Quasielastic electron scattering from nuclei: Random-phase vs ring approximations

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We investigate the extent to which the nuclear transverse response to electron scattering in the quasielastic region, evaluated in the random-phase approximation, can be described by ring approximation calculations. Different effective interactions based on a standard model of the type $g'+V_\pi+V_\rho$ are employed. For each momentum transfer, we have obtained the value of g_0' permitting the ring response to match the position of the peak and/or the non-energy-weighted sum rule provided by the random-phase approach. It is found that, in general, it is not possible to reproduce both magnitudes simultaneously for a given g_0' value.
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I. INTRODUCTION

In the past years, much attention has been paid to the study of the electron scattering (nuclear) responses in the quasifree regime. The good description of the cross sections provided by means of a simple Fermi gas model in the former work of Moniz *et al.* [1] was suddenly broken when the longitudinal/transverse experimental separation was performed [2]. After this, many different physical mechanisms, such as, e.g., short- and long-range correlations, meson-exchange currents, final-state interactions, etc., have been argued to be responsible for the observed discrepancies. However, a definite answer to the problem is still not available.

Calculations of the nuclear responses in this energy region can be grouped in two general approaches. A first one considers the nucleus as a finite system [3–8]. The other one uses nuclear matter together with an additional approximation (say, a variable Fermi momentum or the local-density approximation) to obtain the results for finite nuclei [9–13].

Nuclear matter formalism takes advantage of the translational invariance inherent to the infinite systems, something which simplifies considerably the technology to be used (at least, *a priori*). However, most of the calculations done in this approach have been performed in the so-called ring approximation (RA) [9,11–13]. This framework is usually (and incorrectly) called the random-phase approximation (RPA), though the exchange terms are not considered. Curiously, full true RPA nuclear responses have been evaluated only for finite nuclei [6], despite the complexity of the calculations for these systems in comparison with those for nuclear matter. A first attempt to carry out RPA calculations for infinite systems was done in Ref. [14], where the longitudinal response was evaluated by means of the continued-fraction method with exchange terms considered up to first order only. More recently, two different procedures to calculate the nuclear matter responses in a RPA framework have been developed for a general finite-range effective interaction [15,16].

It is commonly assumed [17] that the RA can simulate the effect of RPA exchange terms by an adequate choice of the Landau parameters included in the interaction. In particular, for the transverse responses, in which we are interested in this work, the g_0' parameter will be the important one. However, Shigehara *et al.* [5] have shown that this is true in the $\sigma\tau$ response for finite nuclei when a particular G matrix, which has a weak momentum dependence in the exchange channel is used as an effective interaction. The validity of this hypothesis for the standard $g_0'+V_\pi+V_\rho$ model has not been clarified.

This is precisely the aim of the present investigation: the study of the possibility for the RA to describe RPA calculations with such an interaction. In Sec. II we compare the RPA responses with the RA ones in order to obtain the values of g_0' providing the best agreement between both. In Sec. III we go deeper in the question by analyzing the results obtained for two effective interactions obtained by slightly modifying the one used in the previous section. Finally, we present our conclusions in Sec. IV.

II. RPA vs RA

We start by performing a ''model'' RPA calculation for the quasielastic nuclear response in ^{40}Ca. We are interested in the transverse channel and we have used an effective interaction of the form

$$V^{\mathrm{I}}=V_{\mathrm{LM}}+V_\pi+V_\rho, \tag{1}$$

which includes a zero-range force of Landau-Migdal type, which takes care of the short-range piece of the NN interaction:

$$V_{\mathrm{LM}}=C_0[g_0\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)+g_0'\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)], \tag{2}$$

and a finite-range component generated by the $(\pi+\rho)$-meson exchange potentials. The particular values of the two parameters of the zero-range piece are $g_0=0.47$ and $g_0'=0.76$ (with $C_0=386$ MeV fm^3). These values permit us to reproduce, within the RPA framework, the energies and B values of the two 1^+ states in ^{208}Pb at 5.85 and 7.30 MeV.

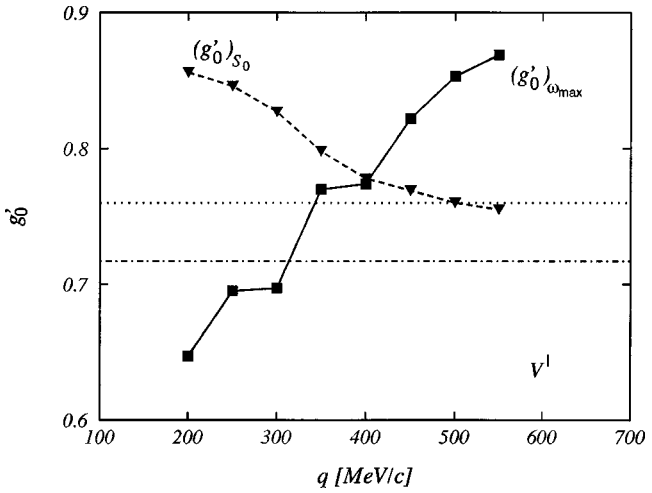

FIG. 1. Dependence with the momentum transfer q of the values of the parameter g_0' to be used in RA calculations in order to reproduce the peak positions (squares) and the the non-energy-weighted sum rule (triangles) corresponding to the RPA responses. The dotted line gives the value $g_0'=0.76$ used in the RPA calculation. The dashed-dotted line shows the value $g_0'=0.717$ which permits us to reproduce the low-energy properties we consider by means of a RA calculation (see Ref. [18]).

These are the (low-energy) observables we consider to fix the different interactions we use throughout this work (see Ref. [18] for details).

For the calculation of the RPA nuclear responses in the quasifree region, we have used the prescription of the scheme developed in Ref. [15] and in which the exchange terms are explicitly taken into account for any interaction. For a pure contact interaction exchange terms can be included up to infinite order, while for a finite-range interaction they must be numerically evaluated for each order.

We want to investigate the conditions under which the RA responses provide a reasonable description of the RPA ones. The difference between both approaches is in the presence (or not) of the exchange terms, which are linked to the finite range piece of the interaction. Then we maintain fixed this part of V^I in the RA calculations and vary the value of g_0' until the required agreement is obtained. This agreement will be ''measured'' by comparing the values obtained in both approaches for two magnitudes derived from the corresponding responses: the position of the peak ω_{max} and the non-energy-weighted sum rule

$$S_0(q)=\int_0^\infty d\omega S_T(q,\omega), \tag{3}$$

where S_T is the structure function corresponding to point nucleons, that is without including the corresponding nucleon form factor. If the full transverse response R_T is used in Eq. (3) instead of S_T, the results quoted below remain unchanged. We call $(g_0')_{\omega_{max}}$ and $(g_0')_{S_0}$, respectively, the values of the parameter g_0' which make the values of ω_{max} and S_0 obtained within the RA equal the RPA ones.

In Fig. 1 we show the results obtained in this procedure for momentum transfers ranging from 200 to 550 MeV/c. Therein, the black squares represent the values $(g_0')_{\omega_{max}}$, whereas the solid triangles correspond to $(g_0')_{S_0}$. The dotted line gives the g_0' value used in the RPA calculation. We have not changed the value of g_0 because, as shown in Ref. [18], its role in the RA is negligible.

These results deserve several comments:

(1) It is clear that the reproduction of the RPA values of ω_{max} and S_0 by means of the RA calculations occurs for values of g_0' which are, in general, quite different from that used for the RPA calculation (dotted line). This is in agreement with the findings of Ref. [18].

(2) The g_0' values permitting the agreement between both types of calculations for the magnitudes taken into account are clearly incompatible. Only the region around $q=400$ MeV/c seems to be ''magic'' in this respect. This is also seen in Fig. 2 where we show the transverse responses for $q=300$ (upper panel), $q=400$ (medium panel), and $q=550$ MeV/c (lower panel) obtained in the RPA (dotted curves), and in the RA with the values $(g_0')_{\omega_{max}}$ (solid curves) and $(g_0')_{S_0}$ (dashed curves) shown in Table I. It is apparent how the three curves overlap in the case of $q=400$ MeV/c, while they differ in the other two cases. This result generalizes those found by Shigehara *et al.* for a G-matrix interaction [5].

(3) The value of the g_0' parameter needed to obtain the agreement between RA and RPA shows a considerably dependence on the momentum transfer q, the range of variation being appreciably large. Besides, the values providing the agreement between both types of calculations are (except for a couple of values around 300 MeV/c) quite different from the value of $g_0'=0.717$ (dashed-dotted line in Fig. 1) found [18] to provide, in the RA framework, the description of the low-energy properties quoted above. This points out even more the difficulties for the RA to reproduce the RPA results in the quasielastic region.

III. ADDITIONAL RESULTS

The results quoted in the previous section show the inability of the RA calculations to describe the responses obtained in the RPA framework. To go deeper in the investigation of the reasons of this situation, we focus our attention on the exchange terms and on those mechanisms providing the more important contributions to them. In particular we will analyze, first, the role of the pion exchange potential and, second, the importance of the tensor piece of the interaction.

As it is known, the contribution of V_π to the RA responses is exactly zero in nuclear matter, while the same does not occur for the RPA because of the presence of the exchange terms. In order to see what is the influence of this piece of the potential, we have performed a new set of cal-

TABLE I. Values of the g_0' parameters used in the RA calculations shown in Fig 2.

q(MeV/c)	$(g_0')_{\omega_{max}}$	$(g_0')_{S_0}$
300	0.697	0.827
400	0.774	0.778
550	0.869	0.755

TABLE II. Adjusted values of the parameters g_0 and g_0' for RPA calculations using the two interactions quoted in the text. With these values and with $C_0=386$ MeV fm^3, the energies and B values of the two 1^+ states in ^{208}Pb at 5.85 and 7.30 MeV, respectively, are reproduced.

Interaction	g_0	g_0'
V^{II}	-0.055	0.64
V^{III}	-0.075	0.60

culations, similar to the previous ones, but considering the effective interaction:

$$V^{II}=V_{LM}+V_{\rho}. \tag{4}$$

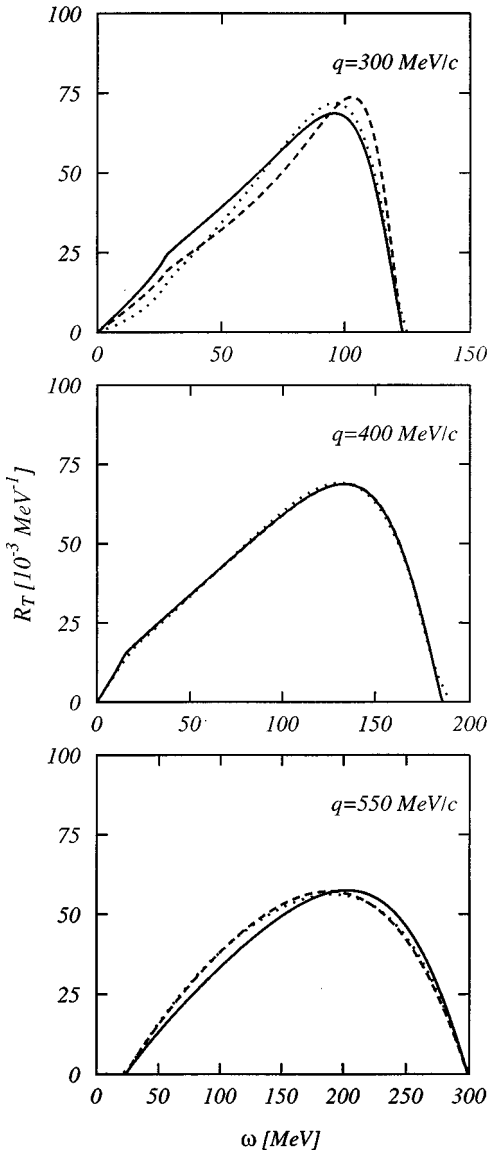

FIG. 2. Transverse responses for ^{40}Ca, calculated for three momentum transfers. Dotted curves are the RPA results. Solid curves represent the RA responses obtained with the values $(g_0')_{\omega_{max}}$. Dashed curves give the same but with the values $(g_0')_{S_0}$ particular values used in the RA calculations are given in Table I.

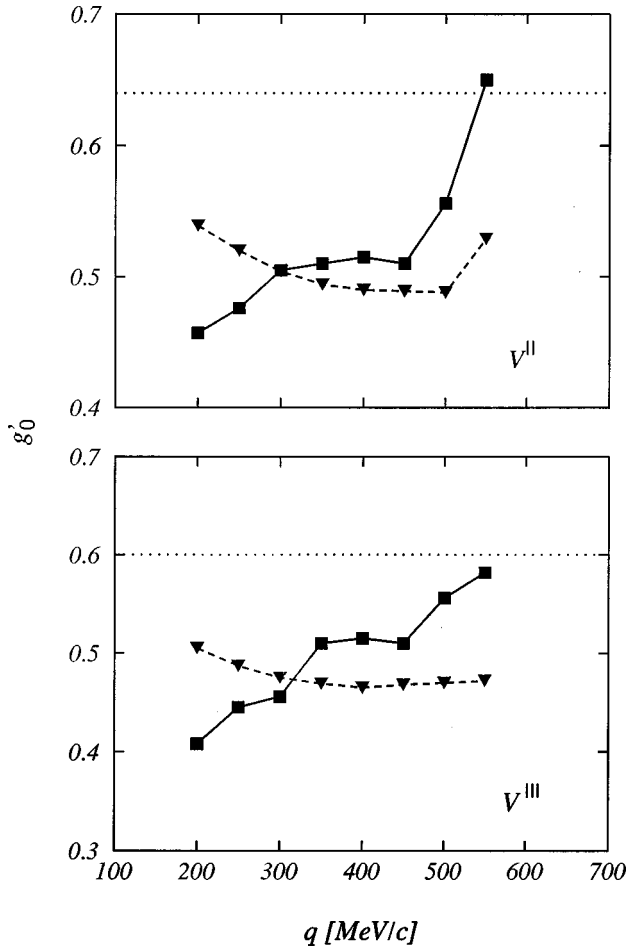

FIG. 3. Same as in Fig. 1 but for the other two interactions considered in this work.

Following the same strategy as for V^I, we have fixed the values of the zero-range Landau-Migdal parameters g_0 and g_0' as indicated above. The results obtained are given in the first row of Table II.

With the interaction fixed in this way we have obtained the corresponding RPA responses and have determined, again, the values of g_0' making the RA results agree with the RPA ones. The results obtained are shown in the upper panel of Fig. 3.

The most important question to be noted is the fact that the absence of the pion exchange potential in the RPA calculations strongly modifies the situation. In fact, it can be seen that, in the q region between 300 and 500 MeV/c, a value for $g_0'\sim 0.5$ would provide RA calculations describing reasonably well and simultaneously, both ω_{max} and S_0 as given by the RPA. This is shown in Fig. 4 where we compare, for the interaction V^{II} we are discussing, the RA responses obtained for $g_0'=0.505$ (solid curves) with the RPA ones (dotted curves). This value of g_0' is the one which makes RA and RPA calculations coincide at $q=300$ MeV/c and it is worth pointing out the big difference with repect to the value $g_0'=0.64$ used for the RPA calculations (see Table II).

In order to know more about the behavior of the important pieces of the interaction, we have repeated the analysis done for V^I and V^{II} for the effective force:

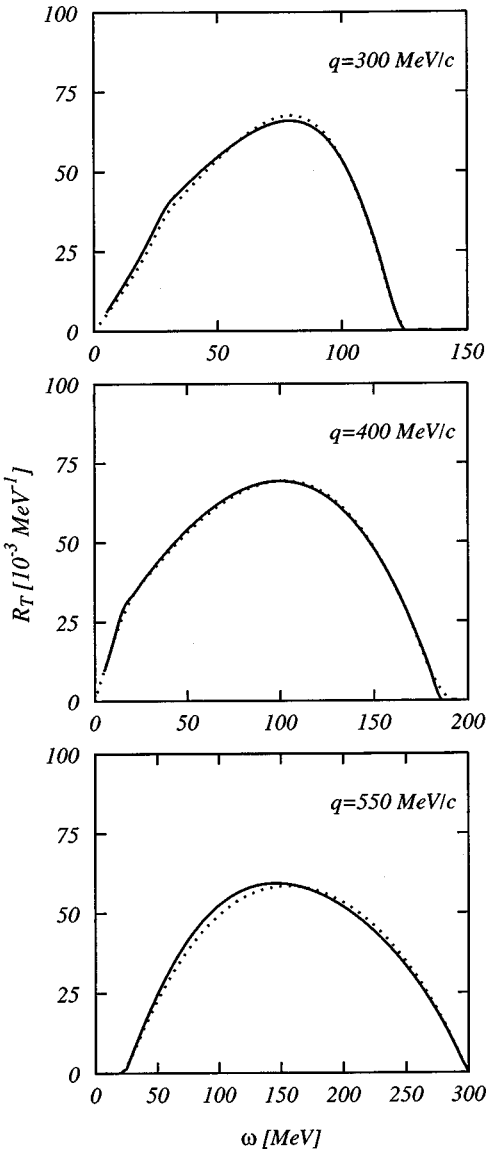

FIG. 4. Transverse responses for ^{40}Ca, calculated for three momentum transfers. Dotted curves are the RPA results. Solid curves represent the RA responses obtained with the value $g_0'=0.505$.

$$V^{\mathrm{III}}=V_{\mathrm{LM}}+(V_\rho)_{\sigma\sigma\tau\tau}, \tag{5}$$

which has been obtained by eliminating the pion exchange and the tensor piece of the ρ exchange from V^{I}. The adjustment of the zero-range parameter at low-energy (as in the two previous calculations) gives the values quoted in the second row of Table II. The results for the values of $(g_0')_{\omega_{\max}}$ and $(g_0')_{S_0}$ are shown in the lower panel of Fig. 3. The situation now is roughly the same as for V^{II}, but for a smaller value of g_0'. These results show the importance of the role of the pion exchange potential in this type of calculations.

It should be also noted that, as it occurs in the case of V^{II}, the g_0' value used for the RPA calculations differs from those needed for the RA ones. This claims again the necessity of changing the values of the zero-range parameters fixed in the RPA framework when performing calculations in a different framework, something which is not usually done in the literature.

IV. CONCLUSIONS

In this work we have addressed the role played by the RPA exchange terms in the (e,e') nuclear response in the quasielastic region. In particular, we have investigated if the RA calculations performed with an effective interaction with a fix g_0' (independent of the momentum transfer) can simulate the results obtained in the RPA. The main findings are the following:

(1) It is not possible to find a single g_0' value permitting the RA to reproduce the RPA responses. The required g_0' shows a strong q dependence. Besides, this dependence is different when different properties of the responses are considered to match the results obtained with the two approaches. As a consequence, it can be concluded that the RA cannot reproduce the RPA responses in a consistent way.

(2) It is important to stress that pion exchange does not contribute to the RA calculations in the transverse channel. It was found that if V_π is arbitrarily turned off in the effective interaction used for RPA calculations, then a reasonable agreement between both approaches is obtained for 300 MeV/$c\leqslant q\leqslant$500 MeV/c. This shows the important role played by this part of the interaction in the type of calculations we have discussed here.

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