Isospin structure of one- and two-phonon giant dipole resonance excitations

A. F. R. de Toledo Piza,[1] M. S. Hussein,[1] B. V. Carlson,[2] C. A. Bertulani,[3] L. F. Canto,[3] and S. Cruz-Barrios[4]
[1]Instituto de Física, Universidade de São Paulo, 01498 São Paulo, SP, Brazil
[2]Departamento de Física, Instituto Tecnológico da Aeronáutica - CTA, 12228-900 São José dos Campos, SP, Brazil
[3]Instituto de Física, Universidade Federal do Rio de Janeiro, 21945-970 Rio de Janeiro, RJ, Brazil
[4]Departamento de Física Aplicada, Universidad de Sevilla, 41080 Sevilla, Spain
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Isospin is included in the description of the Coulomb excitation of multiple giant isovector dipole resonances. In the excitation of even-even nuclei, a relevant portion of the excitation strength is shown to be associated with 1^+ two-phonon states, which tends to be hindered or completely suppressed in calculations in which the isospin degree of freedom is not considered. We find that the excitation cross section is strongly dependent on the ground state isospin. [S0556-2813(99)08105-4]

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I. INTRODUCTION

Coulomb excitation of two-phonon giant resonances in heavy ion collisions at relativistic energies was predicted by Baur and Bertulani in 1986 [1] and generated considerable interest during the last few years [2–4]. The double giant isovector dipole resonance (DGDR) has now been observed in several nuclei: ^{136}Xe [5], ^{197}Au [6], and ^{208}Pb [7,8]. The double giant isoscalar quadrupole resonance has been also observed in the ^{40}Ca(^{40}Ca,^{40}Ca p) reaction, at 44A MeV laboratory energy [9]. Data on the DGDR were confronted with results of coupled channels Coulomb excitation theory [10,11] and the major conclusion reached was that theoretical predictions underestimate the data by a factor of 2–3 in the cases of ^{136}Xe and ^{197}Au. A similar discrepancy, albeit appreciably smaller, was found in ^{208}Pb [12].

Several effects, not taken into account in the coupled channel theory, were considered to explain this discrepancy. Among these are anharmonicities [13,14] and the Brink-Axel mechanism [15], to cite a few. In this paper we examine the relevance of isospin effects in the excitation process relying on the pioneering work of Fallieros and co-workers, in which the isospin splitting of the isovector giant dipole resonance was first analyzed [16–18]. We next present an extension of this work to the double giant isovector dipole states [19], before turning to more technical details of the coupled channel calculation.

The relevant dipole excitation operator is the component $T_3=0$ of an isovector ($T=1$) operator, $D_{T=1,T_3=0}$. Thus, when acting on a target nucleus with isospin quantum numbers T, $T_3=T$, two GDR states will be generated having isospin quantum numbers $T_I=T$ and $T_I=T+1$ ($T_I=T-1$ is forbidden due to charge conservation), where the label I has been introduced to designate the (intermediate) one-phonon state. If the dipole operator is applied again to these one-phonon states, final states with isospin $T_f=T$, $T+1$, and $T+2$ will be generated. In order to take into account the bosonic nature of isovector phonons in these final states, one must keep track of another isospin quantum number, namely, the *total* isospin $\mathfrak{I}$ of the *two* dipole phonon operator, which can take the values $\mathfrak{I}=0,1,2$. These values will constrain the total angular momentum of the two coupled phonons through symmetry requirements. For a nucleus with $J^\pi=0^+$ ground state, the DGDR may have $J^\pi=0^+$, 1^+, and 2^+, and for the state 1^+ one must have $\mathfrak{I}=1$. If isospin is not taken into account, 1^+ states, reached from the 1^- GDR, will be quenched [20], since by itself it is antisymmetric under exchange of the two phonons. However, 1^+ states will in general contribute to the excitation cross section, if explicit reference to its $\mathfrak{I}=1$ nature is made. In this case, the exchange symmetry is odd both in the spin and isospin spaces, so that the product has even symmetry, as required.

The energy splitting of the isodoublet GDR was studied in [18] and was found to be related to the symmetry energy and to the average particle-hole interaction, leading to the estimate

$$\Delta^{(1)}_{T+1}=E^{(1)}_{T+1}-E^{(1)}_{T}\cong 60\frac{T+1}{A}[\mathrm{MeV}]. \tag{1.1}$$

The energy splitting of the isotriplet DGDR can be estimated in a similar way. Since the $T_f=T+2$ state is a double isospin analog state, it involves twice the displacement energy, and we may write

$$\Delta^{(2)}_{T+2}=E^{(2)}_{T+2}-E^{(2)}_{T}\cong 120\frac{T+2}{A}[\mathrm{MeV}]. \tag{1.2}$$

Furthermore, for the $T_f=T+1$ state we may write

$$\Delta^{(2)}_{T+1}=E^{(2)}_{T+1}-E^{(2)}_{T}\cong 60\frac{T+1}{A}[\mathrm{MeV}]. \tag{1.3}$$

We give in Table I the energies of the isospin doublet and triplet states in ^{208}Pb ($T=22$) as obtained from these expressions. Having given the above account on the isospin structure of the one- and two-phonon dipole states, we now turn to the required modifications of the coupling interaction for the coupled channel calculation.

TABLE I. Isospin splitting (in MeV) of one- and two-phonon states in ^{208}Pb and ^{48}Ca.

Energy shift (MeV)	^{208}Pb $(T=22)$	^{48}Ca $(T=4)$
$\Delta^{(1)}_{T+1}$	6.63	6.25
$\Delta^{(2)}_{T+2}$	13.85	15
$\Delta^{(2)}_{T+1}$	6.63	6.25

II. EXCITATION OF MULTIPHONON STATES

A. Coupling interaction

The coupling interaction for the nuclear excitation $i\rightarrow f$ in a semiclassical calculation for an electric $(\pi=E)$ or magnetic $(\pi=M)$ multipolarity, is given by [Eqs. (6) and (7) of Ref. [21]]

$$W_C=\frac{V_C}{\epsilon_0}=\sum_{\pi\lambda\mu} W_{\pi\lambda\mu}(\tau), \tag{2.1}$$

where

$$W_{\pi\lambda\mu}(\tau)=(-1)^{\lambda+1}\frac{Z_1 e}{\hbar v b^{\lambda}}\frac{1}{\lambda}\times\sqrt{\frac{2\pi}{(2\lambda+1)!!}}Q_{\pi\lambda\mu}(\xi,\tau)\mathcal{M}(\pi\lambda,\mu). \tag{2.2}$$

In this expression b is the impact parameter; $\tau=\gamma v t/b$ is a dimensionless time variable with $\gamma=(1-\beta^2)^{-1/2}$ and $\beta=v/c$ being the usual relativistic parameters. The energy scale is set by $\epsilon_0=\gamma\hbar v/b$ and the quantity $Q_{\pi\lambda\mu}(\xi,\tau)$, with ξ defined as the adiabatic parameter $\xi=\xi_{if}=(E_f-E_i)/\epsilon_0$, depends exclusively on the properties of the projectile-target relative motion. The multipole operators acting on the intrinsic degrees of freedom are, as usual,

$$\mathcal{M}(E\lambda,\mu)=\int d^3r\,\rho(\mathbf{r})r^{\lambda}Y_{1\mu}(\mathbf{r}), \tag{2.3}$$

and

$$\mathcal{M}(M1,\mu)=-\frac{i}{2c}\int d^3r\,\mathbf{J}(\mathbf{r})\cdot\mathbf{L}(rY_{1\mu}). \tag{2.4}$$

We treat the excitation problem by the method of Alder and Winther [22]. We solve a time-dependent Schrödinger equation for the intrinsic degrees of freedom in which the time dependence arises from the projectile-target motion, approximated by the classical trajectory. For relativistic energies, a straight line trajectory is a good approximation. The wave function is expanded in the relevant eigenstates of the nuclear Hamiltonian, $\{|k\rangle;k=1,N\}$, N being the number of relevant intrinsic states included in the coupled channel (CC) problem.

In order to simplify the expression for the coupled equations we define the dimensionless parameter $\Gamma^{(\lambda)}_{kj}$ by the relation

TABLE II. Ground-state and one-phonon states with angular momentum and isospin dependence.

n	$\underline{E}$	J^{π}	$\underline{T}$
0	0	0^+	T_0
1	$E_{\rm GDR}$	1^-	T_0
	$E_{\rm GDR}+\Delta^{(1)}_{T_0+1}$	1^-	T_0+1

$$\Gamma^{(\lambda)}_{kj}=(-1)^{\lambda+1}\frac{Z_1 e}{\hbar v b^{\lambda}}\frac{1}{\lambda}\sqrt{\frac{2\pi}{(2\lambda+1)!!}}\mathcal{M}_{kj}(E\lambda). \tag{2.5}$$

The coupled channel equations can then be written in the form [21]

$$\frac{da_k(\tau)}{d\tau}=-i\sum_{r=1}^{N}\sum_{\pi\lambda\mu}Q_{\pi\lambda\mu}(\xi_{kj},\tau)\Gamma^{(\lambda)}_{kj}\exp(i\xi_{kj}\tau)a_j(\tau). \tag{2.6}$$

In what follows we concentrate on the $E1$ excitation mode. In this case, we have

$$Q_{E10}(\xi,\tau)=\gamma\sqrt{2}\left[\tau\phi^3(t)-i\xi\left(\frac{\mathrm{v}}{c}\right)^2\phi(t)\right],$$
$$Q_{E1\pm1}(\xi,\tau)=\mp\phi^3(\tau), \tag{2.7}$$

where $\phi(\tau)=(1+\tau^2)^{1/2}$.

The excitation probability $P_j(b)$ of an intrinsic state $|j\rangle$ in a collision with impact parameter b is obtained from the amplitudes $a_j(\tau)$ at asymptotically large times, in terms of an average over the initial and a sum over the final magnetic quantum numbers. The cross section is then obtained from the classical expression

$$\sigma_j=2\pi\int P_j(b)T(b)b\,db, \tag{2.8}$$

where the impact-parameter-dependent transmission factor $T(b)$ accounts for absorption [21].

TABLE III. Two-phonon states with angular momentum and isospin dependence. The isospin dependence arises from the coupling of the phonon isospins $\mathfrak{I}=1_1\otimes 1_2$.

$\mathfrak{I}$	E	J^{π}	T
0	$E_{\rm DGDR}$	0^+	T_0
	$E_{\rm DGDR}$	2^+	T_0
1	$E_{\rm DGDR}$	1^+	T_0
	$E_{\rm DGDR}+\Delta^{(2)}_{T_0+1}$	1^+	T_0+1
2	$E_{\rm DGDR}$	0^+	T_0
	$E_{\rm DGDR}$	2^+	T_0
	$E_{\rm DGDR}+\Delta^{(2)}_{T_0+1}$	0^+	T_0+1
	$E_{\rm DGDR}+\Delta^{(2)}_{T_0+1}$	2^+	T_0+1
	$E_{\rm DGDR}+\Delta^{(2)}_{T_0+2}$	0^+	T_0+2
	$E_{\rm DGDR}+\Delta^{(2)}_{T_0+2}$	2^+	T_0+2

B. One-phonon states

Consider the excitation of a nucleus with ground state spin zero (any even-even nucleus) and isospin T_0, with its three-component $T_3 = T_0$. In terms of the relevant quantum numbers, these states are written as $|j\rangle = |E_j^{(n)}; J_j M_j; T_j T_{3j}\rangle$, where n is the number of phonons, E the energy, J_j and M_j are, respectively, the spin and its z-component quantum numbers, and T_j and T_{3j} are the isospin and its third component. Note that due to charge conservation, all states have $T_{3j} = T_0$. The ground state and the one-phonon states are given in Table II. The energy splitting $\Delta_{T_0}^{(1)}$, which is assumed to depend exclusively on isospin, is given by Eq. (1.1).

In order to calculate the matrix elements $\mathcal{M}_{ki}(\pi\lambda,\mu)$ between initial, i, and final, k, states we use the Wigner-Eckart theorem in spin-isospin space and (except for the energy dependence) assume that the *reduced matrix elements are isospin independent*. We get

$$\mathcal{M}_{ki}^{(10)}(\pi\lambda,\mu) = \langle E_{T_k}^{(1)}; 1M_k; T_k T_0 | \mathcal{M}(E1,\mu) | E^{(0)}; 00; T_0 T_0\rangle$$
$$= (-1)^{1-M_f+T_k-T_0} \begin{pmatrix} 1 & 1 & 0 \\ -M_k & \mu & 0 \end{pmatrix} \begin{pmatrix} T_k & 1 & T_0 \\ -T_0 & 0 & T_0 \end{pmatrix} \langle 1 || \mathcal{M}(E1) || 0\rangle. \tag{2.9}$$

The value of the reduced matrix element $\langle 1 || \mathcal{M}(E1) || 0\rangle$ can be extracted from the energy-weighted dipole sum rule

$$S = \sum_{M_k T_k} (E_k^{(1)} - E^{(0)}) |\langle E_{T_k}^{(1)}; 1M_k; T_k T_0 | \mathcal{M}(E1,\mu) | E^{(0)}; 00; T_0 T_0\rangle|^2 = \frac{3}{4\pi} \frac{\hbar^2}{2m_N} \frac{NZ}{A} e^2. \tag{2.10}$$

Inserting the matrix elements of Eq. (2.9) into Eq. (2.10), we get

$$S = \sum_{T_k} (E_k^{(1)} - E^{(0)}) \langle 1 || \mathcal{M}(E1) || 0\rangle^2 \times \begin{pmatrix} T_k & 1 & T_0 \\ -T_0 & 0 & T_0 \end{pmatrix}^2 \sum_{M_k} \begin{pmatrix} 1 & 1 & 0 \\ -M_k & \mu & 0 \end{pmatrix}^2. \tag{2.11}$$

Using the relation [23,24]

$$\sum_{M_k} \begin{pmatrix} 1 & 1 & 0 \\ M_k & \mu & 0 \end{pmatrix}^2 = \begin{pmatrix} 1 & 1 & 0 \\ -\mu & \mu & 0 \end{pmatrix}^2 = \frac{1}{3}, \tag{2.12}$$

the sum rule takes the form

$$S = \frac{\langle 1 || \mathcal{M}(E1) || 0\rangle^2}{3} \left\{ \begin{pmatrix} T_0 & 1 & T_0 \\ -T_0 & 0 & T_0 \end{pmatrix}^2 E_{\mathrm{GDR}} + \begin{pmatrix} T_0+1 & 1 & T_0 \\ -T_0 & 0 & T_0 \end{pmatrix}^2 (E_{\mathrm{GDR}} + \Delta_{T_0+1}^{(1)}) \right\}. \tag{2.13}$$

Using the explicit forms of the Wigner coefficients [24],

$$\begin{pmatrix} T_0 & 1 & T_0 \\ -T_0 & 0 & T_0 \end{pmatrix} = \left[\frac{T_0}{(T_0+1)(2T_0+1)} \right]^{1/2},$$
$$\begin{pmatrix} T_0+1 & 1 & T_0 \\ -T_0 & 0 & T_0 \end{pmatrix} = -\left[\frac{1}{(T_0+1)(2T_0+3)} \right]^{1/2}, \tag{2.14}$$

we obtain the reduced matrix element

$$\langle 1 || \mathcal{M}(E1) || 0\rangle = \sqrt{\frac{3S}{E_{\mathrm{GDR}} F(T_0)}}, \tag{2.15}$$

with

$$F(T_0) = \frac{1}{T_0+1} \left\{ \frac{T_0}{2T_0+1} + \left[1 + \frac{\Delta_{T_0+1}^{(1)}}{E_{\mathrm{GDR}}} \right] \frac{1}{2T_0+3} \right\}. \tag{2.16}$$

C. Two-phonon states

The two-phonon states must be symmetric with respect to the exchange of the two phonons in spin and isopin. This symmetry can be tracked by using the coupling scheme $|(1_1 1_2)\mathfrak{I}T_0; T_f T_0\rangle$. One has to distinguish the two cases $\mathfrak{I} = 0$ (isospin even) and $\mathfrak{I} = 1$ (isospin odd). The states which are spin-isospin symmetric correspond then to the combinations

$$(I_f, \mathfrak{I}) = (0,0), (0,2), (2,0), (2,2), (1,1). \tag{2.17}$$

The two-phonon states are represented as $|j\rangle = |E_j^{(2)}; J_j M_j; (1_1 1_2)\mathfrak{I}T_0 T_f T_0\rangle$. We list the main quantum numbers of the two-phonon states in Table III. The isospin shifts $\Delta^{(2)}$ in Table III are given by Eqs. (1.2) and (1.3).

To obtain the excitation amplitudes for the one-phonon to two-phonon transitions we need to calculate the following matrix elements:

$$\mathcal{M}_{fk}^{(21)}(E1,\mu) = \langle E_{T_f}^{(2)}; J_f M_f; (1_1, 1_2)\mathfrak{I}T_0; T_f T_0 | \mathcal{M}(E1,\mu) | E^{(1)}; J_k M_k; T_k T_0\rangle. \tag{2.18}$$

This can be done by changing the final state coupling scheme. We use the relation [Ref. [24], Eq. (6.1.3), p. 91]

$$|JM;(1_1 1_2)\Im T_0;T_f T_0\rangle = \sum_{T_l} (-1)^{T_l+T_f}\sqrt{(2\Im+1)(2T_l+1)}\begin{Bmatrix} 1 & 1 & \Im \\ T_0 & T_f & T_l \end{Bmatrix}|JM;(1_2(1_1 T_0)T_l);T_f T_0\rangle \tag{2.19}$$

and obtain

$$\mathcal{M}^{(21)}_{fk}(E1,\mu) = \sum_{T_l} (-1)^{T_l+T_f}\sqrt{(2\Im+1)(2T_l+1)}\begin{Bmatrix} 1 & 1 & \Im \\ T_0 & T_f & T_l \end{Bmatrix}$$
$$\times\langle E^{(2)}_{T_f};J_f M_f;(1_2(1_1T_0)T_l);T_f T_0|\mathcal{M}(E1,\mu)|E^{(1)};J_k M_k;T_k T_0\rangle. \tag{2.20}$$

We next use the Wigner-Eckart theorem in spin-isospin space, assuming that the reduced matrix elements are spin and isospin independent and vanish unless $T_k = T_l$. We get

$$\langle E^{(2)}_{T_f};J_f M_f;(1_2(1_1T_0)T_l);T_f T_0|\mathcal{M}(E1,\mu)|E^{(1)};J_k M_k;(1_1T_0)T_k T_0\rangle$$
$$=\delta_{(T_l,T_k)}(-1)^{J_f-M_f+T_f-T_0}\begin{pmatrix} 1 & 1 & 0 \\ -M_f & \mu & 0 \end{pmatrix}\begin{pmatrix} T_f & 1 & T_k \\ -T_0 & 0 & T_0 \end{pmatrix}\langle 2||\mathcal{M}(E1)||1\rangle. \tag{2.21}$$

Using $\langle 2||\mathcal{M}(E1)||1\rangle = \sqrt{2}\langle 1||\mathcal{M}(E1)||0\rangle$ in Eq. (2.21), Eq. (2.20) becomes

$$\mathcal{M}^{(21)}_{fk}(E1,\mu) = \sqrt{2}(-1)^{J_f-M_f+2T_f-T_0+T_k}\langle 1||\mathcal{M}(E1)||0\rangle\sqrt{(2\Im+1)(2T_k+1)}$$
$$\times\begin{pmatrix} 1 & 1 & 0 \\ -M_f & \mu & 0 \end{pmatrix}\begin{pmatrix} T_f & 1 & T_k \\ -T_0 & 0 & T_0 \end{pmatrix}\begin{Bmatrix} 1 & 1 & \Im \\ T_0 & T_f & T_k \end{Bmatrix}. \tag{2.22}$$

III. APPLICATIONS

In this section, we apply our results to the excitation of one- and two-phonon states in ^{208}Pb and ^{48}Ca target nuclei, in collisions with relativistic ^{208}Pb projectiles. Before we present the numerical results, we rewrite the matrix elements so that they can be used as input to the coupled channel code RELEX for Coulomb excitation [21]. They become ''effective reduced matrix elements'' that incorporate the isospin dependence of $\mathcal{M}^{(10)}_{ki}(\pi\lambda,\mu)$ [Eq. (2.9)]. Namely,

$$\langle 1||\mathcal{M}(E1)||0\rangle^{(\mathrm{eff})}_{ki} = (-1)^{T_k-T_0}\begin{pmatrix} T_k & 1 & T_0 \\ -T_0 & 0 & T_0 \end{pmatrix}\langle 1||\mathcal{M}(E1)||0\rangle, \tag{3.1}$$

while for Eq. (2.22) it is more convenient to define

$$\langle 2||\mathcal{M}(E1)||1\rangle^{(\mathrm{eff})}_{fk} = \sqrt{2}(-1)^{2T_f-T_0+T_k}\langle 1||\mathcal{M}(E1)||0\rangle\sqrt{(2\Im+1)(2T_k+1)}\begin{pmatrix} T_f & 1 & T_k \\ -T_0 & 0 & T_0 \end{pmatrix}\begin{Bmatrix} 1 & 1 & \Im \\ T_0 & T_f & T_k \end{Bmatrix}. \tag{3.2}$$

The modified reduced matrix elements for one- and two-phonon excitations are presented in Tables IV and V, in the cases of ^{208}Pb and ^{48}Ca, respectively. For comparison the original reduced matrix elements are also given in the last column.

The calculated excitation cross sections for one- and two-phonon states in the ^{208}Pb (650A MeV)+ ^{208}Pb collision are given in Table VI. They are also plotted in Fig. 1(a), as a function of the excitation energy. In this figure, the dominant spin and parity are indicated in each case. For a comparison, corresponding results neglecting isospin are given within parentheses in Table VI and are shown in Fig. 1(b). If isospin is neglected, only $T=22$ (the ground-state isospin of ^{208}Pb) states are populated, namely, the GDR state at 13.5 MeV and 0^+ and 2^+ DGDR states at 27.0 MeV. With the inclusion of isospin, about 3% of the GDR cross section is associated with the population of the $T=23$ state at 20.1 MeV, as can be seen in Table VI and in Fig. 1(a). This corresponds to the exhaustion of about 6.3% of the GDR sum rule. The consequences of the isospin degree of freedom on the DGDR population are more important. Although most of the cross section remains associated with the $T=22$, $J^\pi=0^+$, and 2^+ states at 27.0 MeV, 7% of the total DGDR cross section then arises from the population of the $T=23$ states at 33.6 MeV, of which over 90% corresponds to the non-natural parity $J^\pi=1^+$ state. Thus, 6% of the DGDR cross section goes to the excitation of the $J^\pi=1^+$ state, which would be forbidden in the usual harmonic oscillator picture (without isospin). On the other hand, some calculations using random phase approximation (RPA) descriptions of the giant resonances find nonvanishing population of such states. Lanza *et al.* [13] find negligible populations while Bertulani *et al.* [20] get about half of that of the present work. This result could be traced back to the fact that their second order transitions cancel exactly, so that 1^+ states can only be reached through higher order coupled channel processes.

TABLE IV. Reduced matrix elements for transitions from the ground state to one-phonon states and also from one to two-phonon states, for ^{208}Pb in units of fm e. The numbers within parentheses in the last column are the corresponding reduced matrix elements without consideration of isospin. To avoid ambiguities in the two-phonon final states, we also indicate the isospin of the coupled phonon pair.

E_k	E_f	J_k	J_f	T_k	T_f	$\mathfrak{I}$	$\langle n+1\|\|\mathcal{M}(E1)\|\|n\rangle_{fk}^{(\mathrm{eff})}$
0	13.5	0	1	22	22		7.16 (7.39)
0	20.1	0	1	22	23		1.53
13.5	27	1	0	22	22	0	−5.84 (10.45)
13.5	27	1	2	22	22	0	−5.84 (10.45)
13.5	27	1	1	22	22	1	0.318
13.5	33.6	1	1	22	23	1	−1.46
13.5	27	1	0	22	22	2	8.25
13.5	27	1	2	22	22	2	8.25
13.5	33.6	1	0	22	23	2	1.53
13.5	33.6	1	2	22	23	2	1.53
13.5	40.8	1	0	22	24	2	0
13.5	40.8	1	2	22	24	2	0
20.1	27	1	0	23	22	0	1.25
20.1	27	1	2	23	22	0	1.25
20.1	27	1	1	23	22	1	1.49
20.1	33.6	1	1	23	23	1	−6.85
20.1	27	1	0	23	22	2	0.824
20.1	27	1	2	23	22	2	0.824
20.1	33.6	1	0	23	23	2	−6.56
20.1	33.6	1	2	23	23	2	−6.56
20.1	40.8	1	0	23	24	2	−2.83
20.1	40.8	1	2	23	24	2	−2.83

TABLE V. Same as Table V, for ^{48}Ca.

E_k	E_f	J_k	J_f	T_k	T_f	$\mathfrak{I}$	$\langle n+1\|\|\mathcal{M}(E1)\|\|n\rangle_{fk}^{(\mathrm{eff})}$
0	19.2	0	1	4	4		2.66 (3.00)
0	25.4	0	1	4	5		1.2
19.2	38.4	1	0	4	4	0	−2.17 (4.24)
19.2	38.4	1	2	4	4	0	−2.17 (4.24)
19.2	38.4	1	1	4	4	1	0.594
19.2	44.6	1	1	4	5	1	−1.08
19.2	38.4	1	0	4	4	2	3.01
19.2	38.4	1	2	4	4	2	3.01
19.2	44.6	1	0	4	5	2	1.31
19.2	44.6	1	2	4	5	2	1.31
19.2	53.4	1	0	4	6	2	0
19.2	53.4	1	2	4	6	2	0
25.4	38.4	1	0	5	4	0	1.09
25.4	38.4	1	2	5	4	0	1.09
25.4	38.4	1	1	5	4	1	1.19
25.4	44.6	1	1	5	5	1	−2.15
25.4	38.4	1	0	5	4	2	0.548
25.4	38.4	1	2	5	4	2	0.548
25.4	44.6	1	0	5	5	2	−1.76
25.4	44.6	1	2	5	5	2	−1.76
25.4	53.4	1	0	5	6	2	−1.92
25.4	53.4	1	2	5	6	2	−1.92

TABLE VI. Excitation cross sections of one- and two-phonon states in the collision ^{208}Pb (640A MeV)$+^{208}$Pb.

	E (MeV)	J^{π}	T	$\mathfrak{I}$	σ (mb)
GDR	13.5	1^-	22		2240 (2597)
	20.1	1^-	23		59.86
DGDR	27.0	0^+	22	0	4.489 (15.80)
	27.0	2^+	22	0	31.16 (117.5)
	27.0	1^+	22	1	0.8608
	33.6	1^+	23	1	8.373
	27.0	0^+	22	2	10.85
	27.0	2^+	22	2	69.89
	33.6	0^+	23	2	0.1257
	33.6	2^+	23	2	0.3732
	40.8	0^+	24	2	0.0237
	40.8	2^+	24	2	0.0874

Table VII and Fig. 2(a) give similar results for the excitation of GDR and DGDR states in ^{48}Ca, in the collision ^{208}Pb (650A MeV)$+^{48}$Ca. Corresponding results neglecting isospin are given in the same way as above. For this system, isospin plays a more important role due to the lower isospin quantum number ($T=4$) of the ^{48}Ca ground state. In this case, the dominant $T=4$ GDR state at 19.2 MeV loses more than 10% of its cross section to the $T=5\,1^-$ state at 25.4 MeV, which exhausts about 21% of the GDR sum rule. Moreover, the DGDR cross section is very much affected by isospin. Figure 2(a) indicates that the cross section for $T=5$ DGDR states at 44.6 MeV reaches 32% of that for the dominant $T=4$ DGDR states at 33.6 MeV. It is also important to discuss the J^{π} distribution of the DGDR cross section. Since over 95% of the $T=5$ DGDR cross section corresponds to $J^{\pi}=1^+$, the population of states with this spin and parity is rather large.

IV. CONCLUSIONS

In this paper isospin is taken into account in the excitation of the double giant dipole resonance. We have used a semi-

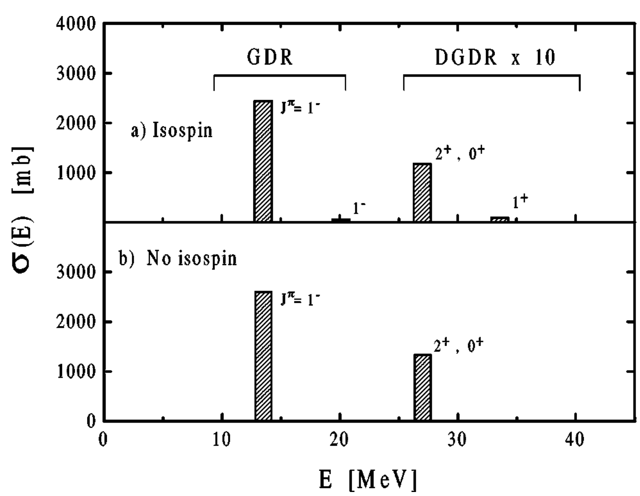

FIG. 1. Excitation of GDR and DGDR states in ^{208}Pb target nuclei, in collisions with ^{208}Pb projectiles at 640A MeV. For details see the text.

TABLE VII. Same as Table VI, for the excitation of ^{48}Ca in the collision ^{208}Pb (640A MeV)+^{48}Ca.

	E (MeV)	J^π	T	$\Im$	σ (mb)
GDR	19.2	1^-	4		318.5 (405.5)
	25.4	1^-	5		33.11
DGDR	38.4	0^+	4	0	0.2842 (2.137)
	38.4	2^+	4	0	0.4453 (3.613)
	38.4	1^+	4	1	0.6695
	44.6	1^+	5	1	1.275
	38.4	0^+	4	2	1.117
	38.4	2^+	4	2	1.640
	44.6	0^+	5	2	0.0155
	44.6	2^+	5	2	0.0366
	53.4	0^+	6	2	0.0268
	53.4	2^+	6	2	0.0375

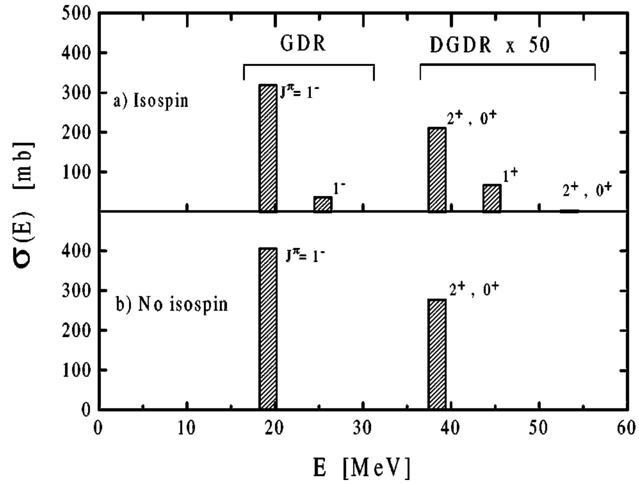

FIG. 2. Same as Fig. 1 but for ^{48}Ca target nuclei.

classical coupled channel formalism to calculate excitation probabilities and cross sections for the collisions ^{208}Pb (640A MeV)+^{208}Pb and ^{208}Pb(640A MeV)+^{48}Ca. We have assumed that the eletromagnetic matrix elements are isospin independent and adopted the isospin splitting of energy levels as given by Akyüz and Fallieros [18]. It has been shown that isospin leads to a redistribution of the strengths of the electromagnetic matrix elements such that the probability for Coulomb excitation of $J^\pi=1^+$ DGDR states is enhanced. This enhancement depends on 3J and 6J coefficients in isospin space so that it becomes more relevant for nuclei with low ground-state isospin. Consequently, this effect turned out to be much stronger in the excitation of ^{48}Ca than in the excitation of ^{208}Pb. One should then expect that isospin splitting should contribute to make the DGDR broader, particularly for nuclei with low ground-state isospin. This result suggests that our formalism should be especially relevant to study the excitation cross sections of a family of isotopes with charge numbers varying from the proton to the neutron drip line. Study along these directions is underway.

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