Coulomb excitation of a damped oscillator and the Brink-Axel mechanism

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Multiple Coulomb excitation of collective nuclear modes is examined in the context of an extremely schematic dynamic model consisting of a harmonic oscillator, taken to represent the relevant collective mode, damped by linear coupling to a ''bath'' consisting also of harmonic oscillators, and forced by an applied external pulse. The (well-known) exact solution of the model allows for an estimate cross section which takes into account the joint effect of *all* excitation mechanisms leading to definite excitation energy domains, including the Brink-Axel mechanism, which occurs very naturally in this type of model. The semiclassical estimate of cross sections leading to the two-phonon domain shows enhancement with respect to the corresponding values obtained in the case of no damping. The magnitude of the enhancement decreases as the beam energy increases (or, equivalently, as the impact parameter averaged time width of the external pulse decreases), for model parameters chosen to conform to the appropriate nuclear orders of magnitude. [S0556-2813(99)07505-6]

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I. INTRODUCTION

Double excitation of the isovector giant dipole mode in Coulomb excitation processes involving heavy-ion collisions at relativistic energies has now been observed in several nuclei: ^{136}Xe [1], ^{197}Au [2], and ^{208}Pb [3,4]. The measured cross sections for double excitation have been compared with results of ''semiclassical'' coupled channel Coulomb calculations [5–7], and it has been found that the theoretical cross sections systematically underestimate the observed ones by factors which are as large as 2–3 for Xe and Au but are reduced to about just 1.3 in the case of Pb [7].

In a frequently used description of giant resonances in atomic nuclei one associates them with special ''doorway'' states in the host nucleus which are spread through the effect of couplings to other nearby noncollective states. A similar picture can be extended also to double excitations such as those that have been observed for the giant isovector dipole mode [5,6]. The convenience of this type of picture stems from the fact that it can be readily implemented in terms of suitable decompositions of the phase space of the nuclear system through the use of appropriate projection operators, leading to the proper sorting of the various dynamical processes involved. In the sorting process one usually introduces energy-averaged amplitudes corresponding to the doorway effects, which require the subsequent calculation of fluctuation contributions to be incoherently added to the results obtained from them. A different approach can be found, however, in the work of Ponomarev *et al.* [8], in which energy-differential cross sections are calculated for the case of Xe, in the framework of a perturbative semiclassical Coulomb excitation scheme, which correspond to the excitation of individual fine structure final states obtained from a nuclear structure calculation taking into account residual nonlinear couplings between one and two random phase approximation phonons. The result again underestimates the observed cross section by a factor of about 4.

When the various doorway states involved imply the presence of a vibrational band, however, as is the case when one considers multiphonon excitations of a giant isovector dipole mode, still another picture suggests itself in which one is led to single out not just a given state, or a group of states, but rather a collective *degree of freedom* [9]. Unlike in the case of a set of doorways, to which one associates a certain subspace of the full quantum phase space which still involves all the nuclear degrees of freedom, in this case one is led to formulate the dynamics in a *factorized* phase space, involving on the one side the relevant collective degree of freedom and, on the other side, the remaining ones, considered as being ''noncollective'' in the adopted sense. The resulting picture is that of a damped oscillator, the noncollective degrees of freedom playing the role of a reservoir which exchanges energy with the collective mode. The dynamics of such a damped oscillator can be represented, albeit somewhat schematically, by a Hamiltonian of the form

$$H_n=\omega_d d^\dagger d\otimes \mathbf{1}_k+\mathbf{1}_d\otimes\sum_k |\epsilon_k\rangle\epsilon_k\langle\epsilon_k|$$
$$+\sum_{kk'}(|\epsilon_{k'}\rangle g_{k'k}d\langle\epsilon_k|+|\epsilon_k\rangle g^*_{k'k}d^\dagger\langle\epsilon_{k'}|),$$

where the first two terms represent the collective linear mode of frequency ω_d and the reservoir with spectrum $\{\epsilon_k\}$ and eigenstates $\{|\epsilon_k\rangle\}$, respectively, while the last term couples these two subsystems. The ground state of the coupled system will have simple properties when the coupling can be seen as restricted so that $g_{k'k}\equiv 0$ unless $\epsilon_{k'}>\epsilon_k$. In this case it reduces in fact to the product state $|0_d\rangle\otimes|\epsilon_0\rangle$, this latter ket being the reservoir ground state. In fact, when this condition does not apply one has a correlated ground state which contains components involving combined excitations of the two subsystems.

II. DRIVEN COUPLED OSCILLATORS

The standard semiclassical picture of the multiple Coulomb excitation of the damped giant isovector mode can be schematically described by a Hamiltonian of this type, to which one adds furthermore an external driving term for the collective mode. This driving term can be represented as

$$V(t)=[f(t)d^{\dagger}+f^{*}(t)d]\otimes\mathbf{1}_{k},$$

where the function $f(t)$ is a scalar ersatz for the Coulomb field of a passing charged projectile. In order to further simplify this model to the point of complete solubility, the reservoir may be further specialized to a set of noncollective linear oscillators, i.e.,

$$\sum_{k}|\epsilon_{k}\rangle\epsilon_{k}\langle\epsilon_{k}|\rightarrow\sum_{k}\omega_{k}b_{k}^{\dagger}b_{k},$$

with a linear coupling of the form

$$\sum_{kk'}(|\epsilon_{k'}\rangle g_{k'k}d\langle\epsilon_{k}|+|\epsilon_{k}\rangle g_{k'k}^{*}d^{\dagger}\langle\epsilon_{k'}|)$$

$$\rightarrow\sum_{k}(g_{k}b_{k}^{\dagger}d+g_{k}^{*}d^{\dagger}b_{k}),$$

which in particular guarantees a simple, factorized ground state. The semiclassical Coulomb multiple excitation process will therefore be studied in the very schematic, soluble model represented by the linear Hamiltonian

$$H=\omega_{d}d^{\dagger}d\otimes\mathbf{1}_{k}+\mathbf{1}_{d}\otimes\sum_{k}\omega_{k}b_{k}^{\dagger}b_{k}+\sum_{k}(g_{k}b_{k}^{\dagger}d+g_{k}^{*}d^{\dagger}b_{k})$$
$$+[f(t)d^{\dagger}+f^{*}(t)d]\otimes\mathbf{1}_{k}. \qquad (2.1)$$

As will be shown below, the effects induced by the presence of the vibrational collective degree of freedom can be studied in this model directly at the level of the fine structure generated by the coupling to the reservoir, independently of the introduction of average amplitudes, so that both energy smooth and fluctuation components are effectively treated together. One possible drawback of this uniform treatment lies actually in the ensuing difficulty of disentangling these two, rather theoretically motivated, types of contribution.

In order to obtain the relevant solutions of the model problem posed by Eq. (2.1), one begins by performing a canonical transformation to the normal modes of the coupled system, which in view of the special nature of the coupling term consists in introducing new bosonic operators c_{ν} related to the d,b_{k} as

$$d=\sum_{\nu}x_{0\nu}c_{\nu},\quad b_{k}=\sum_{\nu}x_{k\nu}c_{\nu}, \qquad (2.2)$$

where the x coefficients are chosen so that the H becomes

$$H=\sum_{\nu}[\omega_{\nu}c_{\nu}^{\dagger}c_{\nu}+x_{0\nu}^{*}f(t)c_{\nu}^{\dagger}+x_{0\nu}f^{*}(t)c_{\nu}]. \qquad (2.3)$$

It is perhaps worth recalling some features of this transformation to normal modes [10]. The eigenfrequencies ω_{ν} are obtained from the dispersion equation

$$\omega_{\nu}-\omega_{d}=\sum_{k}\frac{|g_{k}|^{2}}{\omega_{\nu}-\omega_{k}},$$

while the x coefficients satisfy

$$x_{k\nu}=\frac{g_{k}^{*}}{\omega_{\nu}-\omega_{k}}x_{0\nu},$$

so that normalization gives

$$|x_{0\nu}|^{2}=\frac{1}{1+\sum_{k}[|g_{k}|^{2}/(\omega_{\nu}-\omega_{k})^{2}]}.$$

This last quantity describes the distribution of the collective degree of freedom among the normal modes, as a result of its coupling to the reservoir degrees of freedom, normal mode excitations corresponding to ''fine structure'' states of the damped collective mode. Furthermore, using these relations it is easy to obtain the sum rule relations

$$S_{1}\equiv\sum_{\nu}\omega_{\nu}|x_{0\nu}|^{2}=\omega_{d}\quad\text{and}$$

$$\sum_{\nu}\omega_{\nu}^{2}|x_{0\nu}|^{2}-S_{1}^{2}=\sum_{k}|g_{k}|^{2}. \qquad (2.4)$$

The first of these simply equates the collective frequency ω_{d} to the centroid of the fine structure frequencies ω_{ν}, while the second relates the mean-square deviation of the fine structure frequencies to the sum of the absolute squares of the coupling constants g_{k}. In the case of reservoir oscillator frequencies forming an endless picket fence and k-independent couplings g_{k}, this quantity diverges.

Equation (2.3) describes a set of *independent* driven oscillators, which can thus be dealt with separately, one by one. The relevant initial state in the present context is that in which all oscillators are in their respective ground states. In order to obtain the solution one, e.g., considers the Heisenberg equation of motion for the operator $c_{\nu}(t)=U_{\nu}(t)c_{\nu}U_{\nu}^{\dagger}(t)$, $U_{\nu}(t)$ being the full evolution operator for the corresponding normal mode, which reads

$$i\dot{c}_{\nu}(t)=\omega_{\nu}c_{\nu}(t)+x_{0\nu}^{*}f(t)$$

and is readily solved as

$$c_{\nu}(t)=e^{-i\omega_{\nu}t}c_{\nu}-ix_{0\nu}^{*}e^{-i\omega_{\nu}t}\int_{0}^{t}dt'e^{i\omega_{\nu}t'}f(t'),$$

revealing that, besides acquiring the usual harmonic phase, $c_{\nu}(t)$ undergoes a coherent displacement due to the action of the driving force, the corresponding amplitude being $\exp(-i\omega_{\nu}t)\alpha_{\nu}(t)$, with

$$\alpha_\nu(t) \equiv -i x_{0\nu}^* \int_0^t dt' e^{i\omega_\nu t'} f(t'). \tag{2.5}$$

It is also an easy matter to use the initial condition and write down the state at time t of this oscillator in the Schrödinger picture. One gets

$$U_\nu(t)|0_\nu\rangle = |e^{-i\omega_\nu t}\alpha_\nu(t)\rangle$$
$$\equiv e^{-|\alpha_\nu(t)|^2/2} \sum_{n=0}^{\infty} \frac{[e^{-i\omega_\nu t}\alpha_\nu(t)]^n}{\sqrt{n!}} |n_\nu\rangle, \tag{2.6}$$

where the states $|n_\nu\rangle$ are the usual phonon number eigenstates for the νth normal mode. In the case of short pulses one considers in connection with the semiclassical treatment of Coulomb excitation, letting the pulse start sufficiently late (after $t=0$), one gets for $\alpha_\nu(T)$ at asymptotically large times T essentially the Fourier transform of the pulse, a time-independent quantity.

III. INCLUSIVE EXCITATION PROBABILITY AND THE BRINK-AXEL MECHANISM

The next point to be discussed concerns the quantities, relevant for the Coulomb excitation problem, that one wishes to obtain from this general solution of the driven, damped oscillator model. First and foremost are various excitation probabilities, which eventually become translated to the corresponding cross sections. The ''elementary,'' excitation probabilities that correspond to distinguishable nuclear excitation processes are excitations to a definite phonon number of each of the normal mode oscillators (i.e., definite ''fine structure'' nuclear excitations). Actual measurements are, however, more inclusive, and correspond to the probability of exciting certain energy bands which can be associated with total phonon number in the collection of normal modes. The calculation of these excitation probabilities is completely straightforward in terms of Eq. (2.6), given the fact that the complete state of the model system is simply a product state of coherent states of this sort. The probability for exciting a single phonon in normal mode ν (while all other normal modes remain in their respective ground states) is calculated simply as

$$P_1^{(\nu)}(t) = \left| \langle 0 \ldots 1_\nu 0 \ldots | \left[\prod_{\nu'} |e^{-i\omega_{\nu'} t}\alpha_{\nu'}(t)\rangle \right] \right|^2$$
$$= \exp\left(-\sum_{\nu'} |\alpha_{\nu'}(t)|^2 \right) |\alpha_\nu(t)|^2,$$

so that the corresponding inclusive one-phonon excitation probability is

$$P_1(t) = \sum_\nu P_1^{(\nu)} = \exp\left(-\sum_{\nu'} |\alpha_{\nu'}(t)|^2 \right) \sum_\nu |\alpha_\nu(t)|^2.$$

In order to calculate inclusive probabilities for the excitation of n phonons (of whatever type) one proceeds in the same way, keeping due track of the various possible multiplicity distributions, and obtains the simple result

$$P_n(t) = \sum_{\{\nu\}} P_n^{(\{\nu\})} = \exp\left(-\sum_{\nu'} |\alpha_{\nu'}(t)|^2 \right) \frac{\left[\sum_\nu |\alpha_\nu(t)|^2 \right]^n}{n!}, \tag{3.1}$$

where $P_n^{(\{\nu\})}$ stands for the probability of exciting n phonons of types specified by $\{\nu\}$. This expression works also for $n=0$ (probability of remaining in the product ground state) and makes obvious the overall probability normalization. Note that the inclusive probabilities have been obtained as the *incoherent sum* of ''elementary'' contributions corresponding to the various included fine structure states.

The translation of the $P_n(t)$ (at asymptotically large times T) into cross section language involves in an essential way the customary realization of the external electromagnetic pulse $f(t)$ in the context of the so-called semiclassical calculations of nuclear excitation processes. This pulse, due to the passage of a projectile moving essentially in a straight line trajectory at constant speed, will be parametrized here by the simple scalar ersatz [5]

$$f(t) = f^*(t) = \frac{V_0 b_{\min}}{[\gamma v (t-t_0)]^2 + b^2} \equiv V_0 \frac{\tau_{\min}^2}{(t-t_0)^2 + \tau^2}, \tag{3.2}$$

where V_0 is an overall strength parameter, γ is the usual relativistic factor, and b is the impact parameter of the straight line trajectory, which has to be larger than $b_{\min}$, the value at which ''other processes'' are assumed to take over. The quantity t_0 stands for the time of closest approach. In the last step the definitions

$$\tau \equiv \frac{b}{\gamma v}, \quad \tau_{\min} \equiv \frac{b_{\min}}{\gamma v}$$

have been introduced. Note that the quantity τ corresponds to the (impact parameter dependent) time width of the pulse.

Using this pulse in Eqs. (2.5) and (2.6) we see that the normal mode coherent displacements, and hence also the inclusive transition probabilities, become dependent on the impact parameter b. Cross sections corresponding to the inclusive transition probabilities are then obtained as usual from [6]

$$\sigma_n = 2\pi \int_{b_{\min}}^{\infty} db\, b\, P_n(T;b), \tag{3.3}$$

where $P_n(T;b)$ is the time-independent asymptotic value of the inclusive transition probabilities, obtained in the conditions discussed earlier. Using Eq. (3.2) the required asymptotic values $|\alpha_\nu(T)|^2$ can in fact be obtained in closed form. Assuming that $t_0 \gg \tau$ so that the time integration in Eq. (2.5) can be extended to minus infinity with negligible error, one gets

$$|\alpha_\nu(T)|^2 \to A^2(b) |x_{0\nu}|^2 e^{-2\omega_\nu \tau},$$

with

$$A(b) = \pi V_0 \frac{b_{\min}^2}{\gamma v b}.$$

TABLE I. Multiphonon excitation paths for the collective damped oscillator. The notations f and g refer to the external pulse and to the coupling of the collective mode to the bath oscillators, respectively [see Eq. (2.1)]. The Brink-Axel route linking the ground state (GS) to the two-phonon domain is indicated by the thick arrows.

●●●	. . .		. . .		. . .		. . .
	$f\updownarrow$		$f\updownarrow$		$f\updownarrow$		$f\updownarrow$
		g		g		g	
Three phonons	$\vert 3_d,\{0_k\}\rangle$	$\leftrightarrow$	$\vert 2_d,1_k\rangle$	$\leftrightarrow$	$\vert 1_d,2_k\rangle,\vert 1_d,1_k,1_{k'}\rangle$	$\leftrightarrow$	. . .
	$f\updownarrow$		$f\updownarrow$		$f\updownarrow$		
		g		g			
Two phonons	$\vert 2_d,\{0_k\}\rangle$	$\leftrightarrow$	$\vert 1_d,1_k\rangle$	$\leftrightarrow$	$\vert 0_d,2_k\rangle,\vert 0_d,1_k,1_{k'}\rangle$		
	$f\updownarrow$		$f\Updownarrow$				
		g					
One phonon	$\vert 1_d,\{0_k\}\rangle$	$\Leftrightarrow$	$\vert 0_d,1_k\rangle$				
	$f\Updownarrow$						
GS	$\vert 0_d,\{0_k\}\rangle$						

The asymptotic inclusive transition probabilities relevant for the calculation of cross sections can thus be written explicitly as

$$P_n(b)=\frac{1}{n!}\exp\left(-A^2(b)\sum_\nu |x_{0\nu}|^2 e^{-2\omega_\nu\tau}\right)\times\left(A^2(b)\sum_\nu |x_{0\nu}|^2 e^{-2\omega_\nu\tau}\right)^n, \qquad (3.4)$$

while, for comparison, the corresponding probabilities for the *undamped* collective oscillator reduce to

$$P_n^{(0)}(b)=\frac{1}{n!}e^{-A^2(b)e^{-2\omega_d\tau}}[A^2(b)e^{-2\omega_d\tau}]^n. \qquad (3.5)$$

The various processes which are taken into account in Eq. (3.4) are identified in Table I, in terms of the original, collective and reservoir degrees of freedom. The probabilities $P_n(b)$ are inclusive probabilities for populating each of the n-phonon level groups. For $n\geqslant 2$ this involves several observationally indistinguishable routes, among which is the Brink-Axel [11] collective excitation in the presence of excited reservoir states. Two points are worth stressing here: first, excitation of the *collective* two-phonon state does not correspond to populating an asymptotically well-defined (stationary) state, so that no probability (or cross section) can be unambiguously attributed to this process *per se*, a feature which is in fact independent of the particular realization of the reservoir implied in Eq. (2.1); second, for that particular realization the various processes indicated in Table I are taken into account *to all orders*, both in the damping couplings g_k and in the external pulse $f(t)$.

A qualitative appraisal of the effect of the spreading on the inclusive excitation probabilities, which also serves the purpose of identifying the relevant scale parameters, can be obtained analytically by taking a simple normalized ''box'' distribution of width I for the strengths $|x_{0\nu}|^2$, i.e.,

$$|x_{0\nu}|^2\to\frac{d\omega_\nu}{I}\left[\Theta\left(\omega_\nu-\omega_d+\frac{I}{2}\right)-\Theta\left(\omega_d+\frac{I}{2}-\omega_\nu\right)\right],$$

where $\Theta(x)$ denotes the usual unit step function. Replacing the sum in Eq. (3.4) by an integral one obtains immediately

$$\sum_\nu |x_{0\nu}|^2 e^{-2\omega_\nu\tau}\to e^{-2\omega_d\tau}\frac{\sinh I\tau}{I\tau},$$

which gives for the ratio of the probabilities with and without spreading:

$$\frac{P_n(b)}{P_n^{(0)}(b)}\to\exp\left[-A^2(b)e^{-2\omega_d\tau}\left(\frac{\sinh I\tau}{I\tau}-1\right)\right]\left(\frac{\sinh I\tau}{I\tau}\right)^n.$$

This expression clearly indicates the relevance of the quantity $I\tau$, which corresponds to the ratio of the collision time τ to the characteristic decay time of the collective mode $1/I$. While it approaches unity in the limit of collision times which are very short compared with the collective decay time, it can lead otherwise to important enhancement effects when the squared coherent displacement of the undamped collective mode $A^2(b)e^{-2\omega_d\tau}$ is small, as is in fact the case for typical Coulomb excitation processes of isovector giant dipole modes.

IV. NUMERICAL RESULTS

In order to display the quantitative effects of the damping on multiphonon nuclear excitation processes in a more ''realistic'' situation we next compare results obtained by using Eqs. (3.4) and (3.5) together with a ''typical'' discrete strength distribution such that the quantities $|x_{0\nu}|^2$ are chosen as having the Lorentzian distribution

$$|x_{0\nu}|^2\sim\frac{2}{\pi}\frac{\Gamma\quad\omega_\nu^2\Delta\omega_\nu}{(\omega_\nu^2-\omega_0^2)^2+\Gamma^2\omega_\nu^2},\qquad\sum_\nu |x_{0\nu}|^2=1,$$

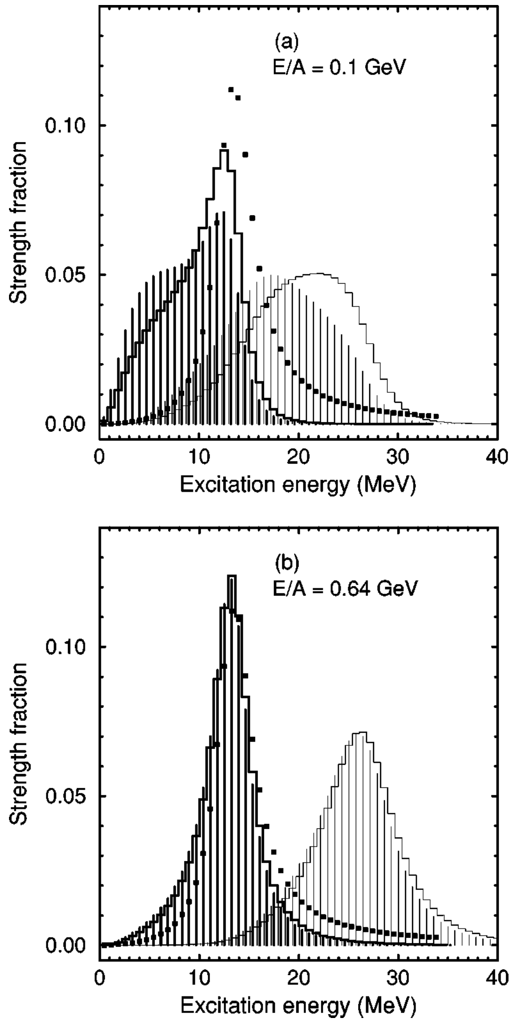

FIG. 1. Strength distribution of the collective mode (solid squares) and one- and two-phonon excitation strengths for $b = 17$ fm (heavy and light vertical lines) and $b = 21$ fm (heavy and light histograms), as functions of excitation energy, for projectile energies $E/A = 0.1$ GeV and 0.64 GeV.

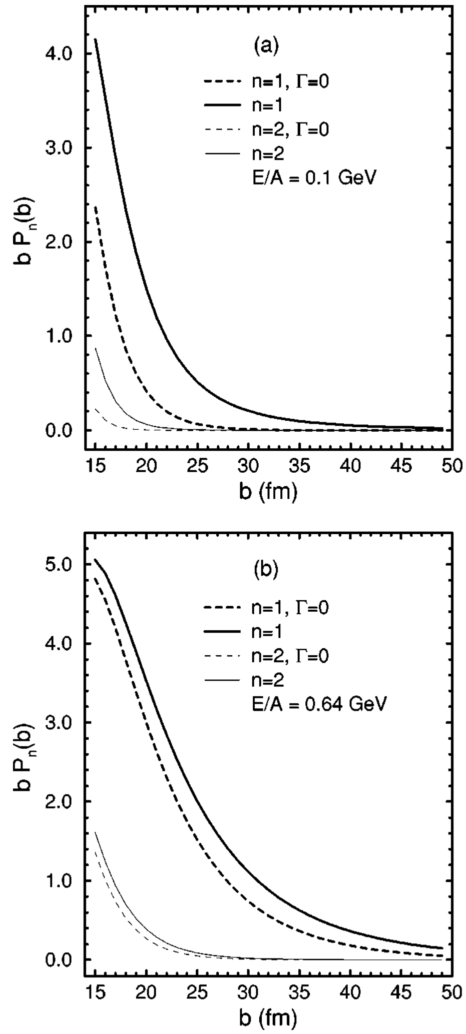

FIG. 3. Impact-parameter-weighted inclusive excitation probabilities for one and two phonons, with and without spreading, as functions of impact parameter for the projectile energies $E/A = 0.1$ and 0.64 GeV.

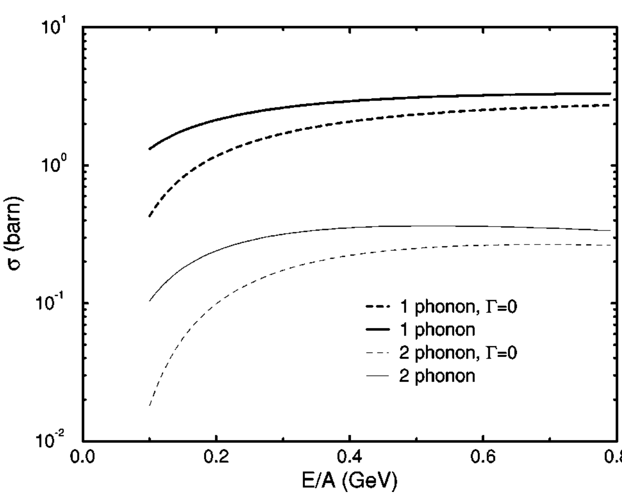

FIG. 2. Enhancement of one- and two-phonon cross sections as a result of the damping of the collective mode. Dashed curves correspond to the undamped collective mode; solid curves correspond to the damping shown in Fig. 1.

with constant spacing $\Delta\omega_\nu$. The resonance parameters $\omega_0 = 13.5$ MeV and $\Gamma = 4.35$ MeV have been chosen so that the resulting strength distribution is a rough fit to the photoexcitation cross section for the (one-phonon) giant dipole resonance in ^{208}Pb. The pulse strength parameter V_0 has been fixed so as to give the observed value [7] of the one-phonon, ^{208}Pb-^{208}Pb Coulomb excitation cross section at $E/A = 0.64$ GeV. The frequency ω_d which appears in the excitation probabilities for the undamped collective oscillator, Eq. (3.5), is obtained using the energy-weighted sum rule S_1, Eq. (2.4), which gives $\omega_d = 15.2$ MeV.

The collective mode strength fractions $|x_{0\nu}|^2$ obtained in this way are shown in Fig. 1, together with the resulting one- and two-phonon excitation strength fractions for different values of the impact parameter and for the projectile energies E/A of 0.1 and 0.64 GeV. The excitation strength fractions are obtained by normalizing to one the excitation energy distributions of the appropriate excitation probabilities. Projectile-energy- and impact-parameter-dependent shape distortions due to different effectiveness of the excitation mechanism are clearly seen, being much more pronounced at lower projectile energies. Inclusive one- and two-phonon,

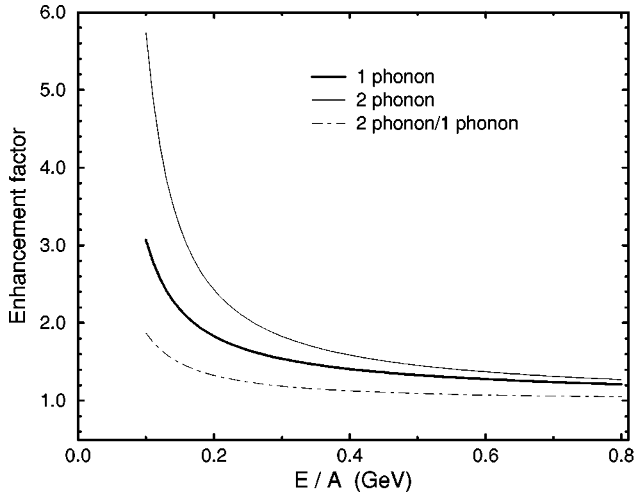

FIG. 4. One- and two-phonon cross-section enhancement factors and their ratio as a function of projectile energy.

damped, and undamped cross sections obtained using Eq. (3.3) are shown in Fig. 2 as a function of incident energy. One- and two-phonon impact-parameter-weighted inclusive excitation probabilities for the projectile energies 0.1 and 0.64 GeV are shown in Fig. 3, as functions of impact parameter, together with the corresponding probabilities for the undamped collective mode.

While both the one- and the two-phonon cross sections are considerably enhanced by the damping of the collective mode, one sees in Fig. 2 that the enhancement factor is consistently larger for the two-phonon domain. This is shown quantitatively in Fig. 4, where the ratio of the two-phonon to the one-phonon enhancement factors is plotted as a function of projectile energy. Furthermore, the enhancement of the two-phonon cross section *relative* to the one-phonon cross section is seen to *decrease* monotonically with projectile energy, consistently with the apparent trend [12,1–4,7] of the ''missing two-phonon strength'' in standard semiclassical calculations carried out in the harmonic limit *or* including spreading effects of the one-phonon collective state treated as an ''exit doorway'' [5,6], which actually *quenches* the cross-section ratio corresponding to the harmonic limit, represented here by the undamped cross sections. It is worth stressing in this connection that one important ingredient which is missing in the latter calculations but is automatically taken into account here is the contribution of Brink-Axel excitation paths (see Table I) to the multiphonon domains.

V. CONCLUSION

In this paper we have discussed an exactly soluble model of multiple giant resonance excitation. The model involves a harmonic oscillator, taken to represent the collective mode, damped through linear coupling to a bath consisting also of harmonic oscillators, and forced by an external pulse which simulates the Coulomb force between two colliding nuclei. The exact solution of the model allows for assessment of the importance of the Brink-Axel phenomenon. It is found that the latter appreciably enhances the excitation cross section when compared to the one obtained in the zero-coupling (harmonic) limit. This enhancement is found to decrease as the interaction time becomes shorter, in qualitative agreement with available data on relativistic Coulomb excitation of double giant dipole resonances in several nuclei.

Because of the schematic character of the model, a full comparison with measured cross section is not warranted at this time. It may be anticipated, however, that treatments including the Brink-Axel path [13,14] may account for the observed Pb cross section in the harmonic limit, while anharmonic effects [15] may be an additional required ingredient in the cases where the discrepancy between calculated and observed cross sections is larger.

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