Giant monopole resonance and nuclear incompressibility within the Fermi-liquid drop model

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We study the important effects of Fermi surface distortion on the isoscalar giant monopole resonance (ISGMR), within a Fermi-liquid drop model, by considering consistently the effects on nuclear incompressibility coefficients and the boundary conditions needed to determine the energy of the ISGMR. There is a significant difference between the static nuclear incompressibility K, derived as a stiffness coefficient with respect to an adiabatic change in the bulk density, and the dynamic one K' associated with the zero sound velocity. We show that the enhancement in the energy of the ISGMR, the lowest breathing mode, which is due to the renormalization of K into K' is strongly suppressed by the effects of the Fermi surface distortion on the boundary condition. This is not the case for higher breathing modes such as the overtone. We also discuss, in particular, the effects of the Fermi surface distortion on energy weighted sums for the monopole mode and on the constrained and the scaling incompressibility coefficients and their relation to the liquid drop one. [S0556-2813(99)04706-8]

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I. INTRODUCTION

The incompressibility coefficient K of finite nuclei provides unique information on the fundamental characteristic of the infinite nuclear matter, the incompressibility coefficient K_∞. However, extraction of the incompressibility coefficient K and the corresponding value of K_∞ from experimental data on isoscalar giant monopole resonance (ISGMR) is not straightforward [1,2]. A commonly used microscopic approach for determining K_∞ is based on the self-consistent Hartree-Fock (HF) random-phase approximation (RPA) method. In the HF-RPA approach one adopts a certain form of effective nucleon-nucleon interaction, such as the Skyrme interaction, involving a set of a few parameters. Carrying out HF calculations, the parameters are determined by a fit to the experimental data on properties of ground states of a wide range of nuclei. It has been found that RPA calculations reproduce known experimental data on the strength distribution of the ISGMR [3,4] when existing effective nucleon-nucleon interactions having $K_\infty = 210 \pm 30$ MeV were used. However, this commonly accepted value of K_∞ was extracted using a limited class of effective interactions.

In this work we mainly consider another commonly used approach which is inspired by the liquid drop mass formula for nuclear binding energy E. This is a semiempirical approach in which one writes [1,3–8] the incompressibility K of a nucleus with mass number A as an expansion in $A^{-1/3}$

$$K = K_{\rm vol} + K_{\rm surf} A^{-1/3} + K_{\rm curv} A^{-2/3} + K_{\rm symm}\left(\frac{N-Z}{A}\right)^2 + K_{\rm Coul}\frac{Z^2}{A^{4/3}} + \cdots. \tag{1}$$

The nuclear matter incompressibility K_∞ is then deduced from Eq. (1) by extrapolation. We first note that this macroscopic approach implies a liquid-drop model type A expansion of K, where K is related to the ISGMR energy by some model assumptions. Different modifications of the Thomas-Fermi approximation were used to estimate the coefficients $K_{\rm vol}, K_{\rm surf}, \ldots$, in Eq. (1) [5,7,8]. However, the incompressibility coefficient K_∞ determined from Eq. (1) at $A \to \infty$ is *a static* stiffness coefficient with respect to an adiabatic change in the bulk density ρ_0. Namely,

$$K_\infty = 9\rho_0^2 \left.\frac{d^2(E/A)}{d\rho_0^2}\right|_{\rho_\infty}, \tag{2}$$

where E and ρ_∞ are the binding energy and the saturation density of the nuclear matter, respectively.

The static incompressibility coefficient K_∞ determines the first sound velocity, $c_1 = \sqrt{K_\infty/9m}$, for propagation of compression waves in nuclear matter. However, the ISGMR corresponds to the *zero sound* mode having the sound velocity $c_0 \approx v_F \approx \sqrt{3} c_1$ and the renormalized incompressibility coefficient $K'_\infty = 9mc_0^2 \approx 3K_\infty$. This strong renormalization of the incompressibility coefficient arises due to dynamical distortion of the Fermi surface.

In contrast to the first sound regime, the eigenfrequency of the zero sound excitations such as the ISGMR cannot, in principle, be used directly to extract the static incompressibility of Eq. (2) because of the additional contribution from the Fermi surface distortion effects which result in the renormalization of the incompressibility K_∞ into K'_∞. However, as will be shown in this work, the Fermi surface distortion effects also affect the boundary condition, needed for determining the energy of the ISGMR of finite nuclei. The enhancement in the energy of the ISGMR, the lowest breathing mode, which is due to the renormalization of K into K' is strongly suppressed by the effects of the Fermi surface distortion on the boundary condition. This is not the case for higher breathing modes such as the overtone. The aim of this work is to study effects of the Fermi surface distortion on the ISGMR and nuclear incompressibility coefficients. We will consider, in particular, the constrained, the scaling and the liquid drop incompressibility coefficients.

In Sec. II we present the model for the ISGMR, introducing the basic equation of motion and the corresponding boundary condition. In Sec. III we provide the solution for the energy of the ISGMR and the energy moments of the distribution of monopole strength. In Sec. IV we present some numerical results and discuss the effects of the Fermi surface distortion on the energies of the ISGMR and the overtone and on the nuclear incompressibility coefficients. Our conclusions are given in Sec. V.

II. FERMI-SURFACE DISTORTION EFFECTS ON MONOPOLE VIBRATIONS

There exists a simple relation between energy E_{0^+} of the ISGMR and the incompressibility coefficient K, if one state exhausts the energy-weighted sum rule. The constrained and scaling derivations of the incompressibility coefficients ($K^{\rm constr}$ and $K^{\rm sc}$, respectively) coincide in this case and E_{0^+} is given by

$$E_{0^+}=\sqrt{\frac{\hbar^2K^{\rm constr}}{m\langle r^2\rangle_{\rm eq}}}=\sqrt{\frac{\hbar^2K^{\rm sc}}{m\langle r^2\rangle_{\rm eq}}}, \tag{3}$$

where

$$K^{\rm constr}=\frac{m\langle r^2\rangle_{\rm eq}}{\hbar^2}\frac{m_1}{m_{-1}},\quad K^{\rm sc}=\frac{m\langle r^2\rangle_{\rm eq}}{\hbar^2}\frac{m_3}{m_1}. \tag{4}$$

Here $\langle r^2\rangle_{\rm eq}$ is the equilibrium root mean square (rms) radius of the nucleus and m_k are the energy moments of the distribution of monopole strength

$$m_k=\sum_n (E_n-E_0)^k|\langle\psi_n|r^2|\psi_0\rangle|^2. \tag{5}$$

The origin of the property manifested in Eqs. (3) and (4) lies in the fact that in the case where the strength is concentrated in one state the dynamical Fermi-surface distortion effects are washed out from the equation of motion and expression (3) represents the excitation energy in first sound description.

To study the Fermi surface distortion effects on the ISGMR energy and the nuclear incompressibility we will start from the zero-sound equation of motion in the nuclear volume in the form (see Appendix)

$$m\rho_0\frac{\partial^2}{\partial t^2}\chi_\alpha=-\sum_\beta \nabla_\beta\mathcal{P}_{\alpha\beta}+\rho_0\nabla_\alpha V_{\rm ext}, \tag{6}$$

where $\vec{\chi}(\vec{r},t)$ is the displacement field, ρ_0 is the equilibrium bulk particle density and $\mathcal{P}_{\alpha\beta}$ is the dynamical part of the pressure tensor

$$\mathcal{P}_{\alpha\beta}=-\mu\rho_0\left(\nabla_\alpha\chi_\beta+\nabla_\beta\chi_\alpha-\frac{2}{3}\vec{\nabla}\cdot\vec{\chi}\delta_{\alpha\beta}\right)-\frac{K}{9}\rho_0\vec{\nabla}\cdot\vec{\chi}\delta_{\alpha\beta}. \tag{7}$$

Here, in general, the nuclear static incompressibility K also includes the surface correction due to the surface tension [3]. We have included the time dependent external field

$$V_{\rm ext}=\lambda(t)\hat{Q}(\mathbf{r}),\quad \hat{Q}(\mathbf{r})=r^2-\langle r^2\rangle_{\rm eq} \tag{8}$$

in Eq. (6) to be able to evaluate the response function and the corresponding energy moments m_k, Eq. (5).

We will assume below a sharp spherical surface of radius $R_{\rm eq}$ for the equilibrium particle density $\rho_{\rm eq}$. The external field $V_{\rm ext}$ induces changes in the bulk density, $\delta\rho(r,t)=\eta(r,t)\rho_0\theta(R_{\rm eq}-r)$, and in the radius $\delta R(t)$. Both of them are related to the displacement field $\vec{\chi}(\mathbf{r},t)$. In the case of a monopole excitation, a general form of the displacement $\vec{\chi}(\mathbf{r},t)$ is given by

$$\vec{\chi}(\mathbf{r},t)=f(r,t)\mathbf{r}. \tag{9}$$

The form factor $f(r,t)$ is found from the continuity equation

$$\delta\rho=-\vec{\nabla}\cdot\rho_{\rm eq}\vec{\chi} \tag{10}$$

to have in the nuclear interior the form

$$f(r,t)=-\frac{1}{r^3}\int_0^r dr_1\ r_1^2\eta(r_1,t). \tag{11}$$

Using Eq. (10), the equation of motion (6) can be transformed in the nuclear interior into an equation of motion for the bulk density parameter $\eta(r,t)$. Namely,

$$m\frac{\partial^2}{\partial t^2}\eta=\frac{1}{9}K'\nabla^2\eta, \tag{12}$$

where K' is given by Eq. (A13).

The external field $V_{\rm ext}$ does not enter Eq. (12). However, the external field affects the boundary condition. This can be taken as a condition for the compensation at the nuclear surface of the compressional pressure $\mathcal{P}_{rr}$ and the pressures generated by the external field $P_{\rm ext}$, and the surface tension forces δP_σ. Namely,

$$P_{\rm ext}+\delta P_\sigma=\mathcal{P}_{rr}|_{r=R_{\rm eq}}, \tag{13}$$

where

$$P_{\rm ext}=\frac{3}{5}\lambda(t)\rho_0R_{\rm eq}^2,\quad \delta P_\sigma=-\frac{2\sigma}{R_{\rm eq}^2}\delta R(t) \tag{14}$$

and σ is the surface tension coefficient. The radial component of the compressional pressure tensor $\mathcal{P}_{rr}$ in Eq. (13) can be derived from Eqs. (7) and (9) and is given by

$$\mathcal{P}_{rr}|_{r=R_{\rm eq}}=\left[\frac{1}{9}\rho_0K'\eta+4\mu\rho_0f\right]_{r=R_{\rm eq}}. \tag{15}$$

The boundary condition given by Eqs. (13)–(15) plays a key role in our consideration. The Fermi surface distortion effects are manifested in both terms on the right-hand side (RHS) of Eq. (15). Usually these effects are missed in the liquid drop model consideration, see, for example, Ref. [3]. We point out also that all original results of the present paper are due to the abovementioned effects in the boundary condition (13).

III. GIANT MONOPOLE RESONANCE IN THE FERMI-LIQUID DROP MODEL

Using Eqs. (9)–(11), the solution to Eq. (12) with spherical symmetry provides the displacement field in the form

$$\vec{\chi}(\mathbf{r},t)=\alpha(t)\frac{j_1(kr)}{kr}\mathbf{r}, \tag{16}$$

where $\alpha(t)$ is a harmonic function of time with a frequency ω which is related to the wave number k by the following dispersion equation:

$$m\omega^2=\frac{1}{9}K'k^2. \tag{17}$$

The amplitude $\alpha(t)$ can be evaluated in terms of the external field parameter $\lambda(t)$ from the boundary condition (13) and is given by

$$\alpha(t)=\frac{-xf_\lambda}{xj_0(x)-(f_\sigma+f_\mu)j_1(x)}\lambda(t), \tag{18}$$

where $x=kR_{\rm eq}$ and

$$f_\sigma=\frac{18\sigma}{\rho_0R_{\rm eq}K'},\quad f_\lambda=\frac{27R_{\rm eq}^2}{5K'},\quad f_\mu=\frac{36\mu}{K'}. \tag{19}$$

The eigenexcitations are obtained as solutions to the secular equation determined by the denominator of Eq. (18). Namely,

$$x_nj_0(x_n)-(f_\sigma+f_\mu)j_1(x_n)=0, \tag{20}$$

and the corresponding eigenfrequencies ω_n are given by [see Eq. (17)]

$$\omega_n=\sqrt{\frac{K'}{9mR_{\rm eq}^2}}\,x_n\,. \tag{21}$$

In a general case of a Fermi-liquid drop with $\mu\neq0$, the eigenfrequency ω_n given by Eq. (21) is renormalized with respect to the one in the traditional liquid drop model [3,9] due to two contributions associated with the Fermi-surface distortion: (1) the direct change in the sound velocity, i.e., K' appears instead of K in Eq. (21); (2) the change of the roots x_n of the secular equation (20) due to the additional contribution from $f_\mu\neq0$ in Eq. (20). These two effects work in opposite directions: K'_∞ increases ω_n while f_μ decreases it, see next section.

The displacement field $\vec{\chi}_n$ associated with the x_n solution of the secular equation (20) allows us to derive the relevant collective mass B_n. The fluid kinetic energy $E_{\rm kin}$ is given by

$$E_{\rm kin}=\frac{1}{2}m\int d\mathbf{r}\sum_n\,\rho_{\rm eq}\dot{\chi}_n^2=\frac{1}{2}\sum_n\,B_n\dot{\alpha}_n^2. \tag{22}$$

Taking into account Eqs. (16), (20), and (22), one obtains

$$B_n=\frac{3}{2}AmR_{\rm eq}^2x_n^{-4}[1-j_0^2(x_n)-(f_\sigma+f_\mu)j_1^2(x_n)]. \tag{23}$$

We would like to stress that, in contrast to the analogical expression for the mass coefficient B_n in the liquid drop model [3], our result, Eq. (23), takes into account the Fermi surface distortion effects because of $f_\mu\neq0$. Moreover, usually one has $f_\mu\geqslant f_\sigma$ and thus the Fermi surface distortion effects are more important than the surface tension effect manifested by the term with f_σ in Eq. (23).

To evaluate the quantum response function $S(\omega)$ with respect to the external field $V_{\rm ext}\sim r^2$ and the corresponding energy weighted sums m_k, one has to evaluate the classical monopole moment $\langle r^2\rangle_n$ for the n mode. Using Eqs. (10) and (20), one obtains

$$\langle r^2\rangle_n=\int d\mathbf{r}r^2\delta\rho_n=\alpha(t)S_n\,, \tag{24}$$

where the particle density variation $\delta\rho_n$ is associated with the displacement field $\vec{\chi}_n$ and

$$S_n=-18AR_{\rm eq}^2x_n^{-3}\left[1-\frac{1}{3}(f_\sigma+f_\mu)j_1(x_n)\right]. \tag{25}$$

Following the quantum correspondency principle and using Eq. (24), we obtain (see also Chap. 6 in Ref. [9])

$$m_k=\sum_{n=1}^{\infty}\,(\hbar\omega_n)^k\alpha_n^{(0)2}S_n^2, \tag{26}$$

where $\alpha_n^{(0)}$ is the zero-point amplitude

$$\alpha_n^{(0)}=\sqrt{\hbar/2B_n\omega_n}. \tag{27}$$

Inserting expressions (21), (27), (23), and (25) into Eq. (26), one obtains

$$m_k=3\times6^2\frac{\hbar^2AR_{\rm eq}^2}{m}\left[1-\frac{1}{3}(f_\sigma+f_\mu)\right]^2\left(\frac{\hbar^2K'}{9mR_{\rm eq}^2}\right)^{(k-1)/2}\widetilde{m_k}, \tag{28}$$

where

$$\widetilde{m_k}=\sum_{n=1}^{\infty}\frac{x_n^{k-3}j_1^2(x_n)}{1-j_0^2(x_n)-(f_\sigma+f_\mu)j_1^2(x_n)}. \tag{29}$$

We would like to stress again that our result Eqs. (28) and (29) provides an account of the Fermi surface distortion effects. These both appear due to the renormalization of the incompressibility coefficient K' and the term f_μ in the boundary condition Eq. (20).

It is necessary to note that the low-energy sums m_{-1}, m_{-3}, . . . , appear in the adiabatic limit $\omega\to0$. This limit corresponds to the first sound regime where the contribution from the Fermi-surface distortion effects is absent. That means that the nonrenormalized incompressibility K instead of K' should be used in Eqs. (21) and (28) with $f_\mu=0$ in Eqs. (20) and (28).

IV. DISCUSSION

The sums $\widetilde{m_k}$ of Eq. (29) and, consequently, energy moments m_k of Eq. (28) can be evaluated analytically in the

TABLE I. Excitation energies (in MeV) of the isoscalar giant monopole resonance E_{0^+} and the overtone isoscalar giant monopole resonance $E_{0_2^+}$ obtained for liquid drop model (LDM) and for Fermi-liquid drop model (FLDM), using Eqs. (20) and (21) (see text).

A	$E_{0^+}^{\rm exp}$	$E_{0^+}^{\rm FLDM}$	$E_{0^+}^{\rm LDM}$	$E_{0_2^+}^{\rm FLDM}$	$E_{0_2^+}^{\rm LDM}$	$E_{0^+}^{\rm FLDM}/E_{0^+}^{\rm LDM}$	$E_{0_2^+}^{\rm FLDM}/E_{0_2^+}^{\rm LDM}$
40	18.0	16.1	13.8	73.0	28.8	1.165	2.533
90	16.2	14.9	12.8	57.5	26.1	1.169	2.202
120	15.2	14.1	12.0	52.6	24.5	1.169	2.145
208	13.7	12.8	10.9	44.6	22.1	1.167	2.015
1000	8.2	7.7	6.6	26.5	13.3	1.166	1.997

case of small perturbation parameters $f_\sigma \ll 1$, $f_\mu \ll 1$. Namely,

$$\widetilde{m_k} = \pi^{k-5}\zeta(5-k) - (k-8)(f_\sigma + f_\mu)\pi^{k-7}\zeta(7-k), \tag{30}$$

where $\zeta(s)$ is the Riemann zeta function

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}.$$

In the limit $f_\sigma \to 0$, $f_\mu \to 0$, i.e., if both additional contributions to the incompressibility K' from the surface tension and from the Fermi-surface distortion are absent, the three lowest sums m_{-1}, m_1, and m_3 can be easily derived from Eqs. (28) and (30) and are given by

$$m_{-1} = \frac{36}{35}\frac{AR_{\rm eq}^4}{K}, \quad m_1 = \frac{6}{5}\frac{\hbar^2 AR_{\rm eq}^2}{m}, \quad m_3 = 2\frac{\hbar^4 AK}{m^2}. \tag{31}$$

In this case of small values for the parameters f_σ and f_μ, one also obtains an analytical solution to the secular equation Eq. (20) in the form

$$x_n \approx n\pi - (f_\sigma + f_\mu)/n\pi. \tag{32}$$

Using Eq. (21), the energy E_{0^+} of the lowest monopole mode $(n=1)$ is then given as

$$E_{0^+} = \hbar\omega_1 = \sqrt{\frac{\hbar^2\pi^2 K'}{9mR_{\rm eq}^2}}\left[1 - \frac{f_\sigma + f_\mu}{\pi^2}\right]. \tag{33}$$

In the limit $f_\mu = 0$ and $K' = K$, Eq. (33) was derived earlier in Ref. [3]. In general, the eigenenergy E_{0^+}, Eq. (33), contains the renormalized incompressibility K' instead of the static one, K. This leads to an increase of the eigenenergy, as can be seen from Eqs. (33) and (A13). However, the additional factor in parenthesis in Eq. (33), which appears due to the Fermi surface distortion effect on the boundary condition, see Eqs. (13) and (20), partly compensates for the Fermi-surface distortion effect in K'.

Both values of K' and f_μ depend on the Landau scattering amplitude F_0. The condition $f_\mu \ll 1$, leading to the approximation Eq. (32) or Eq. (33), holds in the limit of the zero to first sound transition region at $F_0 \gg 1$. However, for realistic nuclear forces, we have $F_0 \sim 0$. Therefore, the general expressions (20) and (21) should be used. For the energy E_{0^+} of the lowest monopole mode $(n=1)$ we have from Eq. (21)

$$E_{0^+} = \hbar\omega_1 = \sqrt{\frac{\hbar^2 K'}{9mR_{\rm eq}^2}}x_1, \tag{34}$$

where x_1 is the lowest $(x_1 > 0)$ solution to the secular equation (20).

We have carried out numerical calculations using the following nuclear parameters: $\rho_0 = 0.17$ fm^{-3}, $\epsilon_F = 40$ MeV, $r_0 = 1.12$ fm, $\sigma = 1.2$ MeV/fm^2, and $F_0 = 0.2$. The static incompressibility K was determined from the experimental energy $E_{0^+}^{\rm exp}$ by using

$$K = \frac{m\langle r^2\rangle}{\hbar^2}(E_{0^+}^{\rm exp})^2. \tag{35}$$

For the case $A = 1000$, we have used the extrapolation formula $E_{0^+}^{\rm exp} \approx 82A^{-1/3}$ MeV. The results of the numerical calculations of the ISGMR energy, E_{0^+} are given in Table I. The liquid drop model (LDM) and Fermi-liquid drop model (FLDM) results were obtained with $f_\mu = 0$ and $f_\mu \neq 0$, respectively. The corresponding values of K and K' are shown in Table II. We point out that the significant renormalization of K into K', which is due to the Fermi surface distortion, increases the value of $E_{0^+}^{\rm FLDM}$. However, the effect of the Fermi surface distortion on the boundary condition (on the value of x_1) acts to decrease the value of $E_{0^+}^{\rm FLDM}$. This com-

TABLE II. Incompressibilities (in MeV) as obtained from different definitions (see text).

A	K	K'	$K_{\rm FLDM}$	$K_{\rm LDM}$	$K^{\rm constr}$	$K_{\rm FLDM}^{\rm sc}$	$K_{\rm LDM}^{\rm sc}$
40	68.8	500.8	54.9	40.5	42.4	57.7	57.6
90	95.7	527.7	81.2	59.4	62.7	87.2	87.1
120	102.0	534.0	87.5	63.9	67.6	94.3	94.2
208	119.6	551.6	103.6	76.1	80.5	113.2	113.0
1000	122.0	554.1	107.9	78.8	83.5	118.2	118.1

pensation of the Fermi-surface distortion effect in the lowest mode ($n=1$) can be seen by comparing the energies $E_{0^+}^{\rm FLDM}$ and $E_{0^+}^{\rm LDM}$ in Table I. We also show in Table I, the energy $E_{0_2^+}$ of the overtone (double) ISGMR which corresponds to $n=2$ in Eqs. (20) and (21). We point out that the effect of the Fermi surface distortion on the boundary condition (on x_n) is rather small for the higher modes with $n\geqslant 2$. As can be seen from Table I, the eigenenergy of the overtone giant monopole resonance, $E_{0_2^+}^{\rm FLDM}$, is shifted up significantly with respect to the energy $E_{0_2^+}^{\rm LDM}$. This effect can be used for the extraction of the Fermi-surface distortion effect on the dynamic nuclear incompressibility K' from an experimental measurement of the energy of the overtone giant monopole resonance.

We will now examine commonly used definitions of incompressibility coefficients. Adopting the relation given in Eq. (35) we define the corresponding incompressibility coefficients

$$K^{\rm FLDM}=\frac{m\langle r^2\rangle}{\hbar^2}(E_{0^+}^{\rm FLDM})^2 \tag{36}$$

and

$$K^{\rm LDM}=\frac{m\langle r^2\rangle}{\hbar^2}(E_{0^+}^{\rm LDM})^2. \tag{37}$$

We also consider the well-known definitions of the incompressibility coefficients through the energy moments of the distribution of monopole strength m_k. Following Ref. [5], we will define the constrained incompressibility $K^{\rm constr}\equiv K(1)$ and the scaled incompressibility $K^{\rm sc}\equiv K(3)$ through the mean energies $\widetilde{E}(k)$ of the monopole resonance

$$\widetilde{E}(k)=\sqrt{\frac{m_k}{m_{k-2}}}=\sqrt{\frac{\hbar^2 K(k)}{m\langle r^2\rangle}}. \tag{38}$$

Let us start from the limiting case $f_\sigma\to 0$, $f_\mu\to 0$. Using Eqs. (20) and (21) with $f_\sigma=f_\mu=0$, we obtain the known result for the incompressibility coefficient associated with classical liquid drop model

$$K^{\rm LDM}=\frac{m\langle r^2\rangle}{\hbar^2}(E_{0^+}^{\rm LDM})^2=\frac{\pi^2}{15}K \quad \text{at} \quad f_\sigma=f_\mu=0. \tag{39}$$

Using Eqs. (38) and (31) one obtains the constrained incompressibility

$$K^{\rm constr}=K(1)=\frac{m\langle r^2\rangle}{\hbar^2}\widetilde{E}(1)^2=\frac{m\langle r^2\rangle}{\hbar^2}\,\frac{m_1}{m_{-1}}=\frac{7}{10}K. \tag{40}$$

This result was derived earlier in Ref. [10]. The scaled incompressibility $K^{\rm sc}$ can be also evaluated from Eqs. (38) and (31) and is given by

$$K^{\rm sc}=K(3)=\frac{m\langle r^2\rangle}{\hbar^2}\widetilde{E}(3)^2=\frac{m\langle r^2\rangle}{\hbar^2}\,\frac{m_3}{m_1}=K. \tag{41}$$

In a general case, $f_\sigma\neq 0$, $f_\mu\neq 0$, expression (28) for m_k has to be used. The constrained incompressibility (40) is not affected by the Fermi-surface distortion because the sum m_1 is model independent and the adiabatic sum m_{-1} contains the static incompressibility K (see above). This is not case for the scaled incompressibility $K^{\rm sc}$ because the renormalized K' enters the sum m_3, see Eq. (28). However, as in the case of the lowest giant monopole resonance, there is a significant compensation between the increase of K', appearing in Eq. (28), and the effect of the Fermi surface distortion in the boundary condition, represented in Eqs. (28) and (29) by $f_\mu\neq 0$. In Table II we give the values of the incompressibilities K, K', $K^{\rm constr}$, $K^{\rm FLDM}$, $K^{\rm LDM}$, $K^{\rm sc}_{\rm LDM}$, and $K^{\rm sc}_{\rm FLDM}$ for several nuclei. The incompressibilities K and K' have been obtained from Eqs. (35) and (A13) at $F_0=0.2$. The constrained incompressibility $K^{\rm constr}$ was evaluated from Eq. (4). Both scaled incompressibilities $K^{\rm sc}_{\rm LDM}$ and $K^{\rm sc}_{\rm FLDM}$ were calculated from Eq. (4) taking $f_\mu=0$ for $K^{\rm sc}_{\rm LDM}$ and f_μ from Eqs. (19) and (A6) at $F_0=0.2$ for $K^{\rm sc}_{\rm FLDM}$. We can see from Table II that in the limit $A\to\infty$ we obtain the results of Eqs. (39), (40), and (41). We stress that the general condition [11] $K^{\rm sc}_{\rm FLDM}=K_\infty$ (in the limit $A\to\infty$) can be fulfilled quite well in spite of the fact that the very large renormalized incompressibility K' enters the sum m_3, see Eq. (28).

We also note that the Fermi-surface distortion effects in the general expression (28) appear because the scaling assumption for the displacement field $\vec{\chi}(\mathbf{r},t)$ such as $\vec{\chi}(\mathbf{r},t)=\alpha(t)\mathbf{r}$ (Tassie model) is not used in our consideration, see Eq. (9). This scaling assumption with $\vec{\chi}(\mathbf{r},t)=\alpha(t)\mathbf{r}$ washes out the Fermi surface distortion effects from the pressure tensor $\mathcal{P}_{\alpha\beta}$, Eq. (7), and Eq. (41) is transformed into the hydrodynamical one. Namely,

$$K^{\rm sc}=K\left[1-\frac{1}{3}f_\sigma\right]. \tag{42}$$

Thus, in the limit $A\to\infty$, we have $K^{\rm sc}=K_\infty$ independently of the magnitude of the Landau parameter F_0.

V. CONCLUSIONS

We have shown that the monopole eigenmode in a finite Fermi-liquid drop is renormalized due to two effects associated with the dynamic Fermi-surface distortion: the change in the nuclear incompressibility, see Eqs. (21) and (A13), and the change in the boundary condition, see Eqs. (15) and (20). These two effects work in opposite directions.

There is a significant difference between the static nuclear incompressibility K, i.e., derived as a stiffness coefficient with respect to a change in the bulk density, and the dynamic one, K' associated with the zero sound velocity. The Fermi-surface distortion effects increase the incompressibility. Of course, the increase of incompressibility due to the Fermi-surface distortion effect leads to an increase of the energy of the giant monopole resonance E_{0^+}. However, the consistent presence of the same Fermi-surface distortion effects in the boundary condition strongly suppresses this increase of E_{0^+}. Therefore the energy of the lowest giant monopole resonance in the Fermi-liquid drop is rather close to the one in the usual liquid drop model where the Fermi-surface distortion effects are not taken into account. The Fermi-surface distortion effects depend on the Landau interaction parameter F_l. They disappear in the limit $F_0 \to \infty$. These effects are completely washed out also from the dynamic incompressibility K' and from the corresponding boundary condition for the breathing mode in the case of the scaling assumption for the displacement field taken in the form $\vec{\chi}(\mathbf{r},t) = \alpha(t)\mathbf{r}$ (Tassie model).

We point out that the effect of the Fermi surface distortion in the boundary condition is rather small for the higher modes with $n \geqslant 2$. This fact can be used for the extraction of the Fermi-surface distortion effect on the dynamic nuclear incompressibility K' by measuring the energy of the overtone giant monopole resonance. We have shown also that the commonly used definition of the nuclear incompressibility through the third energy moment m_3 of the distribution of monopole strength (so-called scaled incompressibility) gives a result which is very close to the adiabatic definition of the incompressibility K, in spite of the fact that the strongly renormalized incompressibility K' enters into the energy moments sums $m_{k>1}$. This is a result of the above mentioned consistent account of the Fermi-surface effects in the boundary condition at the evaluation of the energy weighted sum m_k, Eq. (28).

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APPENDIX

The Euler equation of motion (6) can be derived from the variational principle. Let us introduce the conjugate variables χ_ν and $m\dot{\chi}_\nu$, where χ_ν is the displacement field and consider the Lagrangian $\mathcal{L}(\{\chi_\nu,\dot{\chi}_\nu\})$ in the form [12]

$$\mathcal{L}(\{\chi_\nu,\dot{\chi}_\nu\}) = E_{\mathrm{kin}}(\{\dot{\chi}_\nu\}) - E_{\mathrm{pot}}(\{\chi_\nu\}), \tag{A1}$$

where

$$E_{\mathrm{kin}}(\{\dot{\chi}_\nu\}) = \frac{1}{2} m \int d\mathbf{r} \rho_{\mathrm{eq}} \dot{\chi}_\nu^2 \tag{A2}$$

and

$$E_{\mathrm{pot}}(\{\chi_\nu\}) = \frac{1}{2} \int d\mathbf{r} \left[\frac{K_\infty}{9} \rho_{\mathrm{eq}} (\vec{\nabla}\cdot\vec{\chi})^2 + \mu \rho_{\mathrm{eq}} \sum_{\alpha\beta} (\nabla_\alpha \chi_\beta) \times \left(\nabla_\alpha \chi_\beta + \nabla_\beta \chi_\alpha - \frac{2}{3} \vec{\nabla}\cdot\vec{\chi}\,\delta_{\alpha\beta} \right) \right]. \tag{A3}$$

The term multiplied by μ in Eq. (A3) is due to the dynamical distortion of the Fermi surface appearing at zero-sound excitations in a Fermi liquid. The constant μ depends on the Landau parameters F_l in the interaction amplitude $F(\mathbf{p},\mathbf{p}')$:

$$F(\mathbf{p},\mathbf{p}') = \sum_{l=0}^{\infty} F_l P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'), \quad \hat{\mathbf{p}} = \mathbf{p}/p. \tag{A4}$$

In the case of an isotropic interaction amplitude, i.e.,

$$F_0 \neq 0, \quad F_{l\neq 0} = 0, \tag{A5}$$

one has [15]

$$\mu = \frac{3}{2} s^2 \epsilon_F \left[1 - \frac{1}{3s^2}(1+F_0) \right], \tag{A6}$$

where $s = \omega / k_F q$ and q is the wave number for the longitudinal zero-sound wave. Note that the assumption Eq. (A5) implies also an effective mass of $m^* = m$. An extension to higher multipolarities $l \geqslant 1$ is straightforward [13].

The variational Lagrange equation reads

$$\frac{\partial}{\partial t} \frac{\delta \mathcal{L}}{\delta \dot{\chi}_\nu} - \frac{\delta \mathcal{L}}{\delta \chi_\nu} = 0. \tag{A7}$$

Substituting Eqs. (A1)–(A3) into Eq. (A7), we obtain the Euler-like equation of motion in the following form:

$$m \frac{\partial^2}{\partial t^2} \chi_\alpha = \sum_\beta \nabla_\beta \left[\mu \left(\nabla_\alpha \chi_\beta + \nabla_\beta \chi_\alpha - \frac{2}{3} \vec{\nabla}\cdot\vec{\chi}\,\delta_{\alpha\beta} \right) + \frac{K_\infty}{9} \vec{\nabla}\cdot\vec{\chi}\,\delta_{\alpha\beta} \right]. \tag{A8}$$

We point out that Eqs. (A8) with μ from Eq. (A6) does not imply any restriction on the multipolarity of the Fermi surface distortion. Equations (A8) give a set of closed equations of motion for the displacement field $\vec{\chi}(\mathbf{r},t)$. The solution of Eq. (A8) implies the previous solution of the dispersion equation for the dimensionless sound velocity s. The value of s depends on the multipolarity l of the Fermi-surface distortion. In two important particular cases of first-sound regime, $l \leqslant 1$, and quadrupole distortion of the Fermi surface, $l \leqslant 2$, one has [14]

$$s^2|_{l\leqslant 1} = \frac{1}{3}(1+F_0), \quad s^2|_{l\leqslant 2} = \frac{1}{3}\left(\frac{9}{5} + F_0 \right). \tag{A9}$$

Taking into account the continuity equation

$$\frac{\partial}{\partial t} \delta\rho + \rho_{\mathrm{eq}} \nabla_\nu \dot{\chi}_\nu = 0, \tag{A10}$$

Eqs. (A8) can be cast in the form of the equation of propagation of the zero sound for the particle density perturbation. Namely,

$$\frac{\partial^2}{\partial t^2} \delta\rho = c_0^2 \nabla^2 \delta\rho, \tag{A11}$$

where c_0 is the zero sound velocity

$$c_0 = \sqrt{\frac{1}{9m} K'_\infty} \tag{A12}$$

and, see also Ref. [15],

$$K'_\infty = K_\infty - 12 \epsilon_F F_0 \Omega_{20}, \quad K_\infty = 6 \epsilon_F (1+F_0). \tag{A13}$$

Here

$$\Omega_{l0}(s)=\frac{1}{2}\int_{-1}^{1}dx\frac{xP_l(x)P_0(x)}{x-s}, \tag{A14}$$

$P_l(x)$ is the Legendre polynomial and s can be obtained from the well-known Landau's dispersion relation [16]

$$-\frac{1}{F_0}=\Omega_{00}(s). \tag{A15}$$

The second term in the RHS of the first equation in Eq. (A13) is due to the dynamical Fermi-surface distortion. In the case $F_0>0$, this term increases the incompressibility because of $\Omega_{20}(s)<0$ if $F_0>0$. In the limiting case $F_0\to\infty$, we have from Eqs. (A14) and (A15) $\Omega_{20}\approx 0$ and $K'_\infty\approx K_\infty$. This means that the Fermi-surface distortion effect disappears from the nuclear matter incompressibility K'_∞ for a strong repulsive interaction with $F_0\gg 1$. In the opposite case, $F_0\to +0$, we find from Eqs. (A14) and (A15) $\Omega_{20}\approx -1/F_0$ and strongly renormalized incompressibility $K'_\infty\approx 3K_\infty$.

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