

## Production mechanism of superheavy nuclei in cold fusion reactions

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(Received 8 September 1998)

A relatively simple model which reproduces the measured formation cross sections of deformed superheavy nuclei synthesized in cold fusion reactions is proposed. Within this model, optimal bombarding energies and formation cross sections of the hypothetical spherical superheavy nuclei are predicted.  
[S0556-2813(99)04305-8]

PACS number(s): 27.90.+b, 24.60.Dr, 24.60.Ky

As early as in the 1970's, cold fusion reactions based on the doubly magic lead target have been proposed by Oganessian for the synthesis of transactinide nuclei [1,2]. In such reactions, the excitation energy of the compound nucleus is reduced due to the large shell effect in  $^{208}\text{Pb}$  and, consequently, the final nucleus is obtained after the emission of only one or two neutrons. Recently this method has been particularly successful in the discovery of deformed superheavy nuclei stabilized by large shell effects predicted for such heavy deformed nuclear systems, for example, Refs. [3–6]. The new elements 110 [7], 111 [8], and 112 [9] have been synthesized at GSI-Darmstadt by Sigurd Hofmann and his co-workers in the cold fusion reactions  $^{208}\text{Pb}(^{62}\text{Ni},1n)^{269}110$ ,  $^{208}\text{Pb}(^{64}\text{Ni},1n)^{271}110$ ,  $^{209}\text{Bi}(^{64}\text{Ni},1n)^{272}111$ , and  $^{208}\text{Pb}(^{70}\text{Zn},1n)^{277}112$ . Moreover, the Berkeley group reported on the possible production of the nucleus  $^{267}110$  in the cold fusion reaction  $^{209}\text{Bi}(^{59}\text{Co},1n)$  [10].

Much more asymmetric combinations of colliding nuclei, based on actinide targets, have been used recently by the Dubna-Livermore Collaboration. Hot fusion reactions with the excitation energy of the compound nucleus equal to several tens of MeV, leading to the final nucleus after the emission of several neutrons, have been carried out at Dubna. These experiments resulted in the discovery of the deformed superheavy nuclei  $^{265}\text{Sg}_{159}$  [11],  $^{266}\text{Sg}_{160}$  [11],  $^{267}\text{Hs}_{159}$ , [12], and  $^{273}110_{163}$  [13]. Moreover,  $^{238}\text{U}$  was bombarded by  $^{48}\text{Ca}$  at Dubna last year [14]. According to the authors of Ref. [14], the results of this experiment may be interpreted as decays of a very heavy nucleus  $^{283}112$ .

The production of the heaviest nuclei is very difficult. The measured formation cross sections reached the very low level of 1 pb [9,13]. Moreover, the measured excitation functions are very narrow [7]. Therefore, the understanding of the production mechanism of superheavy nuclei and, consequently, reliable predictions for the formation cross sections of superheavy nuclear systems are of prime importance for the success of future discovery experiments.

The aim of the present paper is the analysis of the  $1n$ -emission channel of the fusion reactions based on the doubly magic  $^{208}\text{Pb}$  target. Several attempts at an explanation of the production mechanism of deformed superheavy

nuclei in such reactions have been made. Sigurd Hofmann proposed the explanation involving the Bass fusion barrier [15]. Since the Bass barrier is too large and, consequently, the quantal penetrability too low to reproduce the measured formation cross sections, Hofmann suggested on the basis of the paper [16] of von Oertzen that these nuclei were probably formed in the fusion reactions initiated by transfer of a pair of protons from the target to the projectile in a head-on collision [17]. Such a transfer would lead to the decrease of the Coulomb barrier, which might permit fusion. However, it is doubtful whether the use of the Bass barrier, which was developed for the classical description of the hot fusion reactions leading to lighter nuclei, is reliable for the description of the processes in question.

A calculation of the formation cross sections of deformed superheavy nuclei was performed by Adamian *et al.* [18]. These authors describe the cold fusion reactions assuming that after the full dissipation of the collision kinetic energy a dinuclear system is formed. After that such a system evolves to the compound nucleus by nucleon transfer from the lighter nucleus to the heavier one. A good agreement of the calculated cross sections with the experimental data was achieved under an assumption that the ratio of the partial widths for neutron emission and fission for nuclei with the proton number  $108 \leq Z \leq 113$  is constant, i.e., the dependence of this ratio on the neutron separation energy, the fission barrier and the thermally damped shell effects was completely disregarded.

Classical models invented by Władek Świątecki [19–21] predict large energy losses caused by friction and, consequently, suggest a large entrance channel hindrance factor for energies at which deformed superheavy nuclei were synthesized. (The “extra-push” hindrance.) The formation of deformed superheavy nuclei may be explained in the framework of the extra-push models [19–21] by the precompound neutron emission at a configuration close to the deformation of the fission barrier, as proposed by Armbruster [22], or by the thermal fluctuation of the fusion barrier. A model including thermal fluctuation around the mean fusion trajectory and the temperature dependent shell correction was proposed in Refs. [23,24] for the description of the symmetric hot fusion reactions leading to  $Z = 114$  spherical superheavy nuclei. Cross sections of the order of tens of picobarns were calculated. Such symmetric hot reactions involving strongly radioactive target and projectile nuclei are outside the scope of the present work.

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In this paper, we propose another explanation of the production of superheavy nuclei in the cold fusion reactions  $^{208}\text{Pb}(\text{HI}, 1n)$ . In our model, based on simple analytical formulas, we assume that the neutron is evaporated from the compound nucleus formed by quantal tunneling through the fusion barrier.

The formation cross section of a very heavy nucleus in the  $1n$ -emission channel of the fusion reaction is given by the formula

$$\sigma_{1n}(E_{\text{HI}}) = \sum_{l=0}^{l_{\text{max}}} \sigma_l(E_{\text{HI}}) P_{1n,l}(E^*), \quad (1)$$

where  $E_{\text{HI}}$  is the bombarding energy (energy of the projectile in the center-of-mass coordinate system),  $\sigma_l(E_{\text{HI}})$  is the cross section for the  $l$ th partial wave of the incident heavy ion, and  $P_{1n,l}(E^*)$  is the probability of the emission of exactly one neutron from the compound nucleus with the total excitation energy  $E^*$  and angular momentum  $l$  for the case where fission is possible. The total excitation energy  $E^*$  is calculated as a difference  $E^* = E_{\text{HI}} - Q$  between the bombarding energy  $E_{\text{HI}}$  and the ground-state  $Q$  value for the considered reaction. The latter quantity is obtained by subtraction of the masses of the target and the projectile from the mass of the compound nucleus. The summation over  $l$  is cut at  $l_{\text{max}}$  for which the contribution of the term  $\sigma_l(E_{\text{HI}}) P_{1n,l}(E^*)$  to the cross section  $\sigma_{1n}(E_{\text{HI}})$  becomes less than 1%.

The partial cross section  $\sigma_l(E_{\text{HI}})$  is given by

$$\sigma_l(E_{\text{HI}}) = \pi \lambda^2 (2l + 1) T_l, \quad (2)$$

where  $\lambda = \sqrt{\hbar^2 / (2\mu E_{\text{HI}})}$  is the reduced de Broglie wavelength of the projectile,  $\mu$  is the reduced mass, and  $T_l$  is the transmission probability of the wave through the fusion barrier. This probability is calculated by means of the WKB approximation

$$T_l = \frac{1}{1 + \exp(2S_l)}. \quad (3)$$

For low angular momenta, the action integral  $S_l$  between the point of closest approach and the exit point from the barrier may be calculated by using the approximate relation

$$S_l = S_0 \left( E_{\text{HI}} - \frac{\hbar^2 l(l+1)}{2\mu R_{fu}^2} \right), \quad (4)$$

where  $R_{fu}$  is the radial coordinate of the position of the fusion barrier and  $S_0$  is the action integral for the case where the centrifugal barrier is absent.

For the sake of simplicity, we use as the fusion barrier the Coulomb potential cut off at the distance  $R_{fu}$ , which depends on the size and the electric charge of the colliding nuclei. The penetrability factor calculated with such an unphysical barrier happens to lead to a better description of the measured formation cross sections of superheavy nuclei than the use of more realistic one-dimensional potentials discussed in Ref. [25], which is another advantage of the cutoff Coulomb barrier. It is worth mentioning that the measured half-lives for the emission of heavy clusters from nuclei are

also better reproduced—for an unknown reason—by means of a cutoff Coulomb barrier with an adjusted cutoff radius than by using more realistic models [26].

The action integral for the cutoff Coulomb barrier for spherical target and projectile nuclei reads

$$S_0 = \sqrt{\frac{2\mu}{\hbar^2 E_{\text{HI}}}} Z_T Z_P e^2 \left[ \arccos \left( \sqrt{\frac{E_{\text{HI}}}{B_{fu}}} \right) - \sqrt{\frac{E_{\text{HI}}}{B_{fu}}} \sqrt{1 - \frac{E_{\text{HI}}}{B_{fu}}} \right]. \quad (5)$$

Here,  $B_{fu} = Z_T Z_P e^2 / R_{fu}$  is the height of the Coulomb barrier,  $Z_T$  and  $Z_P$  are the proton numbers of the target and the projectile, and  $e$  is the elementary electric charge. It was observed both experimentally [27] and theoretically [28] that the ratio of the radial coordinates of the positions of the fusion barrier  $R_{fu}$  and the touching point of colliding nuclei  $R_{12}$  decreases with increasing proton number of the target or projectile. In order to determine  $R_{fu}$ , we assume that the distance between the touching point and the position of the fusion barrier is inversely proportional to the Coulomb potential at the barrier. This leads to the following expression on the position of the fusion barrier:

$$R_{fu} = \frac{R_{12}}{1 - c/Z_T Z_P} > R_{12}, \quad (6)$$

where  $c$  is a parameter. This parameter describes probably all other effects not taken explicitly into account in such a simple schematic picture. The position of the touching point  $R_{12}$  is the sum of the half-density radii of the target and projectile nuclei. We calculate this quantity by using the equation

$$R_{12} = c_T R_T + c_P R_P. \quad (7)$$

Here,  $R_T$  and  $R_P$  are the nuclear radii of the target and projectile determined from the nuclear mean square charge radii. The former are calculated by means of the Nerlo-Pomorska and Pomorski formula [29]

$$R_T = r_0 (1 - \alpha I_T) A_T^{1/3}, \quad R_P = r_0 (1 - \alpha I_P) A_P^{1/3}, \quad (8)$$

where  $I_T = (N_T - Z_T) / A_T$  and  $I_P = (N_P - Z_P) / A_P$  are the relative neutron excesses (reduced isospins) of the target and projectile,  $A_T$  and  $A_P$  ( $N_T$  and  $N_P$ ) are the mass (neutron) numbers of the target and projectile,  $r_0 = 1.256$  fm [29] is the nuclear radius parameter and  $\alpha = 0.202$  [29]. The coefficients  $c_T$  and  $c_P$  relating  $R_T$  and  $R_P$  to the half-density radii of the target and the projectile, respectively, may be deduced from Ref. [30] and read

$$c_T = 1 - \frac{7}{2} \left( \frac{b}{R_T} \right)^2 - \frac{49}{8} \left( \frac{b}{R_T} \right)^4, \\ c_P = 1 - \frac{7}{2} \left( \frac{b}{R_P} \right)^2 - \frac{49}{8} \left( \frac{b}{R_P} \right)^4, \quad (9)$$

where  $b = 1$  fm is the nuclear surface width.

Assuming that the rotational energy is not available either for neutron evaporation or fission, we express the probability

of the emission of one neutron  $P_{1n,l}(E^*)$  in terms of the intrinsic excitation energy  $E_{\text{int}}^*$  defined as the difference between the total excitation energy  $E^*$  and the rotational energy  $E_{\text{rot}} = \hbar^2 l(l+1)/(2J)$ , where  $J$  is the moment of inertia normal to the symmetry axis of the axially symmetric compound nucleus. For the simplicity, we assume that the moment of inertia of the compound nucleus is equal to the rigid body value which is calculated by means of the formula [31]

$$J = J_0 \left[ 1 + \sqrt{\frac{5}{16\pi}} \beta_2 + \frac{45}{28\pi} \beta_2^2 + \frac{15}{7\pi\sqrt{5}} \beta_2 \beta_4 \right]. \quad (10)$$

Here,  $J_0 = (2/5)A_{CN}mR_{CN}^2$  is the rigid body moment of inertia for the spherical nucleus with mass number  $A_{CN}$  and radius  $R_{CN}$ , and  $m$  is the mass of the nucleon. Again, we express the radius  $R_{CN}$  by the Nerlo-Pomorska and Pomorski formula [29]. The deformation parameters  $\beta_\lambda$  are connected with the parametrization of nuclear shape by the spherical harmonics  $Y_{\lambda 0}(\vartheta)$ . The dependence of the moment of inertia on the higher multipolarities is disregarded.

The competition between neutron evaporation and fission is described by the excitation energy-dependent neutron-to-total width ratio for the compound nucleus before the evaporation of the neutron

$$G(E^*, l) = \frac{\Gamma_n}{\Gamma_f + \Gamma_n} = \frac{(\Gamma_n/\Gamma_f)}{1 + (\Gamma_n/\Gamma_f)}. \quad (11)$$

Here,  $\Gamma_n$  and  $\Gamma_f$  are the partial widths for neutron emission and fission, respectively. The expression (11) is an increasing function of excitation energy and it depends also on the neutron separation energy  $S_n$ , the height of the static fission barrier  $B_f^{\text{stat}}$ , and the thermally damped shell effects. The quantity  $P_{1n,l}$  is equal to  $G(E^*, l)$  if the intrinsic excitation energy at the deformation corresponding to the equilibrium point is larger than  $S_n$  and smaller than the sum of  $S_n$  and the threshold for emission of the second neutron or fission. For larger intrinsic excitation energies,  $P_{1n,l}$  decreases rapidly because of the possibility of emission of the second neutron or fission. This means that for narrow excitation functions the cross section reaches its maximum for the excitation energy close to the sum of the neutron separation energies for the compound nucleus and the evaporation residue or close to the sum of the neutron separation energy for the compound nucleus and the static fission barrier for the evaporation residue, depending which of these sums is smaller. This energy is considered in the present paper as the optimal excitation energy for fusion of a superheavy nuclear system.

The assumption that the level density of the highly excited nucleus is described by the Bethe formula leads to the following expression for the neutron-to-fission width ratio [32,33]:

$$\Gamma_n/\Gamma_f = \exp(2\sqrt{a_n E_n^*} - 2\sqrt{a_f E_f^*}), \quad (12)$$

which is accurate to within the pre-exponential factor. Here,  $a_n$  is the level density parameter for the evaporation residue (synthesized nucleus) with the deformation corresponding to the equilibrium point,  $a_f$  is the level density parameter for the compound nucleus with the deformation corresponding to the saddle point, and  $E_n^* = E_{\text{int}}^*(eq) - S_n$  and  $E_f^*$

$= E_{\text{int}}^*(sd) - B_f^{\text{stat}}$  are the excitation energies in the transition state for neutron emission and fission, where  $E_{\text{int}}^*(eq)$  is the intrinsic excitation energy and  $E_{\text{int}}^*(sd)$  is the difference between the total excitation energy  $E^*$  and the rotational energy at the saddle point configuration of the compound nucleus. Since shell effects are still present for the low excitation energies in question, the expression (12) must be modified in order to take into account these effects. We parametrize the dependence of  $a_n$  on the thermally damped shell effects according to the following formula:

$$a_n = \tilde{a} \left\{ 1 + \frac{E_{\text{micr}}}{E_n^*} \left[ 1 - \exp\left(-\frac{E_n^*}{E_D}\right) \right] \exp\left(-\frac{E_n^*}{E_D}\right) \right\}, \quad (13)$$

where  $E_{\text{micr}}$  is the Strutinsky microscopic energy at the equilibrium point of the evaporation residue with mass number  $A$ ,  $E_D$  is the damping constant, and  $\tilde{a} = A/8 \text{ MeV}^{-1}$  is the asymptotic value of  $a_n$  for high excitation energies. The relation (13) holds also for  $a_f$  after the replacement of  $A$  with the mass number of the compound nucleus  $A_{CN}$ ,  $E_n^*$  with  $E_f^*$ , and  $E_{\text{micr}}$  with the saddle point microscopic energy  $E'_{\text{micr}}$  for the compound nucleus. With a value of 12.5 MeV for  $E_D$  and microscopic energies calculated by means of the macroscopic-microscopic model [6,34], we reproduce very well the excitation energy dependence of the neutron-to-total width ratio reduced to zero angular momentum measured for Nobelium isotopes at Dubna [35]. The formula (13) was obtained by multiplying the excitation energy-dependent term of the commonly used Ignatyuk *et al.* expression [36] by  $\exp(-E_n^*/E_D)$ . We introduced the new formula because the former gives the correction  $\tilde{a}E_{\text{micr}}/E_n^*$  to the asymptotic level density  $\tilde{a}$  for larger excitation energies. Because of this correction, the use of the Ignatyuk *et al.* formula does not lead to the correct reproduction of the experimental data obtained for Nobelium isotopes [35].

For spherical and some transitional nuclei ( $0 \leq \beta_2 \leq 0.14$ ), the  $\Gamma_n/\Gamma_f$  ratio is divided by the collective enhancement factor  $k_{\text{coll}}$  [37,38] describing the decrease of the level density at the equilibrium point relative to the axially deformed saddle configuration. This decrease is caused by the absence of rotational levels at the equilibrium point, which leads to a smaller probability of emission of the neutron. The collective enhancement factor is calculated by means of the formula [37]

$$k_{\text{coll}} = \frac{JT}{\hbar^2}, \quad (14)$$

where  $J$  is the moment of inertia at the saddle point configuration calculated according to Eq. (10) and  $T$  is the temperature of the compound nucleus. In the present calculation, the value of 0.6 MeV for  $T$  was taken. For the  $^{286-290}_{114}$ ,  $^{288-296}_{116}$ ,  $^{290-300}_{118}$ , and  $^{300-304}_{120}$  compound nuclei,  $k_{\text{coll}} = 118-125$  is obtained.

Due to the decreasing transmission probability  $T_l$  and the neutron-to-total width ratio  $G(E^*, l)$ , the contribution of the term  $\sigma_l(E_{\text{HI}})P_{1n,l}(E^*)$  to the cross section  $\sigma_{1n}(E_{\text{HI}})$  becomes less than 1% at a relatively low  $l_{\text{max}} = 24-32$  and the contributions for larger  $l$  decrease rapidly. Since the summa-

TABLE I. Calculated ground-state  $Q$  value, the height of the fusion barrier  $B_{fu}$ , transmission probability through the fusion barrier  $T_0$  for the optimal bombarding energy, and zero angular momentum, neutron separation energy  $S_n$ , the height of the static fission barrier  $B_f^{\text{stat}}$ , neutron-to-fission width ratio reduced to zero angular momentum  $(\Gamma_n/\Gamma_f)_0$ , maximal angular momentum  $l_{\text{max}}$  (see text for definition), optimal excitation energy  $E^*$  and formation cross section  $\sigma$ , as well as measured excitation energy  $E_{\text{exp}}^*$  and formation cross section  $\sigma_{\text{exp}}$  [41,7,42,9,17] of reactions  $^{208}\text{Pb}(\text{HI},1n)\text{ER}$  with the projectiles HI and the evaporation residues ER listed in the first two columns. A value of 500 nb for the formation cross section of  $^{255}\text{No}$  was fixed (see text for motivation).

HI	ER	$Q$ MeV	$B_{fu}$ MeV	$T_0$	$S_n$ MeV	$B_f^{\text{stat}}$ MeV	$(\Gamma_n/\Gamma_f)_0$	$l_{\text{max}}$ $\hbar$	$E^*$ MeV	$\sigma$	$E_{\text{exp}}^*$ MeV	$\sigma_{\text{exp}}$
$^{48}\text{Ca}$	$^{255}\text{No}$	153.56	178.67	$3.1 \times 10^{-3}$	7.39	7.23	$6.5 \times 10^{-3}$	28	13.28	500 nb	16.70	$260_{-30}^{+30}$ nb
$^{50}\text{Ti}$	$^{257}\text{Rf}$	169.55	199.41	$3.6 \times 10^{-4}$	7.90	6.87	$1.3 \times 10^{-3}$	27	14.29	9.4 nb	15.48	$10.4_{-1.3}^{+1.3}$ nb
$^{54}\text{Cr}$	$^{261}\text{Sg}$	187.08	219.13	$1.6 \times 10^{-4}$	8.07	6.30	$2.8 \times 10^{-4}$	26	14.65	730 pb	16.38	$500_{-140}^{+140}$ pb
$^{58}\text{Fe}$	$^{265}\text{Hs}$	204.92	238.48	$7.1 \times 10^{-5}$	8.31	5.70	$3.5 \times 10^{-5}$	26	14.33	33 pb	13.16	$67_{-17}^{+17}$ pb
$^{62}\text{Ni}$	$^{269}\text{110}$	223.18	257.52	$3.2 \times 10^{-5}$	8.56	4.88	$2.1 \times 10^{-6}$	24	13.33	730 fb	13.24	$3.5_{-1.8}^{+2.7}$ pb
$^{64}\text{Ni}$	$^{271}\text{110}$	224.50	256.86	$1.6 \times 10^{-4}$	8.32	5.56	$1.1 \times 10^{-5}$	26	13.70	20 pb	11.74	$15_{-6}^{+9}$ pb
$^{68}\text{Zn}$	$^{275}\text{112}$	241.95	275.60	$6.3 \times 10^{-5}$	8.09	4.42	$1.3 \times 10^{-6}$	25	13.07	750 fb		
$^{70}\text{Zn}$	$^{277}\text{112}$	243.68	274.94	$2.2 \times 10^{-4}$	7.70	4.05	$1.3 \times 10^{-6}$	25	12.62	2.7 pb	10.07	$1.0_{-0.7}^{+1.3}$ pb
$^{74}\text{Ge}$	$^{281}\text{114}$	262.09	293.41	$1.9 \times 10^{-4}$	7.91	3.24	$7.8 \times 10^{-8}$	24	11.89	110 fb		
$^{76}\text{Ge}$	$^{283}\text{114}$	263.92	292.75	$7.0 \times 10^{-4}$	7.76	3.12	$1.1 \times 10^{-7}$	24	11.50	560 fb		
$^{80}\text{Se}$	$^{287}\text{116}$	282.46	310.97	$1.4 \times 10^{-3}$	8.15	3.75	$8.6 \times 10^{-9}$	25	11.78	94 fb		
$^{82}\text{Se}$	$^{289}\text{116}$	284.11	310.32	$6.0 \times 10^{-3}$	7.87	4.51	$1.3 \times 10^{-9}$	27	12.12	7.1 pb		
$^{82}\text{Kr}$	$^{289}\text{118}$	298.92	329.62	$7.3 \times 10^{-4}$	8.53	4.07	$1.0 \times 10^{-8}$	26	12.31	57 fb		
$^{84}\text{Kr}$	$^{291}\text{118}$	301.82	328.96	$6.8 \times 10^{-3}$	8.24	4.91	$1.8 \times 10^{-7}$	28	12.81	11 pb		
$^{86}\text{Kr}$	$^{293}\text{118}$	304.41	328.30	$4.0 \times 10^{-2}$	7.90	5.45	$1.5 \times 10^{-6}$	31	13.31	670 pb		
$^{78}\text{Ge}$	$^{285}\text{114}$	264.79	292.10	$1.5 \times 10^{-3}$	7.67	3.32	$6.2 \times 10^{-9}$	25	11.29	77 fb		
$^{80}\text{Ge}$	$^{287}\text{114}$	264.88	291.45	$2.2 \times 10^{-3}$	7.53	4.08	$7.5 \times 10^{-8}$	26	11.35	1.5 pb		
$^{82}\text{Ge}$	$^{289}\text{114}$	264.18	290.80	$2.6 \times 10^{-3}$	7.21	4.90	$2.6 \times 10^{-6}$	28	11.79	72 pb		
$^{84}\text{Se}$	$^{291}\text{116}$	284.87	309.66	$1.5 \times 10^{-2}$	7.58	5.32	$2.1 \times 10^{-6}$	30	12.59	360 pb		
$^{86}\text{Se}$	$^{293}\text{116}$	282.26	309.01	$7.0 \times 10^{-3}$	7.37	5.94	$1.7 \times 10^{-5}$	31	13.19	1.4 nb		
$^{88}\text{Se}$	$^{295}\text{116}$	278.93	308.36	$1.1 \times 10^{-3}$	7.10	6.34	$4.4 \times 10^{-5}$	32	12.81	580 pb		
$^{88}\text{Kr}$	$^{295}\text{118}$	303.11	327.65	$3.3 \times 10^{-2}$	7.63	5.79	$5.2 \times 10^{-6}$	32	13.58	2.0 nb		
$^{90}\text{Kr}$	$^{297}\text{118}$	301.07	326.99	$1.5 \times 10^{-2}$	7.42	6.08	$1.3 \times 10^{-5}$	32	13.45	2.3 nb		
$^{92}\text{Kr}$	$^{299}\text{118}$	298.23	326.34	$3.1 \times 10^{-3}$	7.10	6.03	$1.7 \times 10^{-5}$	32	12.81	590 pb		
$^{92}\text{Sr}$	$^{299}\text{120}$	321.80	344.42	$5.8 \times 10^{-2}$	7.76	5.62	$2.0 \times 10^{-6}$	32	13.74	1.2 nb		
$^{94}\text{Sr}$	$^{301}\text{120}$	320.44	344.76	$3.7 \times 10^{-2}$	7.43	5.51	$3.5 \times 10^{-6}$	32	13.47	1.4 nb		
$^{96}\text{Sr}$	$^{303}\text{120}$	317.94	344.11	$1.1 \times 10^{-2}$	7.06	5.11	$3.2 \times 10^{-6}$	32	12.75	350 pb		

tion ends at a low value of angular momentum and the rigid body moments of inertia at the equilibrium and the saddle points are not very different due to the small distance between these points for superheavy nuclei, we assume that the equilibrium and saddle point positions are independent of  $l$ .

In the present calculation, measured masses of the target nucleus and the projectiles are used [39]. Masses of the compound nuclei and the evaporation residues, static fission barriers, equilibrium and saddle point microscopic energies, and deformations are calculated by means of the macroscopic-microscopic model [6,34,40]. The neutron separation energies are obtained by subtracting the theoretical mass of the compound nucleus from the sum of the experimentally known mass of the neutron and the theoretical mass of the evaporation residue.

Since the damping constant is determined from the neutron-to-total width ratio measured for Nobelium isotopes, we determine the parameter  $c$  by means of the reaction  $^{208}\text{Pb}(^{48}\text{Ca},1n)$  in which the isotope  $^{255}\text{No}$  was produced. The parameter  $c = 379.17435$  is fixed in such a way to obtain

a cross section equal to 500 nb for the calculated optimal excitation energy equal to 13.28 MeV. The choice of the value of the same order but larger than the measured one of  $260 \pm 30$  nb [41] was motivated by the large systematic uncertainties and the fact that the complete excitation function for this reaction was not measured [41].

The obtained results are listed in Table I. The measured formation cross sections with the statistical errors for the nuclei  $^{255}\text{No}$ ,  $^{257}\text{Rf}$ ,  $^{261}\text{Sg}$ ,  $^{265}\text{Hs}$ ,  $^{269}\text{110}$ ,  $^{271}\text{110}$ , and  $^{277}\text{112}$  [41,7,42,9,17] are also given. The calculated values differ from the experimental data [7,42,9,17] by a factor of 2.2, on average. The values of the measured formation cross sections [7,42,9,17] are also accurate within a factor of about 2 because of systematic uncertainties.

We made the predictions for the formation cross sections of the isotopes of the elements 112–118 which may be produced by using stable neutron-rich projectiles (upper part of Table I), as well as for some isotopes of the elements 114–120 which might be synthesized by using the neutron-rich radioactive ion beams (lower part of Table I). The projectiles

are chosen in such a way to obtain the odd- $N$  neighbors of the even-even nuclei for which we calculated in Refs. [6,34,40] the half-lives  $T_{1/2} \geq 1 \mu\text{s}$ , i.e., to obtain the superheavy nuclei which can be detected in the present-day experimental setup [43].

The predicted formation cross sections of  $^{289}116$ ,  $^{291}118$ , and  $^{293}118$ , which might be produced with the use of stable projectiles, are surprisingly large. They contradict earlier expectations [17] that the cross section decreases exponentially with the increasing proton number of the projectile and that the production of these nuclei will not be possible by using the  $1n$ -evaporation channel. According to the present calculation, in order to synthesize the isotopes of the new elements, one should increase the bombarding energy in such a way to obtain the optimal excitation energy. The increase of the formation cross sections of the hypothetical spherical superheavy nuclei is due to the higher static fission barriers in the proximity of the neutron shell closure at  $N=184$ . The larger fission barrier leads to the larger neutron-to-total width ratio. The increase of this ratio is also due to the decreasing microscopic energy (the decreasing level density) at the saddle point. This is a consequence of the fact that the saddle point deformation of spherical superheavy nuclei is smaller in comparison with the saddle point deformation of deformed superheavy nuclei for which we calculate the saddle point microscopic energies close to zero. Moreover, for the reactions with the projectiles with the neutron numbers approximating the magic number  $N=50$ , the increase of the formation cross section is due to the increase of the  $Q$  value leading to the increase of the transmission probability through the Coulomb barrier.

Doubly magic  $^{208}\text{Pb}$  was already bombarded by  $^{82}\text{Se}$  at GSI-Darmstadt [17], but with lower bombarding energies than the optimal one calculated in the present paper. The calculated value of 7 pb for the cross section for the optimal bombarding energy is very close to the experimental cross section limit.

The calculated large values for the formation cross sections indicate that the synthesis of still heavier and more stable superheavy nuclei might be possible even with the use of the present-day experimental setup and available stable neutron-rich projectiles if the bombarding energy is properly chosen. The most promising reaction with the stable projec-

tile for the synthesis of the hypothetical spherical superheavy nuclei is  $^{208}\text{Pb}(^{86}\text{Kr},1n)^{293}118$  with the optimal excitation energy equal to 13.31 MeV.

The results given in the present paper were obtained with the use of the model exploiting the unphysical cutoff Coulomb barrier which cannot, of course, replace a complete theory of fusion. Since such theory is not available so far, we decided to propose our simple model. This model reproduces the measured formation cross sections of deformed superheavy nuclei. Furthermore, it provides predictions for the formation cross sections of the hypothetical spherical superheavy nuclei. These predictions may be tested experimentally. One should keep in mind, however, that the rather far extrapolation made by the use of the schematic fusion barrier may lead to considerable differences between theoretical and experimental formation cross sections. The increase (decrease) of the radial coordinate of the position of the fusion barrier by 1% leads to the increase (decrease) of the calculated formation cross sections by a factor of 3.3–6.2 (3.8–7.0). The variation of the  $Q$  value by 1 MeV changes the formation cross sections by a factor of 1.5–2.3. The increase (decrease) of the optimal excitation energy by 1 MeV increases (decreases) both the transmission probability and the  $\Gamma_n/\Gamma_f$  ratio and, consequently, increases (decreases) the cross section by a factor of 2.4–5.6 (2.6–6.5). The variation of the damping constant  $E_D$  by 1 MeV changes the calculated formation cross sections by a factor 1.1–1.4. In the present paper the influence of the second hump of the fission barrier on the  $\Gamma_n/\Gamma_f$  ratio is disregarded. The presence of the second hump of the fission barrier for many of the considered nuclei may increase slightly the calculated formation cross sections by a factor very close to the unity. The inclusion of the vibrational collective effects, disregarded also in the presented calculation, may decrease the collective enhancement factor and, consequently, may increase the calculated formation cross sections of spherical superheavy nuclei by a factor of 1–10 [38].

The author thanks S. Hofmann and J. Skalski, as well as P. Armbruster, E.A. Cherepanov, F.P. Hessberger, Z. Janas, Z. Łojewski, G. Münzenberg, Z. Patyk, M. Pfützner, K. Rykaczewski, J. Srebrny, and M. Veselsky for many valuable discussions. Grant No. 2 P03B 099 15 of the Polish Committee for Scientific Research (K.B.N.) is gratefully acknowledged.

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