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Exchange current contributions to the charge radii of nucleons

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The exchange charge-density operators that correspond to the Fermi-invariant decomposition of quark-quark interactions have been constructed. Their effect on the electromagnetic charge radii of the nucleons, in combination with that of the relativistic corrections to the single-quark operator, has been studied with a constituent quark model with a spin and flavor dependent hyperfine and a linear confining interaction, which gives a quantitative description of the spectra for the light and strange baryons. The model gives proton and neutron charge radii in approximate agreement with the empirical results assuming reasonable values for the radii of the constituent quarks. [S0556-2813(99)01505-8]

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I. INTRODUCTION

The charge radii, along with electromagnetic form factors and magnetic moments, are observables that provide insight into the internal structure of the nucleons. From the measured electric form factors G_E of the nucleons the charge radius for the proton is found to be (in fm) 0.805 ± 0.011 [1], 0.81 ± 0.04 [2], 0.862 ± 0.012 [3], and 0.88 ± 0.03 [4]. The study of the charge form factor of the neutron is experimentally difficult due to the absence of pure neutron targets. Electron-deuteron scattering and scattering of slow neutrons off atomic electrons have, however, shown the mean-square charge radius of the neutron to be nonzero and negative. Typical experimental values for the mean-square neutron charge radius are (in fm^2), e.g., $-0.113\pm0.003\pm0.004$ [5] and -0.117 ± 0.002 [6] (when averaging [7] and [8]). Since the electric Sachs form factor G_E is defined as a linear combination of the Dirac and Pauli form factors F_1 and F_2 the charge radius of the nucleon can be divided into an ''intrinsic'' part, coming from F_1, and an ''anomalous magnetic moment'' part, coming from F_2 [9]. In the case of the neutron the main part of the charge radius in fact comes from this ''Pauli'' term, while the ''Dirac'' term will be small and positive [10]. Quark model calculations of the charge radii of the nucleons concern the intrinsic part. Due to the small mass of the (constituent) quarks relativistic corrections to the charge-density operator should also be taken into account in the impulse approximation.

In interquark interaction models that are flavor or velocity dependent (the interaction suggested in Ref. [11] being an example of the former) exchange current contributions will arise as a consequence of the continuity equation. The continuity equation links the terms in the charge-density operator of any given order in (v/c) to terms of the next order in (v/c) in the interaction. When the interaction is expressed only to order $(v/c)^2$ the continuity equation does not put any restraints on the exchange charge operator, which is of order $(v/c)^3$. The most obvious constraint, however, is that two-body contributions in the charge density have to have a vanishing volume integral. Exchange current contributions to the current operator have been included, e.g., in the work related in Ref. [12], where pion and gluon exchange current contributions were calculated and in Ref. [13], where also exchange current contributions from the confinement current were taken into account.

It is possible to express the quark-quark interaction in terms of the five relativistic Fermi spin invariants $SVTAP$ [14], corresponding to effective scalar, vector, tensor, axial vector, and pseudoscalar exchange. The corresponding exchange charge-density operators associated with these invariants have been derived in the present work for light and strange [SU(3)] quarks. The exchange current contributions to the charge radius of the nucleons have then been calculated using a recently developed phenomenological model for the quark-quark interaction, which in combination with a static linear confining interaction gives a good description of the spectra of the light and strange baryons [15].

The possibility that the constituent quarks would have a nontrivial electromagnetic structure, described by constituent quark form factors, has also been explored in this work. The total nucleon charge form factor, and subsequently the charge radius, would then also get contributions from the quark form factors. It is found that the quark model interaction of Ref. [15] leads to exchange current contributions, which, when combined with the impulse approximation and the anomalous magnetic moment part of the nucleon charge radii, while assuming reasonable values for the radii of the constituent quarks, give satisfactory values for the charge radii of the nucleons.

This paper is divided into six sections. Section II defines the charge form factor and the charge radius of the nucleon, taking into account relativistic corrections. In Sec. III the charge form factor and the charge radius of the nucleon is derived with inclusion of exchange current contributions, and in Sec. IV these results are applied to the phenomenological model of Ref. [15]. The possible charge form factor and radius of the constituent quarks, and their effect on the charge radii of the nucleons are discussed in Sec. V, and a summarizing discussion is given in Sec. VI.

II. THE CHARGE FORM FACTOR AND THE CHARGE RADIUS

The Sachs charge form factor $G_E(q^2)$ of a nucleon is defined as

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$$G_E(q^2)=F_1(q^2)-\frac{q^2}{4m_N^2}F_2(q^2), \tag{2.1}$$

where $F_1(q^2)$ is the Dirac form factor and $F_2(q^2)$ is the Pauli form factor. The form factors are normalized as $F_1(0)=Q$, i.e., the electric charge of the nucleon in units of e, and $F_2(0)=\kappa$, i.e., the anomalous magnetic moment of the nucleon in units of $(e/2m_N)$. The (mean square) charge radius of the nucleon can be calculated from the charge form factor as

$$\begin{aligned}\langle r^2\rangle_N&=-6\frac{dG_E(q^2)}{d(q^2)}\bigg|_{q^2=0}\\&=-6\left\{\frac{dF_1(q^2)}{d(q^2)}\bigg|_{q^2=0}-\frac{F_2(0)}{4m_N^2}\right\}\\&=-6\frac{dF_1(q^2)}{d(q^2)}\bigg|_{q^2=0}+\frac{3}{2}\frac{F_2(0)}{m_N^2}.\end{aligned} \tag{2.2}$$

The mean-square charge radius is thus divided into two parts, $\langle r^2\rangle_N=\langle r^2\rangle_{\mathrm{int},N}+\langle r^2\rangle_{\mathrm{an},N}$, where the first term comes from the intrinsic charge distribution of the nucleon while the second term arises from the anomalous magnetic moment of the nucleon, which for the proton is $\kappa_p=1.793$ and for the neutron is $\kappa_n=-1.913$. Using $F_{2,p}(0)=\kappa_p$, $F_{2,n}(0)=\kappa_n$, and $m_N=939$ MeV, the results are $\langle r^2\rangle_{\mathrm{an},p}=0.119\ \mathrm{fm}^2$ and $\langle r^2\rangle_{\mathrm{an},n}=-0.127\ \mathrm{fm}^2$. The remaining (intrinsic) part of $\langle r^2\rangle_N$ is interpreted as coming from the charge density of a system of three constituent quarks.

If a constituent quark (denoted here as i) is treated as a point Dirac particle without anomalous terms, its electromagnetic current operator can be expressed as

$$\langle p_i'|J_\mu(0)|p_i\rangle=i\overline{u}(p_i')\gamma_\mu Q^{(i)}u(p_i), \tag{2.3}$$

when p_i and p_i' denote the initial and the final momenta, respectively, and

$$Q^{(i)}=\frac{1}{2}\left[\lambda_3^{(i)}+\frac{1}{\sqrt{3}}\lambda_8^{(i)}\right] \tag{2.4}$$

(in units of charge e) is the [SU(3)] charge operator of the quark i. An argument for the absence of anomalous current terms is given in [16]. Since $J_\mu=(\mathbf{J},i\rho)$ the electromagnetic charge-density operator is

$$\begin{aligned}\langle p_i'|\rho|p_i\rangle&=\overline{u}(p_i')\gamma_4 Q^{(i)}u(p_i)\\&=\sqrt{\frac{(E_i'+m)(E_i+m)}{4E_i'E_i}}\\&\quad\times\left[1+\frac{\mathbf{p}_i'\cdot\mathbf{p}_i+i\boldsymbol{\sigma}^{(i)}\cdot\mathbf{p}_i'\times\mathbf{p}_i}{(E_i'+m)(E_i+m)}\right]Q^{(i)},\end{aligned} \tag{2.5}$$

where m is the mass of the constituent quark and $E_i=\sqrt{\mathbf{p}_i^2+m^2}$. By introducing a velocity operator $\mathbf{v}_i=(1/2m)(\mathbf{p}_i'+\mathbf{p}_i)$ and the momentum transfer $\mathbf{q}=\mathbf{p}_i'-\mathbf{p}_i$, the momentum operators $\mathbf{p}_i$ and $\mathbf{p}_i'$ can be expressed as

$$\begin{aligned}\mathbf{p}_i&=m\mathbf{v}_i-\frac{\mathbf{q}}{2},\\ \mathbf{p}_i'&=m\mathbf{v}_i+\frac{\mathbf{q}}{2},\end{aligned} \tag{2.6}$$

and the expression in Eq. (2.5) for the charge-density operator will, to lowest order in q^2, be

$$\langle p_i'|\rho|p_i\rangle=\left(1-\frac{q^2}{8m^2}\right)Q^{(i)}. \tag{2.7}$$

The above expression in Eq. (2.7) is then the impulse approximation with the (relativistic) Darwin-Foldy correction. The spin-orbit term in Eq. (2.5) is linear in $\mathbf{q}$ and gives no contribution for the ground-state baryons, and has therefore been left out in Eq. (2.7). A charge form factor (which is interpreted as the Dirac part of the electric Sachs form factor) can now be calculated as the Fourier transform of the charge-density operator.

When using a three-body wave function that is symmetric with respect to the combined spin, flavor, and spatial coordinates, the charge form factor of a system of three (constituent) quarks with the same mass m will then in the impulse approximation be

$$\begin{aligned}F_{C,\mathrm{IA}}(q^2)&=3\langle Q^{(1)}\rangle\int d^3r_1d^3r_2d^3r_3\\&\quad\times|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)|^2e^{i\mathbf{q}\cdot\mathbf{r}_1}\left[1-\frac{q^2}{8m^2}\right]\\&=3\langle Q^{(1)}\rangle\int d^3r_1d^3r_2d^3r_3|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)|^2\\&\quad\times\left[1-\frac{q^2r_1^2}{6}+\mathcal{O}(q^4)\right]\left[1-\frac{q^2}{8m^2}\right],\end{aligned} \tag{2.8}$$

where in the expansion of the factor $e^{i\mathbf{q}\cdot\mathbf{r}_1}$ only the spatial scalar part needs to be taken into account for a ground-state baryon. The terms indicated by $\mathcal{O}(q^4)$ are of higher order in q^2. The impulse approximation (with relativistic corrections) for the corresponding mean-square charge radius can then be calculated as $\langle r^2\rangle_{\mathrm{IA}}=-6[dF_{C,\mathrm{IA}}/d(q^2)]|_{q^2=0}$, giving

$$\langle r^2\rangle_{\mathrm{IA}}=3\langle Q^{(1)}\rangle\int d^3r_1d^3r_2d^3r_3|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)|^2\left[r_1^2+\frac{3}{4m^2}\right]. \tag{2.9}$$

III. EXCHANGE CURRENT CONTRIBUTIONS

The interaction between quarks may be decomposed in terms of Fermi invariants as

$$V=\sum_{j=1}^{5}v_jF_j, \tag{3.1}$$

where $F_j=S,V,T,A,P$, defined as [14]

$$S=1^{(1)}1^{(2)},\quad V=\gamma_\mu^{(1)}\gamma_\mu^{(2)},\quad T=\frac{1}{2}\sigma_{\mu\nu}^{(1)}\sigma_{\mu\nu}^{(2)},$$

$$A=i\gamma_5^{(1)}\gamma_\mu^{(1)}i\gamma_5^{(2)}\gamma_\mu^{(2)},\quad P=\gamma_5^{(1)}\gamma_5^{(2)}. \tag{3.2}$$

Exchange current corrections to the charge-density operator ρ for a two-quark system will arise as contact current terms in the nonrelativistic reduction of the five Fermi spin invariants [17]. The exchange charge-density operators ρ_j, $j=1\ldots5$, corresponding to the $SVTAP$ decomposition, consist of flavor-independent and flavor-dependent parts ρ_j^+ and ρ_j^-, respectively.

The simplest method for deriving the flavor-independent exchange charge-density term ρ_1^+, associated with effective scalar exchange, is the following. If $v_1^+(\mathbf{k})$ is the corresponding flavor-independent potential, performing a mass shift $m\to m^*=m+v_1^+$ in the relativistic Darwin-Foldy correction term in Eq. (2.7) leads, to lowest order in v_1^+, to the exchange charge contribution

$$\begin{aligned}\rho_1^+(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})&=\rho_1^+(\mathbf{k}_2,\mathbf{q})+\rho_1^+(\mathbf{k}_1,\mathbf{q})\\&=\frac{q^2}{8m^2}\cdot\frac{2v_1^+(\mathbf{k}_2)}{m}Q^{(1)}\\&\quad+\frac{q^2}{8m^2}\cdot\frac{2v_1^+(\mathbf{k}_1)}{m}Q^{(2)},\end{aligned} \tag{3.3}$$

where $\mathbf{k}_2$ and $\mathbf{k}_1$ are the momentum transfers from quark 2 to quark 1 and from quark 1 to quark 2, respectively, while $\mathbf{q}$ is the momentum transfer to the two-quark system.

The more general way of constructing all of the exchange charge-density operators ρ_j to lowest order in (v/c) is by decomposing the quark propagators in the (relativistic) Born terms in the $\gamma qq\to qq$ amplitudes into positive and negative energy components, retaining only the negative ones (cf. calculations of exchange current corrections to two-nucleon systems in Refs. [17] and [18]), giving

$$\begin{aligned}\rho_j(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})&=\rho_j(\mathbf{k}_2,\mathbf{q})+\rho_j(\mathbf{k}_1,\mathbf{q})\\&=\frac{1}{8m^3}O_j\Bigg(2Q^{(1)}v_j^+(\mathbf{k}_2)\\&\quad+\left\{Q^{(1)},\sum_{k=1}^{8}\lambda_k^{(1)}\lambda_k^{(2)}\right\}v_j^-(\mathbf{k}_2)\Bigg)+(1\leftrightarrow2),\end{aligned} \tag{3.4}$$

where v_j^+ and v_j^- are flavor-independent and flavor-dependent potentials, respectively, and $(1\leftrightarrow2)$ is a term with the coordinates of quarks 1 and 2 exchanged.

The operators O_j, $j=1\ldots5$, corresponding to effective scalar, vector, tensor, axial vector, and pseudoscalar exchange mechanisms, will be

$$O_1=q^2+2i\boldsymbol{\sigma}^{(1)}\cdot\mathbf{P}_1\times\mathbf{q}, \tag{3.5a}$$

$$O_2=\mathbf{q}\cdot\mathbf{k}_2+(\boldsymbol{\sigma}^{(2)}\times\mathbf{k}_2)\cdot(\boldsymbol{\sigma}^{(1)}\times\mathbf{q})-2i\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}\times\mathbf{P}_2, \tag{3.5b}$$

$$\begin{aligned}O_3&=\mathbf{q}\cdot\mathbf{k}_2+\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\mathbf{q}\cdot\mathbf{k}_2-\boldsymbol{\sigma}^{(2)}\cdot\mathbf{q}\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}_2-\mathbf{q})\\&\quad+2i\boldsymbol{\sigma}^{(2)}\cdot\mathbf{P}_2\times\mathbf{q},\end{aligned} \tag{3.5c}$$

$$\begin{aligned}O_4&=q^2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}+2i\boldsymbol{\sigma}^{(2)}\cdot\mathbf{P}_1\times\mathbf{q}-\boldsymbol{\sigma}^{(2)}\cdot\mathbf{q}\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}\\&\quad+\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{k}_2,\end{aligned} \tag{3.5d}$$

$$O_5=\boldsymbol{\sigma}^{(2)}\cdot\mathbf{k}_2\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}. \tag{3.5e}$$

In the expressions above $\mathbf{P}_1=\frac{1}{2}(\mathbf{p}_1+\mathbf{p}_1{}')$ and $\mathbf{P}_2=\frac{1}{2}(\mathbf{p}_2+\mathbf{p}_2{}')$, where $\mathbf{p}_1$ and $\mathbf{p}_2$ are the initial quark momenta, and $\mathbf{p}_1{}'$ and $\mathbf{p}_2{}'$ are the final quark momenta.

For simplicity the following notations are used: $\{Q^{(1)},\Sigma_{k=1}^8\lambda_k^{(1)}\lambda_k^{(2)}\}\equiv Q^{(12)}$, $\{Q^{(2)},\Sigma_{k=1}^8\lambda_k^{(2)}\lambda_k^{(1)}\}\equiv Q^{(21)}$. The operator $Q^{(ij)}$ can be cast in the form

$$\begin{aligned}Q^{(ij)}&=\frac{2}{3}\left(\lambda_3^{(j)}+\frac{1}{\sqrt{3}}\lambda_8^{(j)}\right)+\frac{1}{\sqrt{3}}(\lambda_8^{(i)}\lambda_3^{(j)}+\lambda_3^{(i)}\lambda_8^{(j)})\\&\quad+\frac{1}{3}\sum_{k=1}^{5}\lambda_k^{(i)}\lambda_k^{(j)}-\frac{2}{3}\sum_{k=6}^{7}\lambda_k^{(i)}\lambda_k^{(j)}-\frac{1}{3}\lambda_8^{(i)}\lambda_8^{(j)}.\end{aligned} \tag{3.6}$$

The general form for an exchange current contribution to the charge form factor can be expressed as

$$\begin{aligned}F_{C,\mathrm{ex}}(q^2)&=3\Bigg\langle\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}e^{i(\mathbf{k}_1\cdot\mathbf{r}_1+\mathbf{k}_2\cdot\mathbf{r}_2)}(2\pi)^3\\&\quad\times\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{q})\rho_{\mathrm{ex}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})\Bigg\rangle\\&=3\left\langle e^{i\mathbf{q}\cdot\mathbf{r}_1}\int\frac{d^3k_2}{(2\pi)^3}e^{-i\mathbf{k}_2\cdot\mathbf{r}_{12}}\rho_{\mathrm{ex}}(\mathbf{k}_2,\mathbf{q})\right\rangle+(1\leftrightarrow2),\end{aligned} \tag{3.7}$$

where ρ_{ex} is the exchange charge-density operator and $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$. The corresponding contribution to the mean-square charge radius is then $\langle r^2\rangle_{\mathrm{ex}}=-6[dF_{C,\mathrm{ex}}/d(q^2)]|_{q^2=0}$. The total (intrinsic) charge form factor is then $F_C=F_{C,\mathrm{IA}}\cdot F_{C,\mathrm{ex}}$, i.e.,

$$\begin{aligned}\left(1-\frac{r^2q^2}{6}+\mathcal{O}(q^4)\right)&=\left(1-\frac{r_{\mathrm{IA}}^2q^2}{6}+\mathcal{O}(q^4)\right)\\&\quad\times\left(1-\frac{r_{\mathrm{ex}}^2q^2}{6}+\mathcal{O}(q^4)\right)\\&=\left(1-\frac{(r_{\mathrm{IA}}^2+r_{\mathrm{ex}}^2)q^2}{6}+\mathcal{O}(q^4)\right),\end{aligned} \tag{3.8}$$

giving an intrinsic mean-square charge radius defined as $\langle r^2\rangle_{\mathrm{int}}=\langle r^2\rangle_{\mathrm{IA}}+\langle r^2\rangle_{\mathrm{ex}}$.

Taking into account only the spatial scalar component of the exchange charge-density operators for a ground-state baryon, and noting that the matrix elements of terms in Eq. (3.5) that contain $\mathbf{P}_1$ or $\mathbf{P}_2$ will be small, the expressions for the charge-density operators ρ_j can be simplified. The contribution to the charge-density operator from scalar meson exchange mechanisms will then be

$$\rho_1(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = \frac{1}{8m^3}q^2 \times[2Q^{(1)}v_1^+(\mathbf{k}_2) + Q^{(12)}v_1^-(\mathbf{k}_2)] + (1\leftrightarrow 2). \tag{3.9}$$

For vector meson exchange mechanisms the corresponding expression is

$$\rho_2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = \frac{1}{8m^3}\left[\mathbf{q}\cdot\mathbf{k}_2 + 2\mathbf{q}\cdot\mathbf{k}_2\frac{\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}}{3}\right] \times[2Q^{(1)}v_2^+(\mathbf{k}_2) + Q^{(12)}v_2^-(\mathbf{k}_2)] + (1\leftrightarrow 2), \tag{3.10}$$

and the exchange charge-density operator for effective tensor exchange will be

$$\rho_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = \frac{1}{8m^3}\left[\mathbf{q}\cdot\mathbf{k}_2 + (2\mathbf{q}\cdot\mathbf{k}_2 + q^2)\frac{\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}}{3}\right] \times[2Q^{(1)}v_3^+(\mathbf{k}_2) + Q^{(12)}v_3^-(\mathbf{k}_2)] + (1\leftrightarrow 2). \tag{3.11}$$

The contribution to the exchange charge-density operator from axial vector exchange mechanisms is derived as

$$\rho_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = \frac{1}{8m^3}[2q^2 + \mathbf{q}\cdot\mathbf{k}_2]\frac{\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}}{3} \times[2Q^{(1)}v_4^+(\mathbf{k}_2) + Q^{(12)}v_4^-(\mathbf{k}_2)] + (1\leftrightarrow 2), \tag{3.12}$$

and, finally, the contribution from effective pseudoscalar meson exchange can be derived as

$$\rho_5(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = \frac{1}{8m^3}\mathbf{q}\cdot\mathbf{k}_2\frac{\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}}{3} \times[2Q^{(1)}v_5^+(\mathbf{k}_2) + Q^{(12)}v_5^-(\mathbf{k}_2)] + (1\leftrightarrow 2). \tag{3.13}$$

IV. NUCLEON CHARGE RADII IN THE CHIRAL CONSTITUENT QUARK MODEL

As an illustration of how exchange currents may contribute to the total charge radius of the nucleon the above formalism has been applied to a quark-quark interaction model that is flavor dependent. In Ref. [15] a phenomenological model for the hyperfine interactions between quarks, combined with a static linear confining interaction, was used to get a satisfactory description of the spectra of the light and strange baryons. The Hamiltonian of the model is of the form

$$H = \sum_{i=1}^{3}\sqrt{\mathbf{p}_i^2 + m_i^2} + \sum_{i<j}^{3} V(\mathbf{r}_{ij}). \tag{4.1}$$

The quark-quark interaction $V(\mathbf{r}_{ij})$ consists of a (linear) confining part $V_{\text{conf}}(\mathbf{r}_{ij})$, and a hyperfine interaction term $V_\chi^{\text{octet}}(\mathbf{r}_{ij})$ of the form

$$V_\chi^{\text{octet}}(\mathbf{r}_{12}) = \left\{\sum_{k=1}^{3} V_\pi(\mathbf{r}_{12})\lambda_k^{(1)}\lambda_k^{(2)} + \sum_{k=4}^{7} V_K(\mathbf{r}_{12})\lambda_k^{(1)}\lambda_k^{(2)} + V_\eta(\mathbf{r}_{12})\lambda_8^{(1)}\lambda_8^{(2)}\right\}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}, \tag{4.2}$$

where λ_k and $\boldsymbol{\sigma}$ are the flavor and spin matrices of the quarks. In the full interquark potential of Ref. [15] also a singlet exchange term was included. This smaller term will not be considered here. The spatial part of V_χ^{octet} for the model is taken to have the form

$$V_\gamma(r_{ij}) = \frac{g_\gamma^2}{4\pi}\frac{1}{3}\frac{1}{4m_im_j}\left\{\mu_\gamma^2\frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - \lambda_\gamma^2\frac{e^{-\lambda_\gamma r_{ij}}}{r_{ij}}\right\}, \tag{4.3}$$

where $\gamma = \pi,K,\eta$, $g_\gamma^2/4\pi = 0.67$ for π,K,η, m_i is the constituent quark mass, corresponding to either $m_u = m_d =$ 340 MeV or $m_s =$ 500 MeV, and $\lambda_\gamma = \lambda_0 + \kappa\mu_\gamma$, with $\lambda_0 = 2.87$ fm^{-1} and $\kappa = 0.81$.

This model (one version of the so-called chiral constituent quark model) will be used below, combined with an approximation of the three-body wave function of Ref. [15]. The wave-function approximation is of the form $\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = (m\omega/\pi)^{3/2}e^{i\mathbf{P}\cdot\mathbf{R}}e^{-(m\omega/2)(r^2+\rho^2)}$, with an effective oscillator frequency $\omega = 1240$ MeV, corresponding to an estimate of the impulse approximation radius without relativistic corrections in the model of Ref. [15]. The coordinates $\mathbf{R}$, $\mathbf{r}$, and $\boldsymbol{\rho}$ are defined as

$$\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3}{\sqrt{3}},$$

$$\mathbf{r} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} = \frac{\mathbf{r}_{12}}{\sqrt{2}},$$

$$\boldsymbol{\rho} = \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}}. \tag{4.4}$$

In the calculations the (renormalized) center-of-mass coordinate $\mathbf{R}$ is removed. With this wave function the impulse approximation (with relativistic corrections) for the (intrinsic) mean-square charge radius can be calculated from Eq. (2.9) as

$$\langle r^2\rangle_{\text{IA}} = 3\langle Q^{(1)}\rangle\left[\frac{1}{m\omega} + \frac{3}{4m^2}\right]. \tag{4.5}$$

The static linear confining interaction of Ref. [15] is spin and flavor independent, and can formally be viewed as a static approximation to a scalar exchange interaction, the sign of which is positive instead of negative as for a conventional scalar exchange interaction [13]. In the model of Ref. [15] the confining potential is

$$\tilde{v}_{\text{conf}}(\mathbf{r}_{12}) = Cr_{12} + V_0, \tag{4.6}$$

where the value of the parameter C is 2.33 fm^{-2} and where $V_0 = -416$ MeV. With the approximated wave function the confinement contribution to the charge form factor can be calculated from Eqs. (3.7) and (3.9), where $v_1^+(\mathbf{k})$ is the inverse Fourier transform of the confining potential. The corresponding mean-square charge radius will then be

$$\langle r^2\rangle_{\mathrm{conf}} = -\frac{9}{m^3}\left[\sqrt{\frac{2}{\pi m\omega}}C + \frac{V_0}{2}\right]\langle Q^{(1)} + Q^{(2)}\rangle R(\omega). \tag{4.7}$$

The factor $R(\omega)$ represents a relativistic correction to the exchange charge-density operator, originating in the spinor normalization factor and the energy denominator in the small component of the quark wave function (see Ref. [19] for a discussion on this factor in the context of exchange magnetic moment operators), the value of which is 0.28 for $\omega = 1240$ MeV. The confinement contribution can, as is readily seen from the above equations, be viewed as a one-body contribution to the charge radius because of the equality $\langle Q^{(1)}\rangle = \langle Q^{(2)}\rangle$.

The potential function $v(\mathbf{k})$ of the hyperfine interaction can be obtained as the inverse Fourier transform of [19]

$$\tilde{v}_\gamma(\mathbf{r}_{12}) = \frac{4m^2}{r_{12}}\int_{r_{12}}^{\infty} dr' \int_{r'}^{\infty} dr'' r'' f_\gamma(r''), \tag{4.8}$$

where $f_\gamma(r)$ is obtained from the relation $V_\gamma(\mathbf{r}_{12}) = \frac{1}{3} f_\gamma(r_{12})$ in combination with Eq. (4.3), thus giving

$$f_\gamma(r) = \frac{g_\gamma^2}{4\pi}\frac{1}{4m^2}\left\{\mu_\gamma^2 \frac{e^{-\mu_\gamma r}}{r} - \lambda_\gamma^2 \frac{e^{-\lambda_\gamma r}}{r}\right\}. \tag{4.9}$$

The hyperfine interaction can in this model be interpreted as coming solely from pseudoscalar exchange mechanisms, since the volume integral of the interaction vanishes. The relativistic correction $R(\omega)$ should also be taken into account and, using Eq. (3.13), one finally gets for the effective pseudoscalar (P) meson exchange contribution to the mean-square charge radius

$$\langle r^2\rangle_P = \frac{\pi}{2m^3}\left(\frac{m\omega}{\pi}\right)^{3/2}\langle \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}Q^{(12)} + \boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\sigma}^{(1)}Q^{(21)}\rangle \times \int_0^\infty dr\, r^3 e^{-m\omega r^2}\frac{\partial}{\partial r}\tilde{v}(\sqrt{2}r)R(\omega). \tag{4.10}$$

In the calculations the spin-flavor part of Eq. (4.10) is divided into terms that correspond to the different potential functions $\tilde{v}_\gamma$ defined in Eq. (4.8). This is done by noting that for nucleons the flavor operator $Q^{(ij)}$ in Eq. (3.6) can be written as $Q^{(ij)} = Q_\pi^{(ij)} + Q_\eta^{(ij)} + Q_{\pi+\eta}^{(ij)}$, where $Q_\pi^{(ij)}$, $Q_\eta^{(ij)}$, and $Q_{\pi+\eta}^{(ij)}$ are associated with the potential functions $\tilde{v}_\pi$, $\tilde{v}_\eta$, and $\frac{1}{2}(\tilde{v}_\pi + \tilde{v}_\eta)$, respectively. The matrix elements of the spin-flavor parts of the exchange current contributions are given in Table I. The numerical values for the mean-square charge radius contributions from the impulse approximation (with relativistic correction), from confinement and from pseudoscalar exchange mechanisms are given for the proton and the neutron in Table II.

TABLE I. The matrix elements of the spin-flavor operators for the proton and the neutron: $Q^{(1)} = \frac{1}{2}[\lambda_3^{(1)} + 1/\sqrt{3}\lambda_8^{(1)}]$, $Q^{(2)} = \frac{1}{2}[\lambda_3^{(2)} + 1/\sqrt{3}\lambda_8^{(2)}]$, $Q_\pi^{(12)} = \frac{2}{3}\lambda_3^{(2)} + \Sigma_{k=1}^3\lambda_k^{(1)}\lambda_k^{(2)}$, $Q_\pi^{(21)} = \frac{2}{3}\lambda_3^{(1)} + \frac{1}{3}\Sigma_{k=1}^3\lambda_k^{(2)}\lambda_k^{(1)}$, $Q_\eta^{(12)} = 2/(3\sqrt{3})\lambda_8^{(2)} - \frac{1}{3}\lambda_8^{(1)}\lambda_8^{(2)}$, $Q_\eta^{(21)} = 2/(3\sqrt{3})\lambda_8^{(1)} - \frac{1}{3}\lambda_8^{(2)}\lambda_8^{(1)}$, $Q_{\pi+\eta}^{(12)} = 1/\sqrt{3}(\lambda_8^{(1)}\lambda_3^{(2)} + \lambda_3^{(1)}\lambda_8^{(2)})$, $Q_{\pi+\eta}^{(21)} = 1/\sqrt{3}(\lambda_8^{(2)}\lambda_3^{(1)} + \lambda_3^{(2)}\lambda_8^{(1)})$, $\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}Q_\pi^{(12)}$, $\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\sigma}^{(1)}Q_\pi^{(21)}$, $\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}Q_\eta^{(12)}$, $\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\sigma}^{(1)}Q_\eta^{(21)}$, $\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}Q_{\pi+\eta}^{(12)}$, and $\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\sigma}^{(1)}Q_{\pi+\eta}^{(21)}$.

	p	n
$\langle Q^{(1)}\rangle = \langle Q^{(2)}\rangle$	$\frac{1}{3}$	0
$\langle\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}Q_\pi^{(12)}\rangle = \langle\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\sigma}^{(1)}Q_\pi^{(21)}\rangle$	$\frac{17}{9}$	$\frac{13}{9}$
$\langle\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}Q_\eta^{(12)}\rangle = \langle\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\sigma}^{(1)}Q_\eta^{(21)}\rangle$	$-\frac{1}{9}$	$-\frac{1}{9}$
$\langle\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}Q_{\pi+\eta}^{(12)}\rangle = \langle\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\sigma}^{(1)}Q_{\pi+\eta}^{(21)}\rangle$	$\frac{2}{9}$	$-\frac{2}{9}$

V. CHARGE FORM FACTORS OF THE QUARKS

In the previous sections the constituent quarks have been treated as pointlike objects without internal structure. If, on the other hand, the constituent quarks have a nontrivial electromagnetic structure, i.e., they are assumed to be dressed by their mesonic (quark-antiquark) polarization clouds, quark form factors can be introduced. The charge operator $Q^{(i)}$ in Eq. (2.4) can be expressed as $Q^{(i)}(0) = \frac{1}{2}F_3(0)\lambda_3^{(i)} + 1/(2\sqrt{3})F_8(0)\lambda_8^{(i)}$, where F_3 and F_8 are possible quark form factors, normalized as $F_3(0) = F_8(0) = 1$, and the expression for $Q^{(i)}(q^2)$ is then

$$Q^{(i)}(q^2) = \frac{1}{2}F_3(q^2)\lambda_3^{(i)} + \frac{1}{2\sqrt{3}}F_8(q^2)\lambda_8^{(i)}. \tag{5.1}$$

A possible quark contribution to the form factors of nucleons can subsequently be expressed as

$$F_{q,p} = 3\langle p|Q^{(1)}|p\rangle = \frac{1}{2}(F_3 + F_8),$$

TABLE II. Numerical values for the different contributions to the mean-square charge radius of the proton (p) and the neutron (n) given in fm^2 in the model of Ref. [15] when the impulse approximation radius of the proton (without relativistic correction) has the value 0.304 fm ($\omega = 1240$ MeV in the harmonic oscillator approximation). Here IA+rel, conf, and P indicate impulse approximation with relativistic correction, confinement, and pseudoscalar exchange mechanisms, respectively. The contribution $\langle r^2\rangle_{\mathrm{an}}$ comes from the anomalous magnetic moment part of the electric Sachs form factor, and $\langle r^2\rangle_q$ has been calculated from Eq. (5.6), assuming $\langle r^2\rangle_u = \langle r^2\rangle_d = 0.133$ fm^2.

	p	n
$\langle r^2\rangle_{\mathrm{IA+rel}}$	0.345	0
$\langle r^2\rangle_{\mathrm{conf}}$	0.161	0
$\langle r^2\rangle_P$	-0.015	-0.009
$\langle r^2\rangle_{\mathrm{an}}$	0.119	-0.127
$\langle r^2\rangle_q$	0.133	0
$\langle r^2\rangle_{\mathrm{tot}}$	0.743	-0.136
$\langle r^2\rangle_{\mathrm{exp}}$	$(0.862)^2$	-0.117

$$F_{q,n}=3\langle n|Q^{(1)}|n\rangle=\frac{1}{2}(-F_3+F_8). \tag{5.2}$$

It is possible to re-express the quark form factors in terms of contributions from up and down quarks. One notes that

$$\frac{2}{3}F_u=\langle u|Q^{(1)}|u\rangle=\frac{1}{2}F_3+\frac{1}{6}F_8,$$

$$-\frac{1}{3}F_d=\langle d|Q^{(1)}|d\rangle=-\frac{1}{2}F_3+\frac{1}{6}F_8, \tag{5.3}$$

with the normalization conditions $F_u(0)=F_d(0)=1$. A combination of Eqs. (5.2) and (5.3) will result in the quark contributions to the charge form factors having the form

$$F_{q,p}=\frac{4}{3}F_u-\frac{1}{3}F_d,$$

$$F_{q,n}=\frac{2}{3}F_u-\frac{2}{3}F_d. \tag{5.4}$$

Since $F_u=1-r_u^2q^2/6+\mathcal{O}(q^4)$ and $F_d=1-r_d^2q^2/6+\mathcal{O}(q^4)$, the quark contributions to the proton and neutron mean-square charge radii can be calculated as

$$\langle r^2\rangle_{q,p}=\frac{4}{3}\langle r^2\rangle_u-\frac{1}{3}\langle r^2\rangle_d,$$

$$\langle r^2\rangle_{q,n}=\frac{2}{3}\langle r^2\rangle_u-\frac{2}{3}\langle r^2\rangle_d. \tag{5.5}$$

If now the u and d quarks are assumed to have the same (mean-square) charge radius Eq. (5.5) can be simplified as

$$\langle r^2\rangle_{q,p}=\langle r^2\rangle_u,$$

$$\langle r^2\rangle_{q,n}=0. \tag{5.6}$$

The total mean square charge radius of a nucleon can now be calculated as $\langle r^2\rangle_{\text{tot}}=\langle r^2\rangle_{\text{IA+ex}}+\langle r^2\rangle_{\text{an}}+\langle r^2\rangle_q$, where $\langle r^2\rangle_{\text{IA+ex}}$ represents the combined contribution from the impulse approximation (with relativistic correction) and the necessary exchange charge contributions. The term $\langle r^2\rangle_{\text{an}}$ comes from the anomalous magnetic moment part of Eq. (2.2) and $\langle r^2\rangle_q$ is given by Eq. (5.6).

If the quark contribution $\langle r^2\rangle_{q,p}$ is chosen so as to obtain a value for the total mean-square charge radius $\langle r^2\rangle_{\text{tot},p}$ as close as possible to the empirical one, an estimate of the mean square charge radius of the u and d quarks can be done. The results in Table II were obtained with the value $\langle r^2\rangle_u=\langle r^2\rangle_d=0.133$ fm^2.

VI. DISCUSSION

In quark-quark interaction models that are flavor or velocity dependent the continuity equation requires the presence of exchange currents. The exchange current operators are usually constructed so as to satisfy the continuity equation to relativistic order $(v/c)^2$. The continuity equation to that order does not constrain the corresponding exchange charge-density operators, apart from the requirement that their contributions vanish with q. In this work the exchange charge-density operators that correspond to the Fermi-invariant $(SVTAP)$ decomposition of the quark-quark interactions for a two-quark system have been constructed (cf. Ref. [18] where the corresponding charge-density operator for the pseudoscalar P is given in the case of nucleon-nucleon interactions). Their contribution to the charge radii of the nucleons has been calculated in a phenomenological model for the interquark interaction (a version of the so-called chiral constituent quark model). The results have then been combined with calculations in the one-body (impulse) approximation where relativistic corrections have been taken into account due to the smallness of the (constituent) quark mass to calculate an ''intrinsic'' charge radius of the nucleon. This ''intrinsic'' part can then be added to the anomalous magnetic moment part of the charge radius to get a total (mean-square) charge radius [10]. The possibility that the constituent quarks are not pointlike but have a nontrivial electromagnetic structure described by quark form factors has also been explored.

A comparison between the results of Table II and the empirical value for the electromagnetic charge radius of the proton ($\langle r^2\rangle_p^{1/2}=0.862$ fm [3]) shows that the predicted proton charge radius for the model of Ref. [15] without any quark radius contribution is 9% smaller than the empirical value, or 18% smaller in the case of the mean-square charge radius. The only exchange charge contributions that are considered are the confinement and pseudoscalar exchange charge contributions.

The result for the neutron electromagnetic mean-square charge radius, on the other hand, has the right sign, and the calculated value of $|\langle r^2\rangle_n|$ is 16% larger than the empirical value of $|\langle r^2\rangle_n|$ (empirically $\langle r^2\rangle_n=-0.117$ fm^2 [6]). Since there will be no contribution to the neutron charge radius neither from the impulse approximation nor from confinement [cf. Eq. (4.5), Eq. (4.7), and Table I], only contributions from pseudoscalar exchange mechanisms and the anomalous magnetic moment can affect the sign and the numerical value of $\langle r^2\rangle_n$.

Without any contributions from the charge radii of the quarks themselves the mean-square charge radius for the proton will thus in this model have a somewhat smaller value and for the neutron a somewhat larger value than the empirical ones. If, on the other hand, an electromagnetic charge form factor is defined for the quarks, the expression for the mean-square charge radius will contain an additional term $\langle r^2\rangle_q$. If the charge radius for the u and the d quark is assumed to be equal the proton charge radius will get a positive contribution, while the neutron charge radius will get no additional contribution, i.e., $\langle r^2\rangle_{q,n}=0$. It is possible to adjust the value for the charge radius of the quarks to get an agreement between the prediction and the empirical data for the proton. When assuming the oscillator frequency to be 1240 MeV (corresponding to an impulse approximation radius without relativistic effects of 0.304 fm [20]), the radius for the u and d quarks is determined to be 0.36 fm ($\langle r^2\rangle_u=\langle r^2\rangle_d=0.133$ fm^2).

This can be compared with similar calculations with another flavor-dependent quark-quark interaction model [21], which leads to a proton mean-square charge radius, which is almost twice as large as the empirical one, and to a positive mean-square charge radius for the neutron if no compensating quark radius term $\langle r^2\rangle_q$ is added. To get a reasonable value for the neutron mean-square radius, i.e., not a positive value, without at the same time causing the proton charge radius to become even larger, would in that model require the *u*- and *d*-quark radii to differ substantially, $\langle r^2\rangle_d$ being larger than $\langle r^2\rangle_u$.

The model discussed in this work gives a good description of both the spectrum and the magnetic moment of the nucleon. The magnetic moments (expressed in nuclear magnetons) for the proton and the neutron, assuming exchange current contributions only from confinement and pseudoscalar exchange mechanisms and using the same methods for the calculations as in Ref. [19], are 2.58 and -1.77. The predicted values for the magnetic moments are thus within 8% of the experimental results (2.79 and -1.91). The parameter describing the confinement strength also seems to be realistic in the model. The generally accepted value for the string constant of $q\bar{q}$ systems (mesons) is $b\sim 1$ GeV/fm, derived from the charmonium spectrum [22] and consistent with Regge phenomenology and numerical lattice QCD results (for a review article, see Ref. [23]). For baryons the string constant is $\frac{1}{2}b$, which corresponds to the value of the string tension parameter $C=2.33\ \mathrm{fm}^{-2}\approx 460$ MeV/fm for the confinement in the model studied.

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