Phonon mechanisms of mixing collective and quasiparticle excitations

A. D. Efimov[1,*] and V. M. Mikhajlov[2]
[1]*Cyclotron Laboratory, Ioffe Physical Technical Institute, 194021 St. Petersburg, Russia*
[2]*Physical Institute of St. Petersburg State University, 198904 St. Petersburg, Russia*
(Received 2 February 1999)

A phonon approach with subsequent mapping onto bosons is developed for describing transitional nuclei spectra including the first backbending energy region. Quasiparticle random-phase approximation and its modification with minimum ground state correlations are employed to find B_J phonons (spin $J \geqslant 4$) and most collective quadrupole D phonons, respectively. Phonon functions and matrix elements of B-D interaction are calculated with factorized quasiparticle forces. Besides the interaction mechanisms known in the interacting boson fermion model (IBFM) a new one is introduced that leads to three into one boson transformation. After mapping onto s-, d-, and b_J-boson space a boson Hamiltonian comprises IBM1 Hamiltonian, whose parameters depend on the presence or absence of b_J bosons and are determined phenomenologically, and b_J-d interaction with constants calculated microscopically inside phonon space. This approach applied to energy spectra and $B(E2)$ values in ^{126,128}Ba and ^{130}Ce gives reasonable agreement with experiment. [S0556-2813(99)05806-9]

PACS number(s): 21.60.Ev, 21.10.−k, 23.20.−g

I. INTRODUCTION

In experimental and theoretical explorations of the last decades it has been established that low energy spectra of nonmagic even-even nuclei are predominantly caused by the quadrupole collectivity leading either to rotational bands in deformed nuclei or to vibrational states in nuclei not so far from magic ones, or to complicated superpositions of rotational and vibrational movements in transitional nuclei. However at energies close to the doubled pairing gap weakly collective two-quasiparticle states come into play and a particular role belongs to quasiparticle pairs on intruder levels of the shell model ($g9/2$, $h11/2$, $i13/2$). The maximum spin of such pairs can be high and frequently, when an intruder level is near the chemical potential, the quasirotational spectrum based on these high spin pairs gets yrast as its energies are less than those of purely collective states with the same spin. In deformed nuclei this band crossing, which manifests itself in backbending, is investigated theoretically within the cranking model. In transitional nuclei this phenomenon has become the subject of the interacting boson fermion model (IBFM) [1].

This model involving collective and quasiparticle excitations is a proper tool for theoretical investigation of quasirotational structures in nuclei up to rather high spins, and its extensions for describing quasirotational bands in even-even nuclei were successfully applied to transitional and deformed nuclei [2–7], allowing one to reasonably explain general trends and peculiarities caused by the influence of two- or four-quasiparticle high spin states.

One of the characteristic features of rotational or quasirotational yrast bands is the irregular behavior of $E2$-transition probabilities that reveal a small decrease in the first backbending region. However $B(E2)$ values calculated by employing IBFM are sometimes considerably less than empirical ones [2–4]. A possible reason for such a discrepancy may consist in a weak coupling between collective and two-quasiparticle high spin states predicted by the existing version of the IBFM Hamiltonian adapted to even-even nuclei.

The interaction of quasiparticles with collective excitations is governed by two mechanisms [8]. One of them, which we will call the direct mechanism, consists in the quasiparticle scattering by the collective quadrupole moment. The second mechanism includes the quasiparticle exchange, in which one quasiparticle enters into the collective fermion pair, D phonon (its boson image is d boson), and the other is either present in the system under consideration or created in the course of the interaction. This mechanism in particular accounts for the well-known ''$(j-1)$-anomaly'' [8]. Both mechanisms are taken into account in IBFM for odd nuclei.

Several ways can be considered to employ these mechanisms to even-even nuclei. Yoshida *et al.* [2] made use, practically, of only the direct mechanism that led to a rather weak coupling of two-quasiparticle and collective d-s excitations. This way was also applied by Hsieh *et al.* [3]. To take into account the exchange mechanism the explicit presence of quasiparticles is required. It can therefore straightforwardly be switched on for the process when a d boson and a quasiparticle pair convert into themselves with increasing the quasiparticle pair spin by two or four units. However the quasiparticle structure of the D phonon should be explicitly represented to start up this mechanism for coupling purely collective configurations and those involving a quasiparticle pair. For this purpose Vretenar *et al.* [6] introduce a special term (V_{mix}) to the boson-fermion Hamiltonian, which transforms a d boson into a superposition of quadrupole quasiparticle pairs, and then make use of the usual exchange interaction of IBFM.

A more natural way to include both coupling mechanisms into the boson description is the consideration of their action on quasiparticle D phonons and weakly collective pairs and the consequent mapping of the fermion processes onto those in boson space. Owing to the interaction of fermion pairs with the same spin, $J \geqslant 4$, it is more realistic to consider their superpositions and treat them as phonons that in turn can be

*Electronic address: efimov@cycla.ioffe.rssi.ru

regarded as fermion counterparts of bosons. Thereby, proceeding from quasiparticle structure of diverse phonons we have to show up mechanisms of their interaction and take them into account in describing excited states in the boson framework. Such an approach, the development of which is our goal in this paper, differs evidently from IBFM for even nuclei since instead of bosons and quasiparticles of IBFM we consider only bosons. However we believe that the physical origins of these approaches are identical because both focus efforts on the important role of the interaction of the collective quadrupole modes and high spin two-quasiparticle excitations.

As the number of high spin phonons under consideration is great and they all interact to some extent with collective phonons and between themselves, the boson Hamiltonian cannot comprise a small quantity of parameters. Therefore we are forced to exploit a microscopic scheme for their determination. However at this stage of investigation we do not try to calculate all parameters and employ the phenomenological approach to the IBM Hamiltonian and $E2$ operator.

Our paper is organized as follows. In Sec. II we discuss the coupling mechanisms, introduce additional terms to the exchange boson interaction and give a microscopical method for estimations of parameters. Section III describes calculations of energy spectra and $B(E2)$ values in ^{126}Ba and ^{130}Ce. Section IV contains our conclusions.

II. DIRECT AND EXCHANGE MECHANISMS FOR THE INTERACTION OF COLLECTIVE AND TWO-QUASIPARTICLE PHONONS

As our objective is describing properties of transitional nuclei with developed superfluidity (or superconductivity) we make use of the quasiparticle approach, which from the very beginning takes into account pairing effects and provides the clear hierarchy of states in quasiparticle numbers that is practically equivalent to the seniority scheme. A quasiparticle Hamiltonian involves a single quasiparticle field and quasiparticle interactions:

$$H=\sum_s e_s a_s^+ a_s + H_{\rm int}, \tag{1}$$

a^+, a being quasiparticle operators. This form arises after minimizing the ground-state energy and for the constant pairing force quasiparticle energies are equal to

$$e_s=\sqrt{(\varepsilon_s-\lambda)^2+\Delta^2}, \tag{2}$$

ε, λ, Δ are, respectively, a particle mean field energy, the chemical potential, and pairing gap. $H_{\rm int}$ does not conserve the quasiparticle number N_q and comprises three terms, two of which change N_q by two (H_{13+31}) and four (H_{40+04}) units

$$\begin{aligned} H_{\rm int}=\sum_{1234} [&H_{22}(1234)a_1^+ a_2^+ a_3 a_4 \\ &+H_{31+13}(1234)(a_1^+ a_2^+ a_3^+ a_4 + \text{H.c.}) \\ &+H_{40+04}(1234)(a_1^+ a_2^+ a_3^+ a_4^+ + \text{H.c.})]. \end{aligned} \tag{3}$$

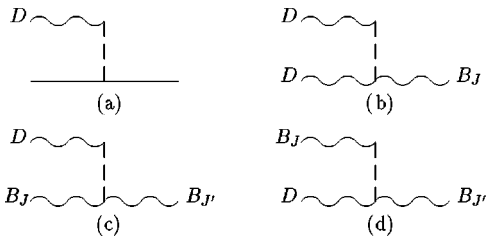

FIG. 1. Diagrammatic representation of the interaction V_1 (1). Wavy and solid lines denote phonons and quasiparticles, respectively. Vertical dashed lines imply an effective internucleon interaction.

Matrix elements, H_{22} and others, are determined by effective forces in the particle-hole (ph) and particle-particle (pp) channels.

The first traditional mechanism of the boson-fermion coupling taken into account in the original work of Yoshida *et al.* [2] corresponds to scattering a quasiparticle with creating (or annihilating) the collective quadrupole phonon D as indicated in Fig. 1(a). In the Bogolubov quasiparticle description this process is governed by the component H_{31+13} [Eq. (3)] of the Hamiltonian. In even-even nuclei this mechanism gives rise to the transformation of two D phonons into a hexadecapole B_4 phonon, then D and B_4 phonons turn into a B_6 phonon and so on [Figs. 1(b)–1(d)]. We use henceforth D, B_J for pair quasiparticle phonons (J is the angular momentum) while we use d, b_J for their boson counterparts. Thus, in perturbation terminology, B_8 appears in third order and B_{10} in fourth one. That can result in a rather weak coupling of the collective space composed of D phonons and the space including a high spin quasiparticle phonon (B_8 or B_{10}), especially when a quasiparticle transition [Fig. 1(a)] proceeds between levels ''1'' and ''2'' near the Fermi level (**F**) (states ''1'' and ''2'' can coincide). In fact, each matrix element in the transformation chain in Fig. 1, for example $\langle DD|H_{31+13}|B_4^+\rangle$, contains a factor $u_1u_2-v_1v_2$ (u,v are Bogolubov parameters) which is small since $u\approx v\approx 1/\sqrt{2}$ for ''1,'' ''2'' $\sim$**F**. However this reduction can be not so pronounced if phonon B_J is distributed over many two-quasiparticle states. Certainly, there may be a singular resonance event of coinciding energies when mixing is maximum (50%) and independent of the interaction strength.

In IBFM the direct mechanism includes also the process with D-phonon scattering [the corresponding diagram can be obtained from Fig. 1(a) if we continue the upper D line to the right from the interaction dashed line]. We omit it as its matrix element in nuclei under consideration is much less than that for Fig. 1(a).

The second mechanism of the B- and D-phonon coupling originates from the well-known exchange quasiparticle-phonon interaction, which can be accompanied not only by D-phonon scattering [Fig. 2(a)] but also by two D-phonon annihilation [Fig. 2(b)]. Therefore the first stage of the second chain of transformations [Figs. 2(c), 2(d)], which was pointed out in our work [5], consists in the conversion of two D phonons into D and B_J phonons with $J=0-6$. Besides, the simultaneous annihilation of three D phonons and creation of B_J is possible as well [Fig. 2(d)]. Here J can take any value accessible for three quadrupole phonons (or bosons), i.e., 0, 2, 3, 4, 6. However for coupling collective

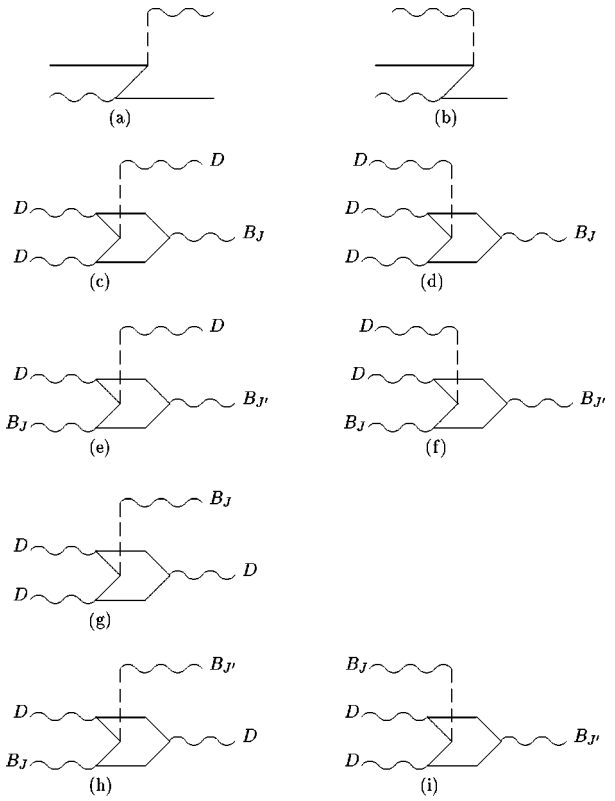

FIG. 2. Diagrammatic representation of the interaction V_2 (2) and V_3 (3).

and high spin states B_4 and B_6 are most important. Starting with the processes in Figs. 2(c) and 2(d) further changes can flow either along the chain in Figs. 1(c) and 1(d) or the chain in Figs. 2(e)–2(i). The second chain may be shorter than the first one because here a phonon B_{10}, for example, is reached already on the second stage. On the first stage we have $(D^2)_{L=4}\rightarrow(DB_6)_{L=4}$ or $(D^3)_{L=6}\rightarrow B_6$, and then $(DB_6)_{L=8}\rightarrow(DB_{10})_{L=8}$ or $(D^2B_6)_{L=10}\rightarrow B_{10}$. Processes in Figs. 2(a), 2(c), 2(e), 2(g), and 2(h) are caused by the component H_{22} [Eq. (3)] of the quasiparticle H whereas transformations in Figs. 2(b), 2(d), 2(f), and 2(i) proceed under the action of H_{40+04} [Eq. (3)]. Ensuing matrix elements of spin independent forces are proportional to a factor $u_1v_2+u_2v_1$ which is ~ 1 if single-particle states ''1,''''2'' are close to **F**.

The transformations conditioned by the processes in Figs. 2(c) and 2(e) can be reproduced by the exchange interaction of IBFM whereas the processes in Figs. 2(d) and 2(f) are not considered in the model. Besides, we allow for the processes in Figs. 2(g), 2(h), and 2(i), which proceed under the impact of the forces with multipolarity equaled to the spin of the created or annihilated B phonon. These processes are outside the existing version of IBFM.

The pair-quasiparticle structure of D and B phonons being determined with some effective forces, matrix elements corresponding to diagrams of Figs. 1 and 2 could be straightforwardly computed in the fermion space. However such a purely fermion approach is pertinent only for low energy simple configurations comprising not more than three D phonons. To pretend to describe the backbending region, any consideration has to involve many phonon excitations, the treatment of which at the fermion level is not practically feasible. The known way to simplify the many fermion problem consists in its transformation into a boson space. As is well known the simplification is attained by means of approximate mappings because the exact boson mapping is completely equivalent to the original fermion problem.

The main trouble of diverse boson mappings is the convergence of the boson series taking account of the Pauli principle for fermion operators. A way to restrict such series is developed in the interacting boson model (IBM) showing itself to advantage in many applications. Therefore we revert to a version of the model, IBM1 [1], where neutron and proton bosons are not distinguished. In the framework of such an approach the vacuum of D and B phonons is mapped onto the normalized state of Ω scalar (s) bosons, where Ω is the total boson number. Other fermion states created by the action of D^+- and B^+-phonon operators on the vacuum are mapped by means of d^+s- and b^+s-boson operators acting on the pure s-boson state. Such a way provides the conservation of Ω and answers approximately the requirement of the exclusion principle. That is more obvious in the generalized Primakoff-Holstein mapping used by Janssen *et al.* [9] where $D^+\rightarrow d^+\sqrt{1-\hat{n}_d/\Omega}$, but both representations (s bosons or square roots) are unitary equivalent and so, for the sake of simplicity, we shall make use of the Arima-Jachello representation.

The correspondence of basic fermion and boson states being obtained, the construction of the boson effective interaction related with Figs. 1 and 2 is implemented so that matrix elements calculated with such functions in the fermion and boson spaces coincide. Thus, we employ the Marumori approach, which generally gives rise to infinite series in bosons. However, following the IBM practice we keep terms with minimum numbers of d and s bosons. Such approximation enables us to represent the fermion processes shown in Figs. 1 and 2 by means of boson interaction operators V_1, V_2, and V_3, respectively:

$$V_1=\frac{1}{\sqrt{\Omega-1}}\sum_{\alpha J}\left[\frac{1}{\sqrt{2}}p_{\alpha J}b^+_{\alpha J}s^+(dd)^{(J)}+\sum_{\alpha' J'}p_{\alpha J\alpha' J'}(b^+_{\alpha J}d^+)^{(J')}(b_{\alpha' J'}s)^{(J')}\right]+\text{H.c.}, \quad (4)$$

$$V_2=\sum_{\alpha J}\left[\frac{1}{\sqrt{2}}\sum_L q^{(L)}_{\alpha J}(b^+_{\alpha J}d^+)^{(L)}(dd)^{(L)}+\sum_{\alpha' J' l}q^{(l)}_{\alpha J\alpha' J'}(d^+d)^{(l)}(b^+_{\alpha J}b_{\alpha' J'})^{(l)}\right]+\text{H.c.}, \quad (5)$$

$$V_3=\frac{1}{\sqrt{(\Omega-1)(\Omega-2)}}\sum_{\alpha J}\left[\frac{1}{N_J}r_{\alpha J}b^+_{\alpha J}s^+s^+(ddd)^{(J)}+\frac{1}{\sqrt{2}}\sum_{\alpha' J' l}r^{(l)}_{\alpha J\alpha' J'}(d^+d^+)^{(l)}(b^+_{\alpha J}b_{\alpha' J'})^{(l)}ss\right]+\text{H.c.}, \quad (6)$$

$$N_J^2=\langle(ddd)^{(J)}(d^+d^+d^+)^{(J)}\rangle,$$

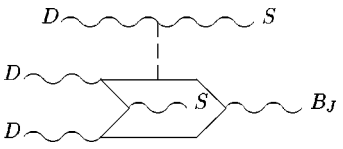

FIG. 3. An example of the three D-phonon conversion into a B_J phonon and two S phonons.

α in Eqs. (4)–(6) distinguishes bosons with the same J. Operator V_3 is a three-boson interaction. However it does not imply that V_3 corresponds to a three-particle force. Figures 2(d), 2(f), and 2(i) indicate that its origin is a two-particle interaction whose boson representation may contain many boson components.

The possibility of the conversion of three D phonon into one B phonon is not a specific feature of the quasiparticle description. In Fig. 3 a diagram of the $(D)^3 \rightarrow BSS$ processes is presented where S is one of the Cooper pairs forming the ground state in the number conserving treatment.

Processes represented in Figs. 1 and 2 are specially picked out among all others since they are enhanced by the D-phonon creation or annihilation and play therefore the decisive role in coupling collective and quasiparticle states. In V_2 and V_3 we exclude the terms with $l=0$ (in the second parts of V_2 and V_3) as they renormalize parameters ε_d and k_1 of H_{IBM1} in the presence of a B phonon. The influence of B phonons on the collective properties of the nucleus is taken into account by introducing several components in the boson Hamiltonian: $H_{\mathrm{C}}(\Omega)$ for description of states without B phonons and $H_{\mathrm{C}}^{(\tau)}(\Omega-1)$ for all states containing a B phonon independently of its spin and other quantum numbers. At the same time we assume that parameters of proton ($\tau=\pi$) and neutron ($\tau=\nu$) Hamiltonians can differ. The choice between π and ν is determined by the microscopical structure of B phonons, e.g., if protons in B are predominant its boson counterpart is regarded as a proton one:

$$H=H_{\mathrm{C}}(\Omega)(1-\hat{n}_b)+\sum_{\tau} H_{\mathrm{C}}^{(\tau)}(\Omega-1)\hat{n}_b^{(\tau)}+H_b+V, \quad (7)$$

$$\hat{n}_b=\sum_{\tau} \hat{n}_b^{(\tau)}, \quad \hat{n}_b^{(\tau)}=\sum_{B} (b_B^+ b_B)^{(\tau)},$$

$$H_b=\sum_{B} \varepsilon_B b_B^+ b_B, \quad V=V_1+V_2+V_3.$$

B as index denotes all b-boson quantum numbers. Our configuration space includes functions comprising not more than one b_J boson ($J=4,6,\ldots$). Therefore $\hat{n}_b$ and $(1-\hat{n}_b)$ play the role of the projection operators.

Both $H_{\mathrm{C}}(\Omega)$ and $H_{\mathrm{C}}^{(\tau)}(\Omega-1)$ are the usual Hamiltonians H_{IBM1} and distinguished by parameters:

$$H_{\mathrm{IBM1}}=\varepsilon_d\hat{n}_d+(k_1 d^+\cdot d^+ ss+k_2[d^+d^+]^{(2)}\cdot ds+\mathrm{H.c.})$$
$$+\frac{1}{2}\sum_{L} C_L[d^+d^+]^{(L)}\cdot[dd]^{(L)}. \quad (8)$$

Each of the collective Hamiltonians of Eq. (7) could comprise its own additional constant depending on boson number. We assume that they can be omitted and when $V=0$ the energy gap between unperturbed states, such as the collective ground state with Ω s and d bosons and any one-b-boson state including also $(\Omega-1)$ s and d bosons, is formed by one boson energy ε_B and by the difference of the correlation energies determined by $H_{\mathrm{C}}(\Omega)$ and $H_{\mathrm{C}}^{(\tau)}(\Omega-1)$. Parameters entering into boson Hamiltonians $[H_{\mathrm{C}},H_{\mathrm{C}}^{(\tau)}(\Omega-1)]$ are fitted phenomenologically whereas matrix elements (p,q,r) in Eqs. (4)–(6) and b-boson energies ε_B are calculated microscopically.

The processes in Figs. 1 and 2 proceed so that at each stage creation or annihilation of the D phonon is caused by the action of the quadrupole component of the quasiparticle forces. The same force together with a single-quasiparticle field leads to forming D phonons that we consider in the framework of the modified random phase approximation (MRPA) [10], which replaces the usual linearization

$$[H,B^+]=\omega B^+ \quad (9)$$

by the modified equation for D-phonons

$$[H,D_\mu^+]=\varepsilon_d D_\mu^+ + 2k_1\Omega D_\mu^-, \quad (10)$$

$$D_\mu^+=\frac{1}{\sqrt{2}}\sum_{1,2;\tau=\pi,\nu}\{[\psi_{12}a_1^+a_2^+ +\varphi_{12}a_{\bar{2}}a_{\bar{1}}]$$
$$\times(j_1j_2m_1m_2|2\mu)\}_\tau. \quad (11)$$

The explicit form of Eq. (10) is a system for D-phonon amplitudes:

$$\sum_{3,4;\tau'=\pi,\nu}\mathcal{H}^{(\eta)}_{(12)\tau,(34)\tau'}Z^{(\eta)}_{(34)\tau'}=[\varepsilon_d+(-1)^{1-\eta}2k_1\Omega]Z^{(1-\eta)}_{(12)\tau}, \quad (12)$$

$$Z^{(\eta)}_{(12)\tau}=[\psi_{12}+(-1)^\eta\varphi_{12}]_\tau, \quad \eta=0;1. \quad (13)$$

$$\mathcal{H}^{(\eta)}_{(12)\tau,(34)\tau'}=\{(e_1+e_2)\delta_{1,3}\delta_{2,4}$$
$$+\mathcal{G}_{12,34}M_{12}^{(1-\eta)}M_{34}^{(1-\eta)}\}_\tau\delta_{\tau,\tau'}$$
$$+\mathcal{V}^{(\eta)}_{(12)\tau,(34)\tau'}(L_{12}^{(1-\eta)})_\tau(L_{34}^{(1-\eta)})_{\tau'}, \quad (14)$$

$$M_{12}^{(1-\eta)}=u_1u_2+(-1)^\eta v_1v_2,$$

$$L_{12}^{(1-\eta)}=u_1v_2+(-1)^{1-\eta}u_2v_1.$$

$\mathcal{G}$ and $\mathcal{V}^{(\eta)}$ are matrices of the quadrupole pp and ph forces, respectively. We take for them the usual factorized forms:

$$\mathcal{G}_{12,34}\rightarrow -G^{(2)}\frac{1}{5}\langle j_1||r^2Y_2||j_2\rangle\langle j_3||r^2Y_2||j_4\rangle,$$

$$\mathcal{V}^{(\eta)}_{(12)\tau,(34)\tau'}\rightarrow\frac{1+(-1)^{1-\eta}}{2}[-\kappa\delta_{\tau,\tau'}-\kappa_{\pi\nu}(1-\delta_{\tau,\tau'})]$$
$$\times\frac{1}{5}\langle j_1||r^2Y_2||j_2\rangle_\tau\langle j_3||r^2Y_2||j_4\rangle_{\tau'}. \quad (15)$$

One can see that Eq. (12) determines only the combination $\varepsilon_d^2-4k_1^2\Omega^2$, which may be positive, equal to zero, or even

TABLE I. Strengths of the quasiparticle interactions and D-phonon energy parameters for ^{126}Ba and ^{130}Ce ($\Delta_\tau, \varepsilon_d, -k_1\Omega$ are in MeV, other values are dimensionless), $\xi_2=(\kappa+\kappa_{\pi\nu}^{(2)})/2\kappa_0(\mathrm{BM})$, $\xi_\lambda=\kappa_\lambda/\kappa_0^{(\lambda)}(\mathrm{BM})$, $\kappa_0^{(\lambda)}(\mathrm{BM})$ is given by Eq. (19).

	Δ_π	Δ_ν	$G_2/\kappa_0(\mathrm{BM})$	ξ_2	ε_d	$-k_1\Omega$	R	ξ_4	ξ_6	ξ_8	ξ_{10}
^{126}Ba	1.32	1.43	1.32	1.10	−0.08	0.49	0.065	0.942	0.926	0.82	0.35
^{130}Ce	1.32	1.34	1.41	1.11	−0.004	0.46	0.067	0.880	0.840	0.70	0.26

negative. That renders MRPA applicable to cases when strength constants are equal to or even more than the critical ones. The relationship of ε_d and k_1 in Eq. (12) influences the ground-state correlations and can be taken such to minimize them

$$R=\sum_{12,\tau}\varphi_{12,\tau}^2\Big/\sum_{34,\tau}\psi_{34,\tau}^2=\min. \tag{16}$$

The absence of the critical strength constants and attenuated ground state correlations make MRPA similar to the Tamm-Dankov approximation (TDA). The difference consists in the commutator with Hamiltonian. In TDA this commutator comprises an infinite set of TDA phonons:

$$[H,B_0^+(TD)]_{20+02}=\varepsilon_0 B_0^+(TD)+\sum_{\nu=0}\tilde{k}_\nu B_\nu(TD),$$

[$B_\nu(TD)$ are TDA phonons, $B_0(TD)$ has the lowest energy ε_0], while for D phonon of MRPA such expansion consists of only two terms given by Eq. (10).

The substitution of ε_d and k_1, found from phenomenological calculations of low-lying collective states, into Eqs. (10) and (12) gives rise to two conditions for three constants $(\kappa,\kappa_{\pi,\nu},G^{(2)})$. Our calculations of 2^+ state energies of semimagic nuclei show that κ can be taken as $\kappa_0(BM)/4$. $\kappa_0(BM)=120A^{-5/3}$ MeV is the isoscalar ph quadrupole strength estimated by Bohr and Mottelson [8]. Fixing the value of κ we obtain $\kappa_{\pi\nu}$ and $G^{(2)}$. Our calculations of D phonon are performed in a wide single particle space involving the valence shell and two shells above and below it. The pairing gaps required for this purpose are equated to pairing energies [11]. The analogous method for determination of the strengths was applied in the TDA in our work [12].

The isoscalar quadrupole strengths $\kappa_0=(\kappa+\kappa_{\pi\nu})/2$ obtained in that way for ^{126}Ba and ^{130}Ce and given in Table I are close to the Bohr-Mottelson estimations. We believe that such a choice is reasonable bearing in mind that we use a simplified form for the interaction matrices and do not take into account higher order processes renormalizing these strengths. We would like to note that we have obtained approximately the same relation between pp and ph strengths as in the works of Tamura *et al.* [13], who studied a microscopical version of the boson expansion technique (BET) with factorized forces.

The D-phonon amplitudes found with Eq. (12) can be applied to calculate IBM1 effective charges e^* determining the $E2$ transition probabilities in IBM1

$$e^*=\frac{1}{\sqrt{\Omega}}\langle 0|\hat{T}_\mu D_\mu^+|0\rangle. \tag{17}$$

$\hat{T}_\mu$ is the $E2$-transition operator in the fermion space

$$\hat{T}_\mu=\sum_{i=1}^{A}\left\{e\left(\frac{1}{2}-\hat{t}_Z+\delta\right)r^2Y_{2\mu}(\theta,\varphi)\right\}_i, \tag{18}$$

e is the proton charge, $t_Z=+1/2,\ -1/2$ for neutrons and protons, respectively, δ is a polarization correction to the charge. With D-phonon amplitudes and empirical values of e^* the quantity of δ is found to be equal to 0.25 for the standard IBM operator of $E2$ transition. That can be regarded as a satisfactory result indicating that the collectivity of the D-phonon operator is sufficiently high.

To calculate parameters in operators V_1, V_2, and V_3 [Eqs. (4)–(6)] one needs the quasiparticle amplitude of D and B phonons. The former is found by means of Eq. (12) for D phonons, the latter together with B-phonon energies (ε_B) is determined by the quasiparticle random-phase approximation (RPA) (9). We solve this equation with ph isoscalar factorized forces. Their strengths are fitted and given in Table I in units of the isoscalar oscillator Bohr-Mottelson strengths [8]:

$$\kappa_0^{(\lambda)}(\mathrm{BM})=\frac{4\pi}{3}\frac{41}{(1.2)^{2(\lambda-1)}}\frac{1}{A^{(\lambda+3)/3}}\ \mathrm{MeV}. \tag{19}$$

Table I indicates that multipole strengths decrease stronger with λ than $\kappa_0^{(\lambda)}(\mathrm{BM})$. This result is in agreement with our estimations of $\kappa_0^{(\lambda)}$, which were performed by comparison of matrix elements of factorized forces with those for Gauss forces [14].

A quasiparticle composition of hexadecapole phonons includes several two-quasiparticle pairs. With growth of the phonon momenta the quasiparticle structure becomes poorer and phonons with $J=8$ and 10 practically consist of one two-quasiparticle component corresponding to the intruder level. Other components are small ($|\psi|<0.1$), nevertheless they influence interaction parameters. The quasiparticle composition [amplitudes ψ in Eqs. (11),(20)] of lowest energy phonons for ^{126}Ba is presented in Table II:

$$B_{J\mu}^+=\frac{1}{\sqrt{2}}\sum_{1,2;\tau=\pi,\nu}\{[\psi_{12}^{(J)}a_1^+a_2^+ + \varphi_{12}^{(J)}a_{\bar{2}}a_{\bar{1}}]\times(j_1j_2m_1m_2|J\mu)\}_\tau. \tag{20}$$

After fixing parameters in the quasiparticle Hamiltonian and computing phonon energies and amplitudes we are able to calculate matrix elements of the Hamiltonian between phonon states and thereby, following the Marumori procedure, to find constants in the boson interaction, Eqs. (4), (5), and (6),

$$p_{\alpha J}=\langle|B_{\alpha J}H(D^+D^+)^{(J)}|\rangle N_1, \tag{21}$$

TABLE II. Phonon energies ε_B and main two-quasiparticle amplitudes ψ ($|\psi| \geq 0.10$) of B_J-phonon wave functions in ^{126}Ba. ν, π denote neutron and proton configurations.

J^π	2^+	4^+	6^+	8^+	10^+
ε_B (MeV)		1.776	2.094	2.551	2.806
$\nu d^2_{5/2}$	0.11				
$\nu g^2_{7/2}$	0.22	−0.13			
$\nu h^2_{11/2}$	−0.54	0.64	−0.63	0.98	1.00
$\nu s_{1/2} d_{5/2}$	−0.16				
$\nu s_{1/2} g_{7/2}$		−0.18			
$\nu d_{3/2} d_{5/2}$		0.17			
$\nu d_{3/2} g_{7/2}$	−0.19	0.14			
$\nu d_{3/2} s_{1/2}$	0.14				
$\nu d^2_{3/2}$	0.13				
$\nu f_{7/2} h_{11/2}$	0.15	−0.10			
$\pi d^2_{5/2}$	0.32	−0.42			
$\pi g_{7/2} d_{5/2}$		−0.249	0.50		
$\pi g^2_{7/2}$	0.29	−0.23	0.17		
$\pi h^2_{11/2}$	−0.21	0.12			
$\pi s_{1/2} d_{5/2}$		−0.15			
$\pi d_{3/2} d_{5/2}$		0.13			
$\pi d_{3/2} g_{7/2}$	−0.11				

$$p_{\alpha J \alpha' J'} = \langle | (DB_{\alpha J})^{(J')} H B_{\alpha' J'} | \rangle N_2 . \tag{22}$$

Hereafter factors N_i provide the normalization of initial and final phonon states. Matrix elements in Eqs. (21) and (22) are determined by the quasiparticle interaction H_{31+13}, Eq. (3), whereas constants in V_2 and V_3 are caused by H_{22} and H_{40+04}:

$$q^{(L)}_{\alpha J} = \langle | (\widetilde{DB}_{\alpha J})^{(L)} (H - E_0) (D^+ D^+)^{(L)} | \rangle N_3 . \tag{23}$$

L is the two-phonon angular moment, E_0 is the energy of the phonon vacuum $|\rangle$,

$$D|\rangle = B|\rangle = 0.$$

The tilde above the left side of Eq. (22) means the orthogonalization:

$$\langle | (\widetilde{DB})^{(L)} (D^+ D^+)^{(L)} | \rangle = 0, \tag{24}$$

$$\begin{aligned} (\widetilde{B^+ D^+})^{(L)} | \rangle = {} & (B^+ D^+)^{(L)} | \rangle \\ & - (D^+ D^+)^{(L)} | \rangle \langle | (DD)^{(L)} (B^+ D^+)^{(L)} | \rangle \\ & \times (\langle | (DD)^{(L)} (D^+ D^+)^{(L)} | \rangle)^{-1}. \end{aligned}$$

Equation (23) includes a commutator $[H, B^+_{\alpha J}]$ in which we retain three components

$$[H, B^+_{\alpha J}] = \varepsilon_B B^+_{\alpha J} + 2 \tilde{k}_1 B_{\alpha J} \delta_{B,D} + [H, B^+_{\alpha J}]_{31+13}. \tag{25}$$

The second term with $\delta_{B,D}$ takes place if phonon B coincides with D, $\tilde{k}_1 = k_1 \Omega$ enters into Eq. (12). With Eqs. (24) and (25) we can write down Eq. (23) as follows:

$$\begin{aligned} q^{(L)}_{\alpha J} / N_3 = {} & \{ \varepsilon_B - \varepsilon_D - \langle | (D[D)^{(L)}, H]_{31+13} (D^+ D^+)^{(L)} | \rangle \\ & \times (\langle | (DD)^{(L)} (D^+ D^+)^{(L)} | \rangle)^{-1} \} \\ & \times \langle | (DB_{\alpha J})^{(L)} (D^+ D^+)^{(L)} | \rangle \\ & - \langle | (D[B_{\alpha J})^{(L)}, H]_{31+13} (D^+ D^+)^{(L)} | \rangle, \end{aligned} \tag{26}$$

ε_B, ε_D being RPA and MRPA eigenvalues for B and D phonons. The index ''31+13'' points out that we take into consideration those parts of commutators that comprise $a^+ a^+ a^+ a$ and $a^+ aaa$ quasiparticle operators.

A very interesting circumstance, which is displayed in calculations of the matrix elements in Eq. (26), consists in that they are reduced to the same matrix $\mathcal{H}$ entering into the RPA equation, Eq. (14), if we keep only terms linear in amplitudes φ, Eqs. (11) and (20). We would like to give this result in a general form for any phonons:

$$\begin{aligned} & \langle | B_a [B_b, H]_{31+13} B^+_c B^+_d | \rangle \\ & \quad = -2 \sum \{ \mathcal{H}^{(d)}_{12,34} z_{12}(d) \psi_{56}(c) + \mathcal{H}^{(c)}_{12,34} z_{12}(c) \psi_{56}(d) \} \\ & \qquad \times \psi_{35}(b) \psi_{46}(a). \end{aligned} \tag{27}$$

In this equation the upper index d or c in the matrix has to be understood as η_d or η_c [$\eta = 0$ or 1, Eq. (13)]. Now employing the RPA or MRPA equation one can express Eq. (27) through the phonon and quasiparticle energies:

$$\begin{aligned} & \langle | B_a [B_b, H]_{31+13} B^+_c B^+_d | \rangle \\ & \quad = (\varepsilon_c + \varepsilon_d) \langle | B_a [B_b, B^+_c]_{11} B^+_d | \rangle \\ & \qquad - \left\langle \left| B_a B_b \left[\sum_s e_s a^+_s a_s, B^+_c B^+_d \right] \right| \right\rangle . \end{aligned} \tag{28}$$

(Here we have assumed that $\langle | a^+_s a_t | \rangle = 0$.) Thus, the residual forces do not manifest themselves explicitly in matrix elements of Eq. (26) and reveal their role only in energies and amplitudes of phonons:

$$\begin{aligned} q^{(L)}_{\alpha J} / N_3 = {} & \Bigg\{ \varepsilon_B + \varepsilon_D - \Bigg[2 \varepsilon_d \langle | (D[D)^{(L)}, (D^+]_{11} D^+)^{(L)} | \rangle \\ & - \left\langle \left| \left(D[D)^{(L)}, \left[\sum_s e_s a^+_s a_s, (D^+ \right] \right]_{(11)} D^+ \right)^{(L)} \right| \right\rangle \\ & \times (\langle | (DD)^{(L)} (D^+ D^+)^{(L)} | \rangle)^{-1} \Bigg\} \\ & \times \langle | (DB)^{(L)} (D^+ D^+)^{(L)} | \rangle \\ & - \left\langle \left| (DB)^{(L)} \left[\sum_s e_s a^+_s a_s, (D^+ D^+)^{(L)} \right] \right| \right\rangle . \end{aligned} \tag{29}$$

As before, symbol $(\cdots)^{(L)}$ means the angular momentum coupling of phonon operators.

Matrix elements for the processes in Fig. 2 ($B_J D \to B_{J'} D$) are calculated analogously by using Eq. (27),

$$q^{(l)}_{\alpha J, \alpha' J'} \sim \langle | (DB_{\alpha J})^{(L)} (H - E_0) (B^+_{\alpha' J'} D^+)^{(L)} | \rangle N_4 .$$

TABLE III. Wave functions for ^{126}Ba. (I_i) means a purely collective d, s boson state with a spin I_i, $(J^q I_i)$ corresponds to a configuration with a b_J boson under a collective state with spin I_i. Absolute values of amplitudes presented are more than 0.1.

State	Main components of wave functions					
0_1^+	$0.99(0_1)$	$-0.11(6^q6_1)$				
2_1^+	$0.99(2_1)$	$-0.12(6^q4_1)$				
2_2^+	$0.98(2_2)$	$+0.13(4^q2_1)$	$-0.11(6^q6_1)$			
4_1^+	$0.97(4_1)$	$-0.17(6^q2_1)$				
4_2^+	$0.96(4_2)$	$+0.15(4^q0_1)$	$+0.11(4^q2_1)$			
6_1^+	$0.92(6_1)$	$-0.25(6^q0_1)$	$+0.15(6^q2_1)$	$-0.12(6^q2_2)$		
6_2^+	$0.18(6_1)$	$+0.79(6_2)$	$+0.38(4^q2_1)$	$+0.32(6^q0_1)$	$-0.19(6^q2_1)$	$-0.13(6^q2_2)$
8_1^+	$0.86(8_1)$	$-0.41(6^q2_1)$	$+0.10(6^q2_2)$	$-0.16(6^q4_1)$	$-0.10(6^q4_2)$	
8_2^+	$0.26(8_1)$	$+0.54(8_2)$	$+0.55(4^q4_1)$	$+0.50(6^q2_1)$	$-0.11(6^q2_2)$	$-0.16(6^q4_1)$
10_1^+	$0.74(10_1)$	$-0.57(6^q4_1)$	$+0.14(6^q4_2)$	$+0.16(6^q6_1)$	$-0.18(8^q2_1)$	
10_2^+	$0.34(10_1)$	$+0.35(10_2)$	$+0.62(4^q6_1)$	$+0.50(6^q4_1)$	$-0.10(6^q4_2)$	$-0.17(10^q0_1)$
12_1^+	$0.27(12_1)$	$-0.42(6^q6_1)$	$-0.15(8^q4_1)$	$+0.20(8^q6_1)$	$+0.81(10^q2_1)$	
12_2^+	$0.45(12_1)$	$+0.15(4^q8_1)$	$-0.45(6^q6_1)$	$-0.54(8^q4_1)$	$-0.49(10^q2_1)$	
12_3^+	$-0.19(12_2)$	$-0.53(4^q8_1)$	$-0.55(6^q6_1)$	$+0.18(6^q8_1)$	$+0.50(8^q4_1)$	$-0.19(10^q2_1)$
14_1^+		$-0.14(6^q8_1)$	$+0.17(8^q8_1)$	$+0.97(10^q4_1)$		
14_2^+	$-0.23(14_1)$	$-0.13(4^q10_1)$	$+0.40(6^q8_1)$	$+0.86(8^q6_1)$		
14_3^+	$-0.14(14_1)$	$+0.40(4^q10_1)$	$+0.77(6^q8_1)$	$-0.18(6^q10_1)$	$-0.35(8^q6_1)$	$+0.13(10^q4_1)$
16_1^+		$0.13(8^q10_1)$	$+0.98(10^q6_1)$			

The calculation of matrix elements for processes in Fig. 2(d) $(D^3 \rightarrow B_J)$ is also closely connected to the MRPA equations, Eq. (30),

$$r_{\alpha J}/N_5 = 2\tilde{k}_1 \langle | D_a \bar{B}_{\alpha J} D_b^+ D_c^+ | \rangle + \langle | B_{\alpha J} [H, D_a^+]_{31+13} D_b^+ D_c^+ | \rangle. \quad (30)$$

The replacement of a B_b operator in Eq. (27) by D_a^+ gives rise to an additional factor $(-1)^{\eta_a}$. The summing of the RPA matrix $\mathcal{H}^{(\eta_a)}$ with this factor reduces Eq. (30) to products of the parameter $\tilde{k}_1$ and overlap integrals:

$$r_{\alpha J}/N_5 = 2\tilde{k}_1 \{ \langle | D_a \bar{B}_J D_b^+ D_c^+ | \rangle + \langle | D_b \bar{B}_J D_a^+ D_c^+ | \rangle + \langle | D_c \bar{B}_J D_a^+ D_b^+ | \rangle \}. \quad (31)$$

The analogous method is applied to calculate matrix elements for $(D^2 B_J \rightarrow B_{J'})$ processes, i.e., for $r^{(l)}_{\alpha J, \alpha' J'}$ constants.

Thus Eqs. (28) and (31) indicate that our modification of RPA gains one more useful feature because it gives not only two parameters of the collective Hamiltonian (ε_d, k_1) but also simplifies calculations of interaction parameters.

III. RESULTS OF CALCULATIONS

Among barium and cerium isotopes ^{126}Ba and ^{130}Ce stand out owing to the fact that quasirotational bands and $E2$-transition probabilities in these isotopes are investigated experimentally in detail in the region of the first band crossing. Thereby these nuclei are the most interesting subjects for applying our approach. The character of their spectra testifies that they can be attributed to transitional or weakly deformed nuclei. Such interpretation is confirmed by the IBM analysis performed earlier [15] where low lying states of these nuclei were approximately described in the O_6 limit of IBM1.

These results are incorporated in the phenomenological description of the collective states on the basis of the IBM1 Hamiltonians H_c in Eq. (7). Initial values of parameters in $H_c(\Omega)$ are fitted so as to reproduce energies of low lying states with even spins $I \leqslant 8$ without b-boson mixture. Then, after switching on the b-d boson interaction [Eqs. (4)–(6)], in these states appear small b-boson components (their amplitudes are given in Tables III, IV). Therefore a ''fine tune'' of the parameters in $H_c(\Omega)$ is needed that is achieved in the course of diagonalization. The final values of the parameters are given in Table V. The extent of the closeness of the s-, d-boson wave functions found with such Hamiltonian to the O_6 limit of IBM1 can be characterized by the overlap integrals

$$\mathrm{In}_I = \langle \psi^I | \psi^I(O_6) \rangle,$$

where ψ^I and $\psi^I(O_6)$ are eigenfunctions of $H_c(\Omega)$ and the O_6 Hamiltonian, respectively. For yrast states of ^{126}Ba the values of In_I are $\mathrm{In}_0 = 0.91$; $\mathrm{In}_2 = 0.85$; $\mathrm{In}_4 = 0.85$; $\mathrm{In}_6 = 0.86$; $\mathrm{In}_8 = 0.87$; $\mathrm{In}_{10} = 0.88$.

In calculations of wave functions and matrix elements we keep two b_J bosons with the same spin (b_{J1}, b_{J2}). However components with b_{J2} are so small that their contributions to wave function normalizations for states under consideration are not more than one percent. That is a consequence of the RPA diagonalization in two-quasiparticle space, which leads to B_J phonons.

In Sec. II we postulated that parameters of $H_c^{(\tau)}(\Omega - 1)$, i.e., in the presence of a b boson, have to differ from those of $H_c(\Omega)$ governing purely collective d, s states. The first manifestation of this alteration in the collective properties caused by the B-phonon blocking is revealed in the calculation of the B-phonon energies. The RPA equation with the

TABLE IV. Wave functions for ^{130}Ce. Designations are the same as in Table III. The structure of those non-yrast states $(2_2,4_2,6_2,8_2,10_4,12_5,14_4)$ are presented which are connected by enhanced $E2$-transitions.

State	Main components of wave functions					
0_1^+	$0.99(0_1)$					
2_1^+	$0.99(2_1)$	$-0.11(6^q4_1)$				
2_2^+	$0.98(2_2)$	$-0.10(6^q6_1)$				
4_1^+	$0.98(4_1)$	$-0.16(6^q2_1)$				
4_2^+	$0.97(4_2)$	$+0.12(4^q0_1)$				
6_1^+	$0.93(6_1)$	$-0.24(6^q0_1)$	$+0.15(6^q2_1)$	$-0.12(6^q2_2)$		
6_2^+	$0.13(6_1)$	$+0.86(6_2)$	$+0.24(4^q2_1)$	$+0.28(6^q0_1)$	$-0.17(6^q2_2)$	
8_1^+	$0.83(8_1)$	$-0.13(8_2)$	$-0.46(6^q2_1)$	$+0.16(6^q4_1)$		
8_2^+	$0.32(8_1)$	$+0.59(8_2)$	$+0.45(4^q4_1)$	$+0.11(4^q4_2)$	$+0.45(6^q2_1)$	
10_1^+	$-0.38(10_1)$	$+0.40(6^q4_1)$	$-0.69(10^q0_1)$	$-0.40(10^q2_2)$		
10_4^+	$0.27(10_1)$	$+0.16(10_2)$	$+0.32(4^q6_1)$	$+0.18(6^q4_1)$	$-0.30(8^q2_1)$	$-0.25(8^q4_1)$
		$-0.32(10^q0_1)$	$+0.53(10^q2_1)$	$-0.27(10^q2_2)$	$+0.32(10^q4_1)$	
12_1^+		$-0.16(8^q4_1)$	$+0.15(8^q6_1)$	$+0.85(10^q2_1)$	$+0.35(10^q2_2)$	$+0.29(10^q4_1)$
12_5^+	$-0.15(12_1)$	$+0.20(6^q6_1)$	$+0.33(8^q4_1)$	$+0.23(8^q4_2)$	$+0.24(8^q6_1)$	
		$+0.43(10^q2_1)$	$-0.48(10^q2_2)$	$-0.25(10^q6_1)$		
14_1^+		$0.17(8^q6_1)$	$-0.13(8^q8_1)$	$-0.90(10^q4_1)$	$-0.32(10^q4_2)$	$-0.21(10^q6_1)$
14_4^+		$0.28(6^q8_1)$	$-0.24(8^q6_1)$	$-0.15(8^q6_2)$	$-0.33(10^q4_1)$	$+0.44(10^q4_2)$
		$+0.67(10^q6_1)$	$+0.22(10^q8_1)$			
16_1^+		$0.15(8^q8_1)$	$-0.11(8^q10_1)$	$-0.93(10^q6_1)$	$-0.27(10^q6_2)$	$-0.16(10^q8_1)$
18_1^+		$0.13(8^q10_1)$	$-0.95(10^q8_1)$	$-0.23(10^q8_2)$	$-0.13(10^q10_1)$	
20_1^+		$0.11(8^q12_1)$	$-0.97(10^q10_1)$	$-0.19(10^q10_2)$	$-0.10(10^q12_1)$	
22_1^+		$0.98(10^q12_1)$	$+0.15(10^q12_2)$			
24_1^+		$1(10^q14_1)$				

standard mean field and pairing and with strengths presented in Table I gives the B-phonon energies to be about 0.5 MeV less than needed for the description of the quasirotational bands. This discrepancy according to our assumption [the text just below Eq. (8)] can be removed if we allow for the difference between the ground-state correlation energies determined by $H_c(\Omega)$ and $H_c^{(\tau)}(\Omega-1)$, i.e., in the absence and presence of a B phonon. Thus, with the appearance of a B phonon the collectivity has to be reduced and the corresponding ground state must lie higher. Therefore an initial set of parameters in $H_c^{(\tau)}(\Omega-1)$ can be taken from the IBM1 analysis of adjacent nuclei where the collectivity is less and in particular d-boson energy ε_d is higher than in the nucleus under consideration. For example, in ^{126}Ba we start with parameters of ^{128}Ba for $H_c^{(\nu)}(\Omega-1)$ and with ^{124}Xe for $H_c^{(\pi)}(\Omega-1)$. In ^{130}Ce we use the same set of the parameters for the neutron and proton Hamiltonians $H_c^{(\tau)}(\Omega-1)$. The final values shown in Table V are established by fitting energies in the yrast band after bandcrossing.

Such choice of parameters provides a good description of the yrast bands: in ^{126}Ba the difference (ΔE) between the theory and experiment for $I=20$ is ~ 0.15 MeV, for lower spins $\Delta E \leqslant 0.15$ MeV (Fig. 4), in ^{130}Ce for $I=20$–26 $\Delta E \leqslant 0.18$ MeV and for $I<20$ $\Delta E \leqslant 0.1$ MeV (Fig. 5). However in ^{126}Ba even for $I<20$ the energies of the neighbor bands are described worse, which one can see in Fig. 4. A reason for this may consist in the assumption that $H_c^{(\tau)}(\Omega-1)$ does not depend on the kind of B phonon that blocks the collectivity. While the yrast band after bandcrossing contains practically only one b boson with the maximum spin (wave functions are written out in Tables III, IV), in other bands the important role is played by bosons with lower spins. Therefore the marked discrepancies in energies may indicate that the above-mentioned assumption strongly sim-

TABLE V. Parameters of the IBM1 Hamiltonions (MeV). $H_{\rm C}(\Omega)$, $H_{\rm C}^{(\nu)}(\Omega-1)$, and $H_{\rm C}^{(\pi)}(\Omega-1)$ are determined by Eqs. (7) and (8), Ω is boson number.

	Ω	ε_d	k_1	k_2	C_o	C_2	C_4
				^{126}Ba			
$H_{\rm C}(\Omega)$	9	-0.040	-0.0565	0.0257	0.1955	0.017	0.2018
$H_{\rm C}^{(\nu)}(\Omega-1)$		-0.038	-0.0499	0.0120	0.0747	0.3049	0.2382
$H_{\rm C}^{(\pi)}(\Omega-1)$		0.138	-0.0539	0.0143	0.2364	0.0254	0.1312
				^{130}Ce			
$H_{\rm C}(\Omega)$	9	-0.004	-0.0508	0.0249	0.1425	0.0008	0.2042
$H_{\rm C}^{(\nu)}(\Omega-1)$		0.003	-0.0557	0.0146	0.1927	0.0761	0.1749

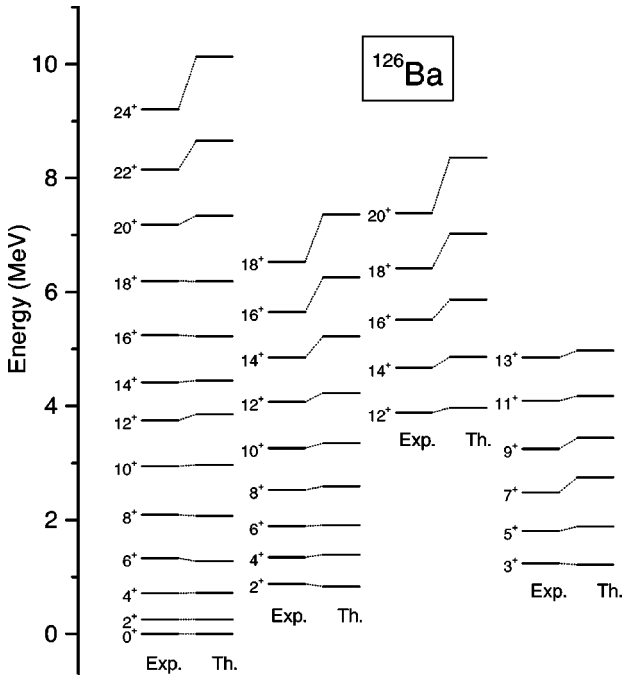

FIG. 4. Experimental [17] and theoretical level scheme of ^{126}Ba.

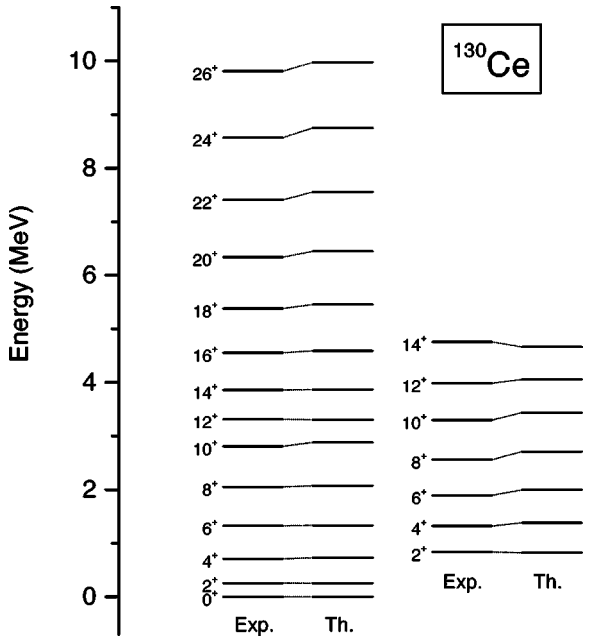

FIG. 5. Experimental [18] and theoretical level scheme of ^{130}Ce. In the figure those states are presented the structure of which are given in Table IV.

plifies the situations and a real improvement in describing energy levels could be attained by the microscopical consideration of blocking effect of each B phonon. Another possible way to give a more exact treatment of high energy levels is the extension of our configuration space by involving states with two b_J bosons especially when they are of the proton and neutron types. The importance of similar configurations (two proton and two neutron quasiparticles) for IBFM was pointed out by Vretenar *et al.* [7].

Another set of parameters [constants of the direct (V_1) and exchange (V_2+V_2) $b-d$ interaction] are calculated microscopically by means of the method expounded in Sec. II. The values of some constant are listed in Table VI. All constants are calculated with B phonon functions determined in Sec. II. The employment of functions of two quasiparticles on the intruder level leads to considerable changes of the constants. For example, in ^{126}Ba, where the intruder $1h_{11/2}$ level energy practically coincides with the chemical potential, the calculations with the quasiparticle functions would give zero values for p constants of the direct mechanism (V_1). Though the phonon calculations remove this result, nevertheless a sign of such level arrangement can be discovered in p constants since the first five of them involving phonons B_4 and B_6 are clearly more in the average (we mean absolute values) than the rest of them. That is a consequence of the wider spread of these phonons over two-quasiparticle states in comparison with phonons B_8 and B_{10}. The p constants are subjected to an impact of the processes determined by forces with multipolarity more than 2 [Fig. 1(d)]. In particular allowing for such force along with small components of phonons B_8 and D gives a marked value of $p_{8,8}$ in ^{126}Ba. For example, the small components $\psi_{2f5/2,1h11/2}(B_8)=0.05$, and $\psi_{2f7/2,1h11/2}(D)=0.15$ are connected with main phonon components by the multipole force with $\lambda=8$, moreover the smallness of amplitudes is compensated by $(u_1u_2-v_1v_2)$ factors that are $\geqslant 0.6$ in these cases. ($p_{10,10}$ constant in ^{130}Ce stands out against others because of a geometric factor.)

One more detail of Table VI draws attention: $q_6^{(4)}$ constant, corresponding to the process $(dd)^{(4)}\rightarrow(db_6)^{(4)}$, dominates over others q in V_2 [Eq. (5)]. That results in a substantial contribution of b_6 boson to wave functions (Tables III, IV).

TABLE VI. Constants of the direct and exchange b-d boson interaction for ^{126}Ba and ^{130}Ce. Constants are presented for processes involving lowest energy B phonons only. V_2 and V_3 interactions [Eqs. (5),(6)] include a great number of constants, of which are given $q_J^{(L)}$ for V_2 [Fig. 2(c)] and r_J [Fig. 2(d)] and $r_{J,J'}^{(l)}$, $J'-J=2$ and 4, $J\geqslant 6$ [Fig. 2(f)].

	V_1									
	p_4	p_{44}	p_{46}	p_{64}	p_{66}	p_{68}	p_{86}	p_{88}	$p_{8,10}$	$p_{10,10}$
^{126}Ba	−0.29	0.29	−0.32	−0.28	0.37	−0.15	−0.12	0.11	−0.04	−0.06
^{130}Ce	−0.13	0.23	−0.12	−0.08	0.14	0.02	0.03	−0.06	0.07	−0.28
	V_2 and V_3									
	$q_4^{(2)}$	$q_4^{(4)}$	$q_6^{(4)}$	r_4	r_6	$r_{68}^{(2)}$	$r_{8,10}^{(2)}$	$r_{68}^{(4)}$	$r_{6,10}^{(4)}$	$r_{8,10}^{(4)}$
^{126}Ba	−0.195	−0.10	0.254	−0.004	0.08	−0.011	−0.004	−0.13	0.08	−0.25
^{130}Ce	−0.145	−0.09	0.206	−0.0006	0.067	−0.008	0.019	−0.11	0.06	−0.22

TABLE VII. The energy shifts δE determined by the interaction mechanisms in the bandcrossing region for ^{126}Ba. E is the theoretical energy calculated taking into account the direct and exchange mechanisms $(V_1+V_2+V_3)$. δE is the difference between energies calculated omitting one $(V_3=0)$, two $(V_2=V_3=0)$, or three $(V_1=V_2=V_3=0)$ mechanisms and E.

	E(keV)	δE(keV)		
I_i^π	$(V_1+V_2+V_3)$	$V_2=V_3=0$	$V_3=0$	$V_1=V_2=V_3=0$
8_1	2074	-6	83	406
10_1	2968	−82	95	693
12_1	3852	−56	83	118
14_1	4450	106	160	91
8_2	2590	122	−136	668
10_2	3345	99	−121	236
12_2	3967	19	84	319
14_2	4865	−71	16	170

The role of the direct and exchange interactions in energies and $E2$ probabilities can be revealed by comparing the results of the calculations with the complete interaction $(V_1+V_2+V_3)$, Eqs. (4)–(6), and with its separate parts (Tables VII, VIII). In these calculations we keep unchanged the parameters of the collective Hamiltonians (Table V) and the interaction constants for V_1, V_2, V_3 (Table VI).

As Table VII shows the energies of the yrast states near the bandcrossing region can be practically reproduced by neglecting both components of the exchange interaction $(V_2=V_3=0)$. Thereby it becomes clear why energy spectra were reasonably described only with the direct mechanism [2,3]. Since omitting V_3 gives also minor energy shifts ($\sim$100 keV) with opposite signs the energy spectrum calculations do not enable us to reveal the role of V_3.

The structure of the states undergoes more drastic changes when any of the interaction mechanisms is switched off. As expected the action of V_1, as the only b-d interaction $(V_2=V_3=0)$, leads to a considerable contribution of mixtures with b_4 boson while (V_2+V_3) intensifies the role of b_6 boson. The comparison of the wave functions created by the direct or exchange mechanisms or by the complete set $(V_1+V_2+V_3)$ indicates that the role of (V_2+V_3) is essential since the functions of $(V_1+V_2+V_3)$ in many details are like the functions of (V_2+V_3) (Table VIII). One can note that this mechanism gives rise to a rather rapid change in the structure of the states under consideration: the state with $I^\pi=10^+$ is collective on the whole whereas the main component of state 12^+ includes the high spin quasiparticle state $J^q=10$. On the contrary, with the direct mechanism (V_1) the functions of states $10^+, 12^+$ are distributed over many configurations so that the transition from collective (8^+) to quasiparticle (14^+) structure is realized through rather washed out states $(10^+, 12^+)$, the composition of which does not practically include b_{10} boson prevailing in state 14^+. Therefore, when either the direct mechanism (V_1) or the exchange one (V_2+V_3) act separately, respectively, $B(E2;14\rightarrow12)$ or $B(E2;12\rightarrow10)$ is suppressed (Table IX). However the combination of both mechanisms $(V_1+V_2+V_3)$ smoothes out the difference between collective and quasiparticle states in the bandcrossing region that results in $B(E2;12\rightarrow10)$, which is about three times more than $B(E2)$ for the pure exchange mechanism. Columns $V_3=0$ in Tables VIII and IX give convincing examples of the essential influence of the new three-boson exchange interaction V_3 on the wave func-

TABLE VIII. The wave functions of the yrast states near bandcrossing produced by the direct interaction mechanism $(V_2+V_3=0)$, the exchange one $(V_1=0)$, and by both $(V_1+V_2+V_3)$. (The absolute values of the amplitudes presented are more than 0.1.)

State	$V_1+V_2+V_3$	$V_1=0$	$V_2=V_3=0$	$V_3=0$
8_1^+	$0.86(8_1)$	$0.89(8_1)$	$0.76(8_1)$	$0.85(8_1)$
	$-0.41(6^q2_1)$	$-0.37(6^q2_1)$	$+0.49(4^q4_1)$	$-0.42(6^q2_1)$
	$-0.16(6^q4_1)$	$-0.11(6^q4_2)$	$+0.31(6^q2_1)$	$+0.18(6^q4_1)$
				$-0.11(6^q4_2)$
10_1^+	$0.74(10_1)$	$0.81(10_1)$	$0.58(10_1)$	$-0.53(10_1)$
	$-0.57(6^q4_1)$	$-0.53(6^q4_1)$	$+0.56(4^q6_1)$	$0.17(4^q6_1)$
	$+0.16(6^q6_1)$		$-0.15(6^q6_1)$	$+0.69(6^q4_1)$
			$+0.17(8^q2_1)$	$-0.17(6^q4_2)$
				$-0.23(6^q6_1)$
				$+0.26(8^q2_1)$
12_1^+	$0.27(12_1)$	$0.14(12_1)$	$0.37(12_1)$	$-0.13(12_1)$
	$-0.42(6^q6_1)$	$-0.22(6^q6_1)$	$+0.51(4^q8_1)$	$+0.29(4^q8_1)$
	$+0.20(8^q6_1)$	$+0.20(8^q6_1)$	$-0.14(6^q8_1)$	$+0.67(6^q6_1)$
	$+0.81(10^q2_1)$	$+0.94(10^q2_1)$	$+0.38(8^q4_1)$	$-0.12(6^q6_2)$
			$+0.12(10^q2_1)$	$-0.16(6^q8_1)$
				$+0.53(8^q4_1)$
				$+0.29(10^q2_1)$
14_1^+	$-0.14(6^q8_1)$	$-0.12(6^q8_1)$	$0.19(8^q4_1)$	$0.19(8^q6_1)$
	$+0.17(8^q8_1)$	$+0.16(8^q8_1)$	$+0.97(10^q4_1)$	$+0.98(10^q4_1)$
	$+0.97(10^q4_1)$	$+0.98(10^q4_1)$		

TABLE IX. Theoretical $B(E2)$ values caused by the different interaction mechanisms in ^{126}Ba.

	$(V_1+V_2+V_3)$	$V_1=0$	$V_2=V_3=0$	$V_3=0$
$10_1\rightarrow 8_1$	7510	8020	6980	5280
$12_1\rightarrow 10_1$	2300	671	6650	4190
$14_1\rightarrow 12_1$	3770	4430	425	1020

tion structure and $E2$-transition probabilities. One can see that excluding V_3 ($V_3=0$) changes the composition of the transitional states especially those with spins 10^+ and 12^+, which considerably redistributes $E2$-transition probabilities.

The effects of each mechanism are markedly pronounced in $B(E2)$ ratios (Table X for ^{126}Ba). Only one or two of them lead to values deviating considerably from the experimental ratios satisfactorily explained with $V_1+V_2+V_3$. Thus, our calculations of these ratios stress once more that all mechanisms have to be taken into account simultaneously and the new exchange mechanism V_3 plays here not the least role.

All calculations of $E2$ probabilities, presented in Tables IX and X as well as in Table XI embracing transitions in the yrast bands of ^{126}Ba and ^{130}Ce, are performed with a collective $E2$ operator constructed in the same manner as the collective Hamiltonian [Eq. (7)]:

$$T_\mu(E2)=T_\mu(E2)_{\rm C}(1-\hat{n}_b)+\sum_\tau T_\mu(E2)^{(\tau)}_{\rm C}\hat{n}_b. \quad (32)$$

Introducing in Eq. (32) projection operators $(1-\hat{n}_b)$ and $\hat{n}_b$ assumes that parameters in the operators of $E2$ transitions between purely d-s states and those containing a b boson can differ. $T(E2)_{\rm C}$ and $T(E2)^{(\tau)}_{\rm C}$ are standard $E2$ operators of IBM1 complicated by adding a term proportional to the d-boson number $\hat{n}_d$

$$T_\mu(E2)_{\rm C}=e^*[d^+_\mu s+s^+\tilde{d}_\mu+\chi(d^+\tilde{d})^{(2)}_\mu + \xi(d^+_\mu\hat{n}_d s+s^+\hat{n}_d\tilde{d}_\mu)]. \quad (33)$$

TABLE X. $B(E2)$ ratios in ^{126}Ba.

	Exp. [17]	Theory			
		$V_1+V_2+V_3$	$V_1=0$	$V_2=V_3=0$	$V_3=0$
$\frac{12_1\rightarrow 10_2}{12_1\rightarrow 10_1}$	0.13(4)	0.11	4.1	0.03	0.20
$\frac{10_2\rightarrow 8_1}{10_2\rightarrow 8_2}$	0.014(6)	0.056	85	0.25	0.50
$\frac{12_3\rightarrow 10_1}{12_3\rightarrow 10_2}$	0.040(12)	0.042	57	0.25	0.20
$\frac{14_3\rightarrow 12_1}{14_3\rightarrow 12_3}$	0.12(4)	0.08	0.18	0.29	0.37
$\frac{14_1\rightarrow 12_2}{14_1\rightarrow 12_1}$	0.14(3)	0.19	0.01	9.3	3.3
$\frac{14_2\rightarrow 12_1}{14_2\rightarrow 12_2}$	0.13(4)	0.20	3×10^{-4}	20	4.8

The values of e^*, χ, and ξ are fitted so as to attain overall agreement with empirical $B(E2)$ values in the yrast bands. These parameters are given in Table XII.

Analyzing the quasiparticle-boson interaction Vretenar *et al.* [7] also suggested to extend the standard boson quadrupole operator by adding a higher order term. In our case however the need of introducing the term with parameter ξ is dictated by an unusual growth of $E2$-reduced probabilities with spin up to $I=8$ in ^{126}Ba and ^{130}Ce. By using the operator with $\xi=0$ we can render only (0.6–0.7) of the empirical $B(E2;8\rightarrow 6)$. An attempt to increase this value by enlarging the total boson number (Ω) gives not more than 10 percent even for too large Ω. One more possibility to influence $B(E2)$ consists in the consideration of corrections connected with $E2$ transitions between b bosons and with simultaneous transformations of two bosons into a gamma quant and a boson. However these corrections are of the single-particle order and can be neglected. The adopted term ($\sim\xi$) facilitates describing the trends in $E2$ probabilities up

TABLE XI. $B(E2)$ (e^2 fm^4) along the yrast band in ^{126}Ba and ^{130}Ce. $B(E2)$ values in columns Th. I and Th. II are calculated with $E2$ parameters of Table XII.

	^{126}Ba			^{130}Ce			
$I_i^+\rightarrow I_f^+$	Exp. [19]	Th. I	Th. II	Exp. [20]	Exp. [21]	Th. I	Th. II
$2_1^+\rightarrow 0_1^+$	3400(100)	3370	3420	3980^{360}_{305}		3960	3990
$4_1^+\rightarrow 2_1^+$	4830(100)	4750	5570	6500^{900}_{470}		5690	6660
$6_1^+\rightarrow 4_1^+$	6620(300)	4930	6820	7430^{646}_{551}		6020	8290
$8_1^+\rightarrow 6_1^+$	8040^{640}_{550}	4790	7660	$11\,600^{4840}_{2640}$		5780	9060
$10_1^+\rightarrow 8_1^+$	7300^{1000}_{800}	4250	7510	5400^{1760}_{1070}		1800	3020
$12_1^+\rightarrow 10_1^+$	4100^{460}_{380}	1290	2300	6120^{397}_{352}		2200	2410
$14_1^+\rightarrow 12_1^+$	5980^{450}_{400}	2900	3770	9150^{541}_{484}		4810	6160
$16_1^+\rightarrow 14_1^+$	4470^{1800}_{1000}	3850	5970	1270^{4240}_{2540}	$10\,300^{3000}_{1850}$	5470	8630
$18_1^+\rightarrow 16_1^+$		3790	7400		6890^{2100}_{1300}	5450	10700
$20_1^+\rightarrow 18_1^+$					4080^{1020}_{680}	4980	12200
$22_1^+\rightarrow 20_1^+$					5490^{1250}_{860}	4160	12600
$24_1^+\rightarrow 22_1^+$					5180^{1300}_{860}	3010	11200

TABLE XII. Parameters of $E2$ transition operators, Eqs. (32), (33). For ^{130}Ce $T_C^{(\nu)}(\Omega-1)=T_C^{(\pi)}(\Omega-1)$. e^* is in e fm^2, χ and ξ are dimensionless. Ω is the boson number.

		Th. I			Th. II		
	Ω	e^*	χ	ξ	e^*	χ	ξ
				^{126}Ba			
$T_C(\Omega)$	9	11.53	−0.81	0	5.87	−1.20	0.4
$T_C^{(\nu)}(\Omega-1)$		12.08	−0.14	0	7.01	0.39	0.4
$T_C^{(\pi)}(\Omega-1)$		10.12	−0.045	0	6.06	0.16	0.4
				^{130}Ce			
$T_C(\Omega)$	9	12.39	−0.89	0	6.35	−1.32	0.4
$T_C^{(\nu)}(\Omega-1)$		14.34	−0.31	0	8.11	−0.34	0.4

to spin ~ 16, however it overestimates $B(E2)$'s for high spin states in ^{130}Ce (Table XI).

Although we could point out several sources of the origin of this term, nevertheless we believe that the discussion of these speculations is untimely now. On the one hand in early experiments [16] $B(E2)$ values of the bandcrossing region in these nuclei were found to be smaller. On the other hand in such adjacent nucleus as ^{128}Ba so large $B(E2)$ values as in ^{126}Ba and ^{130}Ce are not observed and our approach quite satisfactorily reproduces energies and $B(E2)$'s with the standard $E2$ operator at $\xi=0$; these data are given in Table XIII.

TABLE XIII. Energies of excited states (MeV) and $B(E2)$ (e^2 fm^4) in ^{128}Ba.

I_i	E_i		I_f	$B(E2;I_i\rightarrow I_f)$		
	Exp. [22]	Theor.		Exp. [22]	Exp. [23]	Theor.
2_1	0.284	0.293	0_1	2930(160)	2760(180)	2920
4_1	0.763	0.776	2_1	4370(360)	3940(150)	4100
6_1	1.407	1.397	4_1	5700(1300)	4080(450)	4430
8_1	2.189	2.171	6_1	4700(1600)	3490(870)	4330
10_1	3.082	3.074	8_1	2400(800)	2380(1580)	4010
2_2	0.885	0.886	0_1	120(50)		200
			2_1	1300(500)		2150
4_2	1.372	1.467	2_1	55(16)	30(10)	26
			4_1	<1300	<810	1340
			2_2	4000(1200)	1920(640)	2450
6_2	1.939	2.058	4_1	52(8)	50(10)	0
			4_2	5300(800)	5660(1150)	1900
8_2	2.601	2.597	6_1	37(10)	70(40)	1.4
			6_2	4700(1200)	8520(4480)	2540

IV. SUMMARY

In this paper we consider the interaction of the collective quadrupole excitations with pair fermion modes of higher multipolarities among which there are practically pure quasiparticle pairs occupying the intruder shell model level.

In the fermion space all these modes are treated as phonons; the most collective of them are D phonons. The two-quasiparticle composition of the phonons has been calculated with the help of RPA or its modified version for D phonons to attenuate ground-state RPA correlations. We have found that the interaction is realized through three mechanisms. Two of them (the scattering by the collective quadrupole moment and the exchange interaction with conservation of quasiparticle or phonon number) are employed in IBFM, the third mechanism transforming three phonons into one (and vice versa) has not been considered earlier in such problems.

With the same quasiparticle interaction, which is employed to find the structure and energies of the phonons, we have calculated parameters of the phonon interaction. Then in the spirit of IBM the fermion picture has been mapped onto the boson space where we deal with s, d bosons of IBM and additionally with b_J bosons, $J\geqslant 4$, the boson images of B_J phonons (IBFM considers fermion pairs instead of b_J bosons).

Along with microscopical calculations our approach includes phenomenological description of energies by means of fitting the parameters of the IBM1 operators. We assume that B phonons exert a blocking impact on collective properties. Therefore we consider three sets of the IBM1 parameters for purely d-s collective states and for states with neutron and proton b_J bosons. Such phenomenology enables the yield of a reasonable agreement with experiment for the energies of the b_J-boson states calculated with strength constants close to the Bohr-Mottelson estimates.

Our approach has been applied to analysis of the energies and $E2$-transition probabilities in ^{126}Ba and ^{130}Ce. Special attention has been paid to the role of each interaction mechanism. We have found that energy spectra are weakly sensitive to the choice of the mechanism. However, the $B(E2)$ values and particularly their ratios point out unambiguously that all three types of the interaction have to be allowed for simultaneously. If each of them taken separately gives a sharp transition from a purely collective d-s boson state to that containing a high spin boson, their unified action smooths out this sharpness and leads to a relatively small decrease of $B(E2)$ in the bandcrossing region and correctly reproduced $B(E2)$ ratios. In all these processes the new three-boson exchange interaction plays the important role.

To reproduce a rather strong growth of $E2$ probabilities before decreasing in the backbending region in ^{126}Ba and ^{130}Ce we have added to the standard $E2$ operator of IBM1 a term proportional to the d-boson number. Nevertheless we are not sure of its necessity as $E2$ data in an adjacent nucleus ^{128}Ba at the same spins and energies can be satisfactorily described without this term.

The authors wish to express their gratitude to I. Kh. Lemberg and V. G. Kiptily for discussions on various aspects of this study. The help given by D. N. Dojnikov and Yu. N. Lobach is gratefully acknowledged.

[1] F. Iachello and P. Van Isaker, *The Interacting Boson-Fermion Model* (Cambrige University Press, Cambridge, 1991).
[2] N. Yoshida, A. Arima, and T. Otsuka, Phys. Lett. **114B**, 86 (1982).
[3] S. T. Hsieh, H. C. Chiang, and D. S. Chuu, Phys. Rev. C **46**, 195 (1992).
[4] D. Vretenar, G. Bonsignari, and M. Sovoia, *Perspectives for the Interacting Boson Model* (World Scientific, Singapore, 1994), p. 225.
[5] A. D. Efimov and V. M. Mikhajlov [4], p. 714.
[6] D. Vretenar, G. Bonsignori, and M. Savoia, Z. Phys. A **351**, 289 (1995).
[7] D. Vretenar, S. Brant, G. Bonsignori, L. Corradini, and C. M. Petrache, Phys. Rev. C **57**, 675 (1998).
[8] A. Bohr and B. Mottelson, *Nuclear Structure*, Vol. II (Benjamin, New York, 1974).
[9] D. Janssen, R. V. Jolos, and F. Donau, Nucl. Phys. **A224**, 93 (1974).
[10] A. D. Efimov and V. M. Mikhajlov, *Kollektivnaya Yadernaya Dinamika* (Nauka, Leningrad, 1990), p. 120; Izv. Akad. Nauk. SSSR, Ser. Fiz. **56**, n1, 57 (1992) [Bull. Acad. Sci. USSR, Phys. Ser. **56**, 96 (1992)].
[11] G. Audi, O. Bersillon, J. Blachot, and A. H. Wapstra, Nucl. Phys. **A624**, 1 (1997).
[12] D. N. Dojnikov, K. I. Erochina, A. D. Efimov, and V. M. Mikhajlov, Nucl. Phys. **A531**, 326 (1991).
[13] K. Weeks and T. Tamura, Phys. Rev. C **22**, 888 (1980).
[14] A. D. Efimov and V. M. Mikhajlov, Proceedings of the International Conference on Nuclear Spectroscopy and Nuclear Structure, Dubna, 1993, p. 167.
[15] O. Castaños, P. Federman, and A. Frank, *Interacting Bose-Fermi Systems in Nuclei*, edited by F. Iachello (Plenum, New York, 1981).
[16] K. Schiffer, S. Harissopulos, A. Dewald, A. Gelberg, K. O. Zell, P. von Brentano, P. J. Nolan, A. Kirwan, D. J. G. Love, D. J. Thornley, and P. J. Bishop, J. Phys. G **15**, L85 (1989).
[17] D. Ward, V. P. Janzen, H. R. Andrews, D. C. Radford, G. C. Ball, D. Norn, J. C. Waddington, J. K. Jonansson, F. Banville, J. Gascon, S. Monaro, N. Nadon, S. Pilotte, D. Prevost, P. Taras, and R. Wyss, Nucl. Phys. **A529**, 315 (1991).
[18] D. M. Todd, R. Aryaeinejad, D. J. G. Love, A. H. Nelson, P. J. Nolan, P. J. Smith, and P. J. Twin, J. Phys. G **10**, 1407 (1984).
[19] A. Dewald, D. Weil, R. Krücken, R. Kuhn, R. Peusquens, H. Tiesler, O. Vogel, K. O. Zell, P. von Brentano, D. Bazzacco, C. Rossi-Alvarez, P. Pavan, D. De Acuña, G. de Angelis, and M. De Poli, Phys. Rev. C **54**, R2119 (1996).
[20] A. Dewald, P. Sala, R. Wrzal, G. Böhm, D. Liebenz, G. Siems, R. Wirowski, K. O. Zell, A. Gelberg, P. von Brentano, P. Nolan, A. J. Kirwan, A. J. Bishop, R. Julin, A. Lampinen, and J. Hattula, Nucl. Phys. **A545**, 822 (1992).
[21] P. Tikkanen, J. Keinonen, A. Lampinen, R. Julin, A. Pakkanen, P. Ahonen, J. Hatula, S. Juutinen, S. Törmänen, P. J. Nolan, P. von Brentano, A. Dewald, A. Gelberg, G. Siems, and R. Wirowski, Phys. Rev. C **42**, 2431 (1990).
[22] P. Petkov, S. Harissopulos, A. Dewald, M. Stolzenwald, G. Böhm, P. Sala, K. Schiffer, A. Gelberg, K. O. Zell, P. von Brentano, and W. Andrejtscheff, Nucl. Phys. **A543**, 589 (1992).
[23] W. Andrejtscheff, P. Petkov, A. Dewald, G. Böhm, and P. von Brentano, Phys. Rev. C **53**, 1606 (1996).