Hadronic phase space density and chiral symmetry restoration in relativistic heavy ion collisions

Scott Pratt and Kevin Haglin*
Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

(Received 15 October 1998)

The effect of altered hadron masses is studied for its effect with regard to final-state hadronic observables. It is shown that the final phase space densities of pions and kaons, which can be inferred experimentally, are sensitive to in-medium properties of the excited matter at earlier stages of the collision, but that the sensitivity is significantly moderated by interactions that change the effective numbers of pions and kaons during the latter part of the collision. [S0556-2813(99)07105-8]

PACS number(s): 25.75.Gz, 25.75.Dw

Relativistic heavy ion collisions at the CERN SPS (160*A* GeV Pb+Pb) or the Brookhaven AGS (11*A* GeV Au+Au) produce a mesoscopic region where initial energy densities are in the neighborhood of a few GeV/fm^3 [1,2], several times the energy density of a proton. As the system expands and cools normal hadronic degrees of freedom become justified, but the system might still have novel properties when energy densities are in the region of 500 MeV/fm^3. Most notably, as a consequence of restoring chiral symmetry, the masses of heavier hadrons might fall as much as 50% [3]. Evidence of falling masses has appeared in dilepton measurements of the ρ peak in Pb+Pb collisions performed at the SPS. The observed lack of strength of dilepton pairs in the region of the ρ meson mass, combined with the observed additional strength for invariant masses near 400 MeV, suggests either that the ρ meson was altered [4], dissolved [5], or broadened beyond recognition by collisions [6].

Dilepton measurements provide a transparent probe for investigating hadronic properties during the most interesting stages of the collision, but only of those hadrons with the quantum numbers of the photon. Final-state measurements of hadrons—pions, kaons, protons, and hyperons—provide a rich chemistry as nearly 10^3 hadrons are commonly produced in a single event. However, hadrons interact several times after the system has expanded beyond the energy densities of greatest interest when temperatures are near or above 150 MeV, and before the breakup density is reached, when temperatures are approximately 110 MeV. In the high density hadronic state, equilibrium chemical abundances can easily change by factors of 2 if masses are adjusted by several hundred MeV as predicted by some chiral models. In this paper we investigate whether such mass changes, and the corresponding changes in hadronic chemical abundances, survive and produce a signal in the measured hadrons. We model the chemical development of a kinetically equilibrated gas and find that falling hadron masses, on the order of 50%, result in increased final phase space densities for both pions and kaons which can be inferred by measuring spectra and two-particle correlations. We show that if reactions that preserve the effective pion and kaon numbers are ignored, a strong signal survives the final expansion, and that if such reactions are included, the manifestations of the original novel chemistry survive, but are strongly moderated. We also find that hadronic observables are sensitive to the issue of whether both baryon and vector mesons scale or whether only the baryon masses fall [7,8].

In Pb+Pb collisions at the SPS or in Au+Au collisions at the AGS, ≈ 7 fm/c after initial contact, matter has expanded and cooled to the point where a model based on binary interactions of hadrons is justified. We take the onset of binary modeling to occur at a temperature of 160 MeV. We assume that the system is both kinetically and chemically equilibrated at this point and is characterized by three numbers: a baryon chemical potential μ_b, a strangeness chemical potential μ_s, and a temperature T. The strangeness chemical potential is chosen such that the net strangeness is zero. It would be zero if the baryon density were zero. The baryon chemical potential is chosen to match the effective baryon to pion ratios, unity for the AGS example and 1/5 for the SPS example. The assumption of equilibration when $T=160$ MeV is an ansatz, justified only by the fact that the hadrons significantly overlap at higher temperature, suggesting rapid equilibration of the strongly interacting system. At later times and lower temperatures, kinetic equilibrium is approximately maintained [9], but some aspects of chemical equilibrium are lost. When the system expands and cools to approximately $T=110$ MeV, kinetic equilibrium is lost as well and the system dissolves.

As the system expands, the effective numbers of kaons and pions must adjust to the rapidly changing environment. The effective number of pions is defined as the number of pions plus the sum of other hadrons weighted by their effective pionic content. For instance, a ρ meson counts as two pions since it decays principally into two pions, while the effective pionic content of a Δ baryon is one. Reactions such as $\rho\leftrightarrow\pi\pi$ are rapid and can keep the number of ρ mesons in equilibrium but do not change the overall effective pion number [10,11]. Reactions that change the overall pion number, such as $\pi\pi\leftrightarrow\rho\rho$, are not sufficiently fast and allow the system to lose chemical equilibrium. One can characterize the unequilibrated state by an effective chemical potential μ_π that corresponds to the ''conserved'' pion number. We similarly assign an effective chemical potential to the effective kaon number. Although the net strangeness is zero, the net number of strange quarks must adjust during the expan-

*Present address: Department of Physics, Astronomy and Engineering Science, St. Cloud State University, St. Cloud, MN 56301.

sion. Reactions that conserve the net number of strange quarks occur rapidly, e.g., $K^-p \leftrightarrow \Lambda\pi^+$, but reactions that change the number of strange quarks, e.g., $\pi\pi \leftrightarrow KK$, are slow as they require a strange and an antistrange hadron to either interact or be produced jointly. We again describe this lack of chemical equilibrium via an effective chemical potential μ_K [12] that corresponds to the effective kaon number, where a kaon or Λ baryon counts as one kaon as they each have one strange quark. Either an Ω baryon (with quark content sss) or an $\overline{\Omega}$ would count as three kaons. Both μ_K and μ_π become zero in chemically equilibrated systems.

When a system is perturbed from chemical equilibrium, μ_K and μ_π approach equilibrium exponentially with characteristic times τ_K and τ_π. Our analysis is similar to that of Song and Koch [10], but includes a greater variety of hadrons: the spin 1/2 baryon octet, the baryon spin 3/2 dectet, the pseudoscalar meson nonet, and the vector meson nonet, but not those resonances which correspond to orbital excitations of mesons in the framework of the constituent quark model. Given the cross sections for creating pions in individual reactions and time-reversal arguments we calculate the net rates

$$\frac{dN_\pi}{d^3x dt} = \sum_i \Delta N_{i,\pi} R_i (1 - e^{-\Delta N_{i,\pi}\mu_\pi/T}),$$

$$R_i = \frac{(2J_a+1)(2J_b+1)}{(2\pi)^6} \int \frac{E_{\text{c.m.}} d^3P}{E} \times e^{-[E-\mu_a N_a - \mu_b N_b]/T} \int d^3q\, \sigma(E_{\text{c.m.}}) v_{\text{rel,c.m.}}, \quad (1)$$

where the sum is over all reactions $(ab \rightarrow X)$ that increase the effective pion number by $\Delta N_{i,\pi}$. Here, q and v_{rel} are the relative momentum and relative velocity in the center-of-mass frame. The energy of the pair is E in the laboratory frame and $E_{\text{c.m.}}$ in the center-of-mass frame. The integral over the center-of-mass momentum P is performed analytically, while the integral over the relative momentum q is performed numerically.

As mentioned above, time-reversal arguments are used to motivate the factor $(1 - e^{-\Delta N_{i,\pi}\mu_\pi/T})$ in Eq. (1). Time-reversal arguments imply that the inverse reaction rate is equal to the forward rate multiplied by the ratio of the density of states, $e^{\Delta S/T}$, where ΔS is the change of entropy, $\Delta S = -\mu \Delta N/T$.

The effective pion density

$$n_\pi = \sum_a N_{a,\pi}(2J_a+1) \int \frac{d^3p}{(2\pi)^3} e^{-(E-\mu_\pi N_{a,\pi})/T} \quad (2)$$

can also be found analytically given the effective pionic content $N_{a,\pi}$ of the species a. One can expand the above expressions for small μ_π to find the rate at which μ_π exponentially returns to equilibrium,

$$\frac{1}{\tau} = \frac{\sum_i (\Delta N_{i,\pi})^2, R_i(\mu_\pi = 0)}{n_\pi(\mu_\pi = 0)}. \quad (3)$$

Unfortunately, experimental cross sections are not available for the majority of the combinations ab of the 26 mass states used in this analysis, and perturbation theory is not particularly reliable at these energies. We have therefore instituted simplified expressions for the cross sections,

$$\sigma(E_{\text{c.m.}}) = \sigma_0 \theta(E_{\text{c.m.}} - E_{\text{th}} - N_{\pi,X}\Delta_E \text{ MeV}), \quad (4)$$

where $N_{\pi,X}$ is the number of pions in the final state, the threshold E_{th} is the minimum energy of a state with the quantum numbers of the initial state, and $\Delta_E = 350$ MeV. This behavior is motivated by the observed behavior of the inelastic pp and $p\pi$ cross sections [13] with σ_0 being 25 mb if both incoming particles are baryons, 20 mb if one is a baryon, and 15 mb if neither is a baryon. When the production of more pions becomes available, the entire cross section is then devoted to that number of pions. For the case where no baryons are present in the initial state, only even numbers of pions are allowed in the final state as required by conservation of g parity. For the production of strangeness, only pairs of strange-antistrange hadrons are allowed and the steps are in units of $\Delta_E = 1.2$ GeV if no baryons are present in the initial state and 800 MeV if baryons are present. The cross section for producing strangeness is then

$$\sigma(E_{\text{c.m.}}) = \sigma_0 \theta(E_{\text{c.m.}} - E_{\text{th}} - N_{K,X}\Delta_E). \quad (5)$$

The term σ_0 for strangeness production is taken as one-fourth that of the pion production cross section. These values were motivated by measurements of Λ production [14,15], with the value of σ_0 being multiplied by 2.5 to account for the production of other hyperons. Although the prescription for determining cross sections is phenomenologically motivated, given the large amount of experimentally unknown information, the cross sections must not be taken seriously beyond the 50% level, especially for the case of strangeness-producing rates.

Furthermore, one must not forget that the underlying ansatz—that the entire system is described by temperature and chemical potentials—is an approximation. Although certain species, like the ρ, rapidly equilibrate with the pion density, other species like the η which has a lifetime of thousands of fm/c or the ω with a lifetime of 20 fm/c can be slow to equilibrate due to their three-pion decay modes. Although the lifetime of the ω is not so out of scale with typical expansion times, the η lifetime is far longer. Although we include the η in our treatment, it may be better to ignore it. Such a choice might affect the phase space density on the 10% level, since roughly a tenth of the pions originate from η decays. Such considerations underline the importance of not trusting equilibration arguments for calculating chemical abundances to a better than 20% level.

Characteristic chemical equilibration times are displayed in Fig. 1. Characteristic expansion times are ≈ 5 fm/c when $T = 160$ MeV and ≈ 20 fm/c when the system is near breakup at $T \approx 110$ MeV. From viewing Fig. 1 one expects pionic chemical equilibrium to be lost when the temperature approaches 150 MeV while strangeness equilibration can be

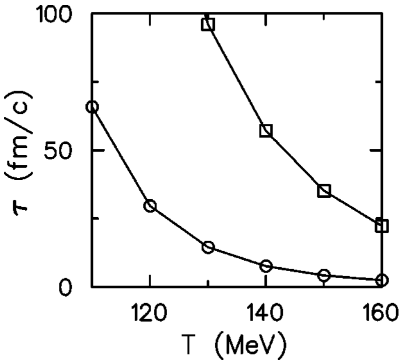

FIG. 1. Characteristic times for chemical potentials returning to zero are shown for μ_π (circles) and μ_K (squares). The decay time corresponding to μ_π becomes larger than characteristic expansion times when $T \approx 150$ MeV, while equilibration times for μ_K are longer than characteristic expansion times even for $T = 160$ MeV.

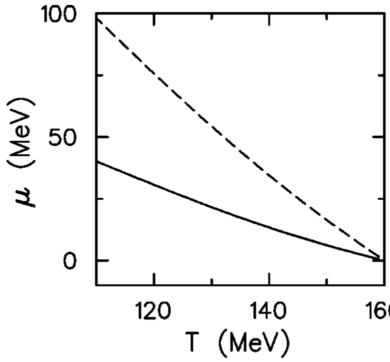

FIG. 2. The evolution of μ_π (solid line) and μ_K (dashed line) is shown for an expanding hadron gas that began at a 160 MeV temperature. This calculation has incorporated pion- and kaon-number-changing rates.

hardly justified even at $T = 160$ MeV. We should point out that if masses fall due to restoration of chiral symmetry, hadron densities rise accordingly and characteristic times are considerably shorter. The role of baryons in maintaining chemical equilibrium is crucial as they catalyze mesonic equilibrium. Even though baryons comprise only a fifth of the produced hadrons from SPS collisions, reactions involving baryons represent the majority of the rate. This comes from the fact that reactions involving baryons do not need to conserve g parity and pions need not be produced pairwise.

We calculate chemical potentials for systems which cool from an equilibrated state at $T = 160$ MeV to a chemically unequilibrated state at $T = 110$ MeV with chemical potentials μ_π and μ_K. If number-changing rates are neglected, the four final chemical potentials μ_π, μ_K, μ_s, and μ_b can be found by the following four constraints: (1) The net strangeness is zero. (2) The baryon to pion ratio is fixed. (3) The entropy per pion is fixed. (4) The net number of strange quarks per pion is fixed. If rates that change the pion number and strange quark number are included, the latter three constraints must be modified by integrating the time development of the system

$$\frac{d}{dt}\frac{n_b}{n_\pi} = -\frac{n_b}{n_\pi^2}\frac{d}{dt}n_\pi, \tag{6}$$

$$\frac{d}{dt}\frac{n_S}{n_\pi} = -\frac{n_S}{n_\pi^2}\frac{d}{dt}n_\pi - \frac{\mu_\pi}{n_\pi}\frac{d}{dt}n_\pi - \frac{\mu_K}{n_\pi}\frac{d}{dt}n_K, \tag{7}$$

$$\frac{d}{dt}\frac{n_K}{n_\pi} = -\frac{n_K}{n_\pi^2}\frac{d}{dt}n_\pi + \frac{1}{n_\pi}\frac{d}{dt}n_K, \tag{8}$$

where n_S is the entropy density. Thus, if one knows the temperature as a function of time, one can integrate these equations forward in time, using the four aforementioned quantities to determine the four unknown chemical potentials at any time. We assumed a simplified behavior of the temperature T as a function of time, $dT/dt = -6.5$ MeV/(fm/c), which was motivated by the behavior observed in cascade simulations [9].

The resulting evolutions of μ_π and μ_K as a function of temperature are illustrated in Fig. 2. The baryon to pion ratio was chosen to be 0.2, which is relevant for Pb+Pb collisions at the SPS. The large strangeness chemical potentials are characteristic of all the calculations we have performed. They can be attributed to two factors. First, the chemical rates for adjusting the strangeness content are slow since they require the production of pairs of strange hadrons, which carries a large penalty in energy. Second, the equilibrated number of kaons falls very sharply with temperature, as compared to pions, due to their relatively large mass. Thus, even if equilibration rates were similar to kaons, they would have more difficulty in maintaining equilibrium in a rapidly cooling system. Attempts have been made to extract temperatures from chemical ratios that involve kaons [16], e.g., K/π. These considerations emphasize the inherent difficulty in correctly performing such extractions.

Chemical evolutions were also calculated with the assumption that the hadronic masses varied as a function of the temperature. The hadrons, aside from the pseudoscalar mesons which are Goldstone bosons, were assumed to scale linearly with the temperature from their vacuum mass when $T = 110$ MeV to a fraction m/m_0 at $T = 160$ MeV. The assumption of a linear scaling of mass with temperature is certainly crude. In finite-temperature field theories the scaling factor typically changes little from the vacuum value, and then changes suddenly for temperatures near 150 MeV. The scaling also strongly depends on the baryon density which also changes rapidly as a function of temperature. If the mass restoration were forced to occur at earlier times and higher temperatures, the system would have more of a chance to chemically equilibrate, while if restoration were to occur at lower temperatures, the out-of-equilibrium effects would be stronger than those reported here. A more sophisticated form of the mass scaling with temperature and density could be easily implemented, although one would still have to emphasize the wide range of theoretical predictions.

The lower panels of Fig. 3 show the value of μ_π at breakup for four cases. The final baryon to pion ratio was chosen to be 0.2 or 1.0, roughly appropriate for SPS or AGS conditions, respectively. If no number-changing rates are included, the generated chemical potential is large, approaching 100 MeV for the SPS case and 150 MeV for the AGS case, when the mass reduction factor falls below 0.5. However, inclusion of the rates reduces the resulting chemical potential to near 50 MeV. Since the falling of the ρ mass is controversial, the calculations were repeated with the as-

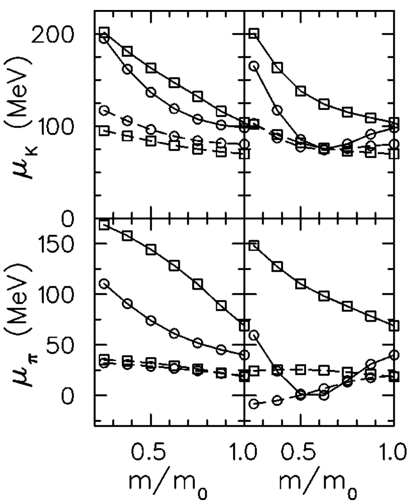

FIG. 3. Final chemical potentials are shown as a function of the mass reduction factor. Calculations are shown for the cases where number-changing processes are neglected (solid lines) or included (dashed lines), and for when vector mesons scale with the baryon masses (left panel) or remain fixed (right panel). Calculations are performed for two choices of the baryon to pion ratio, 1.0 which is relevant for AGS measurements (squares) and 0.2 which is relevant for measurements at SPS (circles). Number-changing rates significantly dampen the effect of altering hadron masses, and the chemistry is less affected if vector meson masses are unchanged.

sumption that only the baryon masses scaled (right panels of Fig. 3), leaving the vector meson masses fixed. In this case the resulting chemical potentials were far lower for the SPS case, and in fact chemical potentials were smaller for increasingly small mass reduction factors. This owes itself to the fact that the entropy per pion due to the presence of baryons is rather high, compared to the entropy per pion in the mesonic sector.

Calculations of μ_K as a function of the mass reduction factor are displayed in the upper panels of Fig. 3. The resulting μ_K is much larger than μ_π, surpassing 100 MeV, even for the case where rates were included. Since the phase space density is proportional to $e^{\mu/T}$, the strangeness phase space density is nearly doubled compared to the $\mu_K=0$ case if masses do not scale and more than doubled when mass scaling occurs. The number-changing rates that most strongly affected μ_K were not those that changed the effective kaon number but those that affected the net pion number. The kaon-number-changing rates are sufficiently small that they had a relatively small effect toward the final outcome.

Chemical potentials can be inferred from hadronic measurements via correlation measurements [17,18]. Combining two-particle correlations measurements, which are sensitive to the breakup volume, and spectra one can infer phase space densities, which should be $\approx e^{\mu/T}$, at low momentum. Strangeness chemical potentials can also be inferred from ratios involving strange or antistrange hadrons, e.g., $\overline{\Lambda}/\overline{p}$ [19]. Combined with careful modeling of the breakup stage of the reaction, one can infer chemical potentials to an accuracy of ± 30 MeV.

The chemical evolution of a heavy ion collision is a complex process that depends on hundreds of cross sections, most of which are not well understood. Although the reduction to a thermal picture as performed here represents a crude approximation to reality, we are nonetheless able to learn several valuable lessons. We see that chemical properties of the hottest stages of relativistic heavy ion collisions can indeed manifest themselves in the final hadronic measurements. In particular, chiral restoration can affect the final phase space densities of both pions and kaons. However, chemical processes, which drive the system towards equilibrium, significantly mitigate such signals. Altering chiral properties of the hottest stages results in a difference of the final chemical potentials of the order of 50 MeV or less, while the ability of an experiment to determine the chemical potentials, is probably in the 30 MeV range at best. Thus, measuring hadrons can be expected to provide information regarding the chiral transition, but would probably not supply a ‘‘smoking gun’’ signal, unless the chemical rates employed here are overestimated. A stronger conclusion can be made with respect to strangeness equilibration. Since strangeness equilibration has longer equilibration times, and since the equilibrium density of kaons is a rapidly changing function of temperature due to the larger mass, chemical equilibration of kaons does not seem possible.

This analysis has relevance beyond the issue of dropping hadron masses. Effective chemical potentials and phase space densities are of crucial importance in understanding the entropy of the reaction, which can signal the presence of the deconfinement transition. Second, the conditions of the final state provide boundary conditions necessary for modeling the earlier stages of the collision. For instance, dilepton production from $\pi\pi$ annihilation would scale as $\exp(2\mu_\pi/T)$. Finally, the sensitivity of the observables to the inclusion of pion-number-changing and kaon-number-changing chemical processes demonstrates the importance of including such reactions, along with the corresponding time-reversed processes, in any modeling of the final-state expansion.

The authors thank Yang Pang for pointing out the importance of baryons in hadronic chemistry. This work was supported by the National Science Foundation under Grant No. PHY-9605207.

[1] T. Peitzmann *et al.*, Nucl. Phys. **A610**, 200c (1996).
[2] S. Margetis *et al.*, Nucl. Phys. **A590**, 355c (1995).
[3] G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 272 (1991).
[4] G. Q. Li, C. M. Ko, and G. E. Brown, Phys. Rev. Lett. **75**, 4007 (1995); Nucl. Phys. **A606**, 503 (1996).
[5] P. J. Siemens and S. A. Chin, Phys. Rev. Lett. **55**, 1266 (1985).
[6] G. Chanfray, R. Rapp, and J. Wambach, Phys. Rev. Lett. **76**, 368 (1996).
[7] R. Pisarski, Phys. Rev. D **52**, R3773 (1995).
[8] V. Koch, Int. J. Mod. Phys. E **6**, 203 (1997).
[9] S. Pratt and J. Murray, Phys. Rev. C **57**, 1907 (1998).
[10] C. Song and V. Koch, Phys. Rev. C **55**, 3026 (1997).

[11] S. Gavin and V. Ruuskanen, Phys. Lett. B **262**, 326 (1991); S. Gavin, Nucl. Phys. **B351**, 561 (1991).
[12] J. Rafelski, Acta Phys. Pol. B **27**, 1037 (1996).
[13] Particle Data Group, ‘‘The 1998 Review of Particle Properties,’’ www-pdg.lbl.gov.
[14] A. Sibirtsev, Phys. Lett. B **359**, 29 (1995).
[15] V. Flaminio *et al.*, CERN Report No. CERN-HERA 01/84, 1984.
[16] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. B **365**, 1 (1996).
[17] G. F. Bertsch, Phys. Rev. Lett. **72**, 2349 (1994).
[18] J. Barette *et al.*, Phys. Rev. Lett. **78**, 2916 (1997).
[19] T. A. Armstrong *et al.*, Phys. Rev. Lett. **79**, 3351 (1997).