Empirical correlation between two-phonon $E1$ transition strengths in vibrational nuclei

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The correlation between the $E1$ transition strengths of the $1_1^- \rightarrow 0_1^+ \gamma$ transition and the $3_1^- \rightarrow 2_1^+ \gamma$ transition in heavy vibrational nuclei is investigated. For a given nucleus the $B(E1)$ values are equal within a factor of 2, although for different nuclei these values can differ by two orders of magnitude. This correlation points to a common origin of these $E1$ strengths, which supports the quadrupole-octupole coupled two-phonon interpretation of the 1_1^- state in vibrators. The observations can be accounted for within a factor of 2 by the simple bosonic phonon picture in the collective model. [S0556-2813(99)03705-X]

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Electric dipole ($E1$) transitions could principally be the most efficient way for the electromagnetic decay of excited quantum systems, which are formed by charged particles. The best-known examples are the electronic transitions in atoms. In complex atomic nuclei, however, $E1$ decay transitions of bound states are strongly hindered [1,2]. This fact is attributed [3,4] to the repulsive character of the dipole-dipole residual interaction between nucleons in nuclear matter, which leads to a concentration of $E1$ strength in the high-lying, unbound giant dipole resonance (GDR). $E1$ transitions between low-lying states are very weak on the scale of the GDR strength: less than 10^{-3} [5]. Therefore, $E1$ transitions between low-lying states are considered second-order effects due to the fermionic structure of phonons [6,7] or they are attributed [8–11] to small admixtures in the wave functions, which allow for collective $E1$ transitions. Even a small fraction of a collective $E1$ transition can lead to a much larger transition probability than a collective transition of other multipolarities. Therefore, $E1$ transitions can still dominate the decay behavior of low-lying states, although the responsible part of the wave function may be very small. Consequently, the theoretical prediction of absolute $E1$ transition strengths between low-lying states requires a precise knowledge of the wave functions, leaving the understanding of $E1$ decay properties of low-lying states one of the most complicated problems in nuclear structure physics [12]. Recently, mean-field calculations [13,14] have questioned the importance of a coupling of the GDR to the low-lying states with respect to the $E1$ transition strengths between low-lying states.

Understanding the low-lying $E1$ transition spectrum in heavy nuclei can gain much simplification if experimental information shows certain systematic relations between the strengths of different low-lying $E1$ transitions. Such relations are the more interesting if the strongest low-lying $E1$ transitions are concerned. In this Brief Report I report on the fact that in heavy even-even vibrational nuclei the $E1$ transition strength from the lowest $J^\pi = 1^-$ state to the ground state is correlated to the $E1$ transition strength between the $J^\pi = 3_1^-$ octupole phonon state and the $J^\pi = 2_1^+$ quadrupole phonon state. This correlation is particularly interesting because the 1_1^- state in vibrational nuclei is considered a quadrupole-octupole coupled two-phonon state (see, e.g., [15–26]).

During the last years the level width of the 1_1^- state, and hence the strength of the $1_1^- \rightarrow 0_1^+$ $E1$ transition, has been measured in systematic photon scattering experiments [27,28] on many vibrational, closed shell, or nearly closed shell nuclei. A quadrupole-octupole coupled two-phonon structure has been assigned to the 1_1^- states in many semimagic and nearly semimagic vibrational nuclei. The two-phonon character of the 1_1^- states was concluded from three hitherto known facts: (i) the experimental systematics for excitation energies and $E1$ transition strengths shows a smooth mass dependence, which hints at a collective (two-phonon?) nature, (ii) the excitation energy closely correlates to—and within 10% equals—the sum energy of the 2^+ and 3^- phonons [29], and finally (iii) the $1_1^- \rightarrow 0_1^+$ transition has a relatively large $E1$ strength of about a milli single-particle unit [milli Weisskopf units (mW.u.)], which on the one hand is three orders of magnitude smaller than the strength of the GDR but on the other hand is one to three orders of magnitude stronger than typical low-lying $E1$ transitions, which again could originate in a certain collectivity. Recently, the two-phonon interpretation of the 1_1^- states in the semimagic nuclei ^{142}Nd and ^{144}Sm has got strong support from $(p,p'\gamma)$ coincidence experiments: The collectivity of the $1_1^- \rightarrow 3_1^-$ one-quadrupole-phonon annihilating $E2$ transition to the octupole phonon state was measured [23,29] to be equal to the $2_1^+ \rightarrow 0_1^+$ one-quadrupole-phonon annihilating $E2$ transition from the quadrupole phonon state to the ground state. This observation agrees with the prediction expected for a two-phonon structure.

However, the relatively strong $1^- \rightarrow 0_1^+ E1$ transition, which is by far the strongest decay channel of the two-phonon 1^- state, is often considered to lie outside the two-phonon interpretation. In a purely bosonic picture, where additionally the $E1$ transition operator is assumed to be a one-body operator, the $1^- \rightarrow 0_1^+$ transition vanishes if the 1^- state is a two-phonon state. Microscopic calculations [11] can trace back the $E1$ decay strength of the two-phonon 1_1^- state to small admixtures of the GDR to the dominant two-phonon part of the wave function. Values of the necessary mixing matrix elements have been determined from data on vibrational and deformed rotational nuclei [9].

Another $E1$ transition between low-lying states in vibrational nuclei is the transition between the 3_1^- octupole pho-

TABLE I. Measured $E1$ transition strengths between low-lying states in vibrational nuclei. Displayed are the excitation energies of the two-phonon 1^- state and the 3^- octupole phonon state and the corresponding $E1$ transition strengths to the ground state and to the 2^+ quadrupole phonon state.

Nuclide	$E(1^-)$ [keV]	$B(E1;1^-\rightarrow 0_1^+)$ [10^{-3} e^2 fm^2]	Ref.	$E(3^-)$ [keV]	$B(E1;3^-\rightarrow 2_1^+)$ [10^{-3} e^2 fm^2]	Ref.
^{144}Sm	3225	6.5(9)	[23]	1810	5.0(7)	[31]
^{144}Nd	2185	3.2(2)	[24]	1511	1.8(2)	[20]
^{142}Nd	3425	5.8(12)	[29,32]	2084	7.5(35)	[33]
^{140}Ce	3643	5.6(3)	[22]	2464	6.6	[34]
^{124}Sn	3490	2.0(2)	[21]	2614	2.0(2)	[35]
^{122}Sn	3359	2.4(1)	[36]	2493	2.2(2)	[35]
^{120}Sn	3279	2.5(1)	[36]	2401	2.0(2)	[35]
^{118}Sn	3271	2.4(1)	[36]	2325	2.3(4)	[35]
^{116}Sn	3334	2.2(2)	[21]	2266	1.7(6)	[35]
^{106}Pd	2485	0.42(2)	[37]	2084	0.20(9)	[38]
^{88}Sr	4744	0.9(2)	[39]	2734	0.764(4)	[40]
^{52}Cr	5544	0.7(1)	[26]	4563	0.36(6)	[41]

non state and the 2_1^+ quadrupole phonon state. With the exception of a few doubly closed shell nuclei, such as, e.g., ^{208}Pb and ^{146}Gd, the 3_1^- state has a larger excitation energy than the 2_1^+ state. For many vibrational nuclei the lifetime of the 3^- octupole phonon state [30] is known. A lifetime measurement of the octupole phonon state is usually considered a measurement of the octupole collectivity of a nuclide. This is only true if a decay transition to the ground state has been observed. Usually, the most intense decay channel of the 3_1^- state is the $E1$ decay transition to the 2_1^+ state. In most heavy vibrators the $3_1^-\rightarrow 2_1^+$ transition carries more than 90% of the total decay intensity. Therefore, a lifetime measurement of the 3_1^- state is moreover a measurement of the two-phonon $E1$ transition strength between the quadrupole phonon and the octupole phonon.

For those even-even vibrational nuclei, for which I have information about the $E1$ strength of the $1_1^-\rightarrow 0_1^+$ transition and the $3_1^-\rightarrow 2_1^+$ transition, the $B(E1)$ values are compared in Table I. Here 3^- states tabulated by Spear [30] were considered as octupole phonon states. Moreover, the lowest-lying strong $E1$ excitation was regarded for the analysis. This state is usually the first $J_i^\pi = 1_1^-$ state known experimentally. For some nuclei there may exist lower-lying 1^- states not yet observed. For clarity the excitation energies of the negative parity states, considered in this Brief Report, are given in Table I and the numeration indices will be dropped in the following. Semimagic even-A nuclei and even-even nuclei with two or four nucleons outside a closed shell were considered for the analysis. These nuclei are more or less vibrational and can at least qualitatively be understood in a harmonic phonon picture. In Fig. 1 the $B(E1;3^-\rightarrow 2_1^+)$ value is plotted versus the $B(E1;1^-\rightarrow 0_1^+)$ value observed in the same nuclide. A double-logarithmic scale is used because the $B(E1)$ values vary by approximately two orders of magnitude for the different nuclides and the data have relative errors of comparable size. Obviously, the plotted $B(E1)$ values are closely correlated.

Although these $E1$ transition strengths can differ by about two orders of magnitude for different nuclides, the ratio of the $B(E1;3^-\rightarrow 2_1^+)$ value to the $B(E1;1^-\rightarrow 0_1^+)$ value is constant within a factor of 2. Moreover, these $E1$ strengths are approximately equal. This fact is shown in Fig. 2, where the ratio $B(E1;1^-\rightarrow 0_1^+)/B(E1;3^-\rightarrow 2_1^+)$ is plotted versus the nuclear mass number A. From the systematic, approximate equality of the $E1$ strengths one must conclude a common origin. This correlation of $E1$ strengths can be considered as an additional support [6,31] for the quadrupole-octupole coupled character of the 1^- states, which is independent from the arguments reiterated above. In turn, a correct explanation for one of the $E1$ transitions must be able to explain the other, as well.

In the ideal phonon picture it is obvious what the $1^-\rightarrow 0_1^+$ transition and the $3^-\rightarrow 2_1^+$ transition have in common: The $3^-\rightarrow 2_1^+$ transition annihilates the octupole phonon and creates the quadrupole phonon. The $1^-\rightarrow 0_1^+$ transition annihilates both the octupole phonon and the quadrupole phonon. In both cases two phonons are changed simultaneously.

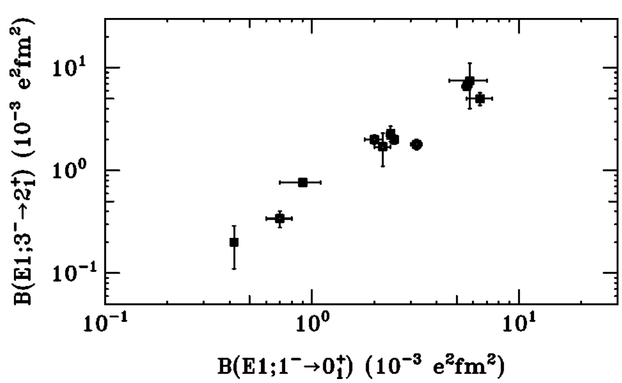

FIG. 1. Comparison of measured low-lying $E1$ transition strengths in vibrators. For each nuclide included in Table I the $B(E1;3^-\rightarrow 2_1^+)$ value is plotted versus the $B(E1;1^-\rightarrow 0_1^+)$ value. The scale is chosen as double logarithmic because the $B(E1)$ values cover about two orders of magnitude and their relative errors are comparable in size. There exists a close correlation between the $E1$ transition strengths of the $1^-\rightarrow 0_1^+$ transition and the $3^-\rightarrow 2_1^+$ transition.

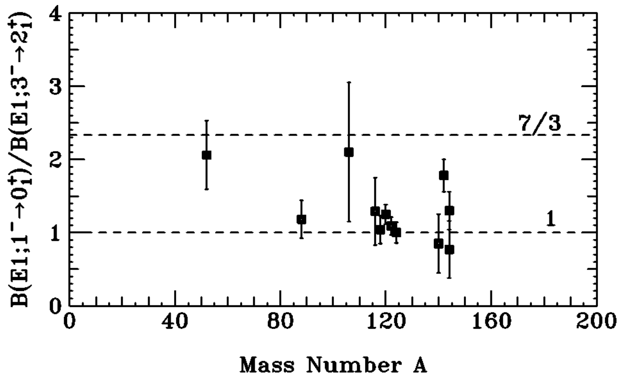

FIG. 2. Ratio of measured low-lying $E1$ transition strengths in vibrators. For each nuclide included in Table I the ratio of the $B(E1;1^-\rightarrow 0_1^+)$ value to the $B(E1;3^-\rightarrow 2_1^+)$ value is plotted versus the nuclear mass number A. For all heavy vibrational nuclei, for which sufficient data are available, the considered $E1$ strengths are equal within a factor of 2.

Two-phonon $E1$ transition operators have been considered before [11,31,42–44]. In the following these $E1$ transitions will be discussed quantitatively in terms of the simple bosonic phonon model [15–17] using the bosonic formulation of the quadrupole-octupole coupled $E1$ transition operator proposed by Strutinsky [45] and Bohr and Mottelson [46].

The basic low-lying phonons [47], considered here, are the quadrupole phonon and the octupole phonon. In the formalism of the second quantization [15] the operators b_λ^+ ($b_\lambda = b_\lambda^{+\dagger}$) denote the phonon creation (annihilation) operators for $\lambda=2$ and 3. It is assumed that the phonons are bosons, i.e., that they fulfill the boson commutation relations $[b_{\lambda_\mu}, b^+_{\lambda'_{\mu'}}] = \delta_{\lambda\lambda'}\delta_{\mu\mu'}$. The subscript μ denotes the z component of the spherical tensor $b^+_{\lambda_\mu}$. If $b^+_{\lambda_\mu}$ is a spherical tensor of rank λ, then $\tilde{b}_{\lambda_\mu} = (-)^{\lambda+\mu} b_{\lambda_{-\mu}}$ is a spherical tensor of rank λ, as well. To lowest order in terms of phonon operators the electric quadrupole and octupole transition operators are defined as

$$T(E\lambda) = e_\lambda (b_\lambda^+ + \tilde{b}_\lambda) \quad \text{for} \quad \lambda = 2,3. \tag{1}$$

where e_2 and e_3 denote effective quadrupole and octupole phonon charges. Without the assumption of an $E1$ phonon the electric dipole transition operator can be formed to lowest order from the basic quadrupole and octupole phonon operators as a two-phonon tensor product [45], which reads, in boson formulation,

$$T(E1) = e_1 [(b_2^+ + \tilde{b}_2)(b_3^+ + \tilde{b}_3)]^{(1)}. \tag{2}$$

The square brackets denote tensor coupling. The transition operators from Eqs. (1) and (2) transform under Hermitian conjugation as usual, $T(E\lambda)^\dagger_\mu = (-1)^{\lambda+\mu} T(E\lambda)_{-\mu}$ for $\lambda = 1,2,3$, and they commute, $[T(E\lambda)_\mu, T(E\lambda')_{\mu'}] = 0$.

The four states, which are involved in the $B(E1)$ values discussed above, have the following simple structure in the harmonic limit:

$$|0_1^+\rangle = |0\rangle, \tag{3}$$

$$|2_1^+\rangle = b_2^+|0\rangle, \tag{4}$$

$$|3_1^-\rangle = b_3^+|0\rangle, \tag{5}$$

$$|1_1^-\rangle = [b_2^+ b_3^+]^{(1)}|0\rangle. \tag{6}$$

Here $|0\rangle$ denotes the boson vacuum. Using these wave functions and the $E1$ transition operator from Eq. (2) one easily calculates the model predictions for the $B(E1)$ values considered here. One obtains

$$B(E1;3_1^-\rightarrow 2_1^+) = \frac{3}{7} e_1^2 \tag{7}$$

and

$$B(E1;1_1^-\rightarrow 0_1^+) = e_1^2. \tag{8}$$

In particular, the $B(E1)$ ratio

$$\frac{B(E1;1_1^-\rightarrow 0_1^+)}{B(E1;3_1^-\rightarrow 2_1^+)} = \frac{7}{3} \tag{9}$$

is a parameter-free prediction of this simple bosonic phonon model. This prediction is indicated as the upper dashed line in Fig. 2. Within a factor of 2, Eq. (9) accounts for the observed ratio of the $B(E1)$ strengths, which correspond to the simultaneous change of two phonons.

In the light of the fact that the simple phonon model can describe the ratio of the two strongest low-lying $E1$ transitions it is worthwhile to critically reflect on the collectivity of these transitions: The 2_1^+ state, the 3_1^- state, and the two-phonon 1^- state are collective states. In particular, the 1^- state is quadrupole collective and octupole collective but *not* $E1$ collective in the sense that an electric dipole phonon would form a large part of the wave function. From the correlation of the $E1$ transitions discussed above one concludes that these $E1$ transitions are generated from the coupling of the low-lying, isoscalar quadrupole and octupole phonons. Therefore, these $E1$ transitions have a collective origin and might be called *quadrupole-octupole collective* in contrast to the $E1$ collectivity of the GDR. The quadrupole-octupole collectivity of the $E1$ transition, which depopulates the two-phonon 1^- state, is in contrast to the noncollective character of $E1$ transitions, which depopulate some 1^- states observed [48,49] in vibrators near 6 MeV and which have dominantly a one-particle–one-hole ($1p$-$1h$) character [48–50] and, hence, gain their comparable large $E1$ strengths mainly from a fragmentation of the GDR. The quadrupole-octupole collectivity makes the $2^+\times 3^-$ two-phonon states interesting and unique objects for the study of low-lying $E1$ transitions.

To summarize, the empirical correlation between the $E1$ transition strengths of the $1_1^-\rightarrow 0_1^+$ transition and the $3_1^-\rightarrow 2_1^+$ transition in heavy vibrational nuclei was reported. For different nuclides the corresponding $B(E1)$ values can vary by about two orders of magnitude. For a given nucleus, however, these $B(E1)$ values are equal within a factor of 2. From this fact one can arrive at the common origin of these $E1$ strengths, which supports independently from hitherto used arguments the quadrupole-octupole coupled two-phonon nature of the 1_1^- state in vibrators. From the properties of the 2_1^+ quadrupole phonon and the 3_1^- octupole phonon, the bosonic phonon model predicts correctly the decay transition strengths (including the $E1$ decay) of the 1_1^- two-phonon state in vibrators within a factor of 2.

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