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Reply to ''Comment on 'Question of low-lying intruder states in ^{8}Be and neighboring nuclei' ''

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In the preceding Comment, our calculations based on the shell model and on the anisotropic harmonic oscillator model for ^{8}Be are criticized. In this Reply we argue that our calculations and remarks on intruder states are relevant. [S0556-2813(99)01905-6]

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The main criticism of the preceding Comment [1] is that the models we used in Ref. [2] are not sufficiently realistic to settle the question of the existence of intruder states in ^{8}Be, formerly predicted in Ref. [3] to be at 8 and 9 MeV. Admittedly, the anisotropic harmonic oscillator model is a schematic model and one may not expect it to be accurate at a quantitative level. However, at the qualitative and semiquantitative levels the predictions of this model should not be dismissed. Gross features of realistic self-consistent mean field calculations can be well understood and accounted for with this model, especially for states which are dominantly spin $S=0$ states. We therefore feel that we made in Ref. [2] a very solid case to the effect that there are no low-lying intruders in ^{8}Be despite the fact that they are present in ^{10}Be, ^{12}C, and ^{16}O. Not only do we present calculations with realistic and schematic interactions, but we also provide physical arguments to support this. Our basic point is that ^{8}Be differs from the other nuclei (^{10}Be, ^{12}C, and ^{16}O) because to get intruder states in ^{8}Be one must excite particles from a lower Nilsson orbit, and this costs a lot more energy. In what follows we respond to specific points raised in Ref. [1].

In the opening paragraphs of the preceding Comment, Barker implies that he did not suggest any analogy between the energies of low-lying intruder states in ^{10}Be, ^{12}C, and ^{16}O and his proposed low-lying intruders (at 6 and 9 MeV) in ^{8}Be. But in 1988 [3] he does present a table of all these energies, and we are sure that any reader would conclude, as we have, that by so doing he is lending support for his ideas in ^{8}Be.

We feel that our conclusion that ''the presence of a low-lying intruder state in ^{10}Be does not imply that there should be a low-lying intruder state in ^{8}Be'' is irrefutable and stands beyond any argument on the realistic character of the models. Indeed, the main role of the models used was to help to understand that this indeed happens: we find intruder states in ^{10}Be, but not in ^{8}Be. At no point in Ref. [2] do we say or imply that the question of the existence of low-lying intruder states is settled by our results. It is our view, however, that those calculations contain a great deal of physical insight and the results are to be taken into account. This view is supported by several facts: (a) whereas intruder states advocated by Barker as *R*-matrix states in ^{8}Be at 6 and 10 MeV appear as possible (dashed line states) in the seventh edition of *Table of Isotopes* (1978), they have been removed from the latest edition (1996). (b) Recent more realistic shell-model calculations using the Arizona interaction [4] give results that completely agree with ours. (c) There is still much controversy about how the *R*-matrix parameters should be chosen [5] and, in particular, a thorough analysis by Humblet *et al.* [6] comparing *R* and *K* parametrizations for the elastic α-α scattering finds no evidence for the existence of a resonance near 9 MeV.

It seems that our argument based on the Nilsson diagram was not transparent enough and needs further clarification. Since the argument was qualitative, we drew the Nilsson diagram as a function of $|\beta|$ to indicate that the main effect of deformation (whether prolate or oblate) is to mix l waves splitting the Ω^π levels. It is well known that this splitting is different on the oblate and on the prolate sides, in particular in the prolate case the levels with larger Ω go up and those with smaller Ω go down in energy, while the reverse is true in the oblate case. This is so well known that we could not expect anyone to think that we implied that the Nilsson diagram is symmetric about $\beta=0$. Clearly the qualitative argument given in Ref. [2] is equally valid for $\beta>0$ as for $\beta<0$. There is no reason to say that this argument is valid only for the prolate side. Our main point in Fig. 1 of Ref. [2] was to show graphically why, *a priori*, there is no analogy in the behavior of ^{8}Be and that of ^{10}Be or ^{12}C. To form an intruder state in ^{8}Be one must remove nucleons from the lowest Nilsson orbit in the *p* shell. This costs a lot of energy. In ^{10}Be and ^{12}C one can remove them from higher Nilsson orbits. This is the crux of the reason why there are no low-lying intruders in ^{8}Be.

Further insight as to why there are no low-lying intruders in ^{8}Be has been given to us by Vogt [5]. In the α-particle model the ground state of ^{12}C would consist of three α particles in a triangle. The intruder state is formed by moving one of the α particles so as to form a linear chain. In ^{8}Be one has only two α particles. One can rotate them around each other, but this would only give us higher angular momentum states of the ground state rotational band. To form an intruder state, we have to excite one of the α particles. But the lowest excitation energy in ^{4}He is at 20.1 MeV.

We strongly disagree with the paragraphs on ''errors and omissions in Table VIII of Fayache *et al.*'' In the preceding Comment it is said that ''for ^{10}Be$\langle J_y^2\rangle$ for the $(0\text{p-0h})_{\text{triaxial}}$ state should be 6.35 (rather than 2.3).'' Here is the proof that the value we give in Table VIII (2.3) is correct. The explicit calculation of $\langle J_y^2\rangle$ in the $(0\text{p-0h})_{\text{triaxial}}$ configuration ($\Sigma_x=7, \Sigma_y=5, \Sigma_z=9$) gives

$$\begin{aligned}\langle J_y^2\rangle &= 4|\langle 101|l_y|000\rangle|^2 + 2|\langle 100|l_y|001\rangle|^2 \\ &\quad + 4|\langle 102|l_y|001\rangle|^2 + 2|\langle 201|l_y|100\rangle|^2 \\ &= 4\left[\frac{\Sigma_x}{\Sigma_z} + \frac{\Sigma_z}{\Sigma_x} - 2\right] + \frac{1}{2}\left[\frac{\Sigma_x}{\Sigma_z} + \frac{\Sigma_z}{\Sigma_x} + 2\right] = 2.3, \quad \text{Q.E.D.}\end{aligned}$$

Concerning the factor of 1/2 in Eq. (16) of Ref. [2], it may look reasonable at first sight. However, we have counterarguments that we will explore in more detail elsewhere. A brief comment on our ideas is as follows. Expression (16) of Ref. [2] was derived using quantum mechanical methods based on variation after angular momentum projection (see in particular Ref. [7]), taking into account that $(2\mathcal{I}_i^{\mathrm{Yoc}})^{-1} \simeq (\mathcal{I}_i^{\mathrm{cr}})^{-1}$ for the anisotropic harmonic oscillator, where $\mathcal{I}_i^{\mathrm{Yoc}}$ and $\mathcal{I}_i^{\mathrm{cr}}$ denote Yoccoz and cranking moments of inertia, respectively. With this factor one gets a continuous transition from the triaxial to the axial case. Since the zero-point energy is a pure quantum mechanical correction, our classical intuition may fail. Thus, a discontinuity in going from the triaxial to the axial case may have to do with the fact that in the axial case rotations around x and y axes are equivalent quantum mechanically (lead to the same rotational states). One possible way of seeing that there could be a discontinuity in going from the axial to the triaxial case is as follows. In the axial case the intrinsic state will correspond to the $K=0$ band in ^{10}Be, the members of which have angular momenta $J=0$, 2, and 4. The triaxial intrinsic state contains both this $K=0$ band but also a $K=2$ band with angular momenta $J=2$, 3, and 4.

In any case we note that the consequence of putting in the 1/2 factor in Eq. (16) would be that the ground state of ^{10}Be would not be pushed down so much. This could result in about 5 MeV lower excitation energies of the (2p-2h) and (0p-0h) axial configurations. To summarize, putting in this factor would reduce by 5 MeV the excitation energies given in Table VIII for ^{10}Be, But none of these will have any effect on the intruders in ^{8}Be, which is the main point of our paper and of these comments. On the contrary, it would make the point even more dramatically that one can have low-lying intruders in ^{10}Be, but not in ^{8}Be.

Barker claims that other 0^+ low-lying nonintruder states can be found with the anisotropic harmonic oscillator model in ^{12}C and ^{8}Be that are in disagreement with experiment. We think that if spin and isospin symmetries are taken into account, there are no such states at these low energies. In any event, we would like to stress that the shell-model calculations in Ref. [2] stand by themselves and are independent of the deformed oscillator model. Indeed, we introduced this model in an effort to look for intruder states in ^{8}Be within a complementary scheme. We would add that we have previously studied the deformed oscillator model and compared it with the Nilsson model and Skyrme-Hartree-Fock model, especially in the context of intruder states. We find that these models track very nicely in light nuclei [8].

In conclusion, Barker and we should focus on the main point: are there low-lying intruder states in ^{8}Be? As far as we can tell he describes our calculations as ''unrealistic'' simply because he does not like our conclusions. Indeed, Barker had been sent calculations by other physicists, using the Arizona interaction which gives results that completely agree with ours [4]. Perhaps something good will come out of this controversy. Barker has been correct over the years in emphasizing the importance of using the *R*-matrix theory for various problems involving the continuum. But one is now realizing that one cannot take the *R*-matrix theory ''off the shelf.'' No one is more suited than Barker to lead the way to showing what changes need to be made in the application of this theory so that it can become a reliable tool in dealing with fundamental problems in nuclear physics. But the use of the *R*-matrix theory cannot be separated from nuclear structure at both the technical and intuitive level.

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