

Analysis of particle multiplicities in Pb+Pb collisions at 158A GeV/c within hadron gas models

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The preliminary data on hadron multiplicities measured in central Pb+Pb collisions at 158A GeV/c are analyzed. The ideal hadron gas model fails to give a reasonable explanation to the Pb+Pb data sets. We study the possible effects of pion enhancement due to different hard-core repulsion for pions and other hadrons and strangeness suppression because of incomplete chemical equilibrium. Each of these two modifications improves the results. The combined effect of these two mechanisms leads to an extremely good agreement with the data. An interpretation of the obtained results in terms of the possible quark-gluon plasma formation at the early stage of the collision is also discussed. [S0556-2813(99)00905-X]

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I. INTRODUCTION

Different versions of thermal hadron gas (HG) models have recently been used to fit the data of particle number ratios in high energy nucleus-nucleus ($A+A$) collisions at BNL AGS and CERN SPS (see, e.g., [1–8]). These particle number ratios were measured in fixed rapidity intervals. Particle abundances in HG models are described in terms of two *chemical* freeze-out parameters, temperature T and baryonic chemical potential μ_B . However, the restriction on hadron yields in a fixed rapidity interval causes some problems. The universal energy distributions of all hadrons in HG models are transformed into different rapidity distributions for different hadron species — the width of rapidity distribution is decreased with increasing particle mass. Therefore, to calculate these particle number ratios measured in the fixed rapidity interval, additional model assumptions on collective flow effects and specific values of *thermal* freeze-out parameters are required.

In the present work, we analyze the data on hadron multiplicities in central Pb+Pb collisions at 158A GeV/c measured at CERN SPS. The aim of our thermal HG analysis of hadron multiplicities in a full 4π -geometry is to avoid the above-mentioned complications. The basic assumption here is rather strong that the values of the chemical freeze-out parameters T and μ_B in different elements of the system are equal. However, the collective flow effects are still permitted and the validity of a pure thermal (“fireball”) model is not necessarily required.

II. THE MODELS

The ideal gas equation of state for a mixture of different particle species $i=1, \dots, h$ with chemical potentials μ_1, \dots, μ_h is given by the pressure function in the grand canonical ensemble as

$$p^{id}(T, \mu_1, \dots, \mu_h) = \sum_i^h p_i^{id}(T, \mu_i) \\ = \sum_i^h \frac{d_i}{6\pi^2} \int_0^\infty dk \frac{k^4}{(k^2 + m_i^2)^{1/2}} f_i(k), \quad (1)$$

where the integration variable k is the momentum, and d_i and m_i are the degeneracy factor and the mass of the i th hadron, respectively. The distribution function is written as

$$f_i(k) = \left[\exp\left(\frac{(k^2 + m_i^2)^{1/2} - \mu_i}{T}\right) + \eta_i \right]^{-1}. \quad (2)$$

The statistic factor η_i is -1 for bosons and $+1$ for fermions. The thermal number of i th particle species is expressed as

$$N_i^{id}(T, \mu_i) \equiv V \frac{\partial p^{id}}{\partial \mu_i} = \frac{d_i V}{2\pi^2} \int_0^\infty dk k^2 f_i(k), \quad (3)$$

in which V is the total volume of the hadron system assumed to be equal to the sum of the proper volume elements at the chemical freeze-out. Hadron chemical potentials μ_i are expressed as

$$\mu_i = b_i \mu_B + s_i \mu_S \quad (4)$$

in terms of baryonic (μ_B) and strange (μ_S) chemical potentials with hadron charges $b_i=0, \pm 1$ and $s_i=0, \mp 1, \mp 2, \mp 3$. Direct (thermal) particle yields are calculated according to Eq. (3) for all known stable hadrons and resonances with mass up to 2 GeV. The total multiplicity of i th hadron is the sum of its direct yield (3) and all possible strong decay contributions from resonances. This is true for all HG models discussed in the present paper.

There are serious theoretical doubts in the validity of the ideal HG model at the chemical freeze-out. Particle number densities and total energy density of the HG for chemical freeze-out parameters found from fitting experimental data look artificially large. This is hardly consistent with the pic-

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ture of a gas of pointlike noninteracting hadrons. The introduction of the ‘‘hard-core’’ repulsion in the HG model has been widely discussed in the literature (e.g., [1–8]). We follow the excluded volume procedure of Ref. [9] with the assumption of substituting the total volume V by $V - \sum_i^h v_i N_i$. The proper volume of each hadron v_i is expressed in terms of its ‘‘hard-core’’ radius r_i as $v_i = 16/3r_i^3$ [8]. The HG equation of state with Van der Waals (VDW) repulsion can then be written as

$$p^{VDW}(T, \mu_1, \dots, \mu_h) = p^{id}(T, \tilde{\mu}_1, \dots, \tilde{\mu}_h),$$

$$\tilde{\mu}_i \equiv \mu_i - v_i p^{VDW}(T, \mu_1, \dots, \mu_h), \quad (5)$$

and particle multiplicities are given as

$$N_i^{VDW}(T, \mu_1, \dots, \mu_h) \equiv V \frac{\partial p^{VDW}}{\partial \mu_i}$$

$$= \frac{N_i^{id}(T, \tilde{\mu}_i)}{h + \sum_{j=1}^h v_j N_j^{id}(T, \tilde{\mu}_j)/V}. \quad (6)$$

All HG multiplicities (6) are suppressed in comparison with the ideal HG results (3). The same values of N_i in the VDW HG as in the ideal HG correspond to a larger value of volume V in the VDW HG than in the ideal HG. An artificially high energy density in the gas of noninteracting hadrons is therefore removed [8,10].

The ideal HG model cannot fit existing data of hadron multiplicities. Numerous attempts to fit data on particle number ratios in $A+A$ collisions at AGS and SPS energies have revealed two possible ways to modify the ideal HG model: pion enhancement or strangeness suppression. In other words, in order to fit data, one usually needs either more pions or fewer strange particles in comparison with the ideal HG results. The introduction of the phenomenological parameters r_i could change particle number ratios in comparison with ideal HG results. To enhance pion/hadron ratios we assume a smaller ‘‘hard-core’’ radius for pion than those for all other hadrons, $r_\pi < r_i = r$ [7,8]. The alternative dynamical explanations of the observed pion excess are presented in Refs. [11,12] in the framework of hydrodynamical and transport approaches.

A chemical nonequilibrium HG with a strangeness suppression was suggested in Ref. [13]. It is usually formulated in terms of a phenomenological parameter $\gamma_s < 1$, which can be connected to additional chemical potential regulating the absolute value of total number of strange quarks and anti-quarks in the HG system [14]. The form of Eqs. (1) and (3) remains the same in this approach, but the distribution functions are modified as

$$f_i^s(k) = \left[\gamma_s^{-|S_i|} \exp\left(\frac{(k^2 + m_i^2)^{1/2} - \mu_i}{T}\right) + \eta_i \right]^{-1}, \quad (7)$$

where $|S_i|$ is the sum of the number of strange quarks and antiquarks in hadron i . Hence Eq. (3) becomes

$$N_i^s(T, \mu_i, |S_i|) = \frac{d_i V}{2\pi^2} \int_0^\infty dk k^2 f_i^s(k). \quad (8)$$

Hadron multiplicity data for CERN SPS nucleus-nucleus collisions were recently fitted with Eq. (8) in Ref. [15].

Hadron multiplicities (3), (6), and (8) in HG models depend on 3 thermodynamical parameters, V , T , and μ_B (strange chemical potential μ_s is always defined by the requirement of zero total strangeness). The model with strangeness suppression (8) introduces an additional phenomenological parameter γ_s . Meanwhile, in the VDW HG model (6), one new parameter r is added to the ideal HG formalism— $r_\pi = 0$ is assumed for simplicity [7,8]. Note that, by putting $\gamma_s = 1$ and $r = 0$, the ideal HG model is recovered: Eqs. (8) and (6) are reduced to Eq. (3).

We consider also the HG model which combines both strangeness suppression and hard-core repulsion effects. The pressure function p in this model is defined by the equation

$$p = p^s(T, \tilde{\mu}_1, \dots, \tilde{\mu}_h), \quad \tilde{\mu}_i \equiv \mu_i - v_i p, \quad (9)$$

where p^s in Eq. (9) is given by Eq. (1), but with distribution functions f_i^s (7) instead of f_i (2). Thermal hadron multiplicities N_i are then given by

$$N_i = \frac{N_i^s(T, \tilde{\mu}_i, |S_i|)}{h + \sum_{j=1}^h v_j N_j^s(T, \tilde{\mu}_j, |S_j|)/V}. \quad (10)$$

III. FITTING PROCEDURE AND RESULTS

The data for hadron multiplicities in Pb+Pb collisions at 158A GeV/c and references to the original papers [16–21] are presented in the last two columns of Table I. Because the measurement of hadron multiplicities in Pb+Pb collisions at SPS energies are not yet complete, we also include several hadron ratios in the central rapidity region, K^+/K^- , \bar{p}/p , $\bar{\Xi}/\Xi$, and $\bar{\Omega}/\Omega$ (the controversial $\bar{\Lambda}/\Lambda$ ratio is ignored) in our fitting. Due to equal mass, the kinematic complications mentioned in previous paragraphs are not expected to be very important. We assume, therefore, that these ratios in the central rapidity region are approximately equal to the corresponding ratios of the total hadron multiplicities.

The fitting procedure is to minimize

$$\chi^2 = \sum_{k=1}^N \frac{(y_k^{exper} - y_k^{theor})^2}{(\Delta y_k^{exper})^2}. \quad (11)$$

To compare the fitting of different models to the data, we normalize χ^2 by

$$\frac{\chi^2}{\text{DOF}} \equiv \frac{\chi^2}{N-n}, \quad (12)$$

where N is the number of experimental data points fitted and n is the number of parameters in each model. The best fit with ideal HG model is presented in the first column (ID) in Table I. It has been known that the ideal HG model cannot fit

TABLE I. Particle multiplicities and ratios for central Pb+Pb SPS collisions in different HG models.

	ID	VDW	SS	SS+VDW	Data	Reference
T [MeV]	165	165	165	165		
μ_B [MeV]	235	235	235	235		
V [fm ³]	2490	6060	3150	6640		
r [fm]	0.00	0.46	0.00	0.45		
γ_s	1.00	1.00	0.55	0.62		
N_B	347	337	382	374	372 ± 10	[16]
h^-	514	615	566	674	680 ± 50	[17]
K_s^0	89.3	86.4	60.5	65.3	68 ± 10	[17]
ϕ	11.7	11.3	4.46	5.42	5.4 ± 0.7	[18]
$p-\bar{p}$	124	120	155	149	155 ± 20	[17]
K^+/K^-	1.78	1.78	1.74	1.75	1.8 ± 0.1	[19]
\bar{p}/p	0.060	0.060	0.059	0.060	0.07 ± 0.01	[20]
$\Xi/\bar{\Xi}$	0.242	0.244	0.223	0.227	0.249 ± 0.019	[21]
$\bar{\Omega}/\Omega$	0.496	0.502	0.441	0.452	0.383 ± 0.081	[21]
χ^2/DOF	108/6	95.4/5	12.5/5	3.58/4		

all hadron ratios simultaneously. This problem has been discovered in the previous attempts to fit data on particle number ratios in collisions of S nuclei with different targets at CERN SPS [2,3]. From our fit of the data set in Table I for Pb+Pb it follows that the main problems for the ideal HG model are indeed a deficiency of pions, the major contribution to h^- , and a surplus of strange particles — K_s^0 and ϕ substantially exceed their experimental estimates. It is possible to improve the agreement with h^- and ϕ within the ideal HG model ($h^- = 603$, $\phi = 6.38$) by choosing a smaller temperature ($T = 125$ MeV) and a larger volume ($V = 15900$ fm³). To keep N_B close to data ($N_B = 370$) one needs then a larger value of the baryonic chemical potential ($\mu_B = 300$ MeV). These “small” T and “large” μ_B parameters in ID HG lead, however, to completely wrong anti-baryon to baryon ratios: \bar{p}/p , $\Xi/\bar{\Xi}$, and $\bar{\Omega}/\Omega$ all become an order of magnitude smaller than their measured values.

The agreement is improved if one adopts either the strangeness suppression scheme (SS) or hadron “hard-core” repulsion scheme (VDW) with $r_\pi = 0$ and $r > 0$ which leads to the pion enhancement as mentioned before. The VDW results are shown in the second column of Table I. Note that h^- becomes closer to data, but K_s^0 and ϕ are still too large. The SS results are shown in the third column of Table I. The quality of the fit is significantly improved. However, there is still a deficiency of pions in the SS HG model. Finally, the results of taking both SS and VDW together into consideration are given in the fourth column of Table I. The χ^2 of this fit is remarkably small as shown in the table. In addition, we present in Table II all relevant total hadron multiplicities calculated with parameters found from SS+VDW. Similar to Table I the hadron multiplicities presented in Table II include both direct (thermal) yield (10) and all possible contributions from strong resonance decays.

IV. DISCUSSION

Let us discuss a possible physical interpretation of the obtained results in terms of the quark-gluon plasma (QGP)

formation at the early stage of $A+A$ collisions. The “enhancement” in strange hadron productions was suggested as a signal for the formation of QGP in $A+A$ collisions (see e.g., Ref. [22]). One expects that the strangeness equilibration time in the QGP is much shorter than in the HG. Therefore, rapid production and equilibration of strangeness is expected in the QGP initial state, and the strangeness suppression factor ($\gamma_s < 1$) is usually associated with an incomplete strangeness equilibrium in the pure hadron matter.

A real picture could be, however, very different. We found that, for the chemical freeze-out parameters T and μ_B shown in Table I, the strangeness to entropy ratio is larger in the equilibrium HG than in the equilibrium QGP. Therefore, $\gamma_s < 1$ strangeness *suppression* in the HG would become a signal for the formation of QGP at the early stage of $A+A$ collisions at CERN SPS energies. The same conclusion was obtained in Ref. [23].

Strange quark-antiquark pairs may be primarily produced by hard (with typical momenta of several GeV) (anti)quark and gluon collisions at the early nonequilibrium stage of $A+A$ at the SPS. If the QGP is formed one can expect the complete strangeness equilibration ($\gamma_s^Q = 1$ in the QGP). To estimate the strangeness/entropy ratio in the QGP, let us use the ideal gas approximation of massless u -, d -(anti)quarks, gluons and strange (anti)quarks with $m_s = 150$ MeV. In the wide range of T and μ_B ($T = 160-300$ MeV, $\mu_B = 0-400$ MeV), one finds an almost constant value of the ratio of total number of strange quarks and antiquarks $N_s + N_s^-$ to the total entropy S :

$$R_s \equiv \frac{N_s + N_s^-}{S} = 0.022 - 0.025. \quad (13)$$

TABLE II. Particle multiplicities for central Pb+Pb SPS collisions predicted in SS+VDW HG model. The values of model parameters are taken from Table I.

π	K^+	K^-	Λ	$\bar{\Lambda}$	Ξ	$\bar{\Xi}$	Ω	$\bar{\Omega}$
1850	83.1	47.6	38.9	4.43	5.86	1.33	0.242	0.110

There is no realistic model for the QGP hadronization. We do not expect, however, essential additional contribution to the strange-antistrange *hadron* production during the QGP hadronization: at this stage the typical thermal (anti)quark and gluon momenta are only a several hundred of MeV. Therefore, we assume the same number of strange quarks and antiquarks in the QGP before hadronization and in the HG after hadronization. This number is not expected to be further changed in the HG with local thermal equilibrium. The typical thermal hadron momenta are too small for strange hadron production. We expect, therefore, the same number of $N_s + N_{\bar{s}}$ in the HG at the chemical freeze-out. The total entropy is also expected to be conserved approximately during the hadronization of QGP and HG expansion. This suggests that the value of R_s at the HG chemical freeze-out should be close to that in the QGP. The value of R_s for different HG model fits are 0.045 (ID), 0.037 (VDW), 0.029 (SS), and 0.026 (SS+VDW). It is remarkable that our best HG model fit, SS+VDW, where both pion enhancement ($r_{\pi}=0$, $r>0$) and strangeness suppression ($\gamma_s<1$) are included, leads to the value of R_s very close, indeed, to the QGP estimate (13).

V. SUMMARY

In summary, we analyzed new Pb+Pb data at CERN SPS on hadron multiplicities in a full 4π -geometry and on antiparticle/particle ratios in the central rapidity region within various HG models. The parameters T and μ_B are found from fitting the data and appear to be the same, $T = 165$ MeV and $\mu_B = 235$ MeV, within four different versions of the HG model. The ideal HG model fails to give a

reasonable explanation to the Pb+Pb data sets. Meanwhile, the obtained particle number density and total energy density of the ideal HG at the chemical freeze-out is extremely large, which is inconsistent with the picture of a gas of pointlike noninteracting hadrons. To improve this situation, the hadron ‘‘hard-core’’ repulsion had been proposed. It removes the artificially high energy and particle number densities in the gas of noninteracting hadrons. The VDW HG model with $r_{\pi}=0$ and $r_i=r>0$ leads to the pion enhancement and better fit of hadron multiplicities is obtained. Chemical nonequilibrium effects with strangeness suppression were also studied. We have found that this modification significantly improves the agreement with data. Furthermore, the combined effect of these two mechanisms results in an extremely good agreement with the data analyzed. We present the SS+VDW HG model predictions for relevant hadron multiplicities in central Pb+Pb at the SPS. Finally, an interpretation of the obtained results in terms of the possible QGP formation at the early stage of A+A collisions at the SPS is also discussed.

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