteron structure effects in both reactions by computing the ratio $R(Q^2)$ defined as

$$R(Q^2)=\frac{(d\sigma/dq^2)(\nu d\rightarrow\mu^-\Delta^{++}n)}{(d\sigma/dq^2)(\nu d\rightarrow\mu^- pp)},\qquad(14)$$

and plotting it as a function of Q^2. In calculating $R(Q^2)$, we use the deuteron wave function obtained from Paris potential. The differential cross sections for the quasielastic reaction is taken from Singh and Arenhoevel [15] for the case where meson exchange currents and final state interaction effects are neglected, in order to be consistent with our present calculation for the inelastic reaction. In Fig. 2, we show $R(Q^2)$ for the range of low Q^2, where deuteron effects are known to be important in the case of quasielastic reactions. We also show in this figure the ratio for the equivalent reactions on the free nucleon, given by

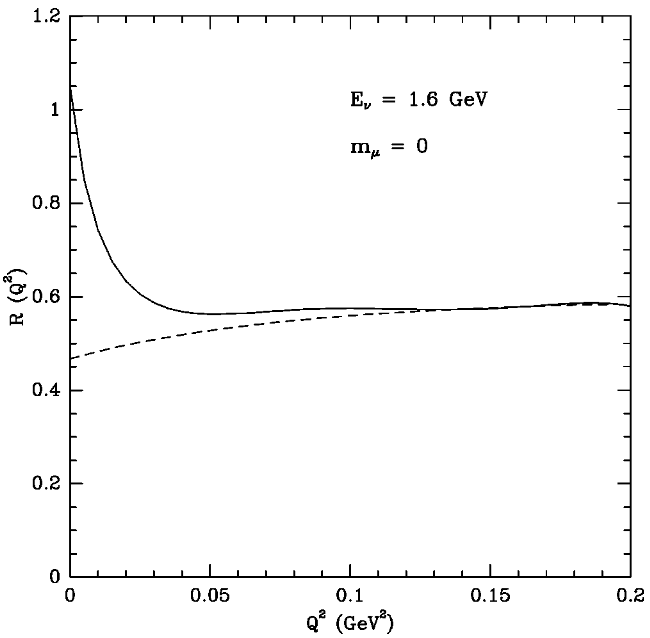

FIG. 2. Ratio of Δ production and quasielastic reactions differential cross sections with (solid line) and without (dashed line) deuteron effects.

$$R_0(Q^2)=\frac{(d\sigma/dq^2)(\nu p\rightarrow\mu^-\Delta^{++})}{(d\sigma/dq^2)(\nu n\rightarrow\mu^- p)};\qquad(15)$$

such a ratio is not directly measurable because of the absence of neutron targets. We see that $R_0(Q^2)$ remains approximately constant for the range of Q^2 considered here. For values of $Q^2<0.05$ GeV^2, the ratio increases; this is mainly due to the decrease in the cross sections of the quasielastic reaction. In the region of $0.05<Q^2<0.10$ GeV^2, the comparison between R and R_0 shows that deuteron effects are always less than 7% according to our calculation. At $Q^2\geqslant 0.1$ GeV^2, the region measured experimentally, $R(Q^2)\approx R_0(Q^2)$; this implies that one can treat the data on $R(Q^2)$ for $Q^2\geqslant 0.1$ GeV^2 obtained in Ref. [5], as if they were data on $R_0(Q^2)$. This fact will be used in Sec. III A to extract the coupling $C_5^A(0)$.

In the region of very low Q^2, the nonzero muon mass may play a role. In order to see its effect, we have evaluated the differential cross section $d\sigma/dq^2$ from Eq. (13), keeping the muon mass term and the induced pseudoscalar form factor $C_6^A(Q^2)$, determined from the PCAC condition and given by Eq. (7). We show our results in Fig. 3 for the case of Paris wave function. The effect of the nonzero muon mass is important in the region of very low Q^2 and is to be noticed in a fast decrease of the differential cross section as Q^2 decreases and reaches a value $Q^2_{\min}$, below which the reaction is kine-

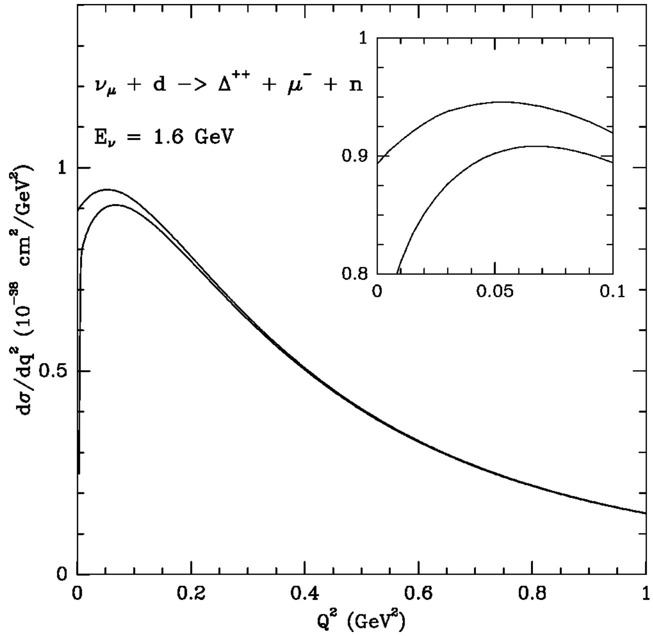

FIG. 3. Effect of the muon mass on the differential cross section for the $\nu d \rightarrow \mu^- \Delta^{++} n$ reaction. In the upper line muon mass is neglected while it is considered in the lower one. Both curves include deuteron effects using the Paris parametrization of deuteron wave function.

matically not allowed. In fact, in an earlier analysis of the Brookhaven experiment [19], this trend is clearly visible (see Fig. 11 of Ref. [19]) but, as no cross sections are quoted in this experiment, a direct comparison with our present theoretical results cannot be made.

Finally, to conclude this section on the effect of deuteron structure in the reaction $\nu d \rightarrow \mu^- \Delta^{++} n$, we would like to elaborate and extend the comments made by Kitagaki *et al.* [5] about these effects and state that, at $E_\nu = 1.6$ GeV: (i) The effects of deuteron structure are small for all Q^2, even for $Q^2 < 0.1\ \mathrm{GeV}^2$, not exceeding 10%, (ii) there is an additional reduction in the cross sections in the region of $Q^2 \sim 0.05\ \mathrm{GeV}^2$ due to the nonzero muon mass, which is about 5%, and could be larger as Q^2 decreases further.

C. Effect of the width of Δ resonance

The analysis of Schreiner and von Hippel [11] uses an S-wave width for the Δ resonance, which has also been used in the ANL and BNL experiments [4,5,8]. The recent paper of HHM [3], dealing with the N-Δ couplings and the extraction of C_5^A, uses an expression for the differential cross section at $Q^2 = 0$, which neglects the width of the Δ resonance. In this situation, it seems worthwhile to examine the effect of the width of the Δ resonance. Therefore, we study the sensitivity of the differential cross section for the process $\nu p \rightarrow \mu^- \Delta^{++}$ to the width of the Δ resonance and its energy dependence. In order to do this, we evaluate the differential cross section given in Eq. (8) with P-wave Δ resonance width given in Eq. (12), S-wave Δ resonance width given by [11]

$$\Gamma = \Gamma_0 \frac{q_{\mathrm{c.m.}}(W)}{q_{\mathrm{c.m.}}(M')}, \tag{16}$$

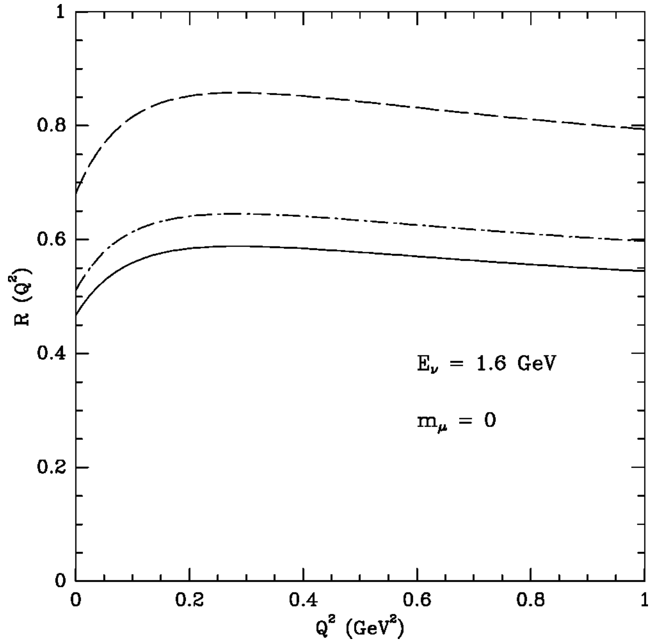

FIG. 4. Effect of Δ width in $R(Q^2)$: the solid line corresponds to a P-wave width, the dash-dotted line to an S-wave width, and the dashed line to the case of zero width resonance. Deuteron effects have been neglected in all curves.

and narrow resonance limit, i.e., $\Gamma \rightarrow 0$, in which the differential cross section is analytically given by

$$\frac{d\sigma}{dq^2} = \frac{1}{64\pi} \frac{1}{(s - M^2)^2} G^2 \cos^2 \theta_c L_{\alpha\beta} J^{\alpha\beta}, \tag{17}$$

obtained from Eq. (8) by integrating over k'^0 after taking the limit $\Gamma_0 \rightarrow 0$.

In Fig. 4, we present the results of $R(Q^2)$ with free nucleon target for the three cases discussed above. We see here that the inclusion of the width gives a considerable reduction of the cross section, but the detailed form of its energy dependence is not very important when an invariant mass of $W \leqslant W_{\mathrm{cut}} = 1.4$ GeV is used. We have also found that the uncertainties in the width at the resonance energy of about 10−15 MeV [14] do not lead to any substantial change in the cross section.

III. AXIAL VECTOR N-Δ COUPLING

A. Neutrino scattering experiments

In this section, we evaluate the value of C_5^A using the data of Kitagaki *et al.* [5] on $R(Q^2)$, and use it later to describe the data of Radecky *et al.* [8] for the differential cross section $d\sigma/dq^2$. In the limit $Q^2 \rightarrow 0$, the cross sections for the quasielastic and inelastic reactions, required to evaluate $R_0(0)$ are [20,21]

$$\frac{d\sigma}{dq^2}(q^2 = 0) = (F_A^2 + F_V^2) \frac{1}{2\pi} G^2 \cos^2 \theta_c \tag{18}$$

and

TABLE I. The numerical values of axial N-Δ coupling C_5^A in various quark model and empirical approaches. The earlier, prior to 1973, evaluations of these couplings in these approaches have been summarized by Schreiner and von Hippel [11] and Llewellyn Smith [9].

	C_5^A
Quark model approaches	0.97 [23,24], 0.83 [25], 1.17 [2], 1.06 [26], 0.87 [3]
Empirical approaches	1.15±0.23 [4], 1.39±0.14 [3], 1.1±0.2 [31], 1.22±0.06[a]

[a] Present result.

$$\frac{d\sigma}{dq^2}(q^2=0)=[C_5^A(0)]^2\frac{1}{24\pi^2}G^2\cos^2\theta_c$$
$$\frac{\times\sqrt{s}(M+M')^2(s-M'^2)^2}{(s-M^2)M'^3}$$
$$\times\int_{k'^0_{\rm min}}^{k'^0_{\rm max}}dk'^0\,\frac{\Gamma(W)}{(W-M')^2+\Gamma^2(W)/4},\tag{19}$$

respectively; $k'^0_{\rm min}$ and $k'^0_{\rm max}$ are given by

$$k'^0_{\rm min}=\max\left(\frac{s-W^2_{\rm cut}}{2\sqrt{s}},0\right),\quad k'^0_{\rm max}=\frac{s-(M+m_\pi)^2}{2\sqrt{s}}.\tag{20}$$

This result depends only on the coupling constant $C_5^A(0)$. In an expansion of R_0 in powers of Q^2, the first term that depends on the axial mass and other couplings is the one proportional to Q^2. Thus, data at low enough Q^2 would allow a model independent extraction of $C_5^A(0)$. The experimental data of Ref. [5] begin at quite low Q^2 ($Q^2\sim0.1$ GeV^2). In the region where the first points lie, we obtain an approximately constant value for $R_0(Q^2)$ with the choice of parameters given in Eqs. (4)–(7), as can be seen in Fig. 2; this behavior remains the same for moderate changes of the form factors. For this reason, we can use a constant value to extrapolate the R_0 data to $Q^2=0$.

Equating the ratio of these two cross sections given in Eqs. (18) and (19), i.e., $R_0(Q^2=0)$ to the extrapolated experimental value of 0.55±0.05 [22], obtained as an average of the data on $R(Q^2)$ for $R(Q^2)\geqslant0.1$ GeV^2 [5] (that is, in the region where we know that $R\approx R_0$). We obtain

$$C_5^A=1.22\pm0.06.\tag{21}$$

Equation (19) could also be used to extract $C_5^A(0)$ from data on the $\nu p\rightarrow\mu^-\Delta^{++}$ reaction. However, the uncertainties, both statistical and related to the neutrino flux, in the existing data do not allow for a better determination of the coupling constants. The quoted error comes exclusively from experiment. It does not include an estimation of the theoretical uncertainties implicit in our approximations, such as the neglect of meson exchange currents and final state interactions, that were discussed in Sec. II A.

In Table I, we compare the values of this coupling constant with the theoretical values obtained in various models. With the exception of the quark model treatment of Liu *et al.* [2], all the quark models underestimate the value of C_5^A when compared to the central values quoted from experimental analyses. On the other hand, it is in good agreement with the prediction of PCAC, which gives $C_5^A=1.15\pm0.01$, when the experimental value of $g_{\Delta N\pi}=28.6\pm0.3$ [2,3] is used. It is expected that the various extensions of the quark models currently proposed to explain the quadrupole moment of Δ, and the $E2/M1$ ratio in the photoproduction and electroproduction of the Δ resonance will be applied to the problem of explaining C_5^A and other N-Δ couplings in these models.

Using our value of C_5^A, at $Q^2=0$, its Q^2 behavior and other form factors as given in Eqs. (4)–(7), we calculate the flux averaged differential cross section for the neutrino energy spectrum of the Argonne experiment of Radecky *et al.* [8] and show this in Fig. 5. We see that the inclusion of deuteron and mass effects lead to a better description of the data. It is to be emphasized that a small reduction in the differential cross section due to these effects is quite important in bringing out a good agreement with the experimental data, especially in the low q^2 region.

B. Electron scattering experiments

It is possible to get information about the axial vector coupling C_5^A from the observation of the parity violating

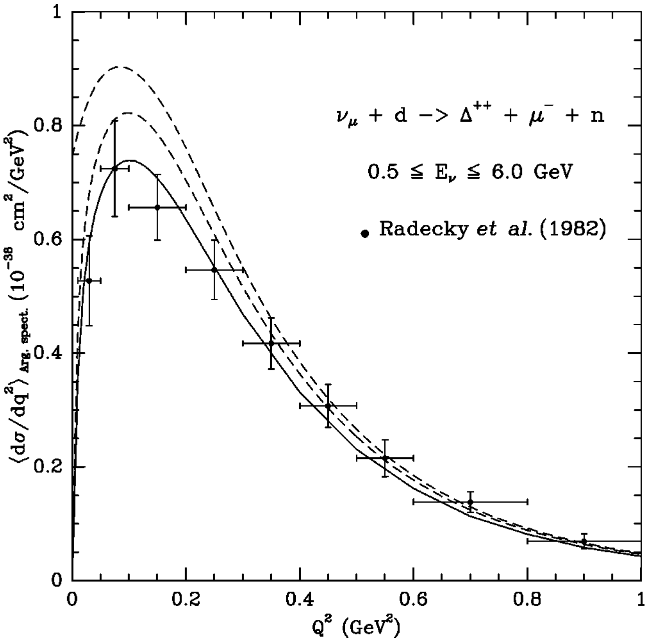

FIG. 5. Differential cross section for weak charged current neutrino production of Δ on deuteron, averaged over the spectrum of ANL experiment, compared to the experimental results given in Ref. [8]. The solid curve includes both nonzero muon mass and deuteron effects. The upper dashed curve neglects muon mass and deuteron effects. The lower dashed curve neglects only deuteron effects.

asymmetry in the polarized electron scattering experiments performed in the Δ region. The feasibility of doing such experiments was discussed in past by many authors [27], but it seems now possible to do these experiments at the high intensity electron accelerators [6,7,28]. In the neutral current reaction $e^- + p \rightarrow e^- + \Delta^+$ with polarized electron the asymmetry $A(Q^2)$ is defined as

$$A(Q^2) = \frac{(d\sigma/dq^2)(+1) - (d\sigma/dq^2)(-1)}{(d\sigma/dq^2)(+1) + (d\sigma dq^2)(-1)}, \tag{22}$$

where $d\sigma(\lambda)/dq^2$ is the differential cross section for an electron with helicity λ. It has been calculated to be [28]

$$A(Q^2) = -\frac{G}{2\sqrt{2}\pi\alpha}|Q^2|\Bigg[(1-2\sin^2\theta_W) + (1-4\sin^2\theta_W) \times \frac{C_5^A}{C_3^V}\left(1 + \frac{M'^2 + Q^2 - M^2}{2M^2}\frac{C_4^A}{C_5^A}\right)P(Q^2,s)\Bigg] + \text{nonresonant contribution}, \tag{23}$$

where α is the fine structure constant and $P(Q^2,s)$ a purely kinematical factor.

In principle, one can determine the value of C_5^A/C_3^V from the asymmetry measurements by selecting the kinematics where nonresonant contributions are negligible. However, as we see from Eq. (23), the hadronic axial vector current contribution containing C_5^A is multiplied by a factor $(1 - 4\sin^2\theta_W)$, which reduces the sensitivity of this term to the asymmetry $A(Q^2)$. This makes the extraction of C_5^A from a measurement of the asymmetry very difficult. Even in the favorable kinematical region of $0.5 < E_e < 1$ GeV and $Q^2 < 1.0$ GeV^2, this term contributes only $(10-20)$ %, as emphasized by Mukhopadhyay *et al.* [28]. This requires very precise measurements of $A(Q^2)$ for a determination of C_5^A from parity violating asymmetry measurements.

There is also a possibility of observing the charged current reaction $e^- + p \rightarrow \Delta^0 + \nu$ with unpolarized electrons through the detection of the protons and pions from the decay of the Δ resonance [29]. At the incident electron energy of 4 GeV, the differential cross section $d\sigma/dq^2$ in the forward direction near $Q^2 = 0$ is estimated to be 2×10^{-39} cm^2/GeV^2. For an incident intensity of about 2×10^{38} cm^2/sec [28] and Q^2 bin of 0.05 GeV^2, one would expect 72 events per hour for the production of Δ^0, assuming 100% efficiency of the detector. One third of these Δ's will produce negatively charged pions and protons, which can be easily observed. Since in the region of $Q^2 \sim 0$, C_5^A gives the dominant contribution, its determination from the weak charged current experiment of Δ production seems feasible. Note, however, that in the analysis of this process, a theoretical study of the nonresonant background is required to extract the resonant contribution from the data, which would lead to further uncertainties. In the case discussed in Sec. III A (Δ^{++} production in deuterium), the nonresonant background was found to be around 1%, whereas for other isospin channels, it was found to be considerably larger [19].

C. Pion photoproduction and electroproduction experiments

It is well known that in the threshold region of photopion and electropion production from the nucleon, the matrix element of these processes in the soft pion limit is related with the nucleonic matrix element of the axial vector current using the methods of current algebra and the PCAC. This relation has been exploited to obtain information about the axial vector form factor of the nucleon [30]. In a similar way, threshold pion production in the processes $e^- + p \rightarrow e^- + \Delta^+ + \pi^0$ and $\gamma + p \rightarrow \Delta^{++} + \pi^-$ is related, in the soft pion limit, with the N-Δ transition matrix element of the axial vector current. The axial vector transition form factors can, in principle, be determined from these processes in the limit of soft pions. Such attempts have been made in past and they yield $C_5^A = 1.1 \pm 0.2$ [31].

However, in this case, the treatment of higher resonances and their effective couplings used for evaluating the matrix elements of the time ordered product of the vector and axial vector current operators occurring in the LSZ reduction involve many approximations, which need further justification. Recently, there has been some progress in calculating the contribution of higher resonances to the production of two pions in the photoproduction and electroproduction processes using effective Lagrangians [32]. It should be possible to isolate the dominant contributions from higher order resonances, which are relevant for the $\Delta\pi$ production in the soft pion limit. This will help to reduce the theoretical uncertainties in the application of the methods of PCAC and current algebra to the processes where a Δ resonance is produced. In addition, when dealing with the Δ resonance, its width has to be properly taken into account as remarked by Bartl *et al.* [31], and also shown by us in the weak charged current production of the Δ resonance. The analysis of Bartl *et al.* [31] uses the older data which suffers from poor statistics. When the results of a recent experiment proposed at TJNAF [33] become available in the near future, it will be possible to get precise information about the axial vector coupling C_5^A and its momentum dependence.

IV. SUMMARY AND OUTLOOK

We have calculated the effect of deuteron structure and width of the Δ resonance in the differential cross section for the reaction $\nu d \rightarrow \mu^- \Delta^{++} n$ and found that these effects are small, but important in order to explain the experimental results at low q^2, where they were initially expected to be important. Furthermore, in the region of very low q^2, the muon mass, which is usually neglected in the calculations, also reduces the cross section.

The effect of the width of the Δ resonance on the cross section is important and plays a crucial role in bringing out good agreement with the experimental data. The detailed shape and 10–15 % uncertainty in the width of the resonance does not affect the cross sections very much.

The axial vector N-Δ coupling C_5^A is extracted from the BNL data on $\nu d \rightarrow \mu^- \Delta^{++} n$, incorporating the effect of the deuteron structure and the width of Δ resonance. This value of C_5^A is found to be larger than the values predicted in most of the quark models and is consistent with the prediction of PCAC and Adler's model. Finally, we have discussed the possibility of determining this coupling from electron scattering experiments, and find that electroproduction and weak charged current of Δ resonance are an interesting alternative to asymmetry measurements in the polarized electroproduction of Δ.

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