Microscopic study of the properties of identical bands in the $A=150$ mass region

C. Rigollet*
CRN, IN2P3-CNRS, F-67037 Strasbourg, France

P. Bonche
SPhT, CEA Saclay, F-91191 Gif-sur-Yvette Cedex, France

H. Flocard
IPN, IN2P3-CNRS, F-91406 Orsay Cedex, France

P.-H. Heenen
Physique Nucléaire Théorique et Physique Mathématique, Université Libre de Bruxelles - CP 229, B1050 Brussels, Belgium

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In this work, we investigate the properties of superdeformed identical bands in the $A\sim150$ mass region by microscopic calculations based on the new SLy4 parametrization of the Skyrme force and on a density-dependent zero-range pairing interaction. Many experimental properties are correctly reproduced, with however some qualitative differences in the description of identical bands. [S0556-2813(99)03306-3]

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I. INTRODUCTION

The first evidence of superdeformed (SD) rotational bands has been observed in the $A\sim150$ mass region [1]. A striking feature is the existence in some nuclei of excited SD bands with transition energies equal to those of the yrast band of a neighboring nucleus within only one or two keV; e.g., in ^{150}Gd* and ^{151}Tb or in ^{151}Tb* and ^{152}Dy [2]. These pairs of identical bands thus have the same dynamical moments of inertia. Moreover, Nazarewicz *et al.* [3] pointed out that the averages of the transition energies in bands 2 and 3 of ^{153}Dy are equal to the transition energies of the yrast band of ^{152}Dy, with an accuracy of the order of one keV. In this case also, these three bands have identical moments of inertia and are called identical. Many other cases of identical bands have been found since these first discoveries.

Following these experimental findings, significant efforts have been made in order to understand the nature and the physical origin of this phenomenon, both experimentally and theoretically. Cranked shell model calculations, based on a Woods-Saxon potential [5], show that large shell gaps occur at $Z=66$ and $N=86$ for a quadrupole deformation β_2 of about 0.6. These large gaps are responsible for the formation at each angular momentum of pronounced secondary SD minima in the potential energy surfaces of this nucleus.

Therefore, it was natural to consider ^{152}Dy as a doubly magic SD nucleus. Accordingly, most SD configurations in neighboring nuclei have been analyzed in terms of the single particle spectrum of ^{152}Dy at superdeformation.

It has also been shown that the behavior of SD bands is strongly influenced by the number of nucleons occupying high-j, high-N intruder orbitals [6]. For this reason, it is now customary to label SD bands with the symbol $\pi6^n\nu7^m$, where n is the number of protons occupying the $N=6$ intruder states (from the $i_{13/2}$ subshell) and m is the number of neutrons occupying the $N=7$ intruder levels (from the $j_{15/2}$ subshell). As the dynamical moment of inertia $\mathcal{J}^{(2)}$ is very sensitive to the occupation of intruder orbitals, it has been conjectured that identical bands have the same intruder configuration assignment [4]. The similarity between SD bands is not only limited to transition energies and dynamical moment of inertia as identical bands also have comparable intensity patterns and quadrupole moments, though in the latter case there are rather large experimental error bars.

Data show that the yrast band in ^{152}Dy is identical in terms of $\mathcal{J}^{(2)}$ moments of inertia to several other SD bands belonging to nuclei differing by one up to four nucleons. Furthermore, because of the ''magic character'' of this nucleus, we will use the yrast SD band of ^{152}Dy as a reference case against which we will compare properties of SD bands of neighboring nuclei such as $\mathcal{J}^{(2)}$, the charge quadrupole moment Q_0, and pairing energies. We will restrict this comparison to SD bands with the same occupancy of intruder orbitals.

The method used in this study has been presented in a series of previous publications [7–9]. It is based on the cranked Hartree-Fock-Bogoliubov (HFB) method with pairing correlations treated dynamically by means of the Lipkin-Nogami prescription. This formalism has been applied to the study of the lowest SD bands of ^{150}Gd, ^{151}Tb, ^{151}Dy, and ^{152}Dy [10]. Most results were obtained with a seniority pairing interaction. However, in ^{150}Gd, an exploratory calculation with a density-dependent zero-range pairing interaction showed that such structure for the pairing field improves greatly the agreement of the dynamical moment of inertia with experimental data. In the mean-field channel, the Skyrme parametrization Skm* was used.

In the present paper, we study isotopes and isotones of ^{152}Dy, focusing on the ground SD bands of these neighboring nuclei and on the bands identical to ^{152}Dy lowest band.

*Present address: National Accelerator Centre, Faure P.O. Box 72, ZA-7131, South Africa.

For this purpose, we use a parametrization of the Skyrme effective force, SLy4 [11], which has been recently derived with special emphasis on its isospin properties. In particular, this interaction gives an improved description of nuclei far from stability as compared to previous Skyrme parametrizations. Thus our study provides a test of this force in the regime of high rotational frequencies, a region for which Sly4 was not *a priori* intended. As in Ref. [9], a density-dependent zero-range interaction has been used in the particle-particle channel.

II. DETERMINATION OF THE PAIRING STRENGTHS

The behavior of the dynamical moment of inertia along a rotational band depends strongly on pairing correlations. As the rotational frequency increases, these correlations weaken, leading to an increase of the moment of inertia. Such a decrease of the pairing correlations puts a severe constraint on the pairing part of the interaction. The strengths must be adjusted to account for the interplay between rotation and pair correlations. The adjustment of the strength of the pairing interaction to the dynamical moments of inertia of SD bands thus provides a natural way to incorporate this constraint in the interaction [7,9,10].

In the $A=150$ mass region, no bands have been detected at low spins. Pairing correlations are thus expected to be weaker than in the $A=190$ region. However, they are responsible for the low spin behavior of the ^{150}Gd SD band, where the moment of inertia shows a rapid decrease for $\hbar\omega$ between 200 and 300 keV. This rapid variation is absent in Hartree-Fock calculations and is related to a crossing of two neutron quasiparticles [5]. As in a previous analysis of this mass region [10], we have studied the influence of the pairing on the ^{150}Gd moment of inertia. The pairing interaction is density-dependent and has a zero-range:

$$V_P=\frac{V_0}{2}(1-P_\sigma)\left(1-\frac{\rho(\vec{r}_1)}{\rho_c}\right)\delta(\vec{r}_1-\vec{r}_2). \qquad (1)$$

In definition (1), V_0 and ρ_c are two parameters and $\rho(\vec{r})$ is the total local single-particle density in coordinate space.

The value of the critical density ρ_c for which the pairing strength is equal to zero has been fixed to 0.16 fm^{-3}, which is the saturation density of nuclear matter for SLy4. Pairing is therefore mainly active on the nuclear surface. An energy cutoff is introduced that limits the active single particle space to an energy window of 5 MeV above and below the Fermi energy.

In Fig. 1, the experimental data are compared to the dynamical moments of inertia obtained with three different strengths for the pairing interaction. In all calculations, the dynamical moment of inertia is overestimated. On the other hand, at low frequencies, the rapid decrease of the moment of inertia is better reproduced by the intermediate pairing strength. The agreement with the data is very similar to the agreement obtained in our previous work [10]. As in this case, the moment of inertia obtained in a pure HF calculation is flat and does not show a rapid variation at low frequencies. One should however note that the plot of the experimental moment of inertia has a shoulder around $\hbar\omega$ equal to 500 keV that is not present in our calculation and does not seem to be directly related to pairing correlations. In the following, we will use the value of 1250 $\mathrm{MeV\,fm}^{-3}$ for both neutron and proton pairing strengths.

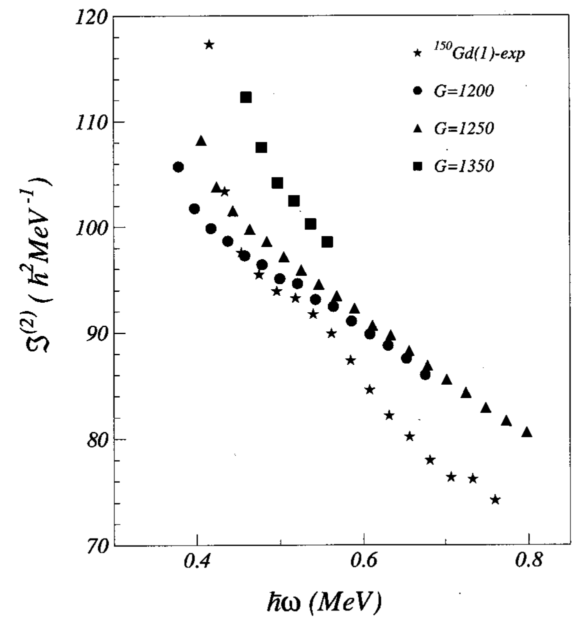

FIG. 1. Dynamical moment of inertia of ^{150}Gd calculated with the Skyrme interaction SLy4 [11] and with three different strengths for the pairing interaction. The experimental data are taken from Ref. [14].

To check that this value also satisfactorily reproduces data at normal deformations, we have determined the one-neutron S_n and one-proton S_p separation energies in the first well for isotopes and isotones around ^{152}Dy. The odd nuclei have been calculated with the same method as the one used for rotating nuclei, i.e., the effects of the time odd terms and of self-consistency due to the creation of a qp have been fully taken into account. Our results are compared in Table I to the experimental data and to the values obtained by Möller and Nix [12]. This table indicates that the accuracy of both theoretical calculations is similar for the six isotopes studied in this paper. A further check of the adjustment of the pairing strength has been done by calculating the moments of inertia of SD bands in the $A=190$ mass region [13]. The agreement with the experimental data is quite good. In particular, the increase of the moment of inertia versus angular momentum is very well reproduced without any further adjustment.

III. THE NUCLEUS ^{152}Dy

The qp Routhians E_i^ω of the ^{152}Dy yrast SD band are shown on Fig. 2 as a function of $\hbar\omega$. All the qp excited bands studied in this paper are based on these diagrams. The

TABLE I. One neutron (three left columns) and one proton separation energies (right).

Nucleus	^{152}Dy	^{153}Dy	^{154}Dy	^{151}Tb	^{152}Dy
This work	8.82	6.90	9.24	3.85	5.42
Experiment	9.59	7.11	9.31	3.13	5.80
Möller and Nix	9.60	7.41	9.52	3.62	5.46

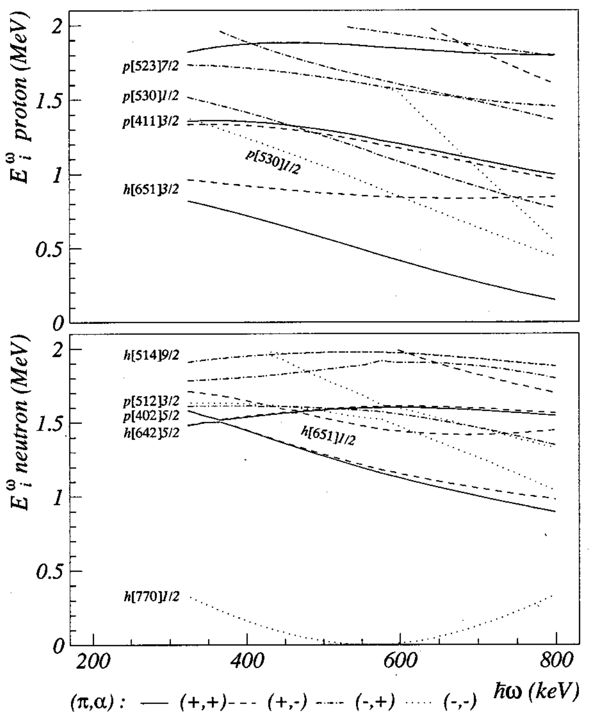

FIG. 2. Quasiparticle Routhians of ^{152}Dy calculated with the Skyrme interaction SLy4 [11].

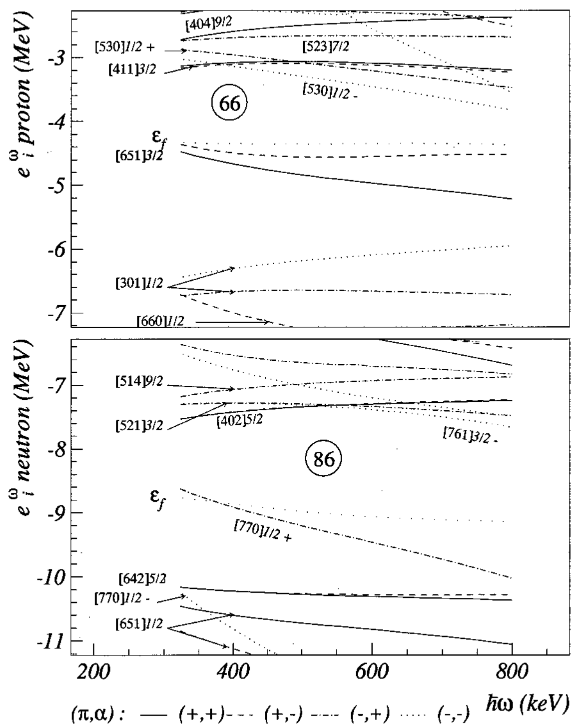

FIG. 3. Single particle Routhians of ^{152}Dy calculated with the Skyrme interaction SLy4 [11].

quasiparticles are labeled by a letter p (particle) or h (hole), which indicates whether their main component in the HF basis has an energy above or below the Fermi level.

The lowest proton qp orbitals correspond to two $N=6$ intruder states. It is usually considered that the lowest and first excited bands of ^{151}Tb are built on these 1qp excitations with respect to ^{152}Dy. These qp become of the hole-type in the lowest band of ^{150}Gd. For both nuclei, bands with the same number of intruders as the ground SD band of ^{152}Dy will be excited bands. One can also construct a 2qp proton excited band in ^{150}Gd with a single $N=6$ orbital, which has the same number of intruders as the ^{151}Tb ground band.

The lowest neutron qp has an intruder $N=7$ orbital as its main component. For all frequencies, this qp has a very small excitation energy. One can therefore expect that the lowest bands in ^{151}Dy and ^{153}Dy will have different numbers of neutron intruders than ^{152}Dy. Above an excitation energy of 1.0 MeV, several qp have similar behaviors and will lead to configurations with properties similar to the ^{152}Dy lowest band.

The particle Routhians in the HF basis e_i^ω are shown on Fig. 3 as a function of $\hbar\omega$. Large gaps are obtained above the proton number 66 and the neutron number 86, making ^{152}Dy a doubly magic nucleus in its SD state.

On the basis of calculations neglecting pairing correlations, rotational bands identical to the lowest band of ^{152}Dy have been predicted in isotones of ^{152}Dy to be constructed by depopulation of the [301]1/2 level. However, since pairing correlations push the corresponding qp energy to a value of the order of 2 MeV, excitations based on other qp's seem to be more favorable. For neutrons, the lowest excitation corresponds to the intruder orbital [770]1/2. Bands identical to ^{152}Dy will in principle require the population of the [651]1/2 or the [642]5/2 orbitals for isotopes with less neutrons and of [402]5/2 or [514]9/2 for ^{153}Dy.

The ordering of the [651]1/2 and [642]5/2 neutron levels is strongly dependent on deformation and is predicted differently in ^{152}Dy by various models. The [651]1/2 level is predicted to be above the [642]5/2 level by mean-field calculations based on Nilsson [6] and Woods-Saxon [5] potentials and by our previous HFB calculation using the Skm* interaction. Both orderings are obtained in a recent RMF calculation, depending on the effective Lagrangian [15].

The configurations that have been calculated in the present work are summarized in Table II.

The experimental $\mathcal{J}^{(2)}$ moment of inertia of the ^{152}Dy yrast band has a smooth behavior. It is well reproduced by our calculation. However, the magnitude of $\mathcal{J}^{(2)}$ is overestimated. This seems to be a general feature of Skyrme parametrizations and of fully microscopic studies, whose origin has not been understood up to now [16]. As we shall see, most of our calculations overestimate moments of inertia compared to the data, the overestimation being the largest in ^{152}Dy. A renormalization by a factor 0.9 applied to all our results would yield a good overall agreement.

IV. THE YRAST SD BANDS

The dynamical moment of inertia of yrast bands is shown in Fig. 4 for isotopes and isotones of ^{152}Dy. As a function of the number of neutrons, a change in trends is visible on the experimental data. It reflects the behavior of the second and third $N=7$ intruder orbitals, which are strongly downsloping as a function of the rotational frequency. Therefore as neutrons are added, the slope of $\mathcal{J}^{(2)}$ decreases and be-

TABLE II. Quasiparticle configurations studied in this paper. The last two columns give the calculated and experimental [25,21,26,19,24] quadrupole moments obtained around $50\hbar$.

Nucleus (band)	Configuration	E^* (MeV)	Q_0^{cal} (e b)	Q_0^{exp} (e b)
^{152}Dy(1)	$\pi 6^4 \nu 7^2$	-	17.5	$17.5^{+0.4}_{-0.2}$
^{151}Dy(1)	$[770]1/2+^{-1}$	-	16.8	$16.9^{+0.2}_{-0.3}$
^{151}Dy(2)	$[642]5/2+^{-1}$ (n)	1.25	17.0	$18.2^{+0.4}_{-0.4}$
^{151}Dy(4)	$[651]1/2+^{-1}$ (n)	1.20	16.9	$17.5^{+1.1}_{-0.7}$
^{153}Dy(1)	$[761]3/2-^{+1}$ (n)	-	17.5	16.8 ± 0.6
^{153}Dy(2,3)	$[402]5/2^{+1}$ (n)	0.40	17.2	16.6 ± 0.5
^{154}Dy(1)	$[402]5/2^{+2}$ (n)	-	17.4	16.9 ± 0.6
^{150}Gd(1)	$[651]3/2^{-2}$ (p)	-	16.1	$17.0^{+0.5}_{-0.4}$
^{150}Gd(2)	$[651]3/2^{-1}[301]1/2^{-1}$ (p)	0.43	16.9	$17.4^{+0.5}_{-0.4}$
^{150}Gd(5)	$[301]1/2^{-2}$ (p)	2.59	18.5	16.8 ± 1.2
^{151}Tb(1)	$[651]3/2^{-1}$ (p)	-	16.7	$16.8^{+0.7}_{-0.6}$
^{151}Tb(2)	$[301]1/2^{-1}$ (p)	0.43	17.8	18.4 ± 6

come negative. Theoretically this trend is rather well reproduced, except for ^{153}Dy. Let us note that the ^{151}Dy band is not correctly described by the HF approximation or with a seniority pairing [5,10].

To analyze the behavior of the theoretical moment of inertia of ^{153}Dy, let us recall how a qp excited state $|K\rangle$ is constructed from the Bogoliubov vacuum $|0\rangle$:

$$|K\rangle = a_K^\dagger |0\rangle, \tag{2}$$

and that the wave function $|K\rangle$ is optimized for each excitation. In particular, the vacuum $|0\rangle$ is different and self-consistent for each excited state and therefore does not correspond to an even integer number of neutrons. Only the one-quasiparticle density matrix of the odd nucleus is constrained in order to have the mean right number of particles. It can be written as

$$\rho^{(K)}_{\alpha,\beta} = \rho^{(o)}_{\alpha,\beta} - [V^*_{\alpha,K}V_{\beta,K} - U_{\alpha,K}U^*_{\beta,K}], \tag{3}$$

where the density matrix $\rho^{(o)}_{\alpha,\beta}$ corresponds to the self-consistent vacuum $|0\rangle$:

$$\rho^{(o)}_{\alpha,\beta} = \sum_L V^*_{\alpha,L}V_{\beta,L}. \tag{4}$$

Similar equations were used in Ref. [17] to decompose the angular momentum into core and qp contributions. Table III gives the occupations at $30\hbar$ and $60\hbar$ of the $N=7$ neutron orbitals in the Dy isotopes yrast bands. The particle numbers of the ^{151}Dy vacuum are different at the two spins. However, in both cases, the result of the qp excitation is similar: the first $N=7$ orbital is fully populated whereas the second is empty. At $60\hbar$, the vacuum is close to that of ^{152}Dy, as one would expect in a situation of weak pairing correlations that are close to the HF limit.

The structure of the yrast state is different in ^{153}Dy. At low spins, the vacuum on which the excitation is constructed has a structure rather similar to that of ^{154}Dy. The second $N=7$ orbital is depopulated and the moment of inertia has a value close to the one obtained in ^{151}Dy. At high spins, the mean number of neutrons in the vacuum is close to 87; the second $N=7$ orbital is already largely depopulated in this vacuum while pairing correlations are responsible for a significant population of the levels above the $N=86$ gap. Since

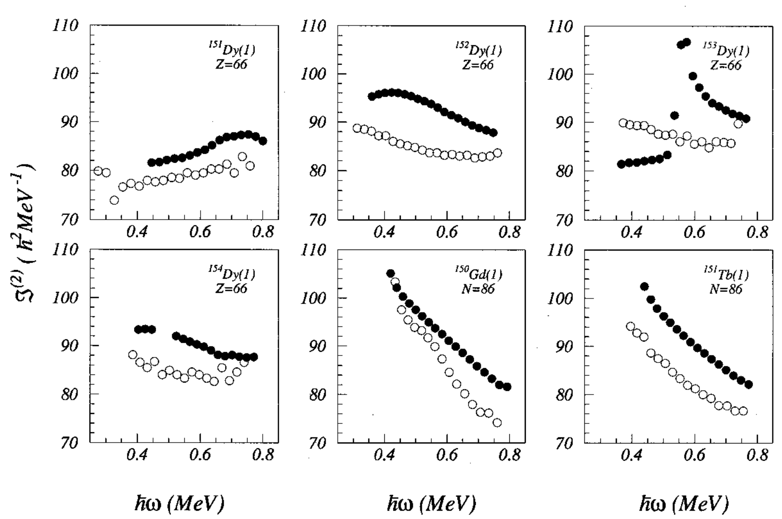

FIG. 4. Dynamical moments of inertia of yrast SD bands in isotopes and isotones of ^{152}Dy. The experimental data are represented by open dots and our results by black dots.

TABLE III. The mean number of neutrons in the self-consistent vacuum (second column) for Dy isotopes. The last three columns give the occupations of the $N=7$ intruder orbitals in the vacuum and (for odd isotopes) in the qp state. The upper lines correspond to an angular of $30\hbar$ and the lower lines to $60\hbar$.

Nucleus	Vacuum	$N=7_1^-$	$N=7_1^+$	$N=7_2^-$
^{151}Dy(1)	84.6	0.3→1.0	0.3→0.0	0.0
	85.8	1.0	1.0→0.1	0.0→0.1
^{152}Dy(1)	86	0.8	0.8	0.0
	86	1.0	1.0	0.0
^{153}Dy(1)	87.6	0.8→1.0	0.84→0.04	0.0
	86.7	1.0→0.64	0.11→0.26	0.01→0.51
^{154}Dy(1)	88	0.96	0.96	0.07
	88	1.0	1.0	0.05

at this high spin, none of these intruder orbitals still have a pure character, pairing correlations lead to a complicated occupation scheme. The situation is different in ^{154}Dy because the number of particles in the vacuum is higher so that the first two $N=7$ orbitals are more bound and fully occupied.

The origin of the discrepancy found in ^{153}Dy is not easy to ascertain. Small modifications in the order of levels would affect the population of the levels that we have obtained with the SLy4 Skyrme parametrization. In particular, the frequency at which we obtain a change of configuration is closely related to the position of the third $N=7$ intruder level.

Our ^{153}Dy result confirms the influence of intruder orbitals on the dynamical moment of inertia. The values found at low and high frequencies are close to the ones of the isotopes with the same number of intruders, ^{151}Dy and ^{154}Dy, respectively.

For isotones, the evolution with the proton number of the moments of inertia is mostly related to the decrease of pairing correlations when approaching the $Z=66$ gap.

V. BANDS WITH SIMILAR NUMBER OF INTRUDERS

Bands with moments of inertia identical to the one of ^{152}Dy have been found in ^{150}Gd (band 5), ^{151}Tb (band 2), ^{151}Dy (bands 2 and 4), ^{153}Dy (bands 2 and 3), and ^{154}Dy (yrast band). They have all been assigned a $\pi 6^4 \nu 7^2$ configuration. The calculated bands are compared with data in Fig. 5. The values of the moments of inertia of the neutron bands at $\hbar\omega$ equal to 0.6 MeV are given in Table IV.

A. Neutron bands

^{151}Dy. Experimentally, the bands labeled 2 and 4 in this nucleus have moments of inertia identical to the yrast SD band of ^{152}Dy [18]. The γ-ray energies of band 2 are close to the 3/4 point transition energies of the ^{152}Dy SD band while those of band 4 are very close to midpoint. Band 2 has been interpreted as based on one of the [642]5/2 orbitals, with a still undetected signature partner. On the basis of the WS level scheme of Ref. [5], an excitation involving the [411]1/2 orbital has been assigned to band 4.

The qp scheme of Fig. 2 indicates that the [642]5/2 excitations are indeed favorable. On the other hand, the [411]1/2 orbital lies at a rather high excitation energy and an excitation of the [651]1/2 orbital seems more plausible. The moment of inertia that we obtain for band 2 differs from that of ^{152}Dy by 1 or 2 percent for 6 transitions; the similarity is slightly better for band 4, although the agreement between the theoretical bands is not as good as in experiment. Around $60\hbar$, band 2 is excited by 600 keV with respect to the yrast SD band. Its signature partner has an excitation energy of 1300 keV while that of band 4 is 1200 keV.

In our results, the transition energies of band 2 are 3/4 of a point of the ^{152}Dy ones over 4 transitions only with a mean deviation of 3 keV. Band 4 energies are midpoint with the same mean deviation over 7 transitions. Experimentally, the mean deviation is 1.7 keV.

The experimental quadrupole moments of bands 2 and 4, determined by lifetime measurements [19], are larger than that of the lowest band of ^{152}Dy. Our calculation does not reproduce this result: the quadrupole moments give in Table II for bands 2 and 4 being closer to the lowest bands of

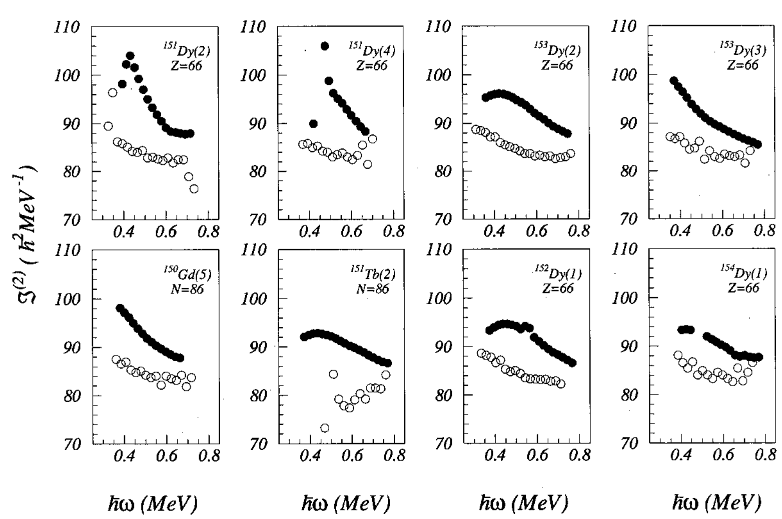

FIG. 5. Dynamical moments of inertia of SD bands in isotopes and isotones of ^{152}Dy to which a $\pi 6^4 \nu 7^2$ configuration has been attributed on the basis of their identity to the yrast SD band of ^{152}Dy. The experimental data are represented by open dots and our results by black dots.

TABLE IV. Dynamical moments of inertia around $\hbar\omega \sim 0.6$ MeV of the SD bands which are experimentally identical to ^{152}Dy.

Nucleus	^{152}Dy	^{151}Dy(2)	^{151}Dy(4)	^{153}Dy	^{154}Dy	^{150}Gd(5)
Moment of inertia	91.8	90.1	91.8	88.9	90.0	90.0

^{151}Dy than of ^{152}Dy. These differences between the experimental quadrupole moments have been taken as evidence for the occupation of a [411]1/2 orbital in the excited ^{151}Dy bands [19], this orbital having a strong deformation driving effect. At present, it seems difficult to reconcile all the experimental data within a single theoretical scenario.

^{153}Dy. The bands labeled 2 and 3 in this nucleus are identical to the yrast band of ^{152}Dy [20]. Based on the absence of a visible band interaction on the experimental $\mathcal{J}^{(2)}$ moment of inertia, configurations involving the [402]5/2$\pm$ orbitals have been suggested as the most probable ones.

Looking to Fig. 3, several neutron orbitals lie just above the ^{152}Dy Fermi level, the [402]5/2$\pm$ orbitals being the most probable ones. The bands based on these orbitals have an excitation energy of 600 keV at high spins with respect to the yrast SD band. In Fig. 5, we show the corresponding moments of inertia. They are remarkably similar, although smaller than that of ^{152}Dy, in contrast with experiment. Above $50\hbar$, the transition energies in these bands are close to the 1/4 and 3/4 point of the ^{152}Dy energies, respectively, as in the experimental data, with however a larger mean deviation of 2.5 keV.

The quadrupole moments that we obtain are very similar for the three bands and close to the ^{152}Dy value, while experimentally [21] the three ^{153}Dy bands seem less deformed than the ^{152}Dy lowest band.

^{154}Dy. Since the first intruder orbital above the $N=86$ gap is predicted to be very excited for values of $\hbar\omega$ smaller than 0.7 MeV, the same number of intruder orbitals should be present in the ^{154}Dy yrast SD band and in ^{152}Dy. The data [22] indicates that indeed the only band known in ^{154}Dy is identical to ^{152}Dy. Unexpectedly, the ^{154}Dy transition energies lie close to the 3/4 point of the ^{152}Dy. Theoretically, the transition energies are identical for the same spins with a mean deviation of 2 keV.

B. Proton bands

^{150}Gd. To construct a band in ^{150}Gd with the same number of proton intruders as in the ^{152}Dy yrast band, one must promote two protons above the ^{150}Gd Fermi level. At the HF level of approximation, this can be done by an excitation of the two [301]1/2 orbitals to the two [651]3/2 ones. Band 5 has been interpreted in this way [23]. Experimentally, its last 6 transition energies are identical to that of the ground SD band of ^{152}Dy. At low spins, a backbend at $\hbar\omega \sim 0.5$ MeV destroys this similarity. It has been interpreted as the alignment of a pair of $N=6$ intruder protons [23].

When these two qp are excited, pairing correlations vanish even at the Lipkin-Nogami approximation. We therefore do not reproduce the backbend in the $\mathcal{J}^{(2)}$ moment of inertia of band 5, as can be seen in Fig. 5. At high frequencies, the moment of inertia of this band is lower than that of ^{152}Dy by 1 or 2 percent, the similarity being slightly less good than in the experimental data. In our calculation, band 5 lies ~ 2.5 MeV above the yrast line at $60\hbar$. This unrealistically high energy may be partly due to the lack of pairing correlations at our level of approximation.

A band with the same number of intruders as the ^{151}Tb ground SD band can be obtained by exciting a [301]1/2 orbital to a [651]3/2 one. It lies ~ 1.3 MeV above the yrast band, and is a candidate for the band labeled 2, experimentally identical to the yrast SD band of ^{151}Tb. In our calculation, as shown in Fig. 6, these bands are only identical above $\hbar\omega \sim 0.5$ MeV.

^{151}Tb. The nucleus ^{151}Tb has one proton less than the doubly magic nucleus ^{152}Dy and exhibits an excited SD band (band 2) that is identical to the ^{152}Dy yrast band [2]. A candidate for band 2 built on the $[301]1/2^+$ proton qp excitation is shown in Fig. 5. Its $\mathcal{J}^{(2)}$ moment of inertia is in good agreement with the experimental data. The discontinu-

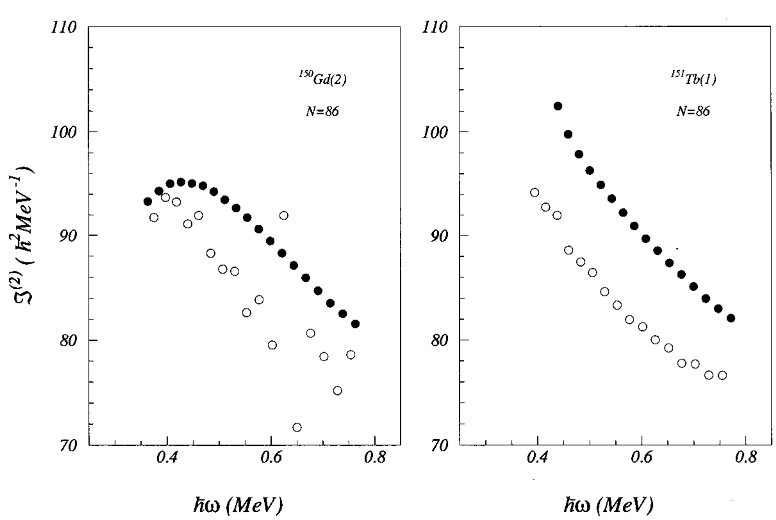

FIG. 6. Dynamical moments of inertia of SD bands in ^{150}Gd and ^{151}Tl to which a $\pi 6^3 \nu 7^2$ configuration has been attributed. The experimental data are represented by open dots and our results by black dots.

ity obtained in the $\mathcal{J}^{(2)}$ moment of inertia at $\hbar\omega \sim 0.55$ is related to a near collapse of the proton pairing correlations. These nearly vanishing pairing correlations may explain the unrealistically large excitation energy of 1.4 MeV that we obtain for this excited band. The use of the Lipkin-Nogami prescription is indeed less justified in the weak pairing regime of the excited band than in the description of the yrast band.

VI. DISCUSSION AND CONCLUSION

The overall agreement between our calculations and experiment appears satisfactory. Although, compared to HF, the effect of pairing correlations is generally small, the introduction of these correlations allows in some cases a better reproduction of data as, for instance, the behavior of the ^{151}Dy moment of inertia as a function of rotational frequency. On the other hand, pairing does not substantially modify some general conclusions that have been drawn previously on the basis of pure mean-field calculations. The polarization effects induced by most orbitals remain small and the effects of neutron intruders are larger than that of proton intruders. However, observed bands differ less than calculated bands, even if the latter exhibit very similar patterns for their dynamical moments of inertia.

The limitations of our treatment of pairing correlations by means of the Lipkin-Nogami prescription is manifest in the calculation of bands of ^{151}Dy, ^{153}Dy, and ^{150}Gd. It is a well-known fact that this prescription is not well suited to very weak pairing regimes (see for instance [28]). A difficult but tractable way to go beyond this approximation would consist in the projection on good particle number of the wave functions obtained with the Lipkin-Nogami method. It has indeed been explored by Magierski *et al.* [28] who have shown that such projection gives a much better approximation of the energy obtained by a variation after projection method. It is interesting to note that an approximate variation after projection calculation of the yrast SD band of ^{152}Dy has very recently been performed by Anguiano *et al.* [29] with the Gogny force. They find that the moments of inertia calculated in this mass region with this force and either the HF or the HFB method overestimate the experimental data by an amount similar to what we have calculated. On the other hand, an approximate variation after projection has corrected this problem and leads to moments of inertia much closer to the data.

In several cases, our calculations agree with experiment for the relative position of gamma-ray energies between ^{152}Dy and other nuclei, over a number of transitions that is large enough not to be fortuitous. However there exist also strong disagreements, like in ^{154}Dy where data are difficult to interpret. An interesting case is band 4 of ^{151}Dy, where the theoretical identity of mid gamma-ray energies is much better than average, although not as good as in the data. In the experimental paper, a configuration [411]1/2 was assigned to this band and its identity with ^{152}Dy was related to the pseudospin model. It is worthwhile to note that we obtain similar properties with a band based on the [651]1/2 orbital.

It is hard to draw definite conclusions from quadrupole moments. The values that we obtain are very often close to experiment, or within the error bars. A positive result is the good agreement for ^{152}Dy, much better than in previous calculations. Excited bands that permit one to test the properties of specific orbitals yield less satisfactory results. Since the quadrupole moments of the three bands in ^{151}Dy and of the band in ^{152}Dy have been determined in the same experiment, one can put a higher degree of confidence on the relative values between the bands. Our results support the differences found between the yrast bands, but there may be some discrepancies concerning the excited bands. Bands 2 and 4 have the same quadrupole moment in our calculations, smaller than the yrast ^{152}Dy, while experimentally, band 2 is significantly more deformed and the deformation of band 4 and of ^{152}Dy yrast band are close. Such a discrepancy may indicate that the SLy4 interaction does not always give the right ordering of single-particle orbits. We have indeed based our calculations on the energetically most favored configurations and identified the bands on this basis. This has led us to make assignments for bands 2 and 4 different from the most usual ones. Unfortunately, bands based on other qp's are too excited to be calculated. Let us finally note that there does not seem to be a strong correlation between the value of the quadrupole moments and the behavior of the moments of inertia.

There has been a recent analysis of quadrupole moments of $A = 150$ SD bands within the HF approximation [27]. Satula *et al.* have shown that the charge moments calculated with respect to ^{152}Dy for many bands in different nuclei can be accurately expressed in terms of single-particle contributions. The quadrupole moments that we give in Table II do not display the same regular behavior. For instance, the addition of the reductions of quadrupole moments obtained in bands 2 and 3 of ^{153}Dy is larger than the calculated reduction in ^{154}Dy. The few other cases (Gd and Tb bands for instance) that can serve as a test of the additivity rule seem also to indicate that this rule becomes less accurate when pairing correlations are introduced.

To conclude on the phenomenon of identical bands, our results are similar to those obtained by Szymański and Nazarewicz [30] in a simple mean-field model. For some orbitals, a cancellation of contributions to the moments of inertia occur in mean-field methods even when pairing correlations are included. However, it is not clear that there exists a ''heroic'' explanation of identity of SD bands based on some symmetry of the mean field.

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