Extension of the chiral low-density theorem

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We show how the linear ''low-density theorem'' of Drukarev and Levin can be extended to arbitrary positive integer power of the baryon density ρ. The nth coefficient in the McLaurin expansion of the fermion condensate's ρ dependence is the connected n-nucleon Σ term matrix element. We calculate the $O(\rho^2)$ coefficient in lowest-order perturbative approximation to the linear σ model and then show how this and other terms can be iterated to arbitrarily high order. Convergence radius of the result is discussed.
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I. INTRODUCTION

Earlier this decade a ''low-density theorem'' (LDT) for the linear term in the baryon density dependence of the quark condensate was formulated [1–4]. This first term in the density power series expansion of the quark condensate is proportional to the nucleon, or the constituent quark Σ term, which leads one to believe that, perhaps, it is only the chiral symmetry breaking of the (strong) hadronic interactions by the current quark masses that controls the baryon density dependence of the quark condensate and thus makes it essentially unique. Soon thereafter it became clear, however, that the extrapolation of this linear formula from the low-density region upwards, to the neutron star densities, would be highly contentious.

In this paper (1) we present an extension of the Drukarev-Levin low-density theorem to terms of arbitrarily high order in the McLaurin expansion of the fermion condensate's dependence on the baryon density ρ. The nth McLaurin coefficient is just the connected n-nucleon Σ term (elastic matrix element), which is a function of the explicit chiral symmetry breaking (χSB) terms in the Hamiltonian of the theory. (2) We show how the ''theorem'' works in an explicit example: We calculate the $O(\rho^2)$ coefficient in the lowest perturbative approximation to the linear σ model and then iterate these diagrams to infinite order. The result can be resummed and we obtain a closed-form solution—we use this result to establish the range of validity of such a diagrammatic calculational scheme.

II. BARYON DENSITY DEPENDENCE OF THE Σ TERM

A. Proposition

We propose an extension of the low-density theorem of Drukarev and Levin [1] to arbitrary (positive integer) powers of the baryon-number–quark density ρ. The nth coefficients in the McLaurin expansion of $\langle\Sigma\rangle_\rho$ is the n-fermion (nucleon or quark) matrix element of the (pion) Σ double commutator

$$\Sigma=\frac{1}{3}\sum_{a=1}^{3}[Q_5^a,[Q_5^a,\mathcal{H}_{\chi\mathrm{SB}}(0)]], \tag{1}$$

is

$$\langle\rho|\Sigma|\rho\rangle=\sum_{j=0}^{\infty}\frac{\rho^j}{j!}\langle jN|\Sigma|jN\rangle_{\mathrm{connected}}, \tag{2}$$

where the sum over j involves *connected matrix elements*. As preconditions we assume that (i) the color symmetry of the theory is neither spontaneously nor explicitly broken, (ii) the theory can be described by a Lorentz invariant, local, approximately chiral-invariant Lagrangian density with a partially conserved axial Nöther current, and (iii) the axial anomaly does not invalidate the relevant axial Ward identities.

The proof is based on the current-algebraic relation or identity

$$\langle\Sigma\rangle_\alpha=\langle\alpha|\Sigma|\alpha\rangle=\frac{1}{3}\sum_{a=1}^{3}\langle\alpha|[Q_5^a,[Q_5^a,\mathcal{H}_{\chi\mathrm{SB}}(0)]]|\alpha\rangle$$
$$=-\frac{1}{3}f_\pi^2\lim_{k\to0}\sum_{a=1}^{3}\langle\pi^a(k)\alpha|S|\pi^a(k)\alpha\rangle \tag{3}$$

between the exact elastic soft-pion $\pi\alpha$ scattering (S) matrix and the corresponding Heisenberg representation (pion) Σ term matrix element. Here a is the flavor index of the external pions, which is averaged over the three varieties of pions, i.e., $a=1,2,3$. This result can be adapted to the kaon Σ term by specifying $a=4$, 5, 6, or 7, or by averaging over some subset thereof. Formula (3) is derived from a chiral Ward identity and the LSZ reduction formula for the case of a nucleon ($\alpha=N$) on pp. 131–137 of Ref. [5]. (This is equivalent to applying Sakurai's ''master formula,'' pp. 111–112 in Ref. [5], twice, once to the initial and once to the final state.) Note that the Σ term is only sensitive to the chiral symmetry breaking terms in the Hamiltonian density $\mathcal{H}=\mathcal{H}_\chi+\mathcal{H}_{\chi\mathrm{SB}}$

$$[Q_5^a,[Q_5^a,\mathcal{H}]]=[Q_5^a,[Q_5^a,\mathcal{H}_{\chi\mathrm{SB}}]], \tag{4}$$

the chiral charges being constants of the motion in the chiral limit

$$[Q_5^a,\mathcal{H}_\chi]=0. \tag{5}$$

Now apply this equation to the nuclear matter state ket $|\alpha\rangle=|\rho\rangle=|nN\rangle$. We shall work in a large but finite box of volume Ω in order to avoid momentum-conserving Dirac δ functions, which are replaced by Kronecker ones. This also means that we can have a finite baryon density $\rho = n/\Omega$ without taking the number of baryons to infinity, $n \to \infty$, which we leave for the very last step. Therefore

$$\begin{aligned}\langle \Sigma \rangle_\rho &= \langle \rho | \Sigma | \rho \rangle \\ &= \lim_{n=\rho\Omega\to\infty} \langle nN | \Sigma | nN \rangle \\ &= \lim_{n=\rho\Omega\to\infty} \frac{1}{3} \sum_{a=1}^{3} \langle nN | [Q_5^a,[Q_5^a,\mathcal{H}_{\chi \mathrm{SB}}(0)]] | nN \rangle \\ &= -\frac{1}{3} f_\pi^2 \lim_{n=\rho\Omega\to\infty} \lim_{k\to 0} \sum_{a=1}^{3} \langle \pi^a(k) nN | S | \pi^a(k) nN \rangle,\end{aligned} \tag{6}$$

where we used the fact that the finite density (zero temperature) ground state wave function is just the wave function of n nucleons at rest enclosed in a box of volume Ω. Formula (6) relates the finite density Σ term matrix element to the *exact* (forward) scattering amplitude of a *soft*, i.e., vanishing four-momentum $(k=0)$ pion from n nucleons at rest.

Here we have merely used the definition of the Σ term to rewrite the object of interest in terms of an *S*-matrix element. To be sure, no new information was gained in this step and at no loss of generality. The result is that we may now use the general results of *S*-matrix theory. That, in turn, allows a Feynman-diagrammatic organization of our problem. The most important property of the *S* matrix in this regard is its decomposability into disconnected and connected parts. The latter part falls further into reducible and one-, two-, three-particle, etc., irreducible classes. Specifically, in our case of pions and nucleons we may separate the exact (Heisenberg representation) *S* matrix into the following distinct categories: (i) the completely disconnected graph (one pion and n baryons all propagating without interaction), (ii) n simply connected πN scattering amplitudes multiplied by $(n-1)$ disconnected baryon lines, and (iii) and

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

simply connected πNN scattering amplitudes multiplied by $(n-2)$ disconnected baryon line, etc. [6]. Each of the disconnected subdiagrams has its own momentum-conserving δ function, which translates into one volume (Ω) factor for each disconnected baryon line[1] [6]. Therefore

[1]This, of course, means that the dimension of the various Σ term matrix elements varies with the number of nucleons (baryons and quarks more generally): the vacuum Σ term has dimension M^4, the (single-)nucleon Σ term has dimension M^1, etc., i.e., M^{4-3n} for the n-nucleon Σ term.

$$\begin{aligned}&\langle nN | \Sigma | nN \rangle \\ &= -\frac{1}{3} \sum_{a=1}^{3} f_\pi^2 \lim_{k\to 0} \Big[\langle \pi^a(k) | S | \pi^a(k) \rangle \langle nN | nN \rangle \\ &\quad + n \langle \pi^a(k) N | S | \pi^a(k) N \rangle_{\mathrm{connected}} \Omega^{-1} \langle nN | nN \rangle \\ &\quad + \binom{n}{2} \langle \pi^a(k)(2N) | S | \pi^a(k)(2N) \rangle_{\mathrm{connected}} \\ &\quad \times \Omega^{-2} \langle nN | nN \rangle + \cdots \Big] \\ &= -\frac{1}{3} \sum_{a=1}^{3} f_\pi^2 \lim_{k\to 0} \sum_{j=1}^{n} \binom{n}{j} \\ &\quad \times \Omega^{-j} \langle \pi^a(k)(jN) | S | \pi^a(k)(jN) \rangle_{\mathrm{connected}} \langle nN | nN \rangle.\end{aligned} \tag{7}$$

Now turn this back into a statement about the Σ terms

$$\frac{\langle nN | \Sigma | nN \rangle}{\langle nN | nN \rangle} = \sum_{j=0}^{n} \binom{n}{j} \left(\frac{1}{\Omega} \right)^j \langle jN | \Sigma | jN \rangle_{\mathrm{connected}}. \tag{8}$$

We are now ready to take the thermodynamic limit $\Omega \to \infty$ at (finite) constant density ρ. Since we assumed a large box to begin with, the baryon number $n = \rho\Omega$ contained therein may be taken to be far larger than any given order in the power expansion $n \gg j$ and in the thermodynamic limit $n \to \infty$ one may write

$$\binom{n}{j} = \frac{n!}{j!(n-j)!} \simeq \frac{n^j}{j!}, \tag{9}$$

which leads to the announced result

$$\begin{aligned}\langle \rho | \Sigma | \rho \rangle &= \lim_{n=\rho\Omega\to\infty} \sum_{j=0}^{n} \frac{1}{j!} \left(\frac{n}{\Omega} \right)^j \langle jN | \Sigma | jN \rangle_{\mathrm{connected}} \\ &= \sum_{j=0}^{\infty} \frac{\rho^j}{j!} \langle jN | \Sigma | jN \rangle_{\mathrm{connected}}, \quad \text{Q.E.D.}\end{aligned} \tag{10}$$

when the nuclear ground state is normalized $\langle \rho | \rho \rangle = 1$. It is manifest that this result does not hold in *finite* systems at orders j comparable to the total baryon number n.

A reminder seems in place that this series need *not* uniquely determine $\langle \Sigma \rangle_\rho$ for arbitrary values of ρ because its radius of convergence may be small, or even zero. One possible reason for such a behavior of a power series is that it really is a Laurent one, i.e., the function may have poles and/or branch cuts in the complex baryon density (ρ) plane. Explicit calculations will show this to be the case.

B. Consequences

Note that the "mechanical" sigma operator

$$\Sigma_{\mathrm{mech}} = \frac{1}{3} \sum_{a=1}^{3} [Q_5^a,[Q_5^a,\mathcal{H}_{\mathrm{mech}}(0)]] = \mathcal{H}_{\mathrm{mech}}(0), \tag{11}$$

equals the ''mechanical'' χSB Hamiltonian

$$\mathcal{H}_{\rm mech}=\bar{\Psi}m_q^0\Psi=m_u^0\bar{u}u+m_d^0\bar{d}d,$$

due to the current quark masses, or the bare nucleon mass term

$$\mathcal{H}_{\rm mech}=\bar{N}M_N^0N=M_p^0\bar{p}p+M_n^0\bar{n}n.$$

Our result (10) holds either for quarks, or for baryons as the fermions. Since the chiral symmetry breaking is defined in terms of quark degrees of freedom, but we do not know the solution to the quark dynamics in baryons, we must first establish, or rather postulate a relation between the quarks' and the observable hadrons' properties. One assumption common in the literature is that $\Sigma_N=3\Sigma_Q$.[2] Assuming, by the same token, effective proportionality of the nucleon and quark condensates, our Eq. (10) leads to a statement about the quark condensate at finite baryon density

$$\frac{\langle\bar{N}N\rangle_\rho}{\langle\bar{N}N\rangle_0}=\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0}=1-\frac{1}{(f_\pi m_\pi)^2}\sum_{j=1}^{\infty}\frac{\rho^j}{j!}\langle jN|\Sigma|jN\rangle_{\rm connected}. \tag{12}$$

The $j=1$ term together with the unity on the right-hand side form the Drukarev-Levin LDT. The rest of this formula appears to be new.[3]

This form of the LDT has been used to argue about the behavior of the pseudoscalar (PS) meson masses at nonzero baryon density. We remind the reader that the quark condensate is not the only density-dependent variable influencing the PS meson mass—one must take into account the (model dependent) PS meson decay constant's density dependence as well, which is not commonly done. Lest this result leads one to believe that the problem of the density dependence of PS meson masses is solved, we remind the reader that neither the radius of convergence of the series is known, nor do we have, as yet, an efficient algorithm for calculating the higher-order coefficients in the expansion. We next present an example of a perturbative calculation in the (chiral) linear σ model and its iteration to arbitrarily high order.

III. AN EXAMPLE

Before we proceed we remind the reader of the assumptions of the LDT: the matrix elements must be either the exact Heisenberg representation ones, or approximate matrix elements that satisfy the underlying chiral Ward identity (master formula). The former is beyond our powers, the latter are available, mostly in the form of perturbative solutions, though solutions to several models have been found that sum certain infinite classes of Feynman diagrams (we shall not consider those approximations here). We shall calculate the terms of $O(\rho^2)$ in the chiral linear σ model, the $O(\rho)$ calculations having been done before [2–4]. This calculation will be with (free) nucleon degrees of freedoms, new methods for calculation of σ terms in interacting nucleon systems having been developed only recently [7].

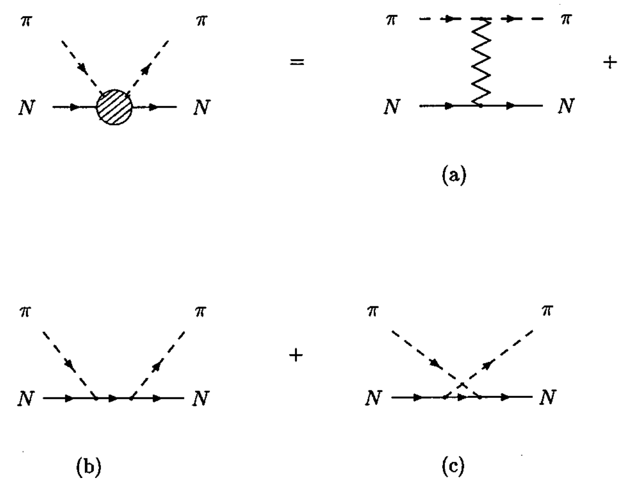

FIG. 1. Feynman diagrams contributing to the ''elementary'' πN elastic scattering amplitude: (a) the σ-pole graph, and (b), (c) the nucleon-pole graphs. The zig-zag line denotes a σ meson, the dashed one a pion and the solid one a nucleon.

A. One- and two-nucleon σ terms in the linear σ model

The one-nucleon connected Σ term can be extracted from the elastic πN scattering amplitude, which in the linear σ model is given by the sum of the three diagrams in Fig. 1: two nucleon-pole diagrams, (b) and (c), and one σ-exchange graph (a). These have been calculated many times [11] and will not be repeated here.

The pion-two-nucleon connected matrix element is substantially more complex: there are more than 50 connected diagrams in the lowest-order perturbative (Born) approximation. All of them are, by definition, reducible diagrams. We separate these graphs into three distinct subsets: (1) pion ''rescattering,'' (2) pion scattering on a σ-meson-in-flight, (3) initial-, final-, and intermediate-state interaction diagrams.

(1) Eighteen $(2\times3\times3)$ of these graphs form the simplest connected, reducible ''pion-rescattering'' amplitude that is built up from two pion-one-nucleon effective vertices, Fig. 1. Their contribution equals

$$\langle\Sigma\rangle_{2N}^{\pi}=-2\left(\frac{\langle\Sigma\rangle_N}{f_\pi m_\pi}\right)^2. \tag{13}$$

The currently accepted value of the nucleon sigma term $\langle\Sigma\rangle_N$ lies between 45 and 65 MeV.

(2) Only the Σ terms of scalar states, discrete or continuum, exchanged between the two nucleons contribute to the two-nucleon Σ term. [This follows from the fact that the Σ term is a Lorentz scalar.] In the first Born approximation to the linear σ model the only such object is the σ meson, so we end up having to calculate its Σ term and its contribution to the two-nucleon Σ term. Four Feynman diagrams, see Fig. 2, contribute to the elastic pion-σ meson scattering amplitude and hence also to its σ term. The result is

[2]It is clear that this amounts to the ''impulse approximation,'' but it was recently shown that the two-quark operator corrections *must* exist [7] if chiral symmetry is to be preserved.

[3]The idea that the elastic π-nN scattering amplitude is related to the ρ dependence of $\langle\bar{q}q\rangle_\rho$ is implicit in Refs. [8–10].

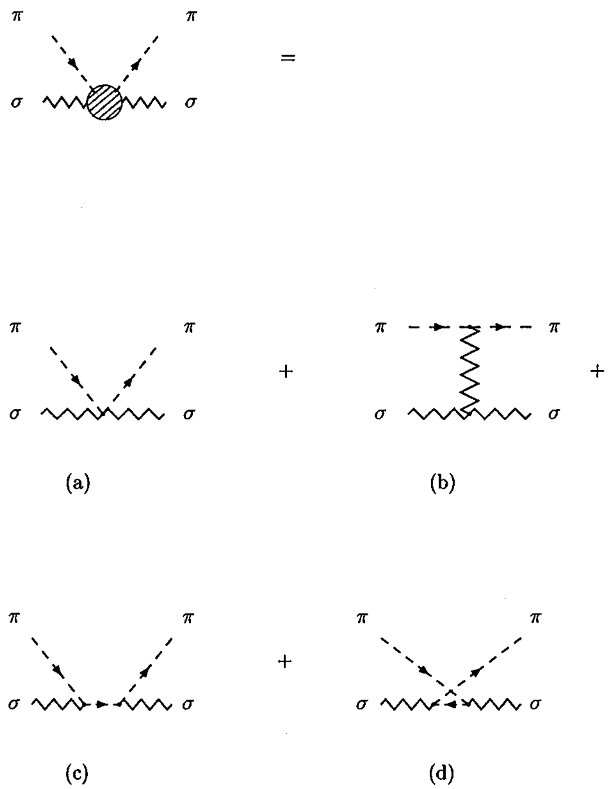

FIG. 2. Feynman diagrams contributing to the ''elementary'' $\pi\sigma$ elastic scattering amplitude: (a) and (b), i.e., to the σ-meson's Σ term.

$$\langle \sigma | \Sigma | \sigma \rangle = 3 m_{\pi}^{2}. \tag{14}$$

Inserting this result into the two-nucleon σ term we find

$$\langle \Sigma \rangle_{2N}^{\sigma} = 3 g_0^2 \left(\frac{m_{\pi}^2}{m_{\sigma}^4} \right), \tag{15}$$

where $g_0 = (g_A M / f_\pi) = 12.6$ is the σ-nucleon coupling constant. The currently accepted value of the σ mass lies between 400 and 1200 MeV [12].

(3) These are *not all* of the reducible graphs at this order, however, all of the remaining graphs can be described as either initial-, intermediate-, or final-state interactions, see Fig. 3. They are formed either by having a complete pion-nucleon scattering amplitude attached to one external nucleon line, or by having a pion-nucleon vertex attached to two external nucleons in the sum of one meson exchanges between the two nucleons (the NN potential). This one-meson exchange potential ought to be summed into an infinite ladder by iteration of the Bethe-Salpeter equation and thus made to form a proper interacting NN (scattering or bound) state. In the end, this summation is equivalent to the introduction of NN correlations into the nuclear matter. These diagrams diverge *in vacuo*, although they are Born diagrams, because the intermediate nucleons are on their mass shells, due to the external soft-pion condition.

Points (1) and (2) put together lead to

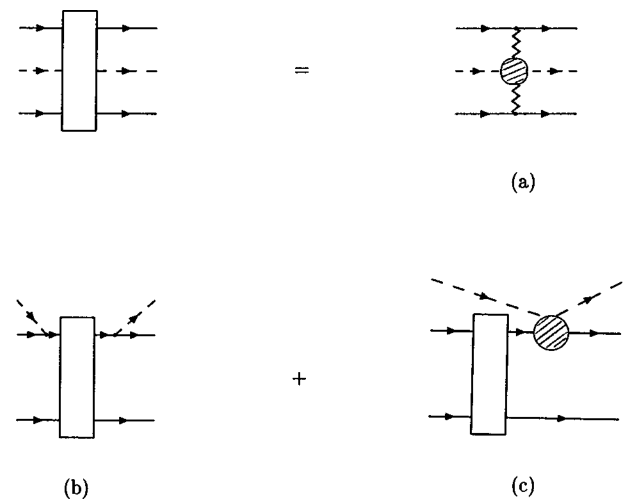

FIG. 3. The pion-two-nucleon scattering amplitude: (a) the σ-meson Σ term effective graph, (b) intermediate-state interaction graphs, (c) initial- and final-state interaction graphs. Graphs (b) and (c) may be termed NN correlation effects. The square box with four external nucleon lines denotes an NN potential due to the exchange of a single pion and σ meson. The hatched ''blob'' in (a) represents the ''elementary'' πN elastic scattering amplitude, such as the one shown in Fig. 1 in the linear Σ model.

$$\begin{aligned} \langle \Sigma \rangle_{2N} &= \langle \Sigma \rangle_{2N}^{\pi} + \langle \Sigma \rangle_{2N}^{\sigma} \\ &= -2 \left(\frac{-\langle \Sigma \rangle_N}{f_\pi m_\pi} \right)^2 + 3 \left(\frac{g_0 m_\pi}{m_\sigma^2} \right)^2, \end{aligned} \tag{16}$$

for the total. With the currently accepted value of the σ mass, the second term exceeds the first one.

One feature of this result stands out: the appearance of the second power of the single-nucleon σ term coming from the repetition of the one-pion reducible diagrams involving the ''primitive'' (elementary) single-nucleon σ term contributions. It has been pointed out by Ericson [8] that this sort of behavior generalizes to higher orders of perturbation theory, Fig. 4, and that the graphs can be resummed into a geometric progression. We shall address that question next.

B. Iteration of primitive contributions in the N-nucleon σ term

In the foregone analysis one may have noticed that certain lower-order graphs are repeated in the higher ones with

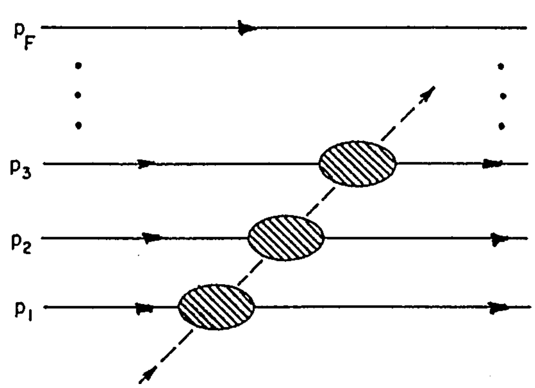

FIG. 4. An example of order $n=3$ reducible single-nucleon pion-rescattering graph.

simple regularity. The multiplicities of these graphs can be figured out and the infinite (sub)series resummed. We shall show that several classes of such diagrams sum into geometric series. That result is particularly useful because its properties, such as the radius of convergence and the analytic structure are well known. Yet, as we shall show, uncritical application may sometimes lead to erroneous conclusions.

Single-nucleon pion rescattering. The connected but reducible diagrams, Fig. 4, consisting of individual πN elastic scattering amplitudes connected by a single pion line carrying zero four-momentum can be written as

$$\langle nN|\Sigma|nN\rangle_{\mathrm{rescatt}}=n!\langle\Sigma\rangle_N^n(-m_\pi^2f_\pi^2)^{(1-n)}, \tag{17}$$

where the factor $1/-m_\pi^2$ comes from the propagation of a soft π, the $\langle\Sigma\rangle_N f_\pi^{-2}$ comes from a single irreducible elastic πN scattering amplitude (vertex) for *soft* pions, and $n!$ is the number of identical graphs. Hence the infinite series of such connected one-pion-reducible graphs is readily summed up as

$$\left.\frac{\langle\bar{N}N\rangle_\rho}{\langle\bar{N}N\rangle_0}\right|_{\mathrm{rescatt}}=\left.\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0}\right|_{\mathrm{rescatt}}=1+\sum_{n=1}^{\infty}\left(\frac{-\rho\langle\Sigma\rangle_N}{(f_\pi m_\pi)^2}\right)^n$$
$$=\left[1+\frac{\rho\langle\Sigma\rangle_N}{(f_\pi m_\pi)^2}\right]^{-1}. \tag{18}$$

Since this is a geometric progression, the series has a finite radius of convergence (in ρ) given by

$$\rho\leqslant|\rho_{c_1}|=\frac{(f_\pi m_\pi)^2}{\langle\Sigma\rangle_N},$$

which equals roughly twice (1.8), or three times the normal nuclear density for $\langle\Sigma\rangle_N=65$, 45 MeV, respectively.

It seems obvious that the complete series including all diagrams, at best can have the convergence properties of the worst-behaved subseries. In other words, our resummation of the one-nucleon pion rescattering diagrams seems to tell us the maximum reliable density calculable with such diagrammatic methods. An important practical consequence of the above reasoning and of the empirical value of the kaon-nucleon sigma term $\langle\Sigma\rangle_{KN}=200$ MeV is that the radius of convergence of the series relevant to the K condensation is at best a fraction of one normal nuclear density. Surprisingly, this conclusion turns out to be only qualitatively correct, due to cancellations from other two-nucleon Σ term contributions. We shall explicitly show that in the linear σ model the inclusion of another two-nucleon Σ term into the infinite series leads to an increase in the number of poles, change of their positions and to an increase in the radius of convergence of the series.

Two-nucleon pion rescattering. The connected two-nucleon ''pion-rescattering'' graphs fall into one of the two categories: (i) even-n and (ii) odd-n, and will be dealt with accordingly.

(i) In the former case, one part of the amplitude is in the form of a product of $n/2$ pion-two-nucleon graphs connected by $n/2$ pion propagators. Since one can select the first pair in $\binom{n}{2}$ many ways, the second pair in $\binom{n-2}{2}$ many ways, etc., one finds

$$\langle nN|\Sigma|nN\rangle|^{n\text{-even}}=-\binom{n}{2}\binom{n-2}{2}\cdots\binom{2}{2}\times\left(\frac{-\langle\Sigma\rangle_{2N}}{m_\pi^2f_\pi^2}\right)^{n/2}(f_\pi m_\pi)^2$$
$$=-n!\left(\frac{-\langle\Sigma\rangle_{2N}}{2m_\pi^2f_\pi^2}\right)^{n/2}(f_\pi m_\pi)^2. \tag{19}$$

The iteration of such two-nucleon pion rescattering diagrams, together with the vacuum term, leads once again to a geometric progression:

$$\left.\frac{\langle\bar{N}N\rangle_\rho}{\langle\bar{N}N\rangle_0}\right|_{\mathrm{irr}}^{n\text{-even}}=\left.\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0}\right|_{\mathrm{irr}}^{n\text{-even}}$$
$$=1+\sum_{n=2,4,\ldots}^{\infty}\left(\frac{\rho}{f_\pi m_\pi}\sqrt{-\frac{1}{2}\langle\Sigma\rangle_{2N}}\right)^n$$
$$=1+\sum_{n=1,2,\ldots}^{\infty}\left(\frac{-\rho^2\langle\Sigma\rangle_{2N}}{2(f_\pi m_\pi)^2}\right)^n$$
$$=\left[1+\frac{\rho^2\langle\Sigma\rangle_{2N}}{2(f_\pi m_\pi)^2}\right]^{-1}. \tag{20}$$

(ii) For n-odd the amplitude is in the form of a product of one πN (or πmN graph, with m-odd) and $(n-1)/2$ [or $(n-m)/2$] two-nucleon irreducible graphs connected by $(n+1)/2$ pion propagators. Since one can select the single πN graph to be any of the n nucleon lines, and the first pair in $\binom{n-1}{2}$ many ways, the second pair in $\binom{n-3}{2}$ many ways, etc., one finds

$$\langle nN|\Sigma|nN\rangle|^{n\text{-odd}}=n\binom{n-1}{2}\binom{n-3}{2}\cdots\binom{2}{2}\times\langle\Sigma\rangle_N\left(\frac{-\langle\Sigma\rangle_{2N}}{m_\pi^2f_\pi^2}\right)^{(n-1)/2}$$
$$=n!\langle\Sigma\rangle_N\left(\frac{-\langle\Sigma\rangle_{2N}}{2f_\pi^2m_\pi^2}\right)^{(n-1)/2}. \tag{21}$$

Hence we conclude

$$\left.\frac{\langle\bar{N}N\rangle_\rho}{\langle\bar{N}N\rangle_0}\right|^{n\text{-odd}}=\left.\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0}\right|^{n\text{-odd}}$$
$$=-\frac{\rho\langle\Sigma\rangle_N}{(f_\pi m_\pi)^2}-\sum_{n=3,5,\ldots}^{\infty}\frac{\rho\langle\Sigma\rangle_N}{(f_\pi m_\pi)^2}\times\left(\frac{\rho}{f_\pi m_\pi}\sqrt{-\frac{1}{2}\langle\Sigma\rangle_{2N}}\right)^{n-1}$$
$$=-\frac{\rho\langle\Sigma\rangle_N}{(f_\pi m_\pi)^2}\sum_{n=0,1,2,\ldots}^{\infty}\left(\frac{-\rho^2\langle\Sigma\rangle_{2N}}{2(f_\pi m_\pi)^2}\right)^n$$
$$=-\frac{\rho\langle\Sigma\rangle_N}{(f_\pi m_\pi)^2}\left[1+\frac{\rho^2\langle\Sigma\rangle_{2N}}{(f_\pi m_\pi)^2}\right]^{-1}. \tag{22}$$

Putting these two together we find

$$\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0} = \frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0}\Bigg|^{n\text{-even}} + \frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0}\Bigg|^{n\text{-odd}}$$
$$= \left[1 - \frac{\rho\langle\Sigma\rangle_N}{(f_\pi m_\pi)^2}\right]\left[1 + \frac{\rho^2\langle\Sigma\rangle_{2N}}{2(f_\pi m_\pi)^2}\right]^{-1}. \quad (23)$$

This result has two conjugate poles in the complex density plane at a distance

$$\rho^2 \leqslant \rho_{c_2}^2 = \left|\frac{(f_\pi m_\pi)^2}{-\langle\Sigma\rangle_{2N}}\right|$$

from the origin. It is clear that the model- and approximation-dependent sign of $\langle\Sigma\rangle_{2N}$ determines the position of the poles in the complex ρ plane, and in particular if they are on the real axis, or not. For instance in the Born approximation to the linear σ model

$$\langle\Sigma\rangle_N = g_0 f_\pi\left(\frac{m_\pi}{m_\sigma}\right)^2,$$

therefore

$$\langle\Sigma\rangle_{2N} = -2\left(\frac{g_0 m_\pi}{m_\sigma^2}\right)^2 + 3\left(\frac{g_0 m_\pi}{m_\sigma^2}\right)^2$$
$$= \left(\frac{g_0 m_\pi}{m_\sigma^2}\right)^2$$
$$= -\frac{1}{2}\langle\Sigma\rangle_{2N}^\pi. \quad (24)$$

Thus in the linear σ model $\langle\Sigma\rangle_{2N}$ is positive, as given in Eq. (14), and the poles are at imaginary densities $\pm i\rho_{c_2}$ a factor $\sqrt{2}$ larger in absolute value than ρ_{c_1} predicted by the one-nucleon rescattering series.

This sort of analysis can and ought to be extended to 3-, 4-, and higher N-pion rescattering graphs. We close with a *conjecture*: In the linear σ model the three-nucleon critical density ρ_{c_3} is larger than the two-nucleon one ρ_{c_2}, the 4-N ρ_{c_4} higher than the 3-N one, etc.

IV. SUMMARY AND CONCLUSIONS

In summary, in this paper we have (i) given an explicit formula for the nth-order term in the powers-of-density (ρ^n) expansion of the nuclear matter sigma term $\langle\Sigma\rangle_\rho$ and of the nuclear matter fermion condensate $\langle\bar{q}q\rangle_\rho$ in particular, (ii) calculated the $\mathcal{O}(\rho^2)$ coefficient in the Born approximation to the linear σ model, (iii) iterated these primitive $\mathcal{O}(\rho^2)$ terms to infinite order, (iv) found poles and radii of convergence associated with two such resummations. Thus we found that the ''larger'' of the two sums actually has the larger radius of convergence.

Perhaps the most important conceptual contribution of this paper is that of putting the older ''intuitive'' calculations onto formally sounder grounds of quantum field theory. In particular, we have shown that the fundamental elements of a calculation of $\langle\Sigma\rangle_\rho$ are the connected n-nucleon Σ term matrix elements. All calculations of $\langle\bar{q}q\rangle_\rho$ can be reduced to these elements. Moreover, this result gives a clear definition of the ''*NN* correlations'' in this context.

Some of the ideas introduced in this paper may have been tacitly assumed in earlier work, most notably that by the Manchester [9] and the Lyon groups [8,10]. Ericson [8] correctly concluded that the resummation of the single-nucleon Σ term leads to a geometric progression in the baryon density. Similarly, Birse and McGovern [9] came close to formulating the correct $O(\rho^2)$ prediction of the LDT in the linear σ model. Finally, some of the diagrammatic methods used here were developed in the 1960's and 1970's [6,13,14] for the description of pion propagation in nuclear matter.

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