Scaling of chiral Lagrangians and Landau Fermi liquid theory for dense hadronic matter

Bengt Friman,[1,*] Mannque Rho,[2,3,†] and Chaejun Song[4,‡]
[1]Theory Group, GSI, Planckstrasse 1, D-64291 Darmstadt, Germany
and Institut für Kernphysik, TU Darmstadt, D-64289 Darmstadt, Germany
[2]Service de Physique Théorique, CEA Saclay, F-91191 Gif-sur-Yvette, France
[3]School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea
[4]Department of Physics and Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea
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We discuss the Fermi-liquid properties of hadronic matter derived from a chiral Lagrangian field theory in which Brown-Rho (BR) scaling is incorporated. We identify the BR scaling as a contribution to Landau's Fermi-liquid fixed-point quasiparticle parameter from ''heavy'' isoscalar meson degrees of freedom that are integrated out from a low-energy effective Lagrangian. We show that for the vector (convection) current, the result obtained in the chiral Lagrangian approach agrees precisely with that obtained in the semiphenomenological Landau-Migdal approach. This precise agreement allows one to determine the Landau parameter that enters in the effective nucleon mass in terms of the constant that characterizes BR scaling. When applied to the weak axial current, however, these two approaches differ in a subtle way. While the difference is small numerically, the chiral Lagrangian approach implements current algebra and low-energy theorems associated with the axial response that the Landau method misses and hence is expected to be more predictive. [S0556-2813(99)04506-9]

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I. INTRODUCTION

At very low energies, the relevant degrees of freedom for strong interactions in nuclear matter are pions, nucleons and other low-mass hadrons identified in the laboratory and the appropriate theory is an effective quantum field theory involving these hadrons even though the fundamental theory is known to be QCD with quarks and gluons [1]. Just which and how many hadronic degrees of freedom must appear in the theory depends upon the energy scale that is probed. Thus for example, if one is probing energies of a few MeV as in the case of low-energy properties of two-nucleon systems, then the nucleon field as a matter field and possibly pions as pseudo-Goldstone bosons would suffice.

In this paper, we would like to extend the strategy of effective field theories to many-body systems and a density regime corresponding to a shorter-length or higher-energy scale than that probed by the low-energy two-nucleon systems [2,3]. This would entail two important changes to the effective Lagrangian: First we need to introduce more massive degrees of freedom (such as vector mesons and/or higher-dimensional operators in the nucleon fields) in the effective Lagrangian and second, we need to take into account the Fermi sea of nucleons in the bound system.

The principal aim in this paper is then to tie in together various results obtained previously in diverse contexts into a unified framework so as to be able to extrapolate our ideas into the kinematic domains that are yet to be explored experimentally. In doing this, we shall be using nonrelativistic arguments which are justified for low-energy and low-density processes we are concerned with here. A relativistic formulation more appropriate for high-energy and high-density heavy-ion processes is in progress and will be presented elsewhere.

The basic strategy we will develop is as follows. First we will present an argument for an effective chiral Lagrangian which in the mean field approximation corresponds to a nontopological soliton describing a lump of nuclear matter. The parameters of this effective Lagrangian will then be identified with the fixed-point quantities in Landau Fermi-liquid theory. Given this identification, one can associate certain mean field quantities of heavy mesons (e.g., the light-quark vector mesons ρ and ω) to Brown-Rho (BR) scaling via Landau parameters. We first illustrate how this chain of arguments works for electromagnetic properties of heavy nuclei. Turning the arguments around, we determine the BR-scaling parameter Φ at nuclear matter density from magnetic moments of heavy nuclei in terms of the Landau parameter F_1^ω associated with massive isoscalar vector meson degrees of freedom that are integrated out from the effective Lagrangian. We then use a similar line of arguments to derive the corresponding formulas for the axial current. In this paper, we shall focus on the processes that are dominated by pionic effects, that is, those to which the ''chiral filter mechanism'' [4] applies, namely, the electromagnetic convection current and the axial charge operator.

This paper is organized as follows. In Sec. II, effective field theories that figure in nuclear physics are described including a brief summary of Landau's Fermi-liquid theory adapted to strongly interacting nuclear systems. The calculation of the electromagnetic current for a particle sitting on top of the Fermi sea in Landau-Migdal theory and in chiral Lagrangian theory is given in Sec. III. The Landau parameter figuring in the nucleon effective mass is determined in terms of the parameter of the chiral Lagrangian that scales as a

*Electronic address: b.friman@gsi.de
†Electronic address: rho@spht.saclay.cea.fr
‡Electronic address: chaejun@fire.snu.ac.kr

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function of density (in the manner of BR scaling). The problem of treating axial charge transitions in heavy nuclei is presented in Sec. IV. The two methods—Fermi liquid and chiral Lagrangian—are found to give almost same numerical results at nuclear matter density but differ in a subtle way due to the intricacy with which chiral symmetry is manifested in nuclear systems. A possible cause of this difference between the electromagnetic current and the weak current is discussed. A summary is given in Sec. V wherein some unresolved/open problems are mentioned. In Appendix A, we show how to compute relativistically the pionic contribution to the Landau parameter F_1 using a Fierz transformation. Appendix B sketches how the particle-hole graph figuring in the ''back-flow'' argument is computed. Remarks not directly relevant to the theme of the paper but helpful for the discussions are relegated to footnotes.

II. EFFECTIVE FIELD THEORY (EFT)

There are two superbly *effective* effective field theories for nuclear physics. One is chiral Lagrangian field theory for low-energy nonperturbative description of hadrons and the other is Landau Fermi-liquid theory applied to nuclear matter.

A. EFT for dilute systems

For two-nucleon systems at very low energies considered in Refs. [2,3], one can integrate out all meson degrees of freedom including the pions and set the cutoff near the pion mass. One then writes an effective Lagrangian in terms of the nucleon field in a systematic (chiral) expansion to compute the irreducible graphs, while summing an infinite set of reducible diagrams to describe the deuteron bound state and scattering states with a large scattering length. Since the system is dilute, the parameters of the theory can be taken from free-space (zero-density) experiments. In principle, we should be able to calculate these parameters from QCD but at present we do not know how to perform this calculation in practice. The results in Refs. [2,3] confirm that the approach works remarkably well. When the pion field is included in addition, it provides a ''new'' degree of freedom and improves the theory even further and allows one to go higher in energy scale [3]. This can be formulated systematically in terms of chiral perturbation theory. But what about heavier (denser) nuclei or higher energy scales?

B. EFT for dense systems

In going to heavier many-nucleon systems, the standard approach has been to start with a Lagrangian whose parameters are defined in free space and then develop perturbative and nonperturbative schemes to account for the complex dynamics involved. Higher-energy scales will be involved since the interactions between nucleons in such systems sample all length scales and hence other degrees of freedom than nucleonic and pionic need be introduced. In doing such calculations, symmetry constraints, such as those of chiral symmetry, are found to be useful but not always properly implementable. Basically phenomenological in character, given a sufficient number of free parameters, such an approach can be quite successful but one cannot check unambiguously that it is consistent with the modern notion of effective field theory. As such, it is difficult to gauge the power of the theory. When something does not work, then there is very little one can do to improve on it since there is no systematic strategy available.

In this paper we will take a different route. Following Lynn [5], we shall assume that a high-order (in chiral counting) effective chiral action supports a nontopological soliton solution that corresponds to a chiral liquid with a given baryon number A. Lynn proposes to construct such an effective action using chiral perturbation theory *to all orders of the chiral expansion*, but up to now explicitly deriving such an action has not been feasible. Lacking such a first-principle derivation, we propose to develop an effective Lagrangian strategy applied to dense many-body systems resorting to certain assumptions based on symmetries which are to be justified *a posteriori*. Given such an action possessing a stable nontopological soliton, we follow Lynn's proposal to identify such a soliton solution as the ground state of a heavy nucleus and to make fluctuations around that ground state. Excitations on top of that state could then be described in terms of *the parameters determined at that minimum*, the bulk properties of which are to be generically characterized by the density of the state that is probed. There have recently been several works along this line. For instance, Furnstahl *et al.* [6] construct such an effective action consisting of ''heavy baryons'' (nucleons) and heavy mesons using arguments based on the ''naturalness condition'' of chiral symmetry of QCD and show that in the mean field the effective Lagrangian quantitatively describes the ground state of nuclear matter as well as the excitation spectra of finite nuclei. The point pertinent to us in the work of Furnstahl *et al.* is that their formulation is basically equivalent to a variant of Walecka mean-field theory. A recent argument by Brown and Rho [7] (and also Ref. [8]) has established that Walecka mean field theory is equivalent to a chiral Lagrangian mean-field theory with the parameters of the Lagrangian scaling in the manner of Brown and Rho (BR scaling) [9]. In Ref. [10], a Walecka-type Lagrangian with BR-scaling parameters was constructed and shown to describe the nuclear matter ground state as successfully as the effective chiral action of Furnstahl *et al.* does. Such a Lagrangian has also been shown to possess thermodynamic properties that are consistent with Landau Fermi-liquid structure of nuclear matter [11]. This is the approach we shall use in this paper. Similar ideas were developed by Brown but using different arguments [12].

C. Fermi-liquid fixed points

A conceptually important point in our arguments is that Landau Fermi-liquid theory [13–16] is an effective field theory with fixed points [17]. In this paper, we will not attempt to show that the Fermi-liquid theory for nuclear matter is also a fixed-point field theory. We will simply take the result established in Ref. [17] and implement its implications in our scheme.

One of the principal consequences of this identification is that the nucleon effective mass (which will be referred to as ''Landau effective mass'') and Landau quasiparticle interactions (defined below) are fixed-point quantities with vanishing β functions. Our goal is to connect these fixed-point quantities to BR scaling parameters that figure in effective chiral Lagrangians appropriate for dense medium. We are thereby combining two effective field theories, the chiral Lagrangian field theory and Landau Fermi-liquid theory, into an effective field theory for dense matter in which BR scaling plays an important role. We believe this ''marriage'' is a successful one for density at least up to that of nuclear matter. Going beyond that and extrapolating into the regime of relativistic heavy-ion collisions involves guesses that need be verified *a posteriori*.

D. A primer on Fermi-liquid theory

Before getting into our main calculation, we give a mini-primer on Landau Fermi-liquid theory to define the quantities involved. We should point out that once the fixed-point quantities are identified in the chiral Lagrangian, then we can use all the standard relations established in Landau's original theory.

Landau Fermi-liquid theory is a semiphenomenological approach to strongly interacting normal Fermi systems at small excitation energies. It is assumed that a one-to-one correspondence exists between the low-energy excitations of the Fermi liquid and that of a noninteracting Fermi gas. The elementary excitations of the Fermi liquid, which correspond to single particle degrees of freedom of the Fermi gas, are called quasiparticles. The quasiparticle properties, e.g., the mass, in general differ from those of free particles due to interaction effects. In addition there is a residual quasiparticle interaction, which is parametrized in terms of the so called Landau parameters.

Fermi-liquid theory is a prototype effective theory, which works because there is a separation of scales. The theory is applicable to low-energy phenomena, while the parameters of the theory are determined by interactions at higher energies. The separation of scales is due to the Pauli principle and the finite range of the interaction. Fermi-liquid theory has proven very useful [16] for describing the properties of, e.g., liquid ^{3}He and provides a theoretical foundation for the nuclear shell model [14] as well as nuclear dynamics of low-energy excitations [18,19].

The interaction between two quasiparticles $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ at the Fermi surface of symmetric nuclear matter can be written in terms of a few spin and isospin invariants [20]

$$
\begin{aligned}
f_{\boldsymbol{p}_1\sigma_1\tau_1,\boldsymbol{p}_2\sigma_2\tau_2}=\frac{1}{N(0)}\Bigg[& F(\cos\theta_{12})+F'(\cos\theta_{12})\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
& +G(\cos\theta_{12})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
& +G'(\cos\theta_{12})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
& +\frac{\boldsymbol{q}^2}{k_f^2}H(\cos\theta_{12})S_{12}(\hat{\boldsymbol{q}}) \\
& +\frac{\boldsymbol{q}^2}{k_f^2}H'(\cos\theta_{12})S_{12}(\hat{\boldsymbol{q}})\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\Bigg], \qquad (1)
\end{aligned}
$$

where θ_{12} is the angle between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ and $N(0)=\lambda k_F m_N^\star/(2\pi^2)$ is the density of states at the Fermi surface (we use natural units where $\hbar=1$ and denote by $m_N^\star$ the (Landau) effective mass of the nucleon to be distinguished from the BR-scaling mass $M_N^\star$). The spin and isospin degeneracy factor λ is equal to 4 in symmetric nuclear matter. Furthermore, $\boldsymbol{q}=\boldsymbol{p}_1-\boldsymbol{p}_2$ and

$$S_{12}(\hat{\boldsymbol{q}})=3\,\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{q}}\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{q}}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \qquad (2)$$

where $\hat{\boldsymbol{q}}=\boldsymbol{q}/|\boldsymbol{q}|$. The tensor interactions H and H' are important for the axial charge, which we consider in Sec. IV. The functions $F,F',\ldots,$ are expanded in Legendre polynomials

$$F(\cos\theta_{12})=\sum_{\ell}F_\ell P_\ell(\cos\theta_{12}), \qquad (3)$$

with analogous expansions for the spin- and isospin-dependent interactions. The energy of a quasiparticle[1] with momentum $p=|\boldsymbol{p}|$, spin σ and isospin τ is denoted by $\epsilon_{p,\sigma,\tau}$ and the corresponding quasiparticle number distribution by $n_{p,\sigma,\tau}$. The effective mass of a quasiparticle on the Fermi surface is defined by

$$\left.\frac{d\epsilon_p}{dp}\right|_{p=k_F}=\frac{k_F}{m_N^\star}. \qquad (4)$$

By using Galilean invariance one finds a relation between the effective mass and the velocity dependence of the quasiparticle interaction

$$\frac{m_N^\star}{m_N}=1+\frac{F_1}{3}=\left(1-\frac{\tilde{F}_1}{3}\right)^{-1}, \qquad (5)$$

where $\tilde{F}_l=(m_N/m_N^\star)F_l$, with analogous definitions for $\tilde{F}_l'$ etc.

III. ELECTROMAGNETIC CURRENT

We will first give a brief derivation of the Landau-Migdal formula for the convection current for a particle of momentum $\boldsymbol{k}$ sitting on top of the Fermi sea responding to a slowly varying electromagnetic (EM) field. We will then analyze it in terms of the specific degrees of freedom that contribute to the current. This will be followed by a description in terms of a chiral Lagrangian as discussed in Ref. [21]. This procedure will provide the link between the two approaches.

A. Landau-Migdal formula for the convection current

Following Landau's original reasoning adapted by Migdal to nuclear systems, we start with the convection current given by[2]

[1]Below we omit the spin and isospin indices σ and τ from our formulas to avoid overcrowding, except where needed to avoid ambiguities. We will also omit the space and time dependence of the quantities, e.g., $\epsilon\equiv\epsilon(\boldsymbol{r},t)$.

[2]More precisely, this is a matrix element of the current operator corresponding to the response of a nucleon (proton or neutron) sitting on top of the Fermi sea to the EM field. The sum over spin and isospin and the momentum integral go over all occupied states up to the valence particle. What we want is a current operator and it is deduced after the calculation is completed. One can of course work directly with the operator but the result is the same.

$$\boldsymbol{J}=\sum_{\sigma,\tau}\int\frac{d^3p}{(2\pi)^3}(\boldsymbol{\nabla}_p\epsilon_p)n_p\frac{1}{2}(1+\tau_3), \tag{6}$$

where the sum goes over the spin σ and isospin τ which in spin- and isospin-saturated systems may be written as a trace over the σ and τ operators. We consider a variation of the distribution function from that of an equilibrium state

$$n_p=n_p^0+\delta n_p, \tag{7}$$

where the superscript 0 refers to equilibrium. The variation of the distribution function induces a variation of the quasiparticle energy

$$\epsilon_p=\epsilon_p^0+\delta\epsilon_p. \tag{8}$$

In the equilibrium state the current is zero by symmetry, so we have

$$\begin{aligned}\boldsymbol{J}&=\sum_{\sigma,\tau}\int\frac{d^3p}{(2\pi)^3}[(\boldsymbol{\nabla}_p\epsilon_p^0)\delta n_p+(\boldsymbol{\nabla}_p\delta\epsilon_p)n_p^0]\frac{1}{2}(1+\tau_3),\\&=\sum_{\sigma,\tau}\int\frac{d^3p}{(2\pi)^3}[(\boldsymbol{\nabla}_p\epsilon_p^0)\delta n_p-(\boldsymbol{\nabla}_p n_p^0)\delta\epsilon_p)]\frac{1}{2}(1+\tau_3)\end{aligned} \tag{9}$$

to linear order in the variation. We consider a proton or neutron added at the Fermi surface of a system in its ground state. Then

$$\delta n_p=\frac{1}{V}\delta^3(\boldsymbol{p}-\boldsymbol{k})\frac{1\pm\tau_3}{2} \tag{10}$$

and

$$\boldsymbol{\nabla}_p n_p^0=-\frac{\boldsymbol{p}}{k_F}\delta(p-k_F), \tag{11}$$

where $\boldsymbol{k}$ with $|\boldsymbol{k}|=k_F$ is the momentum of the quasiparticle. The modification of the quasiparticle energies due to the additional particle is given by

$$\delta\epsilon_{p\sigma\tau}=\sum_{\sigma',\tau'}\int\frac{d^3p'}{(2\pi)^3}f_{p\sigma\tau,p'\sigma'\tau'}\delta n_{p'\sigma'\tau'}. \tag{12}$$

Combining Eqs. (1), (9), (10), and (12) one finds that the first term of Eq. (9) gives the *operator*

$$\boldsymbol{J}^{(1)}=\frac{\boldsymbol{k}}{m_N^\star}\frac{1+\tau_3}{2}, \tag{13}$$

where $\boldsymbol{k}$ is taken to be at the Fermi surface. The second term yields

$$\delta\boldsymbol{J}=\delta\boldsymbol{J}_s+\delta\boldsymbol{J}_v=\frac{\boldsymbol{k}}{m_N}\left(\frac{\tilde{F}_1+\tilde{F}_1'\tau_3}{6}\right), \tag{14}$$

where

$$\delta\boldsymbol{J}_s=\frac{\boldsymbol{k}}{m_N^\star}\frac{1}{2}\frac{F_1}{3}, \tag{15}$$

$$\delta\boldsymbol{J}_v=\frac{\boldsymbol{k}}{m_N^\star}\frac{\tau_3}{2}\frac{F_1'}{3}=\frac{\boldsymbol{k}}{m_N^\star}\frac{\tau_3}{2}\frac{F_1}{3}+\frac{\boldsymbol{k}}{m_N^\star}\frac{\tau_3}{2}\frac{F_1'-F_1}{3}. \tag{16}$$

Putting everything together we recover the well known result of Migdal [14,19]

$$\boldsymbol{J}=\frac{\boldsymbol{k}}{m_N}g_l=\frac{\boldsymbol{k}}{m_N}\left(\frac{1+\tau_3}{2}+\frac{1}{6}(\tilde{F}_1'-\tilde{F}_1)\tau_3\right), \tag{17}$$

where

$$g_l=\frac{1+\tau_3}{2}+\delta g_l \tag{18}$$

is the orbital gyromagnetic ratio and

$$\delta g_l=\frac{1}{6}(\tilde{F}_1'-\tilde{F}_1)\tau_3. \tag{19}$$

Thus, the renormalization of g_l is purely isovector. This is due to Galilean invariance, which implies a cancellation in the isoscalar channel.

We have derived Migdal's result using standard Fermi-liquid theory arguments. This result can also be obtained [22] by using the Ward identity, which follows from gauge invariance of the electromagnetic interaction. This is of course physically equivalent to the above formulation. We shall now identify specific hadronic contributions to the current (17) in two ways: the Fermi-liquid theory approach and the chiral Lagrangian approach.

B. Pionic contribution

1. Fermi-liquid theory approach

In this approach, all we need to do is to compute the Landau parameter F_1 from the pion-exchange interaction. The one-pion-exchange contribution to the quasiparticle interaction is[3]

$$\begin{aligned}f^{\pi\text{-exch}}_{\boldsymbol{p}\sigma\tau,\boldsymbol{p}'\sigma'\tau'}&=-P_\sigma P_\tau V_\pi(q)\\&=\frac{1}{3}\frac{f^2}{m_\pi^2}\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2+m_\pi^2}\\&\quad\times\left(S_{12}(\hat{\boldsymbol{q}})+\frac{1}{2}(3-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')\right)\frac{3-\boldsymbol{\tau}\cdot\boldsymbol{\tau}'}{2},\end{aligned} \tag{20}$$

where $\boldsymbol{q}=\boldsymbol{p}-\boldsymbol{p}'$ and $f=g_{\pi NN}(m_\pi/2m_N)\approx 1$. The one-pion-exchange contributions to the Landau parameters relevant for the convection current are

[3]In a relativistic formulation sketched in Appendix A, we can Fierz the one-pion exchange. Done in this way, the Fierzed scalar channel is canceled by a part of the vector channel and the remaining vector channel makes a natural contribution to the pionic piece of F_1.

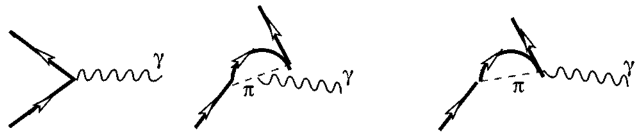

FIG. 1. Feynman diagrams contributing to the EM convection current in effective chiral Lagrangian field theory. (a) is the single-particle term and (b), (c) the next-to-leading chiral order pion-exchange current term. (c) does not contribute to the convection current; it renormalizes the spin gyromagnetic ratio.

$$\frac{F_1(\pi)}{3} = -F_1'(\pi) = -\frac{3f^2 m_N^\star}{8\pi^2 k_F} I_1, \tag{21}$$

where

$$I_1 = \int_{-1}^{1} dx \frac{x}{1-x+m_\pi^2/2k_F^2} = -2 + \left(1+\frac{m_\pi^2}{2k_F^2}\right)\ln\left(1+\frac{4k_F^2}{m_\pi^2}\right). \tag{22}$$

Thus, from Eq. (19), the one-pion-exchange contribution to the gyromagnetic ratio is

$$\delta g_l^\pi = \frac{m_N}{k_F}\frac{f^2}{4\pi^2} I_1 \tau_3. \tag{23}$$

In Sec. III C, we include contributions also from other degrees of freedom.

2. Chiral Lagrangian approach

In the absence of other meson degrees of freedom, we can simply calculate Feynman diagrams given by a chiral Lagrangian defined in the matter-free space. Nonperturbative effects due to the presence of heavy mesons introduce a subtlety that will be treated below.

In the leading chiral order, there is the single-particle contribution Fig. 1(a) which for a particle on the Fermi surface with the momentum $\boldsymbol{k}$ is given by

$$\boldsymbol{J}_{1\text{-body}} = \frac{\boldsymbol{k}}{m_N}\frac{1+\tau_3}{2}. \tag{24}$$

Note that the nucleon mass appearing in Eq. (24) is the free-space mass m_N as it appears in the Lagrangian, not the effective mass $m_N^\star$ that enters in the Fermi-liquid approach, Eq. (13). To the next-to-leading order, we have two ''soft-pion'' terms as discussed in Refs. [4,23,24]. To the convection current we need, only Fig. 1(b) contributes[4]

$$\boldsymbol{J}_{2\text{-body}} = \frac{\boldsymbol{k}}{k_F}\frac{f^2}{4\pi^2} I_1 \tau_3 = \frac{\boldsymbol{k}}{m_N}\frac{1}{6}[\tilde{F}_1'(\pi) - \tilde{F}_1(\pi)]\tau_3. \tag{25}$$

We should emphasize that the Landau parameters $\tilde{F}_1$ and $\tilde{F}_1'$ are entirely fixed by chiral symmetry for any density.

The sum of Eqs. (24) and (25) agrees precisely with the Fermi-liquid theory result (17),(21),(23). This formula first derived in Ref. [25] in connection with the Landau-Migdal parameter is of course the same as the Miyazawa formula [26] derived nearly half a century ago. Note the remarkable simplicity in the derivation starting from a chiral Lagrangian. However, we should caution that there are some nontrivial assumptions to go with the validity of the formula. As we will see shortly, we will not have this luxury of simplicity when other degrees of freedom enter.

C. Vector-meson contributions and BR scaling

So far we have computed only the pion contribution to g_l. In nuclear physics, more massive degrees of freedom such as the vector mesons ρ and ω of mass 700–800 MeV and the scalar meson σ of mass 600–700 MeV play an important role. When integrated out from the chiral Lagrangian, they give rise to effective four-Fermion interactions[5]

$$\mathcal{L}_4 = \frac{C_\phi^2}{2}(\bar{N}N)^2 - \frac{C_\omega^2}{2}(\bar{N}\gamma_\mu N)^2 - \frac{C_\rho^2}{2}(\bar{N}\gamma_\mu \tau N)^2 + \cdots, \tag{26}$$

where the coefficients $C's$ can be identified with

$$C_M^2 = \frac{g_M^2}{m_M^2} \quad \text{with} \quad M = \phi, \rho, \omega. \tag{27}$$

Such interaction terms are ''irrelevant'' in the renormalization group flow sense but can make crucial contributions by becoming ''marginal'' in some particular kinematic situation. A detailed discussion of this point can be found in Ref. [17]. The effective four-Fermion interactions play a key role in stabilizing the Fermi-liquid state and leads to the fixed points for the Landau parameters. In the two-nucleon systems studied in Refs. [2,3], they enter into the next-to-leading order term of the potential, which is crucial in providing the cut-off independence found for cutoff masses $\gtrsim m_\pi$.

1. Fermi-liquid theory approach

Again it suffices to compute the Landau parameters coming from the velocity-dependent part of heavy meson exchanges. We treat the effective four-Fermion interaction (26) in the Hartree approximation. Then the only velocity-dependent contributions are due to the current couplings mediated by ω and ρ exchanges. The corresponding contributions to the Landau parameters are

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[4]We should recall a well-known caveat here discussed already in Ref. [23]. If one were to blindly calculate the convection current coming from Fig. 1(b), there would be a gauge noninvariant term that is present because the hole line is off shell. Figure 1(c) contains also a gauge noninvariant term which is exactly the same as in Fig. 1(b) but with an opposite sign, so in the sum of the two graphs, the two cancel exactly so that only the gauge-invariant term survives. Of course we now know that the off-shell dependence is not physical and could be removed by field redefinition *ab initio*.

[5]For the moment, we make no distinction as to whether one is taking into account BR scaling or not. For the Fermi-liquid approach, this is not relevant since the parameters are not calculated. However, with chiral Lagrangians, we will specify the scaling which is essential.

$$F_1(\omega)=-C_\omega^2\frac{2k_F^3}{\pi^2\mu}\simeq-C_\omega^2\frac{2k_F^3}{\pi^2 m_N} \tag{28}$$

and

$$F_1'(\rho)=-C_\rho^2\frac{2k_F^3}{\pi^2\mu}\simeq-C_\rho^2\frac{2k_F^3}{\pi^2 m_N}, \tag{29}$$

where μ is the baryon chemical potential and the final expressions correspond to the nonrelativistic limit.

Now the calculation of the convection current and the nucleon effective mass with the interaction (26) in the Landau method goes through the same way as in the case of the pion. The net result is just Eq. (17) including the contribution of the contact interactions (28), (29), i.e.,

$$\tilde{F}_1=\tilde{F}_1(\pi)+\tilde{F}_1(\omega), \tag{30}$$

$$\tilde{F}_1'=\tilde{F}_1'(\pi)+\tilde{F}_1'(\rho). \tag{31}$$

Similarly, the nucleon effective mass is determined by Eq. (5) with

$$F_1=F_1(\pi)+F_1(\omega). \tag{32}$$

2. Chiral Lagrangian approach

The most efficient way to bring the vector mesons into the chiral Lagrangian is to implement BR scaling in the parameters of the Lagrangian. We shall take the masses of the relevant degrees of freedom to scale according to the BR scaling[6] [9]

$$\frac{M_N^\star}{m_N}\approx\frac{m_\omega^\star}{m_\omega}\approx\frac{m_\rho^\star}{m_\rho}\approx\frac{m_\phi^\star}{m_\phi}\approx\frac{f_\pi^\star}{f_\pi}\equiv\Phi. \tag{33}$$

Here $M_N^\star$ is a BR-scaling nucleon mass which will turn out to be different from the Landau effective mass $m_N^\star$ [21]. For our purpose, it is more convenient to integrate out the vector and scalar fields and employ the resulting four-Fermi interactions (26). The coupling coefficients are modified compared to Eq. (27), because the meson masses are replaced by effective ones

$$C_M^2=\frac{g_M^2}{m_M^{\star\,2}}\ \text{ with }\ M=\phi,\rho,\omega. \tag{34}$$

The coupling constants may also scale [10] but we omit their density dependence for the moment.

The relation between the BR factor Φ and F_1^ω. The first thing we need is the relation between the BR-scaling factor Φ which was proposed in [9] to reflect the quark condensate *in the presence of matter* and the contribution to the Landau parameter F_1 from the isoscalar vector (ω) meson. For this we first calculate the Landau effective mass $m_N^\star$ in the presence of the pion and the ω fields [21]

$$\frac{m_N^\star}{m_N}=1+\frac{1}{3}[F_1(\omega)+F_1(\pi)]$$
$$=\left(1-\frac{1}{3}[\tilde{F}_1(\omega)+\tilde{F}_1(\pi)]\right)^{-1}. \tag{35}$$

Next we compute the nucleon self-energy using the chiral Lagrangian. Given the single quasiparticle energy ϵ_p, we get the effective mass as in Ref. [21]

$$\frac{m_N^\star}{m_N}=\frac{k_F}{m_N}\left(\frac{d}{dp}\epsilon_p\bigg|_{p=k_F}\right)^{-1}=\left(\Phi^{-1}-\frac{1}{3}\tilde{F}_1(\pi)\right)^{-1}. \tag{36}$$

Comparing Eqs. (35) and (36), we obtain the important result

$$\tilde{F}_1(\omega)=3(1-\Phi^{-1}). \tag{37}$$

This is an intriguing relation. It shows that the BR factor, which was originally proposed as a precursor manifestation of the chiral phase transition characterized by the vanishing of the quark condensate at the critical point [9], is intimately related (at least up to $\rho\approx\rho_0$) to the Landau parameter F_1, which describes the quasiparticle interaction in a particular channel. We believe that the BR factor can be computed by QCD sum-rule methods or obtained from current algebra relations such as the Gell-Mann-Oakes-Renner (GMOR) relation evaluated in-medium. As was shown in Ref. [21], Eq. (37) implies that the BR factor governs in some, perhaps, intricate way low-energy nuclear dynamics. This suggests a possible ''dual'' description at low density between what is given in QCD variables (e.g., quark condensates) and what is given in hadronic variables (e.g., the Landau parameter), somewhat reminiscent of the quark-hadron duality in heavy-light-quark systems [31].

How to calculate the convection current in the presence of BR scaling. In the presence of the BR scaling, a noninteracting nucleon in the chiral Lagrangian propagates with

[6]In this paper, we are not addressing how this relation was arrived at since our main objective is to connect the scaling parameter Φ to many-body interactions and its link to the quark-antiquark condensate in the medium-modified ''vacuum'' does not enter directly into our discussion. But it may be useful for the sake of record to recall that this relation was first written down using the Skyrme Lagrangian embedded in medium with the scaling given by the expectation value of the scalar that figures in the trace anomaly of QCD [9]. Since this relation was first proposed, many authors have attempted to ''derive'' this scaling relation using various QCD-motivated models as well as sum-rule-type arguments. None of them has succeeded to reproduce this relation. The reasons for this are multifold but one of the main reasons is that the scalar field that enters in the scaling has not been correctly identified. As argued in Ref. [10], the scalar field that dials BR scaling is the ''quarkonium'' component of the trace anomaly, not the hard ''gluonic'' component. The latter dominates the trace anomaly but in the effective theory we are considering, this is integrated out with its effects lodged in higher-dimensional operators in the effective Lagrangian. In medium, as density is increased and the chiral transition point is approached, the ''mended symmetry'' argument of Weinberg [27] as interpreted by Beane and van Kolck [28] suggests that the scalar contributing to the trace anomaly that plays an important role in the scaling of hadron properties is the scalar that makes up the fourth component of $O(4)$ in linear σ model. This structure immediately gives, via a Nambu–Jona-Lasinio mechanism developed in Ref. [29], the hadron scaling relation (33). It has been pointed out to us by Brown that this picture is supported by a detailed lattice analysis of Liu *et al.* [30] for the source of the mass of a constituent quark. Indeed most of the mass of the light-quark hadron is shown to arise from the dynamical symmetry breaking and hence is intricately tied to the change of the vacuum implied in Eq. (33).

a mass $M_N^\star$, not the free-space mass m_N. Thus, the single-particle current Fig. 1(a) is *not* given by Eq. (24) but instead by

$$\boldsymbol{J}_{1\text{-body}}=\frac{\boldsymbol{k}}{M_N^\star}\frac{1+\tau_3}{2}. \tag{38}$$

Now the current (38) on its own does not carry conserved charge as long as $M_N^\star \neq m_N$. This means that two-body currents are indispensable to restore charge conservation. Note that the situation is quite different from the case of Fermi-liquid theory. In the latter case, the quasiparticle propagates with the Landau effective mass $m_N^\star$ and it is the gauge invariance that restores $m_N^\star$ to m_N.[7] This clearly indicates that gauge invariance is more intricate when BR scaling is implemented. Indeed if the notion of BR scaling and the associated chiral Lagrangian is to make sense, we have to recover charge conservation from higher-order terms in the chiral Lagrangian. This constitutes a strong constraint on the theory.

Let us now calculate the contributions from the pion and heavy-meson degrees of freedom. The pion contributes in the same way as before, so we can carry over the previous result of Fig. 1(b),

$$\boldsymbol{J}^{\pi}_{2\text{-body}}=\frac{\boldsymbol{k}}{m_N}\frac{1}{6}[\tilde{F}_1'(\pi)-\tilde{F}_1(\pi)]\tau_3. \tag{39}$$

This is of the same form as Eq. (25) obtained in the *absence* of BR scaling. It is in fact identical to Eq. (25) if we assume that one-pion-exchange graph *does not scale* in medium at least up to nuclear matter density. This assumption is supported by observations in pion-induced processes in heavy nuclei.[8] In what follows, we will make this assumption implicitly.

The contributions from the vector-meson degrees of freedom are a bit trickier. They are given by Fig. 2.

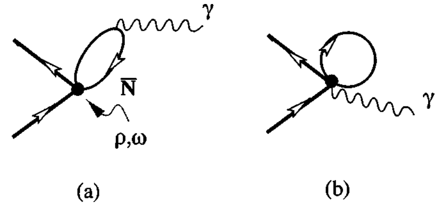

FIG. 2. (a) Feynman diagram contributing to the EM convection current from four-Fermi interactions corresponding to the ω and ρ channel (contact interaction indicated by the closed circle) in effective chiral Lagrangian field theory. The $\overline{N}$ denotes the antinucleon state that is given in the chiral Lagrangian as a $1/m_N$ correction and the one without arrow is a Pauli-blocked or occupied state. (b) The equivalent graph in heavy-fermion formalism with the anti-nucleon line shrunk to a point.

Both the ω (isoscalar) and ρ (isovector) channels contribute through the antiparticle intermediate state as shown in Fig. 2(a). The antiparticle is explicitly indicated in the figure. However in the heavy-fermion formalism, the backward-going antinucleon line should be shrunk to a point as Fig. 2(b), leaving behind an explicit $1/m_N$ dependence folded with a factor of nuclear density signaling the $1/m_N$ correction in the chiral expansion. One can interpret Fig. 2(a) as saturating the corresponding counter term although this has to be yet verified by writing the full set of counter terms at the same order. These terms have been evaluated in Ref. [21] with Fig. 2(a)

$$\boldsymbol{J}^{\omega}_{2\text{-body}}=\frac{\boldsymbol{k}}{m_N}\frac{1}{6}\tilde{F}_1(\omega), \tag{40}$$

$$\boldsymbol{J}^{\rho}_{2\text{-body}}=\frac{\boldsymbol{k}}{m_N}\frac{1}{6}\tilde{F}_1'(\rho)\tau_3, \tag{41}$$

where $\tilde{F}_1(\omega)$ and $\tilde{F}_1'(\rho)$ are given by Eqs. (28), (29) with m_N replaced by $M_N^\star$. The total current given by the sum of Eqs. (38)–(40) and (41) precisely agrees with the Fermi-liquid theory result (17) when we take

$$\tilde{F}_1=\tilde{F}_1(\omega)+\tilde{F}_1(\pi), \tag{42}$$

$$\tilde{F}_1'=\tilde{F}_1'(\rho)+\tilde{F}_1'(\pi). \tag{43}$$

The way in which this precise agreement comes about is nontrivial. What happens is that part of the ω channel restores the BR-scaled mass $M_N^\star$ back to the free-space mass m_N in the isoscalar current. [It has been known for some time that something similar happens in the standard Walecka model (without pions and BR scaling) [35].] Thus, the leading single-particle operator combines with the sub-leading four-Fermi interaction to restore the charge conservation as required by the Ward identity. This is essentially the ''back-flow mechanism'' which is an important ingredient in Fermi-liquid theory. We describe below the standard back-flow mechanism as given in textbooks [15], adapted to nuclear

[7] In condensed matter physics, this is related to a phenomenon associated with the cyclotron frequency which is referred to as the Kohn effect [32]. More on this later.

[8] In the early discussion of BR scaling in Ref. [9], the mass parameter for an in-medium pion $m_\pi^\star$ in the effective chiral Lagrangian was taken to scale down as $\sim\sqrt{\Phi}$. However, chiral perturbation theory in medium predicts the ''pole mass'' of the pion not to scale up to nuclear matter density [33]. In fact a recent analysis of deeply bound pionic states in heavy nuclei [34] shows that the pole mass of the pion could be a few per cent higher than the free-space value at nuclear matter density. The $m_\pi^\star$ in our in-medium effective chiral Lagrangian is not necessarily the pole mass and so it is not clear how to implement this empirical information into our theory. What we shall assume in this paper is that our $m_\pi^\star$ does not scale. This means that the observation that the one-pion-exchange potential does not scale implies that the constant $g_A^\star/f_\pi^\star$ remains unscaling at least up to normal nuclear matter density. At high density above normal nuclear matter density, however, $g_A^\star$ will stabilize to 1 while $f_\pi^\star$ will continue to drop and hence the coupling-constant ratio will increase.

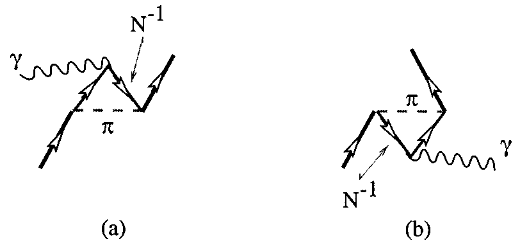

FIG. 3. Particle-hole contributions to the convection current. Here backward-going nucleon line N^{-1} denotes a hole. These graphs vanish in the $q/\omega\to 0$ limit.

systems with isospin degrees of freedom, and elucidate the connection to the results obtained with the chiral Lagrangian in this section.

3. The $\omega/q\to 0$ limit and the ''back-flow current''

The current so constructed is valid for a process occurring very near the Fermi surface corresponding to the limit $(\omega,\boldsymbol{q})\to(0,\mathbf{0})$ where q is the spatial momentum transfer and ω is the energy transfer. In the diagrams considered so far (Figs. 1 and 2) the order of the limiting processes does not matter. However, the particle-hole contribution, which we illustrate in Fig. 3 with the pion contribution,[9] does depend on the order in which $q=|\boldsymbol{q}|$ and ω approach zero. Thus, in the limit $q/\omega\to 0$, the particle-hole contributions vanish whereas in the opposite case $\omega/q\to 0$, they do not. This can be seen by examining the particle-hole propagator

$$\frac{n_k(1-n_{k+q})}{\omega+\epsilon_k-\epsilon_{k+q}+i\delta}-\frac{n_{k+q}(1-n_k)}{\omega+\epsilon_k-\epsilon_{k+q}-i\delta}, \tag{44}$$

where $(\omega,\boldsymbol{q})$ is the four-momentum of the external (EM) field. This vanishes if we set $q\to 0$ with ω nonzero but its real part is nonzero if we interchange the limiting process since for ω we have

$$\frac{\boldsymbol{q}\cdot\hat{\boldsymbol{k}}}{-\boldsymbol{q}\cdot\boldsymbol{k}/m_N}\delta(k_F-k). \tag{45}$$

In the limit $\omega/q\to 0$, the particle-hole contribution to the current is[10]

$$\boldsymbol{J}_{ph}=-\frac{\boldsymbol{k}}{m_N}\left(\frac{\tilde{F}_1+\tilde{F}_1'\tau_3}{6}\right). \tag{46}$$

Adding the particle-hole contribution (46) to the Fermi-liquid result (17) we obtain the current of a *dressed* or localized quasiparticle

$$\boldsymbol{J}_{locQP}=\frac{\boldsymbol{k}}{m_N^\star}\left(\frac{1+\tau_3}{2}\right). \tag{47}$$

Note that $\boldsymbol{J}_{\rm ph}$ precisely cancels $\delta\boldsymbol{J}$, Eq. (14). The current $\boldsymbol{J}_{locQP}$ is the total current carried by the wave packet of a localized quasiparticle with group velocity $\mathbf{v}_F=\boldsymbol{k}/m_N^\star$. However, the physical situation corresponds to homogeneous (plane wave) quasiparticle excitations. The current carried by a localized quasiparticle equals that of a homogeneous quasiparticle excitation modified by the so-called back-flow current [15]. The back-flow contribution $(\boldsymbol{J}_{\rm locQP}-\boldsymbol{J}_{\rm LM})$ is just the particle-hole polarization current in the $\omega/q\to 0$ limit, Eq. (46).

D. The gyromagentic ratio and Kohn's theorem

One of the most important results of this section is that the Landau-Migdal formula (17) does not depend on the Landau mass $m_N^\star$ but depends on the bare mass m_N even though a single quasiparticle responds to the photon with $m_N^\star$. This is completely analogous to the bare-mass dependence of the frequency of the collective excitations of the interacting quasiparticles of the half-filled Landau level, known as Kohn's theorem [32]. That our chiral Lagrangian formulation which starts with a BR-scaled mass $M_N^\star$ satisfies a Kohn theorem is a consistency check of the theory.

It should also be remarked that it is due to this phenomenon that the anomalous gyromagnetic ratio depends on the BR scale factor Φ in a simple way as

$$\delta g_l=\frac{1}{6}(\tilde{F}_1'-\tilde{F}_1)\tau_3=\frac{4}{9}\left[\Phi^{-1}-1-\frac{1}{2}\tilde{F}_1(\pi)\right]\tau_3. \tag{48}$$

This provides the link between the Landau mass $m_N^\star$, BR scaling Φ, and δg_l.

IV. AXIAL CHARGE TRANSITIONS

No one has yet derived the analog to Eq. (17) for the axial current. Attempts using axial Ward identities in analogy to the electromagnetic case have not met with success [36]. The difficulty has presumably to do with the role of the Goldstone bosons in nuclear matter which is not well understood. In this section, we analyze the expression for the axial charge operator obtained by a straightforward application of the Fermi-liquid theory arguments of Landau and Migdal and compare this expression with that obtained directly from the chiral Lagrangian using current algebra. For the vector current we found precise agreement between the two approaches.

A. Applying Landau quasiparticle argument

The obvious thing to do is to simply mimic the steps used for the vector current to deduce a ''Landau-Migdal'' expression for the axial charge operator. We use both methods developed above and find that they give the same result.

In free space, the axial charge operator nonrelativistically is $\sim\boldsymbol{\sigma}\cdot\mathbf{v}$ where $\mathbf{v}=\boldsymbol{k}/m_N$ is the velocity. In the infinite momentum frame, it is the relativistic invariant helicity $\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\nu}}$. It is thus tempting to assume that near the Fermi surface, the

[9]The relation we derive below holds in general regardless of what is being exchanged as long as the exchanged particle has the right quantum numbers.

[10]See Appendix B for a brief derivation of this expression with one pion exchange.

axial charge operator for a local quasiparticle in a wave packet moving with the group velocity $\mathbf{v}_F = \boldsymbol{k}/m_N^\star$ is simply $\sim \boldsymbol{\sigma}\cdot\mathbf{v}_F$. This suggests that we take the axial charge operator for a *localized* quasiparticle to have the form

$$A^i_{0\mathrm{locQP}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N^\star}\frac{\tau^i}{2}. \tag{49}$$

As in the vector current case, we take Eq. (49) to be the $\omega/q \to 0$ limit of the axial charge operator. The next step is to compute the particle-hole contribution to Fig. 3 (with the vector current replaced by the axial current) in the $\omega/q \to 0$ limit. A simple calculation gives

$$A^i_{0\mathrm{ph}} = -g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N^\star}\frac{\tau^i}{2}\Delta' \tag{50}$$

with

$$\Delta' = \frac{f^2 k_F m_N^\star}{4 m_\pi^2 \pi^2}(I_0 - I_1), \tag{51}$$

where I_1 was defined in Eq. (22) and

$$I_0 = \int_{-1}^{1} dx \frac{1}{1 - x + m_\pi^2/2k_F^2} = \ln\left(1 + \frac{4k_F^2}{m_\pi^2}\right). \tag{52}$$

In an exact parallel to the procedure used for the vector current, we take the difference

$$A^i_{0\mathrm{locQP}} - A^i_{0\mathrm{ph}} \tag{53}$$

and identify it with the corresponding Landau axial charge (LAC)

$$A^i_{0\mathrm{LAC}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N^\star}\frac{\tau^i}{2}(1+\Delta'). \tag{54}$$

Let us now rederive Eq. (54) with an argument analogous to that proven to be powerful for the convection current. We shall do the calculation using the pion exchange only but the argument goes through when the contact interaction (26) is included. We begin by assuming that the axial charge—in analogy to Eq. (6) for the convection current—takes the form

$$A^i_0 = g_A \sum_{\sigma\tau}\int \frac{d^3p}{(2\pi)^3}\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}_p \epsilon_p) n_p \frac{\tau^i}{2}, \tag{55}$$

where n_p and ϵ_p are 2×2 matrices with matrix elements

$$[n_p(\boldsymbol{r},t)]_{\alpha\alpha'} = n_p(\boldsymbol{r},t)\delta_{\alpha\alpha'} + \boldsymbol{s}_p(\boldsymbol{r},t)\cdot\boldsymbol{\sigma}_{\alpha\alpha'} \tag{56}$$

and

$$[\epsilon_p(\boldsymbol{r},t)]_{\alpha\alpha'} = \epsilon_p(\boldsymbol{r},t)\delta_{\alpha\alpha'} + \boldsymbol{\eta}_p(\boldsymbol{r},t)\cdot\boldsymbol{\sigma}_{\alpha\alpha'}, \tag{57}$$

with

$$\boldsymbol{s}_p(\boldsymbol{r},t) = \frac{1}{2}\sum_{\alpha\alpha'}\boldsymbol{\sigma}_{\alpha\alpha'}[n_p(\boldsymbol{r},t)]_{\alpha'\alpha}. \tag{58}$$

In general $n=4$ in the spin-isospin space. But without loss of generality, we could confine ourselves to $n=2$ in the spin space with the isospin operator explicited as in Eq. (55). Then upon linearizing, we obtain from Eq. (55)

$$A^i_0 = g_A \sum_{\sigma\tau}\int \frac{d^3p}{(2\pi)^3}[\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}_p \epsilon_p^0)\delta n_{p\sigma\tau} - \boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}_p n_p^0)\delta\epsilon_{p\sigma\tau}]\frac{\tau^i}{2} + \cdots, \tag{59}$$

where

$$\delta n_{p\sigma\tau} = \frac{1}{V}\delta^3(\boldsymbol{p}-\boldsymbol{k})\frac{1+\sigma_3}{2}\frac{\tau^i}{2} \tag{60}$$

and

$$\delta\epsilon_{p\sigma\tau} = \sum_{\sigma',\tau'}\int \frac{d^3p'}{(2\pi)^3} f_{p\sigma\tau,p'\sigma'\tau'}\delta n_{p'\sigma'\tau'} \tag{61}$$

in analogy with Eq. (12). Equation (59) is justified if the density of polarized spins is much less than the total density of particles (assumed to hold here). The first term of Eq. (59) with Eq. (60) yields the quasiparticle charge operator

$$A^i_{0\mathrm{QP}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N^\star}\frac{\tau^i}{2} \tag{62}$$

while the second term represents the polarization of the medium, due to the pion-exchange interaction (20)

$$\delta A^i_0 = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N^\star}\frac{\tau^i}{2}\Delta'. \tag{63}$$

The sum of Eqs. (62) and (63) agrees precisely with the Landau charge (54).

It is not difficult to take into account the full Landau-Migdal interactions (1) which includes the one-pion-exchange interaction as well as other contributions to the quasiparticle interaction. Thus, the general expression is obtained by making the replacement

$$\Delta' \to \frac{1}{3}G_1' - \frac{10}{3}H_0' + \frac{4}{3}H_1' - \frac{2}{15}H_2' \tag{64}$$

in Eq. (63). This combination of Fermi-liquid parameters corresponds to a $\ell = \ell' = 1, J=0$ distortion of the Fermi sea [20]. We will see later that the result obtained with the naive Landau argument may not be the whole story, since the one-pion-exchange contribution disagrees, though by a small amount, with the chiral Lagrangian prediction derived below.

B. Chiral Lagrangian prediction

We now calculate the axial charge using our chiral Lagrangian that reproduced the Landau-Migdal formula for the convection current. Consider first only the pion-exchange contribution. In this case we can take the unperturbed nucleon propagator to carry the free-space mass m_N. The single-particle transition operator corresponding to Fig. 1(a) is given by

$$A^i_{0\,1\text{-body}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N}\frac{\tau^i}{2}. \tag{65}$$

There is no contribution of the type of Fig. 1(b) because of the (G) parity conservation. The only contribution to the two-body current comes from Fig. 1(c) and is of the form [37]

$$A^i_{0\,2\text{-body}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N}\frac{\tau^i}{2}\Delta \tag{66}$$

with

$$\Delta = \frac{f^2 k_F m_N}{2 g_A^2 m_\pi^2 \pi^2}\left(I_0 - I_1 - \frac{m_\pi^2}{2k_F^2} I_1 \right). \tag{67}$$

The factor $(1/g_A^2)$ in Eq. (67) arose from replacing $1/f_\pi^2$ by $g_{\pi NN}^2/g_A^2 m_N^2$ using the Goldberger-Treiman relation.

Now consider what happens when the vector degrees of freedom are taken into account. Within the approximation adopted, the only thing that needs be done is to implement the BR scaling. The direct intervention of the vector mesons ρ and ω in the axial-charge operator is suppressed by the chiral counting, so they will be ignored here. This means that in the single-particle charge operator, all that one has to do is to replace m_N by $M_N^\star = m_N\Phi$ in Eq. (65):

$$A^i_{0\,1\text{-body}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N\Phi}\frac{\tau^i}{2} \tag{68}$$

and that in the two-body charge operator (66), f_π should be replaced by $f_\pi\Phi$ and m_N by $m_N\Phi$:

$$A^i_{0\,2\text{-body}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N\Phi}\frac{\tau^i}{2}\Delta. \tag{69}$$

In the two-body operator, there is a factor (g_A/f_π) coming from the πNN vertex which as mentioned before, is assumed to be nonscaling at least up to nuclear matter density [12,21], in consistency with the observation that the pion-exchange current does not scale in medium. The total predicted by the chiral Lagrangian (modulo higher-order corrections) is then

$$g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N\Phi}\frac{\tau^i}{2}(1+\Delta). \tag{70}$$

which differs from the charge operator obtained by the Landau method, Eq. (54).

C. Some (incomplete) observations on the difference

To expose the differences of the two approaches discussed above, let us look at the effect of the pion field. The cancellation between the two-body current $\boldsymbol{J}^\pi_{2\text{-body}}$ (39) and $\boldsymbol{J}^\pi_{\rm ph}$ (46) leaving only a term that changes $M_N^\star$ to $m_N^\star$ in the one-body operator with a BR-scaling mass, Eq. (38), in the EM case can be understood as follows. Both terms involve the two-body interaction mediated by a pion-exchange. It is obvious how this is so in the latter. To see it in the former, we note that it involves the insertion of an EM current in the propagator of the pion. Thus the sum of the two terms corresponds to the insertion of an EM current in *all internal hadronic lines* of the one-pion exchange self-energy graph of the nucleon. The two-body pionic current—that together with the single-particle current preserves gauge invariance—is in turn related to the one-pion-exchange potential V_π. Therefore what is calculated is essentially an effect of a nuclear force. Now the point is that the density-dependent part of the sum (that is, the ones containing one hole line)—apart from a term that changes $M_N^\star$ to $m_N^\star$ in Eq. (38)—vanishes in the $\omega/q\to 0$ limit. In contrast, the cancellation between Eqs. (63) and (50) in the case of the axial charge, has no corresponding interpretation. While the one-pion-exchange interaction is involved in the particle-hole terms (50),(63) cannot be interpreted as an insertion of the axial vector current into the pion propagator since such an insertion is forbidden by parity. In other words, Eq. (63) does not have a corresponding Feynman graph which can be linked to a potential. We interpret this as indicating that there is no corresponding Landau formula for the axial charge in the same sense as in the vector current case.

In a chiral Lagrangian formalism, each term is associated with a Feynman diagram. There is no contribution to the convection current from a diagram of the type Fig. 1(c) [apart from a gauge noninvariant off-shell term which cancels the counter part in Fig. 1(b)]. Instead this diagram renormalizes the spin gyromagnetic ratio. In contrast, the corresponding diagram for the axial current *does* contribute to the axial charge (66). As first shown in Ref. [4], the contribution from Fig. 1(c) for both the vector current and the axial-vector current is current algebra in origin and constrained by chiral symmetry. Furthermore it does not have a simple connection to the nuclear force. While the convection current is completely constrained by gauge invariance of the electromagnetic field, and hence chiral invariance has little to say, both the EM spin current and the axial charge are principally dictated by the chiral symmetry. This again suggests that the Landau approach to the axial charge cannot give the complete answer even at the level of a quasiparticle description. There is, however, a caveat here: in the Landau approach, the nonlocal pionic and local four-Fermion interactions (26) enter together in an intricate way as we saw in the electromagnetic case. Perhaps this is also the case in the axial charge, with an added subtlety due to the presence of Goldstone pions. It cannot be ruled out that the difference is due to the contribution of the four-Fermion interaction term to Eq. (64) which cancels out in the limit $\omega/q\to 0$ but contributes in the $q/\omega\to 0$ limit. This term cannot be given a simple interpretation in terms of chiral Lagrangians. This point needs further study.

D. Numerical comparison

To compare the two results, we rewrite the sum of Eqs. (62) and (63), i.e., ''Landau axial charge'' (LAC), using Eqs. (5) and (21)

$$A^i_{0\mathrm{LAC}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N \Phi} \frac{\tau^i}{2}(1+\tilde{\Delta}), \tag{71}$$

where

$$\tilde{\Delta} = \frac{f^2 k_F m_N \Phi}{4\pi^2 m_\pi^2}\left(I_0 - I_1 + \frac{3m_\pi^2}{2k_F^2} I_1 \Phi^{-1}\right) \tag{72}$$

and the sum of Eqs. (68) and (69), i.e., the ''current-algebra axial charge'' (CAAC), as

$$A^i_{0\mathrm{CAAC}} = g_A \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{m_N \Phi} \frac{\tau^i}{2}(1+\Delta), \tag{73}$$

where

$$\Delta = \frac{f^2 k_F m_N}{2 g_A^2 m_\pi^2 \pi^2}\left(I_0 - I_1 - \frac{m_\pi^2}{2k_F^2} I_1\right). \tag{74}$$

We shall compare $\tilde{\Delta}$ and Δ for two densities $\rho = \frac{1}{2}\rho_0$ ($k_F = 1.50 m_\pi$) and $\rho = \rho_0$ ($k_F = 1.89 m_\pi$) where ρ_0 is the normal nuclear matter density 0.16/fm^3.

For numerical estimates, we take

$$\Phi(\rho) = \left(1 + 0.28\frac{\rho}{\rho_0}\right)^{-1} \tag{75}$$

which gives $\Phi(\rho_0) = 0.78$ found in QCD sum-rule calculations [10,21]. Somewhat surprisingly, the resulting values for $\tilde{\Delta}$ and Δ are close; they agree within 10%. For instance, at $\rho \approx \rho_0/2$, $\tilde{\Delta} \approx 0.48$ while $\Delta \approx 0.43$ and at $\rho \approx \rho_0$, $\tilde{\Delta} \approx 0.56$ while $\Delta \approx 0.61$. Whether this close agreement is coincidental or has a deep origin is not known.

E. Experimental evidences

The small difference between the two approaches has little effect on the axial charge transition matrix element in heavy nuclei,

$$A(J^+) \leftrightarrow B(J^-) \tag{76}$$

with change of one unit of isospin $\Delta T = 1$. To confront quantitatively our formulas with experiments, we would have to incorporate the finite-size effect for BR scaling which we could do in the local density approximation. This should be a well-defined, though laborious, exercise. However, performing such a calculation is out of scope of this work, so what we will do here is to make a qualitative (yet reliable) estimate to see how things go as a function of density. The quantity of interest is the Warburton ratio ϵ_{MEC} [38]

$$\epsilon_{\mathrm{MEC}} = M_{\mathrm{exp}}/M_{\mathrm{sp}}, \tag{77}$$

where M_{exp} is the measured matrix element for the axial charge transition and M_{sp} is the theoretical single-particle matrix element. It was observed by Warburton that in heavy nuclei,

$$\epsilon_{MEC}^{\text{heavy nuclei}} = 1.9\text{–}2.0 \tag{78}$$

showing that the ''mesonic enhancement'' could be as big as 100% at nuclear matter density. More recent measurements—and their analyses—in different nuclei [39] ranging from $A = 12$ to $A = 205$ nuclei quantitatively confirm this result of Warburton.

To compare our theoretical prediction with the Warburton analysis, we simply take $\rho \approx \rho_0 \approx 0.16\ \mathrm{fm}^{-3}$ and calculate the same ratio using[11] Eq. (73)

$$\epsilon_{\mathrm{MEC}}^{\mathrm{CAAC}} = \Phi^{-1}(1+\Delta). \tag{79}$$

The enhancement corresponding to the Landau formula (71) is obtained by replacing Δ by $\tilde{\Delta}$ in Eq. (79). Using the value for Φ and Δ at nuclear matter density, we find

$$\epsilon_{\mathrm{MEC}}^{\mathrm{th}} \approx 2.1(2.0). \tag{80}$$

Here the value in parenthesis is obtained with the Landau formula (71). The difference between the two formulas (i.e., current algebra vs Landau) is indeed small. It should be noted that this is a check of the scaling of f_π in combination with the assumption that the pion does not scale up to nuclear matter density.

V. SUMMARY AND DISCUSSIONS

In this section we summarize what we have achieved—and failed to achieve—and take up some of the matters inadequately discussed.

By means of nuclear response to electromagnetic convection current, we have identified the BR-scaling parameter Φ with the scaling nucleon mass $M^\star$. The Landau effective mass of the nucleon $m_N^\star$ is in turn given in terms of Φ and the Goldstone boson cloud of the broken chiral symmetry, i.e., pion, through the parameter $\tilde{F}_1^\pi$. The relation between the orbital gyromagnetic ratio δg_l and $m_N^\star$ provides the crucial link between Φ and the Landau parameter F_1^ω coming from the massive degrees of freedom in the isoscalar vector channel dominated by the ω meson.

In the BR-scaling Lagrangian approach, the axial charge transitions in heavy nuclei provides a relation between Φ and the in-medium pion decay constant $f_\pi^\star/f_\pi$. We have, however, failed to link this theory to a corresponding Landau-Fermi-liquid description. This may be due to our poor understanding of the subtle role that Goldstone bosons play in nuclear axial currents in Fermi-liquid theory.

[11]This formula differs from what was obtained in Ref. [40] in that here the nonscaling in medium of the pion mass and the ratio g_A/f_π is taken into account. We believe that the scaling used in Ref. [40] [which amounted to having Δ/Φ in place of Δ in Eq. (79)] is not correct.

A Walecka-type linear model for nuclear matter with the parameters of the Lagrangian scaling in the manner of Brown-Rho consistent with chiral symmetry provides the connection between Φ and the scaling of the vector-meson degrees of freedom (ω and ρ) and scalar-meson degrees of freedom σ in the situation where the mesons are highly off-shell. This relation has been checked against fluctuations into various flavor directions, including strangeness flavor. So far the check is only semi-quantitative and approximate but there is consistency.

A rigorous derivation of BR scaling starting from an effective chiral action via multiple scale decimations required for the problem is yet to be formulated but the main ingredients, both theoretical and phenomenological, seem to be available.

So far, we have succeeded in mapping the chiral Lagrangian theory with BR scaling to nonrelativistic Landau Fermi-liquid theory. This is natural since we worked in heavy-Fermion formalism for the chiral Lagrangian field theory. However, in order to apply the correspondence to dense matter encountered in relativistic heavy-ion collisions and in neutron stars—a nontrivial open problem, we should formulate the mapping relativistically as in Ref. [11] where thermodynamic properties of a BR-scaled chiral Lagrangian in the mean field were shown to be consistent with the relativistic Landau formulas derived by Baym and Chin [41]. This work is in progress.

In discussing properties of dense matter, such as BR scaling of masses and coupling constants, e.g., $f_\pi^\star$, we have been using a Lagrangian which preserves Lorentz invariance. This seems to be at odds with the fact that the medium breaks Lorentz symmetry. One would expect for instance that the space and time components of a current would be characterized by different constants. Specifically such quantities as g_A, f_π, etc., would be different if they were associated with the space component or time component of the axial current. So a possible question is, how is the medium-induced symmetry breaking accommodated in the formalism we have been discussing in this paper? The answer to this question was provided in Ref. [11]. There the argument was given in an exact parallel to Walecka mean field theory of nuclear matter. One writes an effective Lagrangian with all symmetries of QCD which in the mean field defines the parameters relevant to the state of matter with density. The parameters that become constants (masses, coupling constants, etc.) at given density are actually functionals of chiral invariant bilinears in the nucleon fields. When the scalar field ϕ and the bilinear $\psi^\dagger\psi$, where ψ is the nucleon field, develop a nonvanishing expectation value Lorentz invariance is broken and the time and space components of a nuclear current pick up different constants. This is how, for instance, the Gamow-Teller constant g_A measured in the space component of the axial current is *quenched* in dense medium while the axial charge measured in the axial charge transitions is *enhanced* as described above. If one were to calculate the pion decay constant in medium, one would also find that the quantity measured in the space component is different from the time component. The way Lorentz-invariant Lagrangians figure in nuclear physics is in some sense similar to what happens in condensed matter physics. For example, on a lattice where there is not even rotational invariance, one finds a Lorentz-invariant dispersion formula. Another well-known example is the fractional quantized Hall effect which is described by a Lorentz-invariant Lagrangian containing the Chern-Simons term [42].

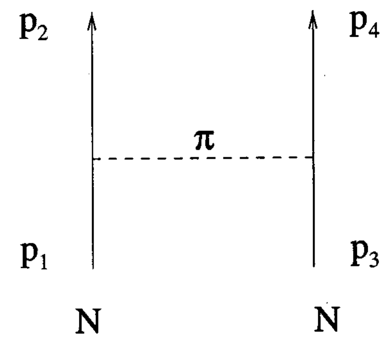

FIG. 4. The-one-pion-exchange diagram that gives rise to F_1^π.

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APPENDIX A: RELATIVISTIC CALCULATION OF F_1^π

In the text, the Landau parameter F_1^π (or f^π) was calculated nonrelativistically via the Fock term of Fig. 4. Here we calculate it relativistically by Fierz transforming the one-pion-exchange graph and taking the Hartree term. This procedure is important for implementing relativity in the connection between Fermi-liquid theory and chiral Lagrangian theory along the line discussed by Baym and Chin [41].

The one-pion-exchange potential in Fig. 4 is

$$V_\pi = -g_{\pi NN}^2(\boldsymbol{\tau}_{21}\cdot\boldsymbol{\tau}_{43})\frac{\overline{u}_2\gamma^5 u_1\overline{u}_4\gamma^5 u_3}{(p_2-p_1)^2-m_\pi^2}. \tag{A1}$$

The Dirac spinors are normalized by

$$u^\dagger(p,s)u(p,s') = \delta_{ss'}. \tag{A2}$$

By a Fierz transformation, we have

$$\boldsymbol{\tau}_{21}\cdot\boldsymbol{\tau}_{43} = \frac{1}{2}(3\,\delta_{41}\delta_{23} - \boldsymbol{\tau}_{41}\cdot\boldsymbol{\tau}_{32}) \tag{A3}$$

and

$$\overline{u}_2\gamma^5u_1\overline{u}_4\gamma^5u_3=\frac{1}{4}[\overline{u}_4u_1\overline{u}_2u_3-\overline{u}_4\gamma^\mu u_1\overline{u}_2\gamma_\mu u_3 +\overline{u}_4\sigma^{\mu\nu}u_1\overline{u}_2\sigma_{\mu\nu}u_3+\overline{u}_4\gamma^\mu\gamma^5u_1\overline{u}_2\gamma_\mu\gamma^5u_3 +\overline{u}_4\gamma^5u_1\overline{u}_2\gamma^5u_3]. \tag{A4}$$

Remembering a minus sign for the fermion exchange, we obtain the corresponding pionic contribution to the quasiparticle interaction at the Fermi surface $f^\pi=-V_\pi(\boldsymbol{p}_1=\boldsymbol{p}_4=\boldsymbol{p},\boldsymbol{p}_2=\boldsymbol{p}_3=\boldsymbol{p}',\boldsymbol{p}^2=\boldsymbol{p}'^2=k_F^2)$ [see Eq. (1)]. Decomposing f^π as

$$f^\pi=\frac{3-\boldsymbol{\tau}\cdot\boldsymbol{\tau}'}{2}(f_S+f_V+f_T+f_A+f_P), \tag{A5}$$

where *S*, *V*, *T*, *A*, and *P* represent scalar, vector, tensor, axial vector, and pseudoscalar channel, respectively, we find

$$f_S=-\frac{m_N^4f^2}{E_F^2m_\pi^2}\frac{1}{q^2+m_\pi^2},$$

$$f_V=\frac{m_N^4f^2}{E_F^2m_\pi^2}\frac{1}{q^2+m_\pi^2}\left(1+\frac{q^2}{2m_N^2}\right),$$

$$f_T=-\frac{m_N^4f^2}{E_F^2m_\pi^2}\frac{1}{q^2+m_\pi^2}\left[\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'\left(1+\frac{q^2}{2m_N^2}\right)+\frac{2\boldsymbol{\sigma}'\cdot\boldsymbol{p}\boldsymbol{\sigma}\cdot\boldsymbol{p}'-\boldsymbol{\sigma}\cdot\boldsymbol{p}\boldsymbol{\sigma}'\cdot\boldsymbol{p}-\boldsymbol{\sigma}\cdot\boldsymbol{p}'\boldsymbol{\sigma}'\cdot\boldsymbol{p}'}{2m_N^2}\right],$$

$$f_A=\frac{m_N^4f^2}{E_F^2m_\pi^2}\frac{1}{q^2+m_\pi^2}\times\left(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'-\frac{2\boldsymbol{\sigma}\cdot\boldsymbol{p}\boldsymbol{\sigma}'\cdot\boldsymbol{p}'-\boldsymbol{\sigma}\cdot\boldsymbol{p}\boldsymbol{\sigma}'\cdot\boldsymbol{p}-\boldsymbol{\sigma}\cdot\boldsymbol{p}'\boldsymbol{\sigma}'\cdot\boldsymbol{p}'}{2m_N^2}\right),$$

$$f_P=0 \tag{A6}$$

with $E_F=\sqrt{k_F^2+m_N^2}$ and $q=|\boldsymbol{p}-\boldsymbol{p}'|$. Thus we obtain

$$f^\pi=\frac{f^2}{m_\pi^2}\frac{m_N^2}{E_F^2}\frac{1}{q^2+m_\pi^2}\left(\boldsymbol{\sigma}\cdot\boldsymbol{q}\boldsymbol{\sigma}'\cdot\boldsymbol{q}-\frac{q^2(1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')}{2}\right)\frac{3-\boldsymbol{\tau}\cdot\boldsymbol{\tau}'}{2} =\frac{1}{3}\frac{f^2}{m_\pi^2}\frac{m_N^2}{E_F^2}\frac{q^2}{q^2+m_\pi^2}\left(3\frac{\boldsymbol{\sigma}\cdot\boldsymbol{q}\boldsymbol{\sigma}'\cdot\boldsymbol{q}}{q^2}-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'+\frac{1}{2}(3-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')\right)\frac{3-\boldsymbol{\tau}\cdot\boldsymbol{\tau}'}{2}. \tag{A7}$$

In the nonrelativistic limit, $E_F\sim m_N$ and we recover Eq. (20). The factor m_N/E_F comes since there is one particle in the unit volume which decreases relativistically as the speed increases. Note that only f_S and f_V in Eq. (A6) are spin-independent and contribute to F_1^π. The f_S is completely canceled by the leading term of f_V with the remainder giving F_1^π. In this way of deriving the Landau parameter F_1, it is the vector channel that plays the essential role.

APPENDIX B: PARTICLE-HOLE CONTRIBUTION TO THE VECTOR CURRENT

The leading contribution of the particle-hole polarization with one-pion exchange is shown in Fig. 3. This graph was computed by several authors (e.g., see Ref. [22]) and is given in the limit $\omega/q\rightarrow 0$ by

$$\boldsymbol{J}_{\rm ph}=-\sum_{\tau'}\left\langle\boldsymbol{\tau}(1)\cdot\frac{1+\tau_3'}{2}\boldsymbol{\tau}(2)\right\rangle\int\frac{d^3p}{(2\pi)^3}\hat{\boldsymbol{p}}\delta(k_F-|\boldsymbol{p}|)f_s^\pi, \tag{B1}$$

where $f_s^\pi\equiv f_S+f_V+f_T+f_A+f_P$. The isospin factor is given by the Fierz transformation

$$\sum_{\tau'}\left\langle\boldsymbol{\tau}(1)\cdot\frac{1+\tau_3'}{2}\boldsymbol{\tau}(2)\right\rangle =\sum_{\tau'}\left\langle\frac{3}{4}-\frac{1}{4}\boldsymbol{\tau}\cdot\boldsymbol{\tau}'+\frac{3}{4}{\rm tr}[\tau_3']-\frac{1}{4}\boldsymbol{\tau}\cdot{\rm tr}[\tau_3'\boldsymbol{\tau}']\right\rangle =\frac{3}{2}-\frac{1}{2}\tau_3. \tag{B2}$$

Note that the factor $\frac{3}{2}$ comes from f_π and $\frac{1}{2}\tau_3$ from f_π'. In the limit that we are concerned with (i.e., $T=0$ and $\omega/q\rightarrow 0$), we find

$$\boldsymbol{J}_{ph}=-\frac{1}{3\pi^2}\hat{\boldsymbol{k}}k_F^2(f_1+f_1'\tau_3) =-\frac{\boldsymbol{k}}{m_N}\frac{\tilde{F}_1(\pi)+\tilde{F}_1'(\pi)\tau_3}{6}. \tag{B3}$$

Contributions from heavy-meson exchanges are calculated in a similar way.

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