New dispersive ΛNN force and s-shell hypernuclei

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Variational Monte Carlo calculations for the ground- and excited-state binding energies of s-shell hypernuclei using a new form of dispersive spin-dependent noncentral ΛNN force have been made to study its effect on the overbinding problem of ${}^5_\Lambda$He and on the spin dependence of ΛN force. A detailed analysis shows that the strength of the dispersive ΛNN force can be adjusted to resolve the overbinding problem using two-body correlations alone. Consequently, the ambiguity in the strength of the dispersive ΛNN force masks the effect of 2π-exchange ΛNN force and ΛNN correlations on the data. The contribution of the dispersive force to the 0^+-1^+ spin-flip splitting of $A=4$ hypernuclei is not uniquely determined. Further B_Λ data favor a small spin dependence of the ΛN potential, a situation characteristically similar to other versions of dispersive ΛNN potentials. [S0556-2813(99)05105-5]

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I. INTRODUCTION

Considerable effort has been put into analyzing the binding energy and other properties of nuclear few-body systems through various methods [1–6], e.g., the Fadeev method, variational Monte Carlo (VMC) method, coupled cluster method, Green function Monte Carlo (GFMC) method, etc. Recently, the cluster Monte Carlo technique [7] has been extended and applied for analyzing B_Λ of the p-shell hypernucleus ${}^{17}_\Lambda$O. This has also been used to analyze ${}^5_\Lambda$He by Usmani [8].

For light systems $A \leqslant 4$ (where A represents the mass number), the nuclear binding energy has been obtained quite reliably using the VMC technique. The results of these calculations are in agreement with the so-called exact GFMC analysis [5]. Consequently Bodmer and Usmani [9] (BU) have used the VMC technique to evaluate the energy expectation values of s-shell hypernuclei. They have been successful in giving a satisfactory account of the B_Λ data of s-shell hypernuclei and Λ binding to infinite nuclear matter with the phenomenological, central, two-pion exchange Urbana type ΛN force and three-body ΛNN forces. Two-pion exchange and strongly repulsive phenomenological ''dispersive''-type three-body ΛNN forces were chosen. The dispersive-type ΛNN force arises from projecting out Σ, Δ,..., etc., degrees of freedom from a coupled channel formalism. The other one, arising due to the mediation of two-pion exchange, is said to be a genuine one. For a ready reference, the ΛN and ΛNN forces used earlier are reproduced in the text.

For the relative s state, the central two-body Urbana-type ΛN potential, having the same form for the singlet and triplet spin state, is given as

$$\tilde{V}_{\Lambda N} = V_{2\pi} = V_0 - \left[\bar{V} - \frac{1}{4} V_\sigma (\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N)\right] T_\pi^2, \qquad (1.1)$$

where

$$V_0 = W_0 \left[1 + \exp\left(\frac{r-R}{d}\right)\right]^{-1}$$

with $W_0 = 2137$ MeV, $R = 0.5$ fm, and $d = 0.2$ fm. T_π is the one-pion exchange (OPE) tensor potential shape modified with a cutoff,

$$T_\pi = \left[1 + \frac{3}{x} + \frac{3}{x^2}\right]\left(\frac{e^{-x}}{x}\right)(1 - e^{-cr^2})^2, \qquad (1.2)$$

where $x = 0.7r$ and $c = 2$ fm^{-2}. The spin-average ($\bar{V}$) and spin-dependent (V_σ) strengths, which contribute to the potential energy of the Λ hypernucleus, are given in terms of singlet and triplet strengths:

$$\bar{V} = \frac{1}{4} V_s + \frac{3}{4} V_t, \quad V_\sigma = V_s - V_t.$$

V_σ does not contribute for a zero-spin core nucleus. The value of V_σ which is to be determined is positive and consistent with hypernuclear spins. Two types of dispersive ΛNN forces consistent with the meson-exchange model were considered. These are spin independent:

$$V^D_{\Lambda NN} = W T_\pi^2(r_{1\Lambda}) T_\pi^2(r_{2\Lambda}), \qquad (1.3a)$$

as well as spin dependent (Fig. 1)

$$V^{DS}_{\Lambda NN} = V^D_{\Lambda NN}\left[1 + \frac{1}{6}\boldsymbol{\sigma}_\Lambda \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\right]. \qquad (1.3b)$$

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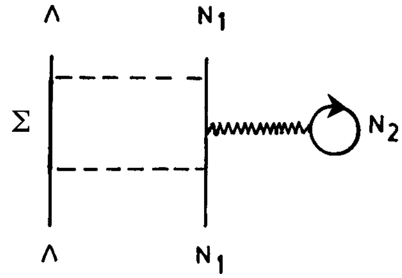

FIG. 1. The dispersive ΛNN interaction of Ref. [9] associated with suppression of the TPE ΛN potential arising from modifications of the intermediate Σ, N, ... by the medium (a second nucleon N_2).

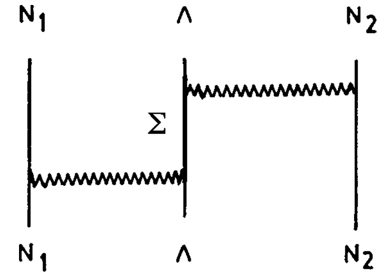

FIG. 2. The two-pion exchange diagram generating the ΛNN interaction. Wavy lines denote the one-pion exchange $\Lambda N \leftrightarrow \Sigma N$ potential.

These have a repulsive contribution for all relative distances of ΛN pairs in the triad ΛNN, thus leaving little room for ΛN or ΛNN correlations to alter its sign. The expectation values for the spin factor within the square bracket in Eq. (1.3b) for s-shell hypernuclei are listed in Table I. The two-pion exchange (TPE) ΛNN force (Fig. 2) for s-shell hypernuclei with an appropriate cutoff is

$$V^{2\pi}_{\Lambda NN} = C_p[1+(3\cos^2\theta-1)T_\pi(r_{1\Lambda})T_\pi(r_{2\Lambda})] \times Y_\pi(r_{1\Lambda})Y_\pi(r_{2\Lambda}), \quad (1.4)$$

where $\cos\theta = \hat{r}_{1\Lambda}\cdot\hat{r}_{2\Lambda}$, T_π is the same as given above except that c is replaced by $\hat{c}$ for the cutoff parameter. The Yukawa function is

$$Y_\pi(r) = \frac{e^{-\mu r}}{\mu r}(1-e^{-\hat{c}r^2}), \quad \mu = 0.7\ \text{fm}^{-1}. \quad (1.5)$$

The value of $\hat{c} = 2\ \text{fm}^{-2}$ and $C_p = 2$ MeV were chosen. The first term in the square bracket of Eq. (1.4) is central and weakly repulsive, whereas the angle-dependent second term makes a repulsive contribution for asymptotic Λ distances and it is strongly attractive for small distances. The presence of a three-body correlation may make its overall contribution [9] attractive or repulsive.

The three-body ΛNN correlations were chosen to be of the form

$$f_{\Lambda NN} = f^D_{\Lambda NN} f^{2\pi}_{\Lambda NN}, \quad (1.6)$$

where

$$f^D_{\Lambda NN} = 1-\alpha\tilde{Y}(r_{1\Lambda})\tilde{Y}(r_{2\Lambda})$$

and

$$f^{2\pi}_{\Lambda NN} = 1-\beta\zeta$$

with

$$\zeta = (3\cos^2\theta-1)\tilde{Y}(r_{1\Lambda})\tilde{Y}(r_{2\Lambda}).$$

$\tilde{Y}(r)$ are the Yukawa functions as defined earlier but with the difference that range ($\tilde{\mu}$) and cutoff ($\tilde{c}$), along with the correlation strengths α and β, are treated as variational parameters.

The main conclusions of their analysis were that the central and spin-dependent dispersive ΛNN force (1.3b) contributes to the 0^+-1^+ spin-flip splitting of $A=4$ hypernuclei $\simeq\frac{1}{3}$ less than the one obtained with spin-independent dispersive ΛNN force and data favors weakly spin-dependent ΛN potential.

The dispersive ΛNN force used by BU is phenomenological in nature and is motivated from the NNN potential used by the Urbana group [10] in the analysis of s-shell nuclei and nuclear matter. The three-body ΛNN potential employed in the recent work of B_Λ analyses [7,8] of hypernuclei is of the same nature as suggested by BU.

Not too long ago a dispersive spin-dependent noncentral ΛNN force $v^{DSN}_{\Lambda NN}$ [vide Eq. (2.3) in the next section] has been derived by Gal [11] whose radial and spin-isospin dependence is radically different from the one used in the literature. The presence of the tensorial term makes its spatial behavior highly nonlinear. This force vanishes identically for

TABLE I. B_Λ, effective potential strengths and expectation values of spin/spin-isospin factors given in Refs. [9] and [11]. The $B_{,}$ values for ${}^4_\Lambda H-{}^4_\Lambda He$ and ${}^4_\Lambda H^*-{}^4_\Lambda He^*$ are the average of the two.

Hyper-nuclei	B_Λ (MeV)	V_A (MeV)	$J^\pi;\ T$	Expectation values of spin-isospin and spin factors given in Ref. [11]	Ref. [9]
${}^3_\Lambda H$	0.13 ± 0.05	$\overline{V}+1/2V_\sigma$	$1/2^+;\ 0$	1/3	1/3
${}^4_\Lambda H-{}^4_\Lambda He$	2.22 ± 0.04	$\overline{V}+1/4V_\sigma$	$0^+;\ 1/2$	0	2
${}^4_\Lambda H^*-{}^4_\Lambda He^*$	1.12 ± 0.06	$\overline{V}-1/12V_\sigma$	$1^+;\ 1/2$	−4/3	10/3
${}^5_\Lambda He$	3.12 ± 0.02	$\overline{V}$	$1/2^+;\ 0$	−2	6

ground state of $A=4$ hypernucleus whereas the one used by BU is nonzero. The expectation values (Table I) of the spin isospin factors, Eq. (2.6) of the new force in the case of ${}^5_\Lambda$He and $A=4$ systems, are qualitatively different from those employed in the earlier analyses, except for ${}^3_\Lambda$H. The dispersive ΛNN interaction of BU is repulsive everywhere for all the systems considered, whereas the dispersive force of Gal [11] for asymptotic Λ distances is repulsive for ${}^3_\Lambda$H and attractive for ${}^4_\Lambda$H*$-{}^4_\Lambda$He* and ${}^5_\Lambda$He. Based on previous work, Gal has made an observation that a short-range correlation of the type $f^{2\pi}_{\Lambda NN}$ could easily reverse this behavior leading to a repulsive contribution for ${}^5_\Lambda$He. In contrast, in the present work we have found that a short-range two-body ΛN correlation changes the overall contribution of the dispersive force to repulsive for ${}^5_\Lambda$He, thus enabling us to resolve the overbinding problem without the three-body correlation.

In the present work, the VMC calculations for the energy of $A=3,4,4^*,5$ hypernuclei (where 4^* represents the spin-flip excited state of the $A=4$ hypernucleus), using the new form of the dispersive ΛNN force, have been made with three objectives: to delineate the role of the central two-body and three-body hyperon-nucleon correlations; to see in what respect it differs from the other dispersive forces (1.3) with regard to the overbinding problem of ${}^5_\Lambda$He; to examine its contribution to the spin-flip excited state of $A=4$ systems and to find V_σ, the spin dependence of the ΛN potential. Initially, the calculation is performed with two-body correlations and later the role of three-body correlations have also been explored to make the analysis comprehensive.

The two-body and three-body potentials as well as appropriate correlation functions, employed here are discussed in the next section. A general Hamiltonian and a brief procedure of the calculation of the energy of s-shell hypernuclei, using the VMC technique [5,9], are discussed in Sec. III. The effect of the $V^{DSN}_{\Lambda NN}$ on the overbinding problem of ${}^5_\Lambda$He and on the V_σ along with the other results of our analysis are discussed in Sec. IV, and Sec. V gives the conclusion.

II. POTENTIALS, CORRELATIONS, AND TRIAL WAVE FUNCTIONS

A. Two-body *NN* potential

For the NN pair, we use the local central, spin-isospin independent Malfliet-Tjon potential [12]:

$$V_{NN}(r)=[7.39\exp(-3.11r)-2.93\exp(-1.55r)]\frac{\hbar c}{r}. \tag{2.1}$$

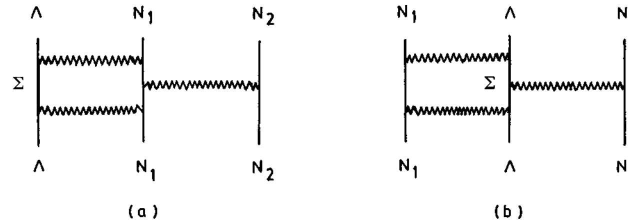

FIG. 3. (a) and (b) Pion-exchange diagrams generating dispersive ΛNN interactions. Wavy lines denote the one-pion exchange.

This potential, besides being simple, gives ground-state binding energies and rms radii for (^{3}H, ^{3}He) and ^{4}H nuclei in reasonable agreement with experiment [10,13] It also reproduces the corresponding data for the ^{2}H nuclei fairly well with a slight adjustment in one of its strength parameters. The coefficient of the attractive part for ^{2}H is 3.201, appropriate to $S=1$, $I=0$. The effect of the Coulomb interaction is small and is neglected as was done earlier [9].

B. Two-body ΛN potential

For the ΛN pair, the central and spin-dependent Urbana-type TPE potential ($\epsilon=0.25$)

$$V_{\Lambda N}=(1-\epsilon+\epsilon P_X)\tilde{V}^0_{\Lambda N}, \tag{2.2}$$

consistent with Λp scattering is employed. P_X is the space exchange operator and $\tilde{V}^0_{\Lambda N}$ has the definition given in Eq. (1.1).

C. Dispersive and 2π-exchange three-body ΛNN potentials

Dispersive ΛNN interaction represents the effect of the nuclear medium via a third baryon on the two-body ΛN interaction [see Figs. 3(a) and 3(b)]. The propagation of the intermediate ΣN pair occuring in the medium generates these interactions. Gal [11] has derived the dispersive, spin-dependent, and noncentral ΛNN forces incorporating the approximations: the dominant tensor term in the transition potentials is retained, the full OPE form is taken in the intermediate NN and ΣN potentials, and assumption of the same closure energy everywhere is made. The potential, when restricted to the s-shell hypernuclei, has the following form:

$$V^{DSN}_{\Lambda NN}=WY(r_{1\Lambda})Y(r_{2\Lambda})\{Y(r_{1\Lambda})T^2(r_{1\Lambda})[T(r_{2\Lambda})(3\cos^2\theta_{1\Lambda 2}-1)-1]$$
$$+Y(r_{2\Lambda})T^2(r_{2\Lambda})[T(r_{1\Lambda})(3\cos^2\theta_{2\Lambda 1}-1)-1]\}\frac{1}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_\Lambda\cdot\mathbf{S}_{12}), \tag{2.3}$$

where angle $\theta_{i\Lambda j}$ has the same definition as θ in Eq. (1.4), the tensor radial shape $T(r)$ and Yukawa function $Y(r)$ modified with a cutoff are

$$T(r)=\left[1+\frac{3}{x}+\frac{3}{x^2}\right](1-e^{-\hat{c}r^2}), \tag{2.4}$$

$$Y(r)=\frac{e^{-x}}{x}(1-e^{-\hat{c}r^2}) \tag{2.5}$$

with $x=m_\pi r$ and $\mathbf{S}_{12}=(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)/2$. The expectation values [11] of the spin-isospin factor

$$\frac{1}{9}\sum_{i<j}^{A-1}[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_\Lambda\cdot\mathbf{S}_{12})] \tag{2.6}$$

for *s*-shell hypernuclei are listed in Table I for comparison along with those of Ref. [9]. It may be noted that the spatial part of the ΛNN force (2.3) is noncentral, a feature resembling the two-pion exchange force (1.4). Thus, it is not unlikely that $V^{DSN}_{\Lambda NN}$ may simulate the behavior of $V^{2\pi}_{\Lambda NN}$. Two-pion exchange ΛNN force [14] Eq. (1.4), averaged over the spin-isospin for *s*-shell hypernuclei, is used in the present work.

D. Correlated wave functions

The calculation of Λ-seperation energy through the variational principle requires the choice of a good trial wave function. This is constructed from a product of two- and three-body correlation functions as

$$\psi^{(A)}=\left\{\prod_{i=1}^{A-1}f_{\Lambda N}(r_{i\Lambda})\prod_{i<j}^{A-1}f_{NN}(r_{ij})\prod_{i<j}^{A-1}f_{\Lambda NN}(r_{ij\Lambda})\right\}\chi^{(A)},$$

$$\psi^{(A-1)}=\left\{\prod_{i<j}^{A-1}f_{NN}(r_{ij})\right\}\eta^{(A-1)}, \tag{2.7}$$

where $\psi^{(A)}$, $\psi^{(A-1)}$ are the wave functions of the hypernucleus of mass number A and of the corresponding core nucleus, respectively, and $\chi^{(A)}$ and $\eta^{(A-1)}$ are the appropriate spin functions.

The two-body correlation functions f_{BN} (where B denotes Λ or N) are calculated using the procedure developed by the Urbana group and these are required to have the following asymptotic form:

$$f_{BN}\sim r^{-\nu_{BN}}\exp(-k_{BN}r) \tag{2.8}$$

with the variational parameters appropriately chosen.

The three-body ΛNN correlations of the dispersive and two-pion exchange-type used in the present work are

$$f^{DS}_{\Lambda NN}=1-\alpha[\tilde{Y}(r_{1\Lambda})+\tilde{Y}(r_{2\Lambda})]\zeta,$$

$$f^{2\pi}_{\Lambda NN}=1-\beta\zeta, \tag{2.9}$$

where the symbols have meaning as explained above. The choice (2.9) is motivated to simulate the desired features of the three-body ΛNN forces.

III. HAMILTONIAN AND ENERGY CALCULATION

The Hamiltonian for the hypernucleus of mass number A is given by

$$H^{(A)}=H^{(A-1)}-\frac{\hbar^2}{2m_\Lambda}\nabla^2_\Lambda+\sum_{i=1}^{A-1}V_{\Lambda N}(i\Lambda)+\sum_{i<j}^{A-1}V_{\Lambda NN}(ij\Lambda), \tag{3.1}$$

where

$$V_{\Lambda NN}(ij\Lambda)=V^{DSN}_{\Lambda NN}(ij\Lambda)+V^{2\pi}_{\Lambda NN}(ij\Lambda)$$

and

$$H^{(A-1)}=-\frac{\hbar^2}{2m_N}\sum_{i=1}^{A-1}\nabla^2_i+\sum_{i<j}^{A-1}V_{NN}(ij) \tag{3.2}$$

is the Hamiltonian of the core nucleus. The Λ-separation energy, with the wave functions and Hamiltonian of the hypernucleus of mass number A and the corresponding core nucleus of mass number $(A-1)$, is written as

TABLE II. Variational results for ${}^3_\Lambda$H. $\langle T\rangle$ and $\langle V_{BN}\rangle$ are the expectation values of the total kinetic and total two-body potential energies, respectively. $E\pm\Delta E$ represents the total energy of the hypernucleus with the corresponding Monte Carlo error. The value marked by an asterisk is that obtained using the dispersive ΛNN correlation given by Eq. (4.1). For all cases $c_{NN}=3.7\ \text{fm}^{-2}$, $a_{NN}=1.60$ fm, $R_{NN}=3.30$ fm, $c_{\Lambda N}=3.70\ \text{fm}^{-2}$, $a_{\Lambda N}=1.60$ fm, $R_{\Lambda N}=3.30$ fm, $\tilde{\mu}=0.7\ \text{fm}^{-2}$, $\tilde{c}=\hat{c}$, and $k_{NN}=0.27\ \text{fm}^{-1}$.

V_3 (MeV)	W (MeV)	$C_p(\hat{c})$ (MeV) (fm^{-2})	$k_{\Lambda N}$ (fm^{-2})	s	α	β	$\langle T\rangle$ (MeV)	$-\langle V_{BN}\rangle$ (MeV)	$\langle V^{DSN}_{\Lambda NN}\rangle$ (MeV)	$\langle V^{2\pi}_{\Lambda NN}\rangle$ (MeV)	$-E\pm\Delta E$ (MeV)
6.250	0.125	0(2)	0.07	1.0	0.00	0.0	16.06	18.38	−0.030	0.0	2.355±0.011
6.255	0.09	0(2)	0.07	1.0	0.00	0.0	15.97	18.30	−0.024	0.0	2.347±0.014
6.255	0.09	0(2)	0.07	1.0	0.25	0.0	15.87	18.20	−0.021	0.0	2.350±0.010*
6.250	0.125	0(2)	0.07	1.0	−0.05	0.0	16.06	18.40	−0.026	0.0	2.349±0.014
6.250	0.125	0(2)	0.07	1.0	0.0	−0.1	16.11	18.42	−0.020	0.0	2.333±0.017
6.255	0.085	2(2)	0.07	1.0	0.00	0.0	15.97	18.28	−0.023	0.001	2.332±0.019
6.255	0.085	2(2)	0.07	1.0	0.00	0.1	16.12	18.44	−0.033	−0.009	2.359±0.009
6.255	0.085	2(2)	0.07	1.0	0.10	0.0	15.73	18.08	−0.027	0.0007	2.366±0.008
6.255	0.085	2(2)	0.07	1.0	0.10	0.1	15.90	18.19	−0.037	−0.018	2.342±0.013

TABLE III. Variational results for ${}^4_\Lambda$H$-{}^4_\Lambda$He. Same as for Table II but with $c_{NN}=2$ fm^{-2}, $a_{NN}=0.6$ fm, $R_{NN}=1.3$ fm, $c_{\Lambda N}=2$ fm^{-2}, $a_{\Lambda N}=0.8$ fm, and $k_{NN}=0.31$ fm^{-1}.

V_4 (MeV)	W (MeV)	$C_p(\hat{c})$ (MeV) (fm^{-2})	$k_{\Lambda N}$ (fm^{-2})	s	α	β	$\langle T\rangle$ (MeV)	$-\langle V_{BN}\rangle$ (MeV)	$\langle V^{DSN}_{\Lambda NN}\rangle$ (MeV)	$\langle V^{2\pi}_{\Lambda NN}\rangle$ (MeV)	$-E\pm\Delta E$ (MeV)
6.183	0.125	0(2)	0.13	1.0	0.00	0.0	45.20	55.65	0.0	0.0	10.453±0.021
6.188	0.090	0(2)	0.13	1.0	0.00	0.0	45.55	56.03	0.0	0.0	10.483±0.020
6.188	0.090	0(2)	0.13	1.0	0.20	0.0	44.02	54.52	0.0	0.0	10.501±0.021*
6.183	0.125	0(2)	0.13	1.0	0.05	0.0	45.53	55.96	0.0	0.0	10.430±0.029
6.183	0.125	0(2)	0.13	1.0	0.00	0.1	45.37	55.80	0.0	0.0	10.437±0.028
6.190	0.085	2(2)	0.12	1.0	0.00	0.0	43.84	54.18	0.0	0.084	10.256±0.152
6.190	0.085	2(2)	0.12	1.0	0.10	0.0	43.74	54.22	0.0	−0.060	10.540±0.026
6.190	0.085	2(2)	0.12	1.0	0.00	0.2	44.21	54.58	0.0	−0.132	10.500±0.040
6.190	0.085	2(2)	0.12	1.0	0.10	−0.1	44.38	54.86	0.0	0.038	10.446±0.025

$$-B_\Lambda={}_\Lambda E^A-E^{(A-1)}=\frac{(\psi^{(A)}|H^{(A)}|\psi^{(A)})}{(\psi^{(A)},\psi^{(A)})}-\frac{(\psi^{(A-1)}|H^{(A-1)}|\psi^{(A-1)})}{(\psi^{(A-1)},\psi^{(A-1)})}. \tag{3.3}$$

The B_Λ is expressed in term of potential parameters V_A (defined in Table I), W, C_p, $\hat{c}$ apart from the variational parameters. The estimate for the energy ${}_\Lambda E^A$ or $E^{(A-1)}$ were made for 100 000 points. The hypernuclei included in the analysis and their B_Λ data along with other relevant information are given in Table I. The B_Λ for ${}^4_\Lambda$H$-{}^4_\Lambda$He and ${}^4_\Lambda$H*$-{}^4_\Lambda$He* are the average values of the two specimens.

The general procedure for calculating the energy using the VMC technique is as follows: For a chosen set of potential parameters the variational parameters corresponding to two- and three-body correlations involved in the wave function are varied to optimize the energy of each hypernucleus. The potential parameters are changed until the optimum energy consistent with the experimental value $-B_\Lambda$ is obtained. Variational parameters in f_{NN} for $A=3,4,4^*,5$ hypernuclei were kept fixed at the optimum values (vide Table captions in Ref. [9]), as experience has shown, these are not expected to change significantly from those of the bare core nuclei. In the case of $f_{\Lambda N}$, variational parameter $k_{\Lambda N}$ alone was varied for all the hypernuclei while other parameters $c_{\Lambda N}$, $R_{\Lambda N}$, and $a_{\Lambda N}$ were fixed at their optimum values [9], because a slight variation of these do not alter the minima of the energy. In the absence of any theoretical estimate of the strength W of $V^{DSN}_{\Lambda NN}$, it is treated as a phenomenological parameter. Therefore, W is adjusted from a fit to B_Λ of ${}^5_\Lambda$He where the needed two-body ΛN part V_5 is fixed at $V=6.15\pm0.05$ MeV, a value determined fairly well from Λp scattering [9] data. The other systems were used to calculate V_3, V_4, and V_4^* which in turn led to the determination of V_σ. A detailed analysis [15] indicates the choice of the 2π-exchange three-body potential parameters $C_p(\hat{c})=2(2)$ MeV(fm^{-2}) to be the most favorable option, therefore, for the sake of academic interest and completeness of the analysis, the effect of $V^{2\pi}_{\Lambda NN}$ on the contribution of $V^{DSN}_{\Lambda NN}$ to B_Λ is also examined.

IV. RESULT AND DISCUSSION

The B_Λ data of s-shell hypernuclei using the two-body ΛN potential (2.2) and the new dispersive ΛNN force (2.3) with or without $V^{2\pi}_{\Lambda NN}$ are analyzed. These two cases are discussed below separately. Only selected results with appropriate combinations of the optimum variational parameters which yield the Λ-binding energies close to the experimental one are shown in the tables.

(i) $C_p=0$ case. Initially, a hypernuclear wave function consisting of the purely central two-body ΛN and NN correlations (i.e., $\alpha=\beta=0$) were used and it was found that B_Λ data are explained for $W=0.09$ MeV. It may be remarked

TABLE IV. Variational results for ${}^4_\Lambda$H*$-{}^4_\Lambda$He*. Same as for Table III.

V_4^* (MeV)	W (MeV)	$C_p(\hat{c})$ (MeV) (fm^{-2})	$k_{\Lambda N}$ (fm^{-2})	s	α	β	$\langle T\rangle$ (MeV)	$-\langle V_{BN}\rangle$ (MeV)	$\langle V^{DSN}_{\Lambda NN}\rangle$ (MeV)	$\langle V^{2\pi}_{\Lambda NN}\rangle$ (MeV)	$-E\pm\Delta E$ (MeV)
6.140	0.125	0(2)	0.09	1.0	0.00	0.00	39.76	49.50	0.708	0.0	9.036±0.145
6.130	0.090	0(2)	0.09	1.0	0.00	0.00	38.94	48.80	0.463	0.0	9.387±0.043
6.130	0.090	0(2)	0.09	1.0	0.10	0.00	39.17	49.82	0.516	0.0	9.334±0.042*
6.140	0.125	0(2)	0.09	1.0	−0.15	0.00	39.32	49.14	0.456	0.0	9.360±0.030
6.140	0.125	0(2)	0.09	1.0	0.00	−0.15	39.43	49.28	0.492	0.0	9.359±0.035
6.140	0.085	2(2)	0.08	1.0	0.00	0.00	38.11	47.78	0.451	0.032	9.191±0.138
6.140	0.085	2(2)	0.09	1.0	−0.05	0.00	38.94	48.84	0.422	0.076	9.401±0.026
6.140	0.085	2(2)	0.09	1.0	0.00	−0.15	39.43	49.28	0.335	0.119	9.398±0.032
6.140	0.085	2(2)	0.08	1.0	−0.05	−0.10	37.54	47.28	0.250	0.127	9.367±0.046

TABLE V. Variational results for ${}^{5}_{\Lambda}$He. Same as for Table II, but with $c_{NN}=1$ fm^{-2}, $a_{NN}=0.5$ fm, $R_{NN}=1.0$ fm, $c_{\Lambda N}=2$ fm^{-2}, $R_{\Lambda N}=1$ fm, $a_{\Lambda N}=0.8$ fm, and $k_{NN}=0.304$ fm^{-1}.

V_5 (MeV)	W (MeV)	$C_p(\hat{c})$ (MeV) (fm^{-2})	$k_{\Lambda N}$ (fm^{-2})	s	α	β	$\langle T\rangle$ (MeV)	$-\langle V_{BN}\rangle$ (MeV)	$\langle V^{DSN}_{\Lambda NN}\rangle$ (MeV)	$\langle V^{2\pi}_{\Lambda NN}\rangle$ (MeV)	$-E\pm\Delta E$ (MeV)
6.15	0.125	0(2)	0.105	0.9	0.00	0.0	82.20	118.04	2.425	0.0	33.412±0.202
6.15	0.090	0(2)	0.125	1.0	0.00	0.0	87.20	123.70	2.172	0.0	34.331±0.092
6.15	0.090	0(2)	0.125	1.0	0.35	0.0	88.19	124.75	2.139	0.0	34.418±0.098*
6.15	0.125	0(2)	0.105	0.9	−0.10	0.0	83.27	119.18	1.598	0.0	34.308±0.083
6.15	0.125	0(2)	0.105	0.9	0.00	−0.2	86.65	122.29	0.594	0.0	35.042±0.107
6.15	0.085	2(2)	0.135	0.9	0.00	0.0	86.67	123.32	2.141	0.224	34.289±0.074
6.15	0.085	2(2)	0.135	0.9	−0.10	0.0	87.52	123.66	1.268	0.495	34.427±0.045
6.15	0.085	2(2)	0.135	0.9	0.00	−0.05	88.63	125.15	1.776	0.343	34.377±0.058
6.15	0.085	2(2)	0.135	0.9	−0.10	−0.05	88.84	124.91	1.026	0.578	34.463±0.054

that freedom in choosing W might have simulated the effect of $V^{2\pi}_{\Lambda NN}$ in the analysis. A further increase in W decreases the energy of the systems, while for ${}^{3}_{\Lambda}$He a marginal increase is noticed. The important point to note is that two-body correlation is capable of making an overall contribution of $V^{DSN}_{\Lambda NN}$ to the repulsive value for ${}^{5}_{\Lambda}$He and the attractive value for ${}^{3}_{\Lambda}$H. The results are presented in Tables II–V. Although the ΛN potential and the dispersive ΛNN force with two-body correlations appear to be adequate to explain the data, nevertheless the effect of ΛNN correlations has also been investigated in light of comments made earlier [9,11]. Therefore, in the hypernuclear wave function, the simplest central correlation for the dispersive ΛNN force of the form [16]

$$f^{D}_{\Lambda NN}=1-\alpha\tilde{Y}(r_{1\Lambda})\tilde{Y}(r_{2\Lambda}), \tag{4.1}$$

was included along with two-body $f_{\Lambda N}$ and f_{NN} correlation functions. Despite that the binding energy data is explained for $W=0.09$ MeV (see Tables II–V), the correlation (4.1) has little effect on the wave function consisting of two-body correlations alone. This is not unexpected in view of the inflexible choice of the three-body correlation.

The remaining results quoted in the tables correspond to the more appropriate form [17] of the ΛNN correlation $f^{DS}_{\Lambda NN}$ [Eq. (2.9)], which may be simulating broad features of $V^{DSN}_{\Lambda NN}$. The effect of $f^{2\pi}_{\Lambda NN}$ is also explored on $V^{DSN}_{\Lambda NN}$, though it was primarily designed for the $V^{2\pi}_{\Lambda NN}$ force. The inclusion of $f^{2\pi}_{\Lambda NN}$ with $f^{DS}_{\Lambda NN}=1$ has an effect on $\langle V^{DSN}_{\Lambda NN}\rangle$ similar to that of $f^{DS}_{\Lambda NN}$ with $f^{2\pi}_{\Lambda NN}=1$. The contribution of $V^{DSN}_{\Lambda NN}$ significantly reduces in the presence $f^{DS}_{\Lambda NN}$ or $f^{2\pi}_{\Lambda NN}$, compared to the case when $\alpha=0$ or $\beta=0$, and consequently, W is increased to 0.125 to explain the B_Λ data.

Since the strength W of the dispersive ΛNN force is not constrained by the theory, the appropriate two-body ΛN correlations seem to be the only ingredient necessary for explaining the B_Λ data within the VMC framework. Therefore, it seems premature to make a definite comment about the role played by ΛNN correlation functions until the arbitrariness in choosing the strength W in explaining the data is removed.

(ii) $C_p(\hat{c})=2(2)$ MeV(fm^{-2}). The motivation for studying the effect of $V^{2\pi}_{\Lambda NN}$ on the data, apart from the academic one, is to see how its presence modifies the dispersive strength to explain the data. The last four entries ($C_p=2$ MeV) in Tables II–V give the variational results without and with ΛNN correlations.

Variation of B_Λ with V_A, W, and C_p shows trends which are somewhat similar to those found by BU. In general, B_Λ increases with V_A but $V^{DSN}_{\Lambda NN}$ reduces the energy for all the hypernuclei considered, except in the case of the hypertriton where it helps in binding. This behavior is exhibited because noncentral $V^{DSN}_{\Lambda NN}$ may give an appropriate correlation with either the repulsive or attractive contribution to the energy, depending on the relative distances in the triad ΛNN. Such is the situation for noncentral $V^{2\pi}_{\Lambda NN}$ in the case of ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H. For systems $A=4^*$ and 5 the energies $\langle V^{DSN}_{\Lambda NN}\rangle$ are repulsive and $\langle V^{2\pi}_{\Lambda NN}\rangle$ takes the attractive or repulsive value for $A=4$. The introduction of $f^{DS}_{\Lambda NN}$ and/or $f^{2\pi}_{\Lambda NN}$ significantly reduces the repulsive contribution $\langle V^{DSN}_{\Lambda NN}\rangle$ and increases that of $\langle V^{2\pi}_{\Lambda NN}\rangle$. However, $f^{2\pi}_{\Lambda NN}$ has the opposite effect for

TABLE VI. The ΛN spin dependences, along with the values of V_A, W, C_p, and $\hat{c}$ consistent with the experimental B_Λ for dispersive spin-dependent ΛNN forces. Here $V_5=\overline{V}=6.15$ MeV. The results marked by an asterisk correspond to the calculations done with $f^{D}_{\Lambda NN}$, given by Eq. (4.1).

$C_p(\hat{c})$ (MeV) (fm^{-2})	W (MeV)	V_3 (MeV)	V_4 (MeV)	V_4^* (MeV)	$V_\sigma^{(3)}$ (MeV)	$V_\sigma^{(4)}$ (MeV)	$V_\sigma^{(4^*)}$ (MeV)
0(2)	0.125	6.250	6.183	6.140	0.200	0.132	0.129
*0(2)	0.090	6.255	6.188	6.130	0.210	0.152	0.174
2(2)	0.085	6.255	6.190	6.140	0.210	0.160	0.135

${}^4_\Lambda$H*$-{}^4_\Lambda$He* and ${}^5_\Lambda$He, i.e., repulsion is slightly increased. $\langle V^{DSN}_{\Lambda NN}\rangle$ and $\langle V^{2\pi}_{\Lambda NN}\rangle$ are found to become progressively more repulsive with increasing A. The contribution of $V^{2\pi}_{\Lambda NN}$ is too small to account for the overbinding of ${}^5_\Lambda$He. The dominant repulsion is provided by $V^{DSN}_{\Lambda NN}$ making it an important component of the potential energy in reducing the overbinding of ${}^5_\Lambda$He. Further analysis shows that strength W in the presence of C_p has to be reduced to fit the data.

The spin-dependent component V_σ of the ΛN force has been deduced for each relevant hypernuclei using standard relations and is listed along with W, V_A, C_p in Table VI. The ΛN spin-dependent strengths $V^{(3)}_\sigma$, $V^{(4)}_\sigma$, and $V^{(4^*)}_\sigma$, are generally of the same order as those found in the previous analysis [9]. The spin dependence of $V^{DSN}_{\Lambda NN}$ in the presence of $C_p(\hat{c})=2(2)$ MeV(fm^{-2}), contributes ~30% to the 0^+-1^+ splitting of 1.1 MeV between the ground and spin-flip excited state of $A=4$ hypernuclei, while for $C_p=0$ its contribution varies approximately between 45 and 70%. Thus, at present, the contribution of the spin-dependent dispersive force to the spin-flip excited state cannot be determined uniquely until the precise information about the strength of the dispersive force is not available, and it is also sensitive to whether $V^{2\pi}_{\Lambda NN}$ is included or not.

V. CONCLUSION

From the discussion given above we find that the new dispersive spin-dependent noncentral three-body ΛNN force, derived by Gal [11], is as effective in interpreting B_Λ data as the one used by BU. However, we may remark that unless the strength of dispersive ΛNN force is not constrained by the theory, the appropriate two-body ΛN correlation functions are enough to explain the B_Λ data and consequently, the importance of the role of three-body correlations and the 2π-exchange ΛNN force cannot be ascertained; the fraction of the amount of the splitting of 1.1 MeV between the ground and spin-flip excited state of $A=4$ hypernuclei due to spin-dependence dispersive ΛNN interaction is not uniquely determined. Further, the version of dispersive ΛNN force used here favors small spin dependence for the ΛN potential in explaining the data.

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