Generalized orthogonality-condition model for the NN interaction

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A complete version of the Moscow NN potential model is presented. The excellent description for all essential partial waves has been found in the energy range 0–350 MeV. The one-channel version of the model includes the orthogonality condition to most symmetric six-quark states in all lowest partial waves and thus, from this point of view, the model generalizes the well-known Saito's orthogonality condition model for the baryon-baryon interaction case. The specific features of the presented model which distinguish it from many conventional force models are discussed in detail. One of them is a specific tensor mixing between nodal and nodeless wave functions which results in very reasonable values of the one-pion exchange cutoff parameter $\Lambda = 0.78$ GeV and the πNN-coupling constant value $f^2 = 0.075$ in nice agreement with modern trends. The model, in case of its confirmation in precise few-nucleon calculations, can lead to noticeable revisions for many nuclear properties given by conventional force models. [S0556-2813(99)03806-6]

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I. INTRODUCTION

In spite of the great progress attained recently in the construction of the modern realistic NN potentials of second generation, based on the concept of improved one- and two-meson exchange [1–4], a large number of unsolved problems are still left in the field. The majority of the problems here are related to the description of the short-range part of the NN interaction and to the quantitative description of few-nucleon systems [5–9]. In particular, one of the basic problems is connected with the consistent incorporation of gluon- and quark-exchange degrees of freedom and their ''matching'' with the meson-exchange concept.

One of the most fundamental difficulties here is how to avoid double counting of the same effects in the gluon- and meson-exchange sectors of the unified interaction. Another basic problem is the fact the true six-quark microscopic Hamiltonian is presently unknown. While some effective three-quark Hamiltonians which include chiral symmetry breaking and confinement [10–12] have been developed to describe the baryon spectra, there is no guarantee that the same Hamiltonians can also be applied for six- (and multi) quark systems.

Recently [5–9] some fundamental problems have also been found in the consistent description of few-nucleon and meson-few-nucleon systems. On the one hand, the numerous calculations made in recent years for quark-effects in various few-nucleon observables [13,14] have shown, in general, quite moderate contributions of such effects at low energies and momentum transfers [13–16]. On the other hand, however, quite remarkable disagreements between the data and the most accurate three- and four-nucleon calculations have been found [5–8]. They probably can be ascribed to an improper treatment of the quark degrees of freedom. This follows from the fact that the above-mentioned few-nucleon calculations do include $3N$-force and Δ-isobar effects together with the most realistic NN interactions, i.e., they include, to our current knowledge, all essential contributions.

Thus, developing a quantitative NN interaction which includes properly both quark- and meson-exchange effects and is, on the other hand, not so difficult to handle and complicated as those NN models derived from multiquark Hamiltonians, is still very topical. In addition, in view of our insufficient knowledge of the accurate six-quark Hamiltonian, the theory must be constructed in a manner to avoid features of QCD which presently cannot be handled with confidence, e.g., the choice for the qq interaction, the form of confinement (e.g., linear or quadratic), etc. At the same time, however, it is highly desirable to incorporate into the interaction model some general reasoning about the preferred symmetry of the six-quark system in various NN channels, about the characteristic size of six-quark states, etc. In such a case the conclusions derived from the model will result mainly from general symmetry requirements and general structure of the model etc. rather than some particular choice of parameters for the qq interaction, or for the particular law of confinement, etc.

We will demonstrate in the present work that only a few basic assumptions are quite sufficient for the derivation of such a hybrid model. On this basis quark effects in the few-nucleon physics can be described more reliably. Our consideration is based on the assumption (which is common for all hybrid models) that both nucleons merge somehow their quark contents at short ranges into different six-quark states dependent on the partial wave, the energy and the total spin. While at intermediate and large distances where the nucleons do not overlap noticeably with each other the interaction mechanism is governed by one-meson exchange (which was just the original Yukawa idea about the origin of strong interaction [17,18]). This is a common basis of all hybrid models and one model is distinguished from other one by the way of matching of inner quark- and external meson-exchange channels. For example, in hybrid models due to Kisslinger [19] and Simonov [20], the matching of both channels is done at some arbitrarily chosen hypersphere with radius R, although the matching conditions in both models [19,20] are quite different.

In a sharp contrast to such hybrid models we prefer to do this matching in the Hilbert space of the six-quark states with different symmetries, where every such a state is constructed from single-particle harmonic oscillator (HO) quark orbits [21–23]. Thus, in accordance to this idea we subdivide a total Hilbert space $\mathcal{H}$ of the six-quark states on two mutually orthogonal subspaces $\mathcal{H}_P$ and $\mathcal{H}_Q$:

$$\mathcal{H}=\mathcal{H}_Q+\mathcal{H}_P\,.$$

In the first subspace $\mathcal{H}_Q$ we include six-quark states with highest possible spatial symmetry, where one has a maximal overlap of the single-quark orbitals. Whereas the six-quark states with a lower spatial symmetry are placed in its orthogonal complementary subspace. The most symmetrical states can be shown to have the structure which is rather similar to compound states in a spherical or weakly deformed bag. On the other hand the states of lower symmetry include a few p-quark orbitals such as s^4p^2, s^3p^3, etc., and these mixed symmetry states, being projected out onto the NN channel (of unexcited nucleons), result into the nodal NN-radial wave functions [21]. Accordingly, their structure is analogous to clusterized peripheral states.

On the basis of above considerations and also of other arguments of symmetry character we have suggested in the previous works [24–28] a two-component model for baryon-baryon interaction with two mutually orthogonal channels. Then, by subsequent exclusion of six-quark compound states one comes to an effective one-channel potential model of Moscow type, in which a deep NN-potential well includes as its eigenstates the most symmetrical six-quark states (to be more precise, their projections onto the NN channel). As a result of combining two different components into one channel for the effective interaction, the orthogonality condition between NN scattering states and localized six-quark states in such a model is satisfied automatically due to the hermiticity of the Hamiltonian. In a consistent realization of such a program the wave functions of the NN-relative motion in the ''external,'' i.e., clusterized channel are generally not to be related to the six-quark wave functions in the ''inner'' channel. Moreover, it is very likely they should be wave functions belonging to quite different Hamiltonians. The underlying dynamics of the most symmetric six-quark states must be very tightly interrelated to specific chromodynamic effects such as quark and gluon condensates, instantons, breaking chiral invariance, etc., whereas the external channel should be describable in terms of meson exchange.

Thus, these two different channels can hardly be described consistently by a unified Hamiltonian (at least, on the up-to-date level of our knowledge for low-energy QCD). The price to perform technically this description is a different dynamics in the multiquark- and meson-exchange channels.[1] And hence it is highly desirable to employ for these two components a two-channel model with mutually orthogonal channels.

This reasoning justifies our model from the physical point of view. Moreover, its real success in description of NN partial amplitudes demonstrated in the present paper allows us to justify the model *a posteriori*.

The structure of the work is as follows. In Sec. II we present our approach for the construction of the two-component hybrid model of NN interaction and its interrelation to possible dibaryons. In Sec. III we present a realization of the generalized orthogonality condition model (GOCM). In Sec. IV the above GOCM is constructed and the structure of the one-channel potential is discussed. Section V is devoted to a quantitative description of NN-phase shifts in the energy range 0–350 MeV. We also give here the effective-range parameters and a detailed discussion of the structure of the deuteron. In Sec. VI we discuss specific nonconventional interference and tensor mixing between nodal and nodeless wave functions and the cutoff parameters for the meson-exchange potentials at short range. And finally, the main results of the work are summarized in Sec. VII. In the appendix we give the formulas for our interaction model in the momentum representation.

II. A HYBRID MODEL WITH ORTHOGONAL COMPONENTS

The nucleon-nucleon interaction at large and intermediate distances is well known and can be described by meson-exchange potentials [2,4]. The internal nucleon degrees of freedom (quark and gluon ones, if we start from the quark model for nucleon) do not show up in this approach. However, when the nucleons come closer than ~ 1 fm, a transition of the NN system into other channels arises, i.e., the internal nucleon degrees of freedom begin to be of crucial importance. As a dynamic model, e.g., a six-quark bag model can be used.

However, we still have no full dynamic model describing all possible states of the two-nucleon system. Therefore we divide the full Hilbert space $\mathcal{H}$ (including both nucleonic and non-nucleonic degrees of freedom) into two orthogonal subspaces [24–28]

$$\mathcal{H}=\mathcal{H}_{NN}\oplus\mathcal{H}_{6q}\,. \tag{1}$$

These subspaces must be orthogonal because the dynamics in them is essentially different: one of them, $\mathcal{H}_{NN}$, includes only nucleonic degrees of freedom and meson-exchange dynamics, whereas the other, $\mathcal{H}_{6q}$, includes $6q$-model dynamics (or QCD-inspired dynamics). Accordingly to Eq. (1) we introduce two mutually orthogonal projection operators P_{NN} and P_{6q}. It is important that the (unknown) full Hamiltonian of the system does not commutate with P_{NN} and P_{6q} and contains transitions between the NN and the $6q$ channels.

If we suppose the existence of a full Hamiltonian H obeying the $6q$ Schrödinger equation

$$H\psi=E\psi$$

one can easily obtain, following Feshbach [29], the effective Hamiltonian for the NN component

[1] It should be emphasized that in currently developed models of baryons in which the qq interaction is described via one-meson exchange [10,21] these one-meson degrees of freedom are nothing else but effective degrees of freedom. Thus, these degrees of freedom in multiquark system will be somehow different from those in three-quark systems.

$$\psi_{NN} \equiv P_{NN}\psi, \tag{2}$$

$$P_{NN}HP_{NN}\psi_{NN} + P_{NN}HP_{6q}[P_{6q}(E-H)P_{6q}]^{-1} \times P_{6q}HP_{NN}\psi_{NN} = E\psi_{NN}, \tag{3a}$$

$$P_{6q}\psi_{NN} = 0. \tag{3b}$$

In accordance with these ideas the effective nucleon-nucleon Hamiltonian $h_{NN} \equiv P_{NN}HP_{NN}$ includes only meson-exchange potentials

$$h_{NN} = P_{NN}HP_{NN} = t + v^{ME}. \tag{4}$$

The second term in Eq. (3a) is an effective potential which couples NN and $6q$ channels and will be designated further as v_{NqN}. With these notations, the effective equation with orthogonality condition (3b) takes the form

$$(h_{NN} + v_{NqN})\psi_{NN} = E\psi_{NN}, \tag{5a}$$

$$P_{6q}\psi_{NN} = 0. \tag{5b}$$

As an effective wave function in NN channel ψ_{NN} one can naturally use the resonating group ansatz (RGA)

$$\psi_{NN} = \mathcal{A}(\varphi_N \varphi_N \tilde{\chi}_{NN}),$$

in which φ_N is the nucleon wave function and $\tilde{\chi}_{NN}$ is the wave function for the NN-relative motion obeying the orthogonality constraint (3b).

The approach formulated above can be considered as a general formal framework for the hybrid model of the NN interaction. The parameters of v_{NqN} can be determined from the underlying six-quark Hamiltonian [21,22], if one assumes that the effective qq interaction is the same in $3q$ and $6q$ systems, or by fitting the NN-scattering phase shifts. (A quite similar procedure has been used, e.g., in the quark compound bag model due to Simonov [20].)

There are two essential differences in our approach from other hybrid models. These are the orthogonality condition in Eqs. (5) and the structure of total Hamiltonian. It should be emphasized that Eqs. (3) and (5) are fundamentally different. Equation (3) is formally derived from the full Schrödinger equation by means of the identity transformations. The orthogonality condition does not play a role in Eq. (3a) except at $E=0$. On the contrary, Eqs. (5) are model equations, which do not involve the full Hamiltonian H. Therefore the presence of the orthogonality condition (5b) is absolutely necessary.

III. TWO-COMPONENT MODEL IN FRAME OF CONSTITUENT QUARK MODEL AND MOSCOW POTENTIAL

To fill the general scheme (5) with a microscopic content it is necessary to use some approximation for the full Hamiltonian H. This can be done in the frame of the constituent quark model (see, e.g., our previous paper [27]). Symmetry considerations allowed to identify the subspace $\mathcal{H}_{6q}$. It consists out of square integrable functions ψ_{6q} describing the lowest $6q$-bag states with maximal spatial symmetry: $|s^6[6]\rangle$ for S waves and $|s^5p[51]\rangle$ for P waves.

This choice of $\mathcal{H}_{6q}$ can be justified by several independent reasons [23–28]. For example, recent chiral model calculations [30] have shown that the structure of fully symmetric $6q$ states $|s^6[6]\rangle$, in contrast to the mixed symmetry states as $|s^4p^2[42]\rangle$, cannot be described by the cluster RGM ansatz and that they are quite similar to the shell-model ground states of magic nuclei. However, the most conclusive argument in favor of the separation of $6q$ states with high and low symmetry arises from our general understanding of quantum chromodynamics, where the effective interactions must essentially depend on number and type of quarks. Moreover, if we assume, that some effective bosonization of initial QCD degrees of freedom occurs in the peripheral area of nucleon and thus this bosonic mode is an important component of interquark interaction [10] one can conclude, taking into account a highly nonlinear character of such bosonization, chiral meson fields must play a crucial role in the dynamics of the six-quark configurations. Such chiral fields should stabilize strongly these six-quark components of interaction. Thus, by approximating the Q-space Green function $[P_{6q}(E-H)P_{6q}]$ with one pole at $E=E_{6q}$ one gets a separable form for the potential v_{NqN}

$$v_{NqN} = P_{NN}|H|\psi_{6q}\rangle(E-E_{6q})^{-1}\langle\psi_{6q}|H|P_{NN}, \tag{6}$$

where

$$E_{6q} = \langle\psi_{6q}|H|\psi_{6q}\rangle.$$

As a projection operator onto the NN channel one can employ a respective operator taken from the resonating group method (RGM)

$$P_{NN} = \mathcal{A}|\psi_N\psi_N\rangle\mathcal{N}^{-1}\langle\psi_N\psi_N|\mathcal{A}, \tag{7}$$

where ψ_N is the three-quark function of the nucleon, $\mathcal{A}$ is the antisymmetrizer, and $\mathcal{N}$ is the overlap kernel:

$$\mathcal{N} = \langle\psi_N\psi_N|\mathcal{A}|\psi_N\psi_N\rangle. \tag{8}$$

With this choice of the projection operator P_{NN}, the model equation (5) becomes a two-body effective Schrödinger equation for the orthogonalized relative motion wave function $\tilde{\chi}(R)$

$$\left(T_R + V^{ME} + 10\frac{|f\rangle\langle f|}{E-E_{6q}}\right)\tilde{\chi} = E\tilde{\chi}, \tag{9a}$$

$$\langle g|\tilde{\chi}\rangle = 0, \tag{9b}$$

in which

$$\langle\mathbf{R}|f\rangle \equiv f(\mathbf{R}) = \langle\psi_{6q}|H|\psi_N\psi_N\rangle, \tag{10}$$

$$\langle\mathbf{R}|g\rangle \equiv g(\mathbf{R}) = \langle\psi_{6q}|\psi_N\psi_N\rangle. \tag{11}$$

In a good approximation one can take a delta function for the overlap kernel $\mathcal{N}(\mathbf{R},\mathbf{R}')$ [21,22]:

$$\mathcal{N}(\mathbf{R},\mathbf{R}') \simeq \frac{1}{10}\delta(\mathbf{R}-\mathbf{R}'). \tag{12}$$

We emphasize once again that Eq. (9a) with the orthogonality condition (9b) is not equivalent to the full six-quark Schrödinger equation $H\psi=E\psi$. Actually we suppose we know only individual parts of the full Hamiltonian.

(i) Sub-Hamiltonian $h_{NN}=t_R+v^{ME}$, acting in the subspace $\mathcal{H}_{NN}$ and describing the meson-exchange interaction between unexcitable nucleons;

(ii) Other sub-Hamiltonian H_{6q}, describing the lowest states in the $6q$ bag (in given case $H_{6q}=\Sigma E_{6q}|\psi_{6q}\rangle\langle\psi_{6q}|$). Thus the full six-quark Hamiltonian is needed only for determination of coupling between the subspaces in Eq. (6). In this model the $6q$-bag functions ψ_{6q} are not eigenfunctions of the full Hamiltonian (otherwise $[P_{6q},H]=0$ and $v_{NqN}\equiv 0$). Moreover, it is obvious that the sum of the projectors P_{NN} (7) and $P_{6q}=\Sigma|\psi_{6q}\rangle\langle\psi_{6q}|$ is not unity in the full six-quark space $\mathcal{H}$. Therefore, Eqs. (9) cannot be formally deduced from the full Schrödinger equation and the orthogonality condition (9b) proves to be necessary.

The effective two-nucleon equation (9a) provides the basis for developing the local and nonlocal parts of NN-interaction models of Moscow type. The main point here is just the orthogonality condition (in S and P waves), which results in appearance of nodes in NN-scattering wave functions, the positions of the nodes not depending on energy (at least up to laboratory energies $E_{NN}\sim 1$ GeV). The term v_{NqN} provides an additional attractive interaction at $E<E_{6q}$. It has been shown in previous papers [24–28], that the phase shifts and nodal behavior of wave functions typical for Eqs. (9) are well reproduced by a deep local attractive potential with an extra bound state and the respective orthogonality condition constraint. So, from this point of view, the NN-interaction model, known today as the Moscow potential, is the simplest local model which ensures the orthogonality between the scattering wave functions and the most symmetric $6q$ states $|s^6[6]\rangle$ projected onto the NN channel. However, the situation for P waves turns out to be different. Attempts to achieve a satisfactory description of the phase shifts by using a local attractive potential failed for these partial waves [25]. Therefore, one needs to use the general orthogonality condition model (GOCM) presented here.

IV. STRUCTURE OF THE POTENTIAL

Here we give the full version of the *NN potential* model with the additional orthogonality condition in S and P waves. The potential is *an effective one-component approximation* to the two-component model, described in the previous section. Actually we have replaced the nonlocal term V_{NqN} (attractive at low energies) in Eq. (9) by an additional local attractive well.

The total interaction is, however, highly nonlocal due to the presence of the S- and P-wave projection operators which are employed in order to take into account the orthogonality condition (9b). As a result we do not require locality, this means we have a weaker interrelation between the orthogonality condition and the form of the attractive well. This decoupling of the attractive potential from the orthogonality condition improves essentially the approach. In particular, the quality of the fits for P waves gets more accurate than in the old-fashioned Moscow model with eigenprojection [24–26]. Besides the matrix eigenstate projector in coupled $^3S_1$-$^3D_1$ channels, as was demonstrated in our previous paper [27], can be replaced quite accurately by a scalar one-channel projector. In order to use a potential with the orthogonality conditions in few-body calculations, one has to add the projection operator with a very large positive coupling constant to the local part of the potential, in all S- and P-partial waves [27].

TABLE I. Parameters of local part of the potential.

spin parity	singlet even	singlet odd	triplet even	triplet odd
α	6.08671	6.08671	6.08671	4.3160
V_0	−4346.19	−1767.26	−4567.12	−223.63
β	3.49366	2.84152	3.81272	2.4959
V_0^{ls}				−591.1
β_1				3.4688

For the sake of uniformity and convenience we include similar separable terms, but with finite coupling constants, also in some other partial waves (D and F). These terms replace the standard spin-orbital part of the interaction (for even-parity waves) and partially reduce the strong attraction due to the central part of the local potential. In fact, these separable terms imitate a short-range repulsion generated by ω-meson exchange.[2] We also include the tensor interaction which couples partial waves with angular momenta l and $l\pm 2$. It can be quite accurately described by a truncated one-pion-exchange (OPE) potential in all partial waves with the channel coupling being determined by a truncation parameter.

In the present version of the Moscow potential we have replaced the Gaussian form of the central potential which has been used in all previous versions of the model [24–26,31] by an exponential one. We have found the exponential form gives a more satisfactory description of the phase shifts, in particularly for the $^3S_1$-$^3D_1$ channel (see also Refs. [32]).

Thus, the model potential consists of three parts

$$v_{NN}=v_M^{\rm loc}+v^{\rm OPE}+v^{\rm sep}, \tag{13}$$

where the local exponent well $v_M^{\rm loc}$ depends on the channel spin and parity

$$v_M^{\rm loc}(r)=V_0\exp(-\beta r)+(\mathbf{sl})V_0^{ls}\exp(-\beta_1 r). \tag{14}$$

In the state-dependent separable part

$$v^{\rm sep}=\lambda|\varphi\rangle\langle\varphi| \tag{15}$$

a Gaussian form factor $\langle r|\varphi\rangle=\varphi(r)$ is used

$$\varphi(r)=Nr^{l+1}\exp\left[-\frac{1}{2}\left(\frac{r}{r_0}\right)^2\right] \tag{16}$$

[2]It should be emphasized here that the ω-exchange terms in traditional meson-exchange models are highly nonlocal due to form factors and energy and momentum dependence.

TABLE II. Parameters of projectors and separable parts of the potential.

State	λ, MeV	r_0, fm
$^1S_0$	∞	0.3943
$^1P_1$	∞	0.5550
$^1D_2$	107.2	0.4527
$^1F_3$	182.6	0.5191
$^3S_1$	∞	0.3737
$^3D_2$	161.2	0.4695
$^3D_3$	588.2	0.3572
$^3G_4$	2.74	0.8077
$^3P_0$	∞	0.3209
$^3P_1$	∞	0.3226
$^3P_2$	∞	0.1632
$^3F_4$	5.447	0.6221

with normalization condition $\int \varphi^2 dr = 1$. The integer l labels the partial waves.

For the one-pion-exchange part of the potential the standard form with a *dipole* form factor is chosen:

$$v^{\mathrm{OPE}}(\mathbf{k}) = \frac{f_\pi^2}{m}\frac{1}{\mathbf{k}^2+m^2}\left(\frac{\Lambda^2-m^2}{\Lambda^2+\mathbf{k}^2}\right)^2 (\boldsymbol{\sigma}_1\mathbf{k})(\boldsymbol{\sigma}_2\mathbf{k})\frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}. \tag{17}$$

With such a form factor choice the OPE tensor potential vanishes at the origin as it should. In the coordinate representation the OPE potential has the form

$$v^{\mathrm{OPE}}(r) = \frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}\frac{f_\pi^2}{4\pi} m[f_C(r)(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2) + f_T(r)\hat{S}_{12}], \tag{18}$$

where the tensor operator

$$\hat{S}_{12} = \frac{(\boldsymbol{\sigma}_1\mathbf{r})(\boldsymbol{\sigma}_2\mathbf{r})}{r^2} - \frac{(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)}{3} \tag{19}$$

and

$$f_C(r) = [\exp(-x)-\exp(-\alpha x)]/x - (\alpha^2-1)\alpha/2 \times \exp(-\alpha x), \tag{20}$$

$$f_T(r) = \exp(-x)/x(1+3/x+3/x^2) - \alpha^3\exp(-\alpha x)/(\alpha x)[1+3/(\alpha x)+3/(\alpha x)^2] - (\alpha^2-1)\alpha/2\exp(-\alpha x)[1+1/(\alpha x)], \tag{21}$$

$$x = mr, \quad \alpha = \Lambda/m. \tag{22}$$

We use here the averaged pion mass $m=(m_{\pi_0}+2m_{\pi_\pm})/3$ and the averaged value of pion-nucleon coupling constant $f_\pi^2/(4\pi)=0.075$ as we do not wish to deal with the difference between np and pp isovector phase shifts in the present work.

Thus only three free parameters V_0, β, and α are left for the local part of interaction for each combination of spin and parity in addition to two parameters r_0 and λ of the separable term in each channel. It should be noted that only some of the values r_0 and λ are independent free parameters (for D and F waves). Values of λ for S and P waves must go to infinity (in real calculations the value of $\lambda \sim 10^5 - 10^6$ MeV is quite enough). Values for r_0 for these channels are related to the local attractive well [for the local potential, the requirement of the best approximation for eigen bound state by Gaussian (16) defines r_0 uniquely]. Thus, we have totally 32 parameters of the potential (and the value of πNN coupling constant) giving a very good description of all N-N partial waves (except some high l channels) in the wide energy range 0–400 MeV. The number of parameters almost coincides with that for most recent version of the Nijmegen N-N potential [2]. The parameters for the present version of our NN potential are given in Tables I and II.

V. DESCRIPTION OF PHASE SHIFTS AND DEUTERON STRUCTURE

The potential parameters as given in Tables I and II were determined by fitting the Nijmegen phase shifts (PWA93) [1]. In Figs. 1–3 the recent SAID phase shifts (SP97) [33] are also presented for comparison. As can be seen from the figures, some discrepancy between the results of both phase shift analyses (PSA) exist, especially for some partial phase shifts. With applications to few-nucleon problems in mind we tried to reproduce with maximal accuracy the $^1S_0$ and $^3S_1$-$^3D_1$ phase shifts and the values of the scattering length and the effective range.

A. Singlet partial wave channels

The description of singlet n-p phase shifts for both even and odd parities is illustrated in Fig. 1. It is evident from the figure that the quality of fit to the data of recent phase shift analysis is quite good, especially for the Nijmegen PSA results. For example, the fits in $^1S_0$ and $^1P_1$ channels are almost perfect. The quality of fits can be estimated quantitatively from Table IV for these channels. The average devia-

TABLE III. Effective-range parameters for the potential variant given in Tables I and II.

	a, fm		r_0, fm	
	theory	experiment	theory	experiment
triplet $^3S_1$	5.422	5.419(7) [a]	1.754	1.754(8) [a]
singlet $^1S_0$	23.74	−23.748(10) [b]	2.66	2.75(5) [b]

[a]Reference [42].
[b]Reference [43].

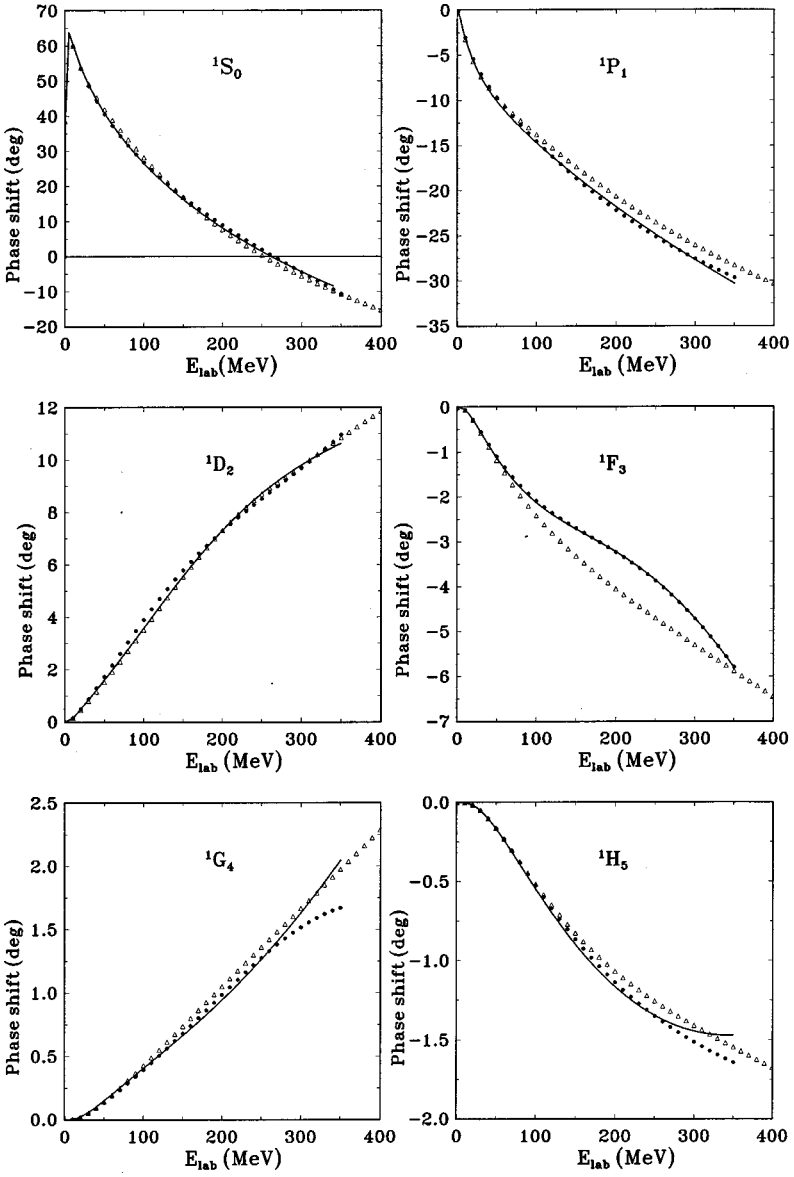

FIG. 1. The comparison of spin-singlet phase shifts for the present version of the Moscow *NN* potential with the data of the recent energy-dependent phase-shift analyses: PWA93 [1] (circles) and SAID97 [33] (triangles).

tions for all singlet channels are only $0.1-0.2\,\%$ excepting the $^1G_4$ channel where the discrepancy with PSA data is largest and around 1%.

Also there is a problem in precise description of the singlet effective range r_0 (see Table III). We used as ''experimental'' the value r_0 presented in compilation of Dumbrajs *et al.* of 1983 (see footnote to Table III). However, in view of the very good agreement of our phase shifts with the Nijmegen PSA for the $^1S_0$-channel one could conclude that the disagreement for r_0 should be really much reduced.

B. The even-parity waves

The *S*-wave potential turns out, as is in the previous versions of Moscow potential [27], to be strongly attractive. The Gaussian (15) with the range parameter r_0, included in the orthogonality condition (9b), is close to the eigenfunction of the ground ''forbidden'' state in the potential. In other words, we obtain for *S* waves almost a local potential. However, for the $^3S_1$-$^3D_1$ channel (and also for all triplet coupled channels) we use, strictly speaking, noneigenstate one-channel projector, as in Ref. [27], in order to avoid a more complicated two-channel eigenprojector.

We do not introduce here a spin-orbital potential for even partial waves in an explicit form because it cannot be determined by PSA data for the $^3S_1$-$^3D_1$ channel, and the role of spin-orbital potential for higher even-parity partial waves is played by the term $v^{\rm sep}$.

It should be kept in mind here that the complete two-channel version of our model includes in the proper *NN* channel one-meson exchange interaction terms (in a subspace orthogonal to symmetric six-quark compound states). Thus, in the two-channel model, the spin-orbit terms should be described by a conventional meson-exchange model. However, in the effective *one-channel* model presented here the separable state-dependent spin-orbit interaction in even-parity channels is inavoidable to compensate partially the strong attractive potential in the *S* wave.

The effective range parameters for singlet and triplet *S*-wave channels are given in Table III. Among all the calculated phase shifts the maximal disagreement with PWA93 (though not large) is observed for the tensor mixing parameter ε_1.

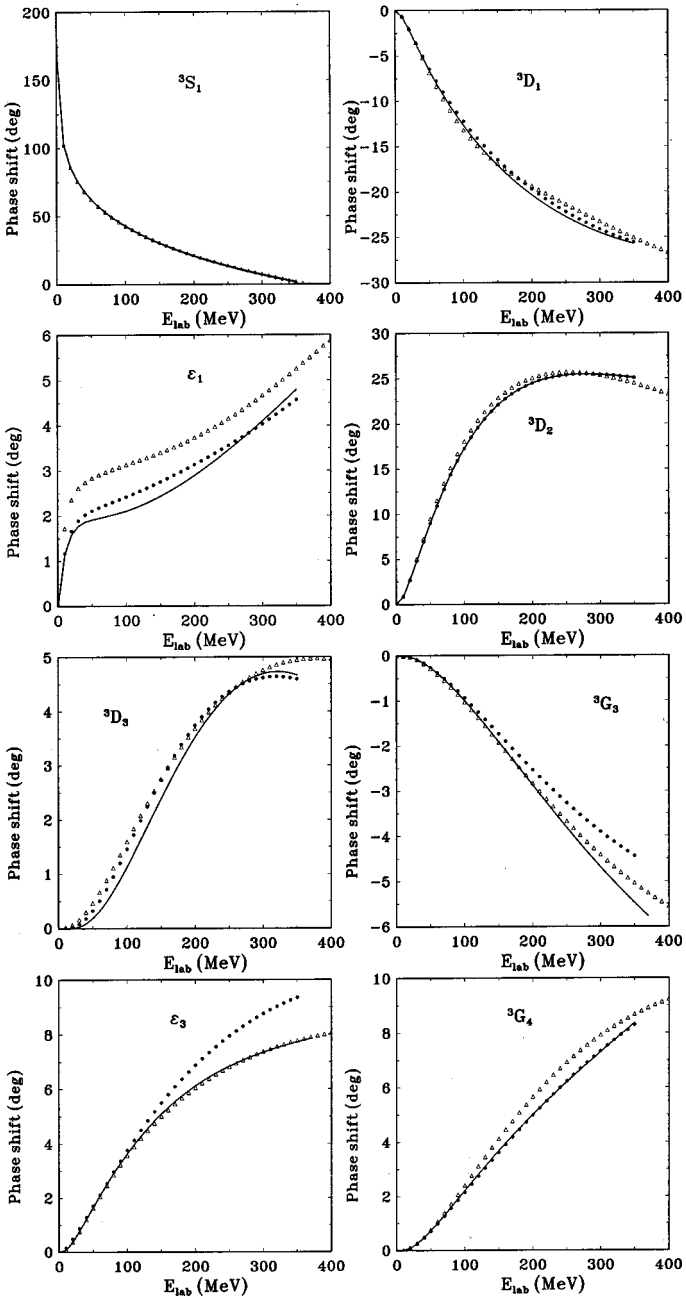

FIG. 2. The spin-triplet even-parity phase shifts for the present version of the Moscow *NN* potential. The data of the energy-dependent phase-shift analyses are PWA93 [1] (circles) and SAID97 [33] (triangles).

C. The triplet odd-parity waves

In the accordance to the microscopic quark picture, the orthogonality to the baglike functions $|s^5p[51]\rangle$ must be included for all *P* waves. However, if we look at the behavior of ''experimental'' *P*-wave phase shifts at energies up to 500 MeV we will not find any repulsion in the $^3P_2$ channel, because the corresponding phase shifts are purely positive until the energies ~ 1 GeV. There is no repulsive core for this channel in the majority of the conventional realistic *NN* potentials either. But if we look to the phase shifts at high energies (see Fig. 4) we can observe a repulsion appearing in all three triplet *P* waves, while $^3P_2$-phase shifts become negative only at energies higher than 1 GeV. From the point of view of the constraints imposed by the orthogonality condition, this means the function to which the $^3P_2$ scattering function is orthogonal is much more narrow than that for the other *P*-wave channels, $^3P_0$ and $^3P_1$.

It is interesting that fitting the $^3P_2$ wave at energies up to 350 MeV enables us to determine the range parameter r_0 of the projector (see Table II). The inclusion of the projector improves appreciably the description of the phase shifts up to 350 MeV. An attempt to reproduce the $^3P_2$-phase shift using a purely attractive potential with an ''extra'' bound state results in a very deep (~ 15 GeV) potential and an unsatisfactory quality of the description. In addition, such a deep potential is not suitable for a description of the $^3P_0$ and $^3P_1$ phase shifts. That is why we have strayed from the concept of a local Moscow model for *P* waves in the present version.

So, for odd partial waves we have a rather small attractive well (~ 220 MeV) and orthogonality to the noneigenbound states for the local part of potential. This might mean that the size of six-quark bag in *P* waves should be smaller than in $^3S_1$ and $^1S_0$ waves. One notices here that the range parameters of the projectors for the $^3P_0$ and $^3P_1$ channels (r_0

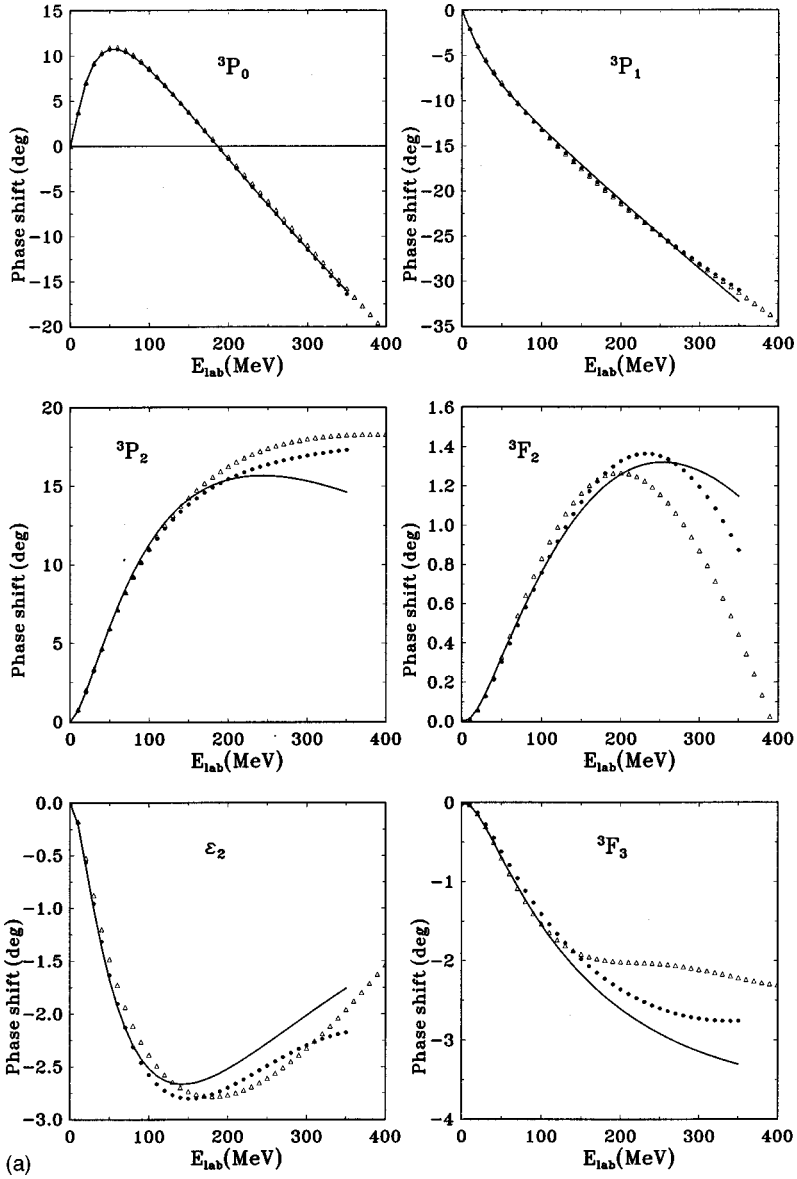

FIG. 3. The spin-triplet odd-parity phase shifts for the present version of Moscow *NN* potential. The data of the energy-dependent phase-shift analyses are PWA93 [1] (circles) and SAID97 [33] (triangles).

≈0.32 fm) almost coincide with each other. As seen from Fig. 4, the attraction for some odd higher partial waves with $L=J(^3F_3$ and $^3H_5)$ is noticeably deficient in the given version of the model. Unlike the even partial waves, we used a more conventional local spin-orbit potential for odd partial waves [see Eq. (14)] because the usage of the separable spin-orbital form is not convenient to describe the splitting of *P*-phase shifts.

It would be rather instructive to estimate the averaged relative difference of phase shifts predicted by the Moscow model and the recent phase shift analysis [1] using the criterion of *relative* difference or the respective absolute difference measured in radians:

$$\varepsilon_{\rm rel}=\frac{1}{N}\sum_{i=1}^{N}\left|\frac{\delta^{\rm pot}_{JSl,i}-\delta^{\rm PSA}_{JSl,i}}{\delta^{\rm PSA}_{JSl,i}}\right|^2 \tag{23}$$

$$\chi^2_{JSl}=\frac{1}{N}\sum_{i=1}^{N}|\delta^{\rm pot}_{JSl,i}-\delta^{\rm PSA}_{JSl,i}|^2, \tag{24}$$

where $\delta^{\rm pot}_{JSl,i}$ and $\delta^{\rm PSA}_{JSl,i}$ are partial phase shifts in the channels JSl at the energy E_i for the Moscow model and Nijmegen phase shift analysis, respectively. Table IV presents the values of $\varepsilon_{\rm rel}$ and χ^2_{JSl} for all considered JSl channels. It is evident from the table the average deviation of phase shifts predicted by the Moscow model and recent PSA is very small and around $0.2-0.4\%$. This means that the description of *NN* observables with the presented force model should be very good.

D. Deuteron structure

The accurate description of the deuteron structure offers an additional strong test for any nuclear force model. Many deuteron properties, even in the static limit, depend sensitively on the behavior of the *NN* force at intermediate and short ranges [15], especially on the *D*-wave contribution. For example, with the first version of the present force model [31], we found an impressive agreement with experimental data for all crucial *D*-wave deuteron observables such as Q_d, A_S, A_D/A_S. But this early model included a node not only in the *S* wave but also in the *D* wave. This extra node in the *D* wave was a consequence of a very short-range truncation of the OPE tensor force [31] which contradicts somehow the microscopic picture of the underlying interactions (e.g., ac-

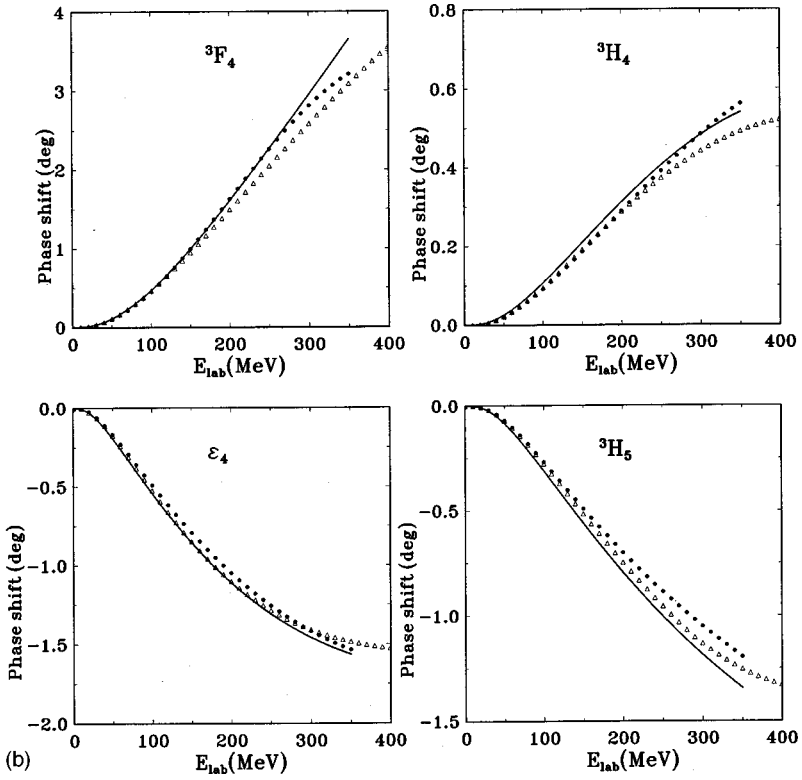

FIG. 3. (*Continued*).

cording to wide-spread opinion [34] the OPE tensor force cannot penetrate deeply inside the two-nucleon overlap region).

Hence, in subsequent versions of the model [26–28], a more soft cutoff factor has been employed which resulted in the disappearance of the *D*-wave node. As an immediate consequence of the softer truncation in the OPE tensor force the *D*-wave deuteron observables have become close to the values predicted by conventional force models, i.e., the values of η and Q_d are a little bit underestimated (see Table V). Nevertheless the node in the *S* wave and the strong attractive *S*-wave potential, tightly related to this, results in a very specific interference between *S*- and *D*-wave components and a specific character of tensor mixing (see Sec. VI).

The values of deuteron observables for three versions of our force model are presented in Table V while the pattern of the deuteron wave functions is displayed in Fig. 5. One can see in the figure the short-range maximum in the *D* wave almost disappears for the current version of the force model while this maximum in the *S* wave gets rather reduced. It is interesting to note the *D*-wave amplitude in the current version of the model (solid line) is a little bit lower than in the previous versions (dashed and dot-dashed lines) due to a smaller value of the derivative of the *D*-wave component near the *S*-wave node ($\sim$0.53 fm). While the asymptotic behavior of the *S* wave looks almost perfect (see values of A_S in Table V).

Thus we can conclude from the deuteron results presented in this section that the short-range part of the tensor force needs to be a bit improved. Careful inspection of Table V shows unambiguously the general good agreement for the deuteron parameters found with the sharply different force models such as Nijmegen and Moscow potentials. The values for the deuteron observables are a result of some general properties (such as the OPE tail) and of the *NN* phase shifts used for fitting only and essentially do not depend on the details of the force at short ranges.[3]

VI. SPECIFIC INTERFERENCE BETWEEN TENSOR AND CENTRAL FORCES AND THE πNN COUPLING CONSTANT IN THE MOSCOW FORCE MODEL

We included this specialized section to the present work in order to emphasize a specific character of interference between tensor and central forces in the Moscow force model. This interference will be shown below to be very advantageous in some aspects as compared to the traditional force models. The main difference between our and traditional models as concerned to wave function form is the nodal character of the *S*-wave deuteron and scattering wave functions and the practically nodeless character of the *D*-wave functions.[4] We will show here that the specific tensor mixing between the *S*-wave state with a node and the almost nodeless *D*-wave state results in a remarkably different ε_1 behavior. We compare the *D*-wave observables with the results of traditional models.

First of all we emphasize here that the best fit for *NN* phase shifts is attained in our case with a very reasonable value for the OPE cutoff parameter $\Lambda_{\rm dip}=0.78$ GeV (we used here the dipole form factor), see Eq. (17) and Fig. 6.

[3]Certainly this conclusion may be invalid for nonstatic, e.g., energy-dependent or multicomponent force models.

[4]The very small inner maximum in the *D*-state wave function in the present version [see the solid lines in Figs. 6(a)–6(b)] can be ignored in any calculation if we do not consider the very high momentum transfer.

TABLE IV. Accuracy of fitting of phase shifts.

channel	$^1S_0$	$^1P_1$	$^1D_2$	$^1F_3$	$^1G_4$	$^1H_5$	$^3S_1$	$^3D_1$
$\varepsilon_{\rm rel}$ [a]	0.007504	0.000527	0.002113	0.000197	0.012030	0.001802	0.004524	0.000867
χ^2 per point[b]	0.005282	0.001731	0.000627	0.000008	0.000200	0.000055	0.005595	0.003922
channel	ε_1	$^3D_2$	$^3D_3$	$^3G_3$	ε_3	$^3G_4$	$^3P_0$	$^3P_1$
$\varepsilon_{\rm rel}$	0.006816	0.000022	0.034310	0.027256	0.023088	0.001451	0.000184	0.000455
χ^2 per point	0.000843	0.000007	0.000856	0.004545	0.013887	0.000029	0.000186	0.003596
channel	$^3P_2$	$^3F_2$	ε_2	$^3F_3$	$^3F_4$	$^3H_4$	ε_4	$^3H_5$
$\varepsilon_{\rm rel}$	0.003874	0.010679	0.007830	0.017762	0.007590	0.021424	0.010617	0.026970
χ^2 per point	0.018245	0.000123	0.000693	0.001413	0.000279	0.000004	0.000042	0.000155

[a] $\varepsilon_{\rm rel}=1/N\Sigma_{k=1}^{N}(\delta_{JSl,k}^{\rm pot}-\delta_{JSl,k}^{\rm PSA}/\delta_{JSl,k}^{\rm PSA})^2$.
[b] $(1/N)\Sigma_{k=1}^{N}(\delta_{JSl,k}^{\rm pot}-\delta_{JSl,k}^{\rm PSA})^2$ (in radians).

This soft cutoff parameter is in nice agreement with both experimental results and with all theoretical estimations made in π-N dynamics [33–37]. It should be contrasted with a statement formulated in Ref. [38], p. 232 for traditional OBE-force model: ‘‘ . . . a value of 1.3 GeV is *a lower limit* for Λ_π.’’ The conventional OBEP model with $\Lambda=0.78$ GeV gives the extremely low values for $Q_d=0.238$ fm^2, the ratio $D/S=0.0233$, and $P_D=2.4\%$ [38] which should be compared to the respective values for our force model (see Table V).

In despite of the ‘‘soft’’ value of Λ, the D-wave deuteron properties in our model (see Table V in Sec. V) are in a rather good agreement with the experimental data, being remarkably better than the respective predictions of the traditional force models with the same Λ value. We note, in passing, that the harder truncation with $\Lambda\simeq 1.3$–1.7 GeV is usually taken in the traditional force model just in order to fit reasonably the deuteron properties and the tensor mixing parameter (see below).

The second important point in the story is related to the mixing parameter ε_1. In fact, in order to reach a reasonable agreement with the recent phase shift analysis data for the ε_1-mixing parameter [1,33] the Λ value must be taken also around 1.5–1.7 GeV [38] (see Fig. 6) while the same agreement with the experimental ε_1 is reached in our model using a much more soft $\Lambda=0.78$ GeV. This sharp difference from the traditional force models can be ascribed to a different

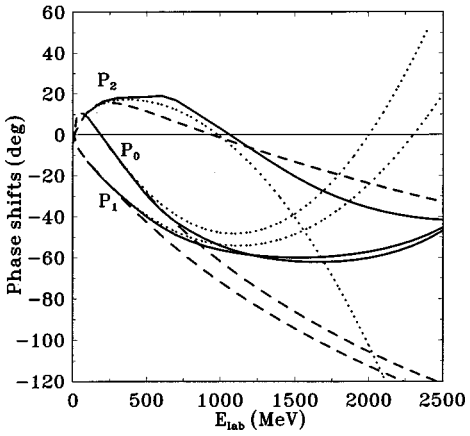

FIG. 4. The spin-triplet P-wave phase shifts in a wider energy region: the data of energy-dependent phase-shift analysis SAID97 [33] (solid lines) and predictions for the present version of the Moscow NN potential (dashed lines) and for the Bonn NN potential (dotted lines).

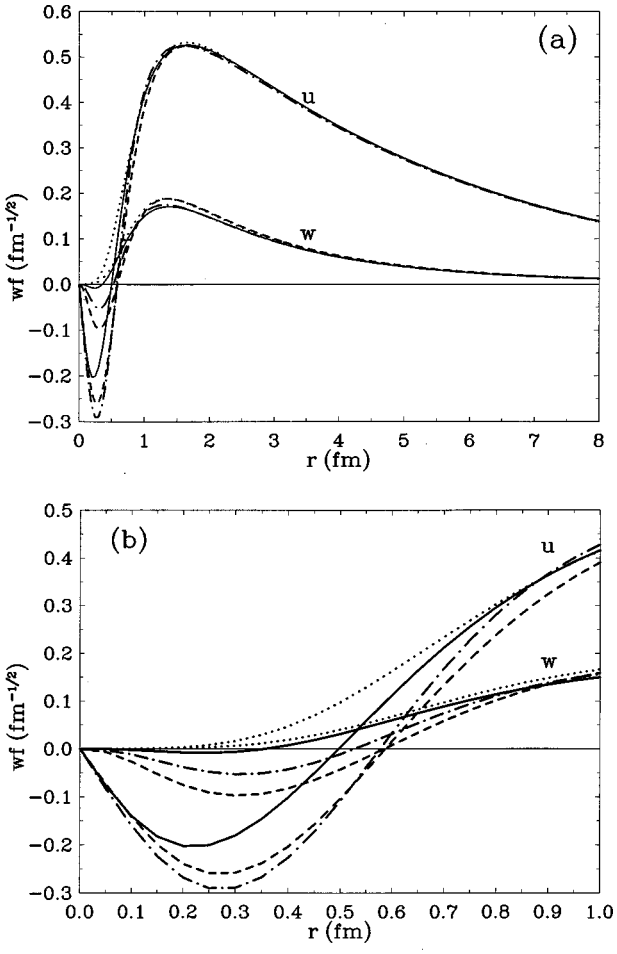

FIG. 5. (a) The deuteron S-wave and D-wave functions for present (solid lines) and previous versions ([25], dashed lines and [27], variant B, dot-dashed lines) of the NN Moscow-type potential. The deuteron wave functions calculated with the RSC potential (dotted lines) are shown for comparison. (b) Short-distance zoom of (a).

TABLE V. Deuteron parameters for conventional and Moscow NN potentials.

model	E_d (MeV)	P_D (%)	r_m (fm)	Q_d (fm^2)	$\mu_d\,(\mu_N)$	A_S (fm$^{-1/2}$)	D/S	$D_{\rm loop}$ [a]
RSC [b]	2.22461	6.47	1.957	0.2796	0.8429	0.8773	0.0262	
Nijm 93	2.224575	5.754	1.966	0.2706	0.8429	0.8844	0.02524	
Moscow 86 [b]	2.22444	6.57	1.966	0.2862	0.8422	0.8838	0.0268	0.53
Moscow 98[b,d]	2.22440	5.75	1.954	0.2708	0.8470	0.8746	0.0259	0.30
present [c]	2.22456	5.65	1.967	0.2731	0.8476	0.8845	0.0255	0.08
experiment	2.224575(9)		1.9660(68)	0.2859(3)	0.857406(1)	0.8846(16)	0.0256(1) [e]	

[a] $D_{\rm loop}$ is the relative amplitude of the D-wave maxima, i.e., the absolute value of the ratio of the first and second maximum of the deuteron D component.

[b] The value $\hbar^2/2m = 41.47$ MeV fm^2 has been used ($m = 938.978$ MeV).

[c] The value $\hbar^2/2m = 41.47107$ MeV fm^2 is used ($m = 938.918$ MeV).

[d] Unfortunately, in our previous work [27] only rounded values for potential parameters are given in Table III. The deuteron parameters cited in Ref. [27] (for variant B) do not correspond to the rounded potential parameters cited in Table III of Ref. [27]. We thank Dr. S. B. Dubovichenko, who has attracted our attention to this disagreement, and give here the exact values for variant (B) of Ref. [27]: $V_O = -1329.18$ MeV, $\eta = 2.2959$ fm^{-2}, $\alpha = 1.8835$ fm^{-1}.

[e] The present value is taken from Ref. [41].

character of mixing between S and D waves in our model.

Some additional confirmation comes from the value of πNN-coupling constant obtained in our model. We choose here the Nijmegen force model as a good representative of the traditional NN potentials (see Table V). The two models above include practically the same values for the πNN (charged) coupling constants ($f^2_{\pi NN} = 0.075$ in our case[5] and $f^2_{\pi^\pm NN} = 0.0748$ for Nijmegen potential). The latter fact is very important because the D-wave characteristics are directly related to the πNN coupling constant. In this respect our model appears to corroborate the smaller value of $g^2_{\pi NN} \simeq 13.60$ advocated by the Nijmegen group [1,2]. The two nice features of our model discussed above, i.e., the soft cutoff parameter Λ and low value of πNN coupling constant, are in agreement with modern trends and lend strong support to our model.

VII. CONCLUSION

The force model presented in this paper differs in a few important aspects from traditional NN interaction models currently in use. First of all the Moscow two-component model includes two mutually orthogonal quark- and meson-exchange channels. This channel orthogonality leads to many differences from the traditional force models. In particular it requires a node in low partial waves with the node position almost independent on the relative energy in a wide energy range ($\leqslant 1$ GeV). The nodal behavior of wave functions is also preserved for the one-channel model presented here. The node in the NN wave functions results in an enhancement of high momentum components and a strong increase of the average kinetic energy in the deuteron and in all few-nucleon systems. This increase of the inner kinetic energy leads to significant enhancement of higher angular momentum components in nuclei and nuclear matter and also for many particular nuclear processes [27,28] such as π-meson absorption and scattering in the Δ-resonance region, etc. This strong enhancement of high-momentum components in the N-N system as compared to any traditional N-N force model may be seen, e.g., in hard bremsstrahlung process $pp \to pp\gamma$ [39] at $E_p = 300$ MeV and higher at small forward and backward angles θ_γ of γ emission. To make the comparison with traditional repulsive core models most unambiguously the authors of Ref. [39] did their bremsstrahlung calculations with both the Moscow model (in its previous version [26]) and its exact phase-shift equivalent supersymmetrical partner. Thus, such a comparison removes any questions on the possible on-shell origin of disagreements observed.

Redistribution of higher partial waves along Jacoby coordinates leads, e.g., to a noticeable enhancement of the P-wave attraction for $N+d$ and $N+2\alpha$ systems [27,40]. The long-standing puzzle of the analyzing power A_y in low energy $N+d$ scattering is explained by insufficient attraction in just the $N-d$ relative motion P wave [5,6]. The apparent discrepancies for $n+{}^3$H elastic and $n+{}^3\text{He} \to d+d$ rearrangement low-energy scattering observed recently [8] also appear to have to be explained by insufficient attraction in the $n+{}^3\text{H}({}^3\text{He})$ P wave [7,8]. Such enhancement of higher partial wave contributions to near-threshold and low-energy processes in few-nucleon and few-cluster physics when replacing the deep Moscow-type potential (including extra bound states) with its SUSY partner potential—which is exactly phase-shift equivalent—is a sequence of some very general algebraic properties of kinetic energy operator in different coordinate systems and is disconnected at all to any small variations in the on-shell properties of various N-N potential models of current use.

The second crucial point in the development of Moscow NN force model is the important role of the six-quark components with maximal possible symmetry. We showed recently that the coupling of the meson-exchange NN channel to the six-quark component can be strong enough to represent adequately the intermediate-range NN attraction. In turn, this fact leads to quite remarkable contributions of such six-quark configurations in nuclear bound and low-excited

[5] The value corresponds to the charged coupling constant because we considered first of all the pn scattering phase shifts.

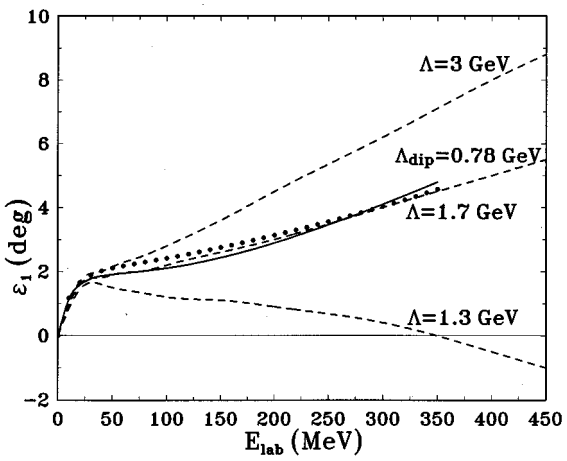

FIG. 6. The energy dependence of the mixing parameter ε_1 for different values of cutoff parameter Λ corresponding to conventional (dashed lines) and present (solid lines) force models. The data of the energy-dependent phase-shift analysis (PWA93 [1]) are shown by circles.

states. If so, it may require some strong revision for many nuclear properties as given by traditional force models (e.g., the meson-exchange current contributions). Thus the strongest test for the new model may offer few-nucleon calculations for the analyzing power A_y in the $n+d$ and $p+d$ low-energy scattering, for the analyzing power A_y in $p+d$ radiative capture reaction and for the $p+d$ intermediate energy elastic scattering cross sections (the so-called Sagara puzzle [6,7]). Hence the careful comparison of the predictions for few-nucleon systems using the Moscow force and more traditional NN interactions may be extremely interesting.

ACKNOWLEDGMENTS

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APPENDIX: MOSCOW POTENTIAL IN MOMENTUM SPACE

The K-matrix defined as

$$2iMq\hat{K}=\frac{1+\hat{S}}{1-\hat{S}} \tag{A1}$$

(M is the reduced mass while q is a linear momentum) obeys the partial-wave Lippmann-Schwinger equation

$$\hat{K}(q',q)=\hat{V}(q',q)+\frac{2}{\pi}P\int k^2dk\frac{\hat{V}(q',k)\hat{K}(k,q)}{E-k^2/2M}, \tag{A2}$$

where P means the principal value integral. The elements of matrix $\hat{V}$ in Eq. (2) are equal to the partial-wave momentum-space potential in the lSJ basis (up to factor $1/4\pi$)

$$V_{l'l}(q',q)=\frac{1}{4\pi}\langle (l'S)J|V(\mathbf{q}-\mathbf{q}')|(lS)J\rangle, \tag{A3}$$

where $V(\mathbf{q})$ is related to $V(\mathbf{r})$ by a standard Fourier transformation

$$V(\mathbf{q})=\int e^{-i(\mathbf{qr})}V(\mathbf{r})d\mathbf{r}. \tag{A4}$$

Here we give explicit formulas for all terms of the present version of the Moscow potential $V_{l'l}(q',q)$ in momentum space (in MeV^{-2}).

1. Local part of Moscow potential $V_{ll'}^{\mathrm{loc}}$

$$V_{ll'}^{\mathrm{loc}}=\delta_{ll'}\left\{\frac{V_0\tilde{\beta}}{2(qq')^2}F_l\left(\frac{q^2+q'^2+\tilde{\beta}^2}{2qq'}\right)\right.$$
$$+\frac{1}{2}[J(J+1)-l(l+1)-S(S+1)]$$
$$\left.\times\frac{V_0^{ls}\tilde{\beta}_1}{2(qq')^2}F_l\left(\frac{q^2+q'^2+\tilde{\beta}_1^2}{2qq'}\right)\right\}, \tag{A5}$$

where the parameters $\tilde{\beta}$ and $\tilde{\beta}_1$ are given in MeV:

$$\tilde{\beta}=\beta\hbar c,\quad \tilde{\beta}_1=\beta_1\hbar c.$$

F_l is the derivative of the second kind Legendre function

$$F_l(x)=-\frac{d}{dx}Q_l(x),\quad Q_l(x)=\frac{1}{2}\int_{-1}^{1}\frac{dz\,P_l(z)}{x-z}. \tag{A6}$$

2. Separable terms of the potential

In momentum space the separable terms with Gaussian form factors (15),(16) have the same form as in the coordinate space:

$$V_{ll'}^{\mathrm{sep}}(q,q')=\delta_{ll'}\lambda\frac{\pi}{2}\varphi_l(q)\varphi_l(q'), \tag{A7}$$

where

$$\varphi_l(q)=\left(\frac{2^{l+2}}{(2l+1)!!\sqrt{\pi}\,\tilde{r}_0^{2l+3}}\right)^{1/2}q^l\exp\left(-\frac{q^2\tilde{r}_0^2}{2}\right). \tag{A8}$$

Here the normalization condition $\int\varphi_l^2(q)q^2dq=1$ is assumed and the factor $\pi/2$ is related to the integration measure used in Eq. (A2), and $\tilde{r}_0$ is given in MeV^{-1}:

$$\tilde{r}_0=r_0/(\hbar c).$$

3. The OPE potential with dipole truncation

For the sake of the reader's convenience we also give the known formulas for OPE matrix elements.

(a) The central part of OPE potential:

$$(V_c^{\rm OPE})_{ll'}(q,q') = \delta_{ll'} \frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\frac{f_\pi^2}{4\pi}\frac{1}{2qq'} \times\left\{ Q_l(x) - Q_l(y) - \frac{\Lambda^2}{m_\pi^2}(y-x)F_l(y)\right\}. \tag{A9}$$

Here and below

$$x = \frac{q^2+q'^2+m_\pi^2}{2qq'}, \quad y = \frac{q^2+q'^2+\Lambda^2}{2qq'}. \tag{A10}$$

(b) The tensor part of OPE potential for triplet uncoupled channels with $l=J$:

$$(V_{\rm ten}^{\rm OPE})_{JJ}(q,q') = \frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}\frac{f_\pi^2}{4\pi}\frac{1}{m_\pi^2}\left\{\frac{q^2+q'^2}{qq'}G_J - \frac{2J+3}{2J+1}G_{J-1} - \frac{2J-1}{2J+1}G_{J+1}\right\}, \tag{A11}$$

where the function G_l is introduced as

$$G_l(q,q') = Q_l(x) - Q_l(y) - (y-x)F_l(y) \tag{A12}$$

and x and y are defined by Eq. (A10).

(c) The tensor part of OPE potential for coupled channels with $l=J\pm1$:

$$(V_{\rm ten}^{\rm OPE})_{J-1,J-1}(q,q') = \frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}\frac{f_\pi^2}{4\pi}\frac{1}{m_\pi^2}\frac{J-1}{2J+1} \times\left\{\frac{q^2+q'^2}{qq'}G_{J-1} - \frac{2J+1}{2J-1}G_{J-2} - \frac{2J-3}{2J-1}G_J\right\}, \tag{A13}$$

$$(V_{\rm ten}^{\rm OPE})_{J+1,J+1}(q,q') = \frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}\frac{f_\pi^2}{4\pi}\frac{1}{m_\pi^2}\frac{J+2}{2J+1}\left\{\frac{q^2+q'^2}{qq'}G_{J+1} - \frac{2J+5}{2J+3}G_J - \frac{2J+1}{2J+3}G_{J+2}\right\}, \tag{A14}$$

$$(V_{\rm ten}^{\rm OPE})_{J-1,J+1}(q,q') = \frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}\frac{f_\pi^2}{4\pi}\frac{3}{m_\pi^2}\frac{\sqrt{J(J+1)}}{2J+1} \times\left\{2G_J - \frac{q'}{q}G_{J-1} - \frac{q}{q'}G_{J+1}\right\}, \tag{A15}$$

$$(V_{\rm ten}^{\rm OPE})_{J+1,J-1}(q,q') = (V_{\rm ten}^{\rm OPE})_{J-1,J+1}(q',q). \tag{A16}$$

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