Model calculations for the two-fragment electro-disintegration of ^{4}He

M. Braun, L. L. Howell, and S. A. Sofianos
University of South Africa, P.O. Box 392, Pretoria, 0003, South Africa

W. Sandhas
Physikalisches Institut, Unversität Bonn, D-53115 Bonn 1, Germany

(Received 6 October 1998)

Differential cross sections for the electro-disintegration process $e+{}^4\mathrm{He}\rightarrow{}^3\mathrm{H}+p+e'$ are calculated, using a model in which the final-state interaction is included by means of a nucleon-nucleus $(3+1)$ potential constructed via Marchenko inversion. The required bound-state wave functions are calculated within the integro-differential equation approach. In our model the important condition that the initial bound state and the final scattering state are orthogonal is fulfilled. The sensitivity of the cross section to the input p^3H interaction in certain kinematical regions is investigated. The approach adopted could be useful in reactions involving few cluster systems where effective interactions are not well known and exact methods are presently unavailable. Although, our plane-wave impulse approximation results exhibit, similarly to other calculations, a dip in the fivefold differential cross section around a missing momentum of ~ 450 MeV/c, it is argued that this is an artifact of the omission of rescattering four-nucleon processes. [S0556-2813(99)02105-6]

PACS number(s): 21.45.+v, 25.10.+s, 25.30.Fj

I. INTRODUCTION

In recent years, electro-disintegration processes in light nuclei have attracted much experimental and theoretical attention. In the three-nucleon case, early calculations of the electron-proton coincidence cross section, i.e., of the scattered electron detected in coincidence with the ejected proton, were performed by Griffy and Oakes using analytical wave functions without inclusion of the final-state interaction (FSI) [1]. Since then, few-body techniques were used to obtain exact wave functions. Using the Faddeev formalism, Lehman [2] calculated the two- and three-fragment electro-disintegration of ^{3}H and ^{3}He for a separable, Yamaguchi-type potential. In the final state the interactions between the ejected nucleon and the spectator pair were neglected.

Heimbach *et al.* [3] also studied the two-fragment electro-disintegration of ^{3}He and ^{3}H, but with the inclusion of the FSI. These calculations were based on a method introduced by Barbour and Phillips [4], and extended by Gibson and Lehman [5], for the photo-disintegration of ^{3}He. The equations in this approach were derived by making use of the Alt-Grassberger-Sandhas (AGS) formalism for the three-nucleon system [6]. As in Ref. [2], a spin-dependent, *s*-wave separable potential of Yamaguchi form was used in these calculations. In contrast, van Meijgaard and Tjon [7] employed the semirealistic Malfliet-Tjon (MT I-III) potential [8] represented in separable form via the unitary pole expansion.

Realistic interactions have also been used to determine the electro-disintegration cross sections. Schiavilla [9], for example, employed the variational Monte Carlo (VMC) method using the Urbana [10] and Argonne [11] nucleon-nucleon (NN) interactions for determining the bound-state wave functions in the electro-disintegration of ^{3}He. His method involved constructing the final scattering states to be orthogonal to the initial bound state. In a completely different method, Laget [12] employed a diagrammatic expansion of the scattering amplitude, which can be used to account for the most important multiparticle effects in the nuclear medium such as FSI and meson exchange currents (MEC). In a more rigorous approach Ishikawa *et al.* [13] calculated the electron-induced two-fragment disintegration of ^{3}He by using a 34-channel treatment based on Faddeev-type equations in momentum space. In their work they employed realistic two-nucleon interactions, namely the Paris [14] and Bonn-B [15] potentials. Later on, Golak *et al.* [16] extended the work of Ref. [13] by including higher angular momentum components of the NN force (up to $j=2$).

In contrast to the three-nucleon electro-disintegration, calculations for the four-nucleon system are computationally much more demanding and to date no calculations have been performed to reliably include scattering states in a rigorous way. One therefore resorts to using the approximation

$$ {}^{(-)}\langle q_\alpha;\Psi_{A-1}| \simeq {}^{(-)}\langle q_\alpha|\langle\Psi_{A-1}|, \tag{1}$$

which is often employed in nuclear physics, and has previously been used in the photo-disintegration [17] and electro-disintegration [18] of ^{4}He. Here ${}^{(-)}\langle q_\alpha;\Psi_{A-1}|$ denotes the full scattering state associated with the channel state $\langle \boldsymbol{q}_\alpha|\langle\Psi_{A-1}|$ in which the ejected proton is moving with momentum $\boldsymbol{q}_\alpha$ relative to the residual nucleus represented by the bound state $\langle\Psi_{A-1}|$. In such a model approach extreme care must be taken to ensure that the approximate scattering states in Eq. (1) are orthogonal to the initial bound state $|\Psi_A\rangle$,

$$ {}^{(-)}\langle q_\alpha|\langle\Psi_{A-1}|\Psi_A\rangle = 0. \tag{2}$$

Nonfulfillment of this condition means that the effective scattering state, although asymptotically correct, can be different from the ''true'' wave function in the interior. This in turn means that the off-shell characteristics of the effective interaction employed can be completely wrong. This point is crucial since in photo- and electro-processes we are concerned with overlap integrals that include also the interior

part of the wave function. That condition is often simply ignored or some kind of an orthogonalization method is employed to ensure that it is fulfilled (see, for example, Ref. [9]). The approximation (1) together with the condition (2) has been used by Schiavilla [18] within the VMC method (in a fashion similar to the one employed in ^{3}He electro-disintegration) using the Argonne two-nucleon and Urbana-VII three-nucleon [19] interaction models. The optical potential of van Oers *et al.* [20] was employed in this approach to describe the $3+1$ interaction. Laget [21] included the FSI and the meson exchange currents (MEC) using a diagrammatic expansion similar to the three-nucleon case. Nagorny *et al.* [22] included the electromagnetic field in the strongly interacting system in a relativistic and gauge invariant way, with the FSI being incorporated via the pole piece of the $p\,^3\mathrm{H}\rightarrow p\,^3\mathrm{H}$ scattering t matrix [23]. In a more recent work [24] the Schiavilla, the Laget, and the Nagorny methods were employed to study newly obtained data by van Leeuwe [25]. The results thus obtained were interpreted as an indication that the dip could be due to FSI effects and contributions from two-body currents.

The main findings in the aforementioned investigations of three- and four-nucleon disintegration processes were that (a) in certain kinematical regions there is an extreme sensitivity to the nuclear forces; (b) there are important differences between the results obtained when using different reaction models, and (c) in the four-nucleon case all model calculations within the plane-wave impulse approximation (PWIA) exhibit a dip in the fivefold differential cross section around a missing momentum of $\sim$450 MeV which, being absent from the experimental data, seems to be an artifact of the models.

In this paper, we investigate the above sensitivities of the cross section to the input nucleon-nucleus interaction in the exclusive electro-disintegration process

$$e+{}^4\mathrm{He}\rightarrow{}^3\mathrm{H}+p+e', \tag{3}$$

where the scattered electron and the ejected nucleon are assumed to be measured in coincidence.

In our model the interaction between the outgoing proton and the residual nucleus is determined by using the nucleon-nucleus scattering data to generate an effective potential via the Marchenko inverse procedure [26–28]. The potential obtained would be unique, provided the phase shifts were known at all energies. Since theoretically the phase shifts can be calculated reliably only below the three-body breakup threshold while experimentally they are available from a limited number of phase shift analyses, extrapolations to higher energies are required. The most obvious choice appears to be the one leading to states which fulfill the orthogonality condition given by Eq. (2). Thus the dependence of the interaction on the extrapolated phase shifts and the resulting effect on the cross section is studied. Such an approach could be useful in reactions involving few cluster systems where the effective interaction needed to construct the scattering wave function, contains a lot of inherent ambiguities mainly stemming from the limited scattering data. In contrast, the bound states of the clusters involved can, nowadays, be calculated quite reliably using a variety of methods (hyperspherical harmonics, variational, etc.) with realistic or semirealistic NN forces. In our case the three-body and four-body bound-state wave functions are calculated in the integrodifferential equation approach (IDEA) [29–31] using the MT I-III potential [8].

This paper is organized as follows: In Sec. II we describe the cross section for the electro-disintegration ^{4}He in configuration space. In Sec. III the electro-disintegration of ^{4}He is discussed and the results of the model described in this paper are compared with the available experimental data and other theoretical results. Our concluding remarks are presented in Sec. IV

II. THE CROSS SECTION

The electron-proton coincidence cross section is given by

$$\frac{d^3\sigma}{dE_f\,d\Omega_p\,d\Omega_e}=\frac{\sigma_{\mathrm{M}}}{(\hbar c)^3(2\pi)^3}\,\frac{\rho_f}{4E_iE_f\cos^2(\theta/2)}\,|\mathcal{M}(\boldsymbol{q}_\alpha)|^2, \tag{4}$$

where σ_{M} is the Mott differential cross section,

$$\sigma_{\mathrm{M}}=\frac{e^4\cos^2(\theta/2)}{4E_i^2\sin^4(\theta/2)}. \tag{5}$$

$E_i(E_f)$ is the energy of the incoming (outgoing) electron and ρ_f is the relativistic density of states. The transition matrix is given by

$$\mathcal{M}(\boldsymbol{q}_\alpha)={}^{(-)}\langle\boldsymbol{q}_\alpha;\Psi_{A-1}|H|\Psi_A\rangle, \tag{6}$$

where H is the effective Hamiltonian describing the interaction between an electron and the nucleons. The fragmentation considered here is of the $3+1$-type while the ejected proton moves away with momentum $\boldsymbol{q}_\alpha$ with respect to the residual bound cluster described by the bound state Ψ_{A-1}.

The Hamiltonian for the interaction between an electron and A nucleons, is that of Mc Voy and van Hove [32] which has been previously employed in the electro-disintegration of the trinucleon system [33,2,3]. This effective Hamiltonian, correct to order $\hbar^2q^2/M^2c^2$, is

$$\begin{aligned}H=-\frac{4\pi e^2}{q_\mu^2}\langle u_f|\sum_{j=1}^{A}\Bigg\{&F_{1N}(q_\mu^2)e^{-i\boldsymbol{q}\cdot\boldsymbol{x}_j}\\&-\frac{F_{1N}(q_\mu^2)}{2M}[(\boldsymbol{p}_j\cdot\boldsymbol{\alpha})\;\;e^{-i\boldsymbol{q}\cdot\boldsymbol{x}_j}+e^{-i\boldsymbol{q}\cdot\boldsymbol{x}_j}(\boldsymbol{p}_j\cdot\boldsymbol{\alpha})]\\&-i\left[\frac{F_{1N}(q_\mu^2)+\kappa F_{2N}(q_\mu^2)}{2M}\right]\boldsymbol{\sigma}_j\cdot(\boldsymbol{q}\times\boldsymbol{\alpha})e^{-i\boldsymbol{q}\cdot\boldsymbol{x}_j}\\&+\frac{q_\mu^2}{8M^2}[F_{1N}(q_\mu^2)+2\kappa F_{2N}(q_\mu^2)]e^{-i\boldsymbol{q}\cdot\boldsymbol{x}_j}\Bigg\}|u_i\rangle,\end{aligned} \tag{7}$$

where $\boldsymbol{x}_j$ and $\boldsymbol{p}_j$ are vectors denoting the position and momentum of the jth nucleon; $\boldsymbol{\sigma}_j$ is the nucleon spin operator, $\boldsymbol{\alpha}$ is the electron's Dirac operator acting on the free electron spinors $|u_i\rangle$ and $|u_f\rangle$, while q_μ^2 is the exchanged four-momentum squared; F_{1N} and F_{2N} are the Dirac and Pauli form factors of the nucleon, κ is the anomalous moment of

the nucleon in nuclear magnetons, and M is the nucleon mass. For a proton knockout the transition matrix, Eq. (6), is written

$$\mathcal{M}=\langle u_f|\langle f|\frac{1}{2}(1+\tau)_j\Bigg[F_{1p}(q_\mu^2)+\frac{q_\mu^2}{8M^2}\times[F_{1p}(q_\mu^2)+2\kappa_pF_{2p}(q_\mu^2)]\Bigg]e^{-i\mathbf{q}\cdot\mathbf{x}_j}|\Psi_A\rangle|u_i\rangle$$
$$-\langle u_f|\langle f|\boldsymbol{\alpha}\cdot\Bigg\{\frac{1}{2}(1+\tau)_j\frac{F_{1p}}{2M}(\mathbf{p}_je^{-i\mathbf{q}\cdot\mathbf{x}_j}+e^{-i\mathbf{q}\cdot\mathbf{x}_j}\mathbf{p}_j)+\frac{1}{2}(1+\tau)_ji\Bigg[\frac{F_{1p}(q_\mu^2)+\kappa_pF_{2p}(q_\mu^2)}{2M}\Bigg]\boldsymbol{\sigma}_j\times\mathbf{q}e^{-i\mathbf{q}\cdot\mathbf{x}_j}\Bigg\}|\Psi_A\rangle|u_i\rangle. \tag{8}$$

Here the subscript p refers to the proton. The transition matrix can be written in the convenient form as

$$\mathcal{M}=-[\langle u_f|u_i\rangle Q-\langle u_f|\boldsymbol{\alpha}|u_i\rangle\cdot\mathbf{J}], \tag{9}$$

where

$$Q=F_{\rm ch}^p(1+q_\mu^2/8M^2)\left\langle f\left|\sum_{j=1}^A e^{-i\mathbf{q}\cdot\mathbf{x}_j}\frac{1}{2}(1+\tau_3)_j\right|\Psi_A\right\rangle,$$
$$\mathbf{J}=\mathbf{J}^{\rm el}+\mathbf{J}^{\rm mag}, \tag{10}$$
$$\mathbf{J}^{\rm el}=\left\langle f\left|\sum_{j=1}^A(F_{\rm ch}^p/2M)(\mathbf{p}_je^{-i\mathbf{q}\cdot\mathbf{x}_j}+e^{-i\mathbf{q}\cdot\mathbf{x}_j}\mathbf{p}_j)\times\frac{1}{2}(1+\tau_3)_j\right|\Psi_A\right\rangle,$$
$$\mathbf{J}^{\rm mag}=(i/2M)F_{\rm mag}^p\left\langle f\left|\sum_{j=1}^A e^{-i\mathbf{q}\cdot\mathbf{x}_j}\boldsymbol{\sigma}\times\mathbf{q}\frac{1}{2}(1+\tau_3)_j\right|\Psi_A\right\rangle.$$

In these equations, τ_3 is the nucleon isospin operator, and $F_{\rm ch}^p$ and $F_{\rm mag}^p$ are the proton charge and magnetic form factors defined by

$$F_{\rm ch}^p=F_{1p}+(q_\mu^2/4M^2)\kappa_pF_{2p}, \tag{11}$$
$$F_{\rm mag}^p=F_{1p}+\kappa_pF_{2p}. \tag{12}$$

The analytical fit to the proton form factors F_{1p} and F_{2p}, given by Janssens *et al.* [34] is used in the calculations.

Squaring the matrix element and summing and averaging over electron spins yields

$$\frac{1}{2}\sum_{\text{electron spins}}|\mathcal{M}|^2=[(4E_iE_f+q_\mu^2)QQ^*-q_\mu^2\mathbf{J}\cdot\mathbf{J}^*+2(\mathbf{k}_f\cdot\mathbf{J})\times(\mathbf{k}_i\cdot\mathbf{J}^*)+2(\mathbf{k}_f\cdot\mathbf{J}^*)(\mathbf{k}_i\cdot\mathbf{J})-2E_f\{(\mathbf{k}_i\cdot\mathbf{J})Q^*+(\mathbf{k}_i\cdot\mathbf{J}^*)Q\}$$
$$-2E_f\{(\mathbf{k}_f\cdot\mathbf{J})Q^*+(\mathbf{k}_f\cdot\mathbf{J}^*)Q\}. \tag{13}$$

Substituting this expansion into Eq. (4) results in

$$\frac{d^3\sigma}{dE_f\,d\Omega_p\,d\Omega_e}=\frac{\sigma_{\rm M}}{(\hbar c)^3(2\pi)^3}\times\frac{|\mathbf{p}_p|E_p}{1-(E_p/E_{A-1})(\mathbf{p}_p\cdot\mathbf{p}_{A-1}/|\mathbf{p}_p|^2)}\times\Bigg\{|Q|^2-\frac{1}{2}\sec^2\frac{\theta}{2}(Q^*\mathbf{J}+\mathbf{J}^*Q)\cdot(\hat{k}_i+\hat{k}_f)+\frac{1}{2}\sec^2\frac{\theta}{2}\ (\mathbf{J}\cdot\hat{k}_i\mathbf{J}^*\cdot\hat{k}_f+\mathbf{J}\cdot\hat{k}_f\mathbf{J}^*\cdot\hat{k}_i)+|\mathbf{J}|^2\tan^2\frac{\theta}{2}\Bigg\}. \tag{14}$$

The evaluation of the coincidence cross section is thus reduced to the evaluation of the nuclear matrix elements Q and $\mathbf{J}$, which depend on the choice of the initial and final wave functions.

The inclusion of the FSI presupposes that the full solution of the scattering state be determined. Although this has been achieved in the three-nucleon case, it has not yet been accomplished in the four-nucleon case beyond the breakup threshold. To circumvent this problem we employ the model outlined in the introduction in which the FSI is taken into account via an optical potential treatment of the relative motion of the outgoing clusters. In this particularly simplifying modification the plane wave $\langle\boldsymbol{q}_\alpha|$ is replaced by scattering states ${}^{(-)}\langle\boldsymbol{q}_\alpha|$ generated by a $p-{}^3$H potential obtained using the Marchenko inverse scattering procedure [26–28]. In other words, instead of solving a four-body integral equation providing us with the full scattering state ${}^{(-)}\langle\boldsymbol{q}_\alpha;\psi_B|$, we use the approximation given by Eq. (1).

In implementing this model, we found it was numerically more stable to solve the integral equations in momentum space, in a similar fashion to the method employed by Fiedeldey *et al.* [35] in the photo-disintegration of ^{4}He. In this method, the scattering state and the channel state can be projected onto each other by means of Møller operators [36].

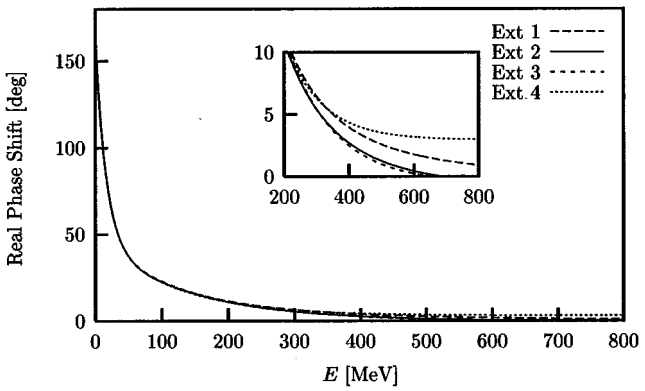

FIG. 1. Typical extrapolations of the high-energy $p+{}^3$H phase shifts used.

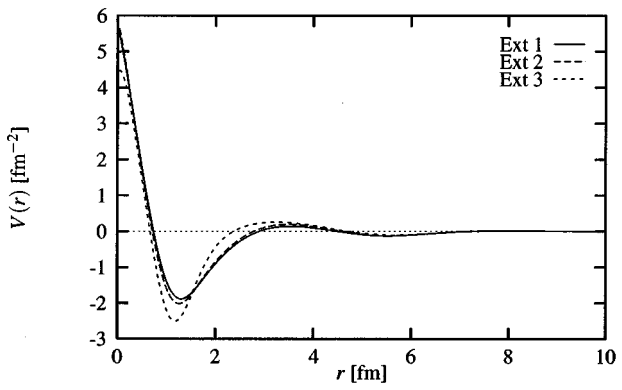

FIG. 2. Real part of the Marchenko potentials corresponding to the phase shifts given in Fig. 1.

The resulting equation is of the Lippmann-Schwinger-type and can be written schematically as

$$|\mathcal{M}\rangle = |\mathcal{B}\rangle + VG_0|\mathcal{M}\rangle. \tag{15}$$

$|\mathcal{B}\rangle$ is the plane-wave (Born) term, which can be evaluated in either configuration or momentum space using the wave functions described below.

The three- and four-nucleon bound-state spatial wave functions are obtained using the IDEA. In this method, the A-nucleon wave function $\Psi(\boldsymbol{x})$ is written as a sum of sub-amplitudes $\Psi(\boldsymbol{x}) = \Sigma_{i<j\leqslant A}\psi(\boldsymbol{r}_{ij}, r)$ obeying the Faddeev-type equation

$$\left[T + \frac{A(A-1)}{2}V_0(r)\right]\psi(\boldsymbol{r}_{ij}, r) = -[V(\boldsymbol{r}_{ij}) - V_0(r)] \times \sum_{i<j\leqslant A}\psi(\boldsymbol{r}_{ij}, r), \tag{16}$$

where r is the hyper-radius of the system, $r^2 = (2/A)\Sigma\boldsymbol{r}_{ij}^2$ and $V_0(r)$ is the hypercentral potential which is the first term of the potential harmonic expansion of the interaction. The effects of the all important higher partial waves are approximately included via the use of the hypercentral potential (for more details of the IDEA see Refs. [29–31]).

III. RESULTS

Reliable experimental phase shifts required in the construction of the effective potential for the $3+1$ fragmentation of ^{4}He are not available. Furthermore, the energy region in which they are available has little or no overlap with the region needed for our calculations. On the theoretical front, phase shifts beyond the three-body breakup have not yet been obtained. Therefore, in implementing our model we employed phase shifts obtained from the optical potential used by Schiavilla [18,20]. Since the phase shifts produced by this potential become negative for energies greater than 200 MeV, we had to extrapolate them to higher energies. In Fig. 1 four such extrapolations of the real phase shifts to higher energies are plotted. The corresponding real parts of the Marchenko potentials that describe the $p + {}^3$H interaction are plotted in Fig. 2. The local potentials produced by the Marchenko procedure are complex and unique for each set of data. They are phase equivalent up to approximately 300 MeV, producing phase shifts with slightly different high-energy behavior. The extrapolation Ext. 2, produced an overlap integral that was, for all practical purposes, zero.

TABLE I. Kinematics R and (A,B,C,D) from Ref. [25], and kinematics S from Ref. [37].

	E_i (MeV)	E_f (MeV)	θ	q (MeV/c)
R	524.9	423.9	49.60°	408
(A,B,C,D)	525.0	310.0	49.60°	401
S	560.0	360.0	25.00°	278

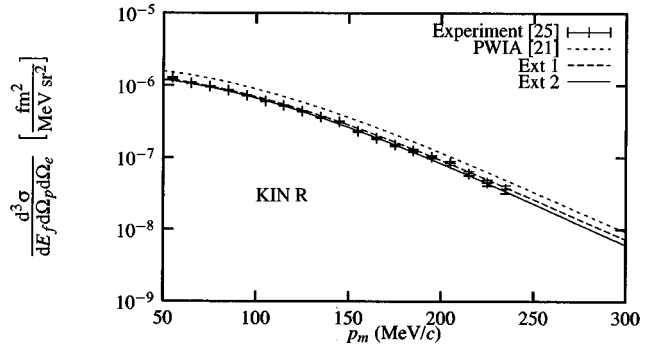

FIG. 3. The two-fragment electro-disintegration cross section of ^{4}He as a function of the missing momentum for kinematics R of Table I.

The various kinematics used are given in Table I. In Fig. 3 we have plotted our cross section results for the kinematics R together with the experimental data of Ref. [25]. These results are very close to those obtained by Laget [21]. It is obvious that the cross section is only weakly dependent on the nucleon-nucleus interaction, all extrapolations yielding virtually identical results.

The effect of the high-energy behavior of the phase shifts on the cross section and the importance of the orthogonality condition given by Eq. (2), is more clearly demonstrated in the plots of the cross sections for the kinematics (A,B,C,D) and kinematics S, shown in Figs. 4–6. The PWIA prediction for kinematics (A,B,C,D) shows a minimum, in fact a zero, in agreement with the calculations of Laget and Schiavilla. At missing momenta below 300 MeV/c, all calculations

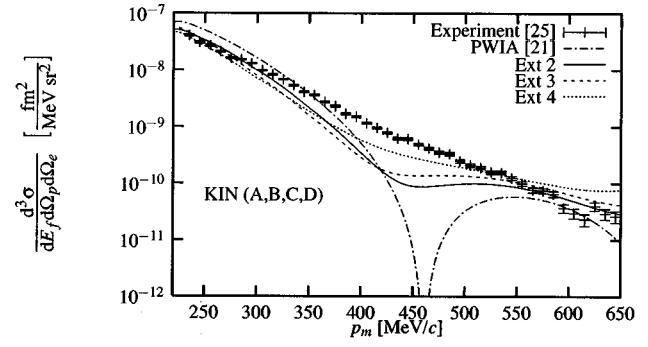

FIG. 4. The two-fragment electro-disintegration cross section of ^{4}He as a function of the missing momentum for kinematics (A,B,C,D) of Table I.

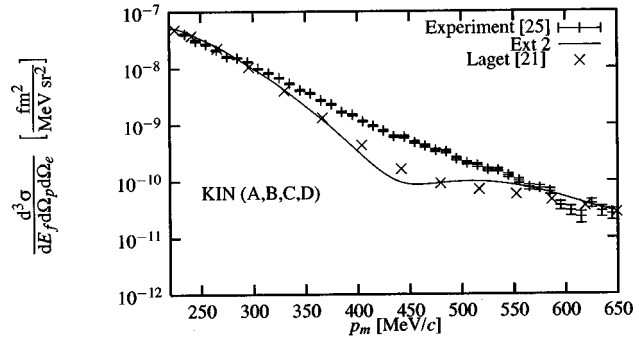

FIG. 5. A comparison of our two-fragment electro-disintegration cross section of ^{4}He with that of Laget as a function of the missing momentum for kinematics (A,B,C,D).

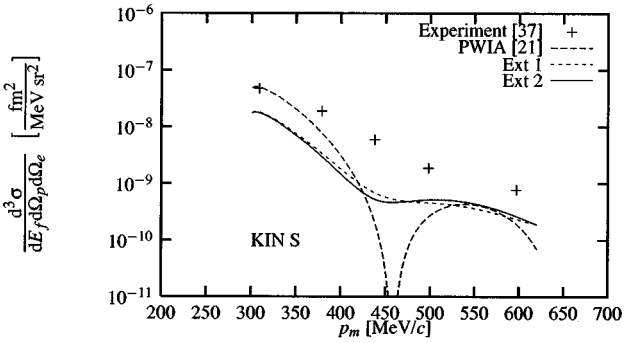

FIG. 6. The two-fragment electro-disintegration cross section of ^{4}He as a function of the missing momentum for kinematics S of Table I.

show good agreement with the data, but in the region of the dip the calculations underestimate the data considerably. Beyond missing momenta of $\sim$550 MeV/c the agreement of the calculations with the data is again fair. The best result is the one obtained for Ext. 2 which fulfills the orthogonality condition (2). In Fig. 5, we compare the cross section produced using this extrapolation with the results of Laget [21]; it is seen that the agreement is good.

The PWIA cross section calculated for kinematics S, shown in Fig. 6, also exhibits a dip at $\sim$450 MeV/c. Including the FSI causes a partial filling of this dip. The position of the minimum and the value of the cross section in this region once again depends strongly on the input nucleon-nucleus and the NN potentials.

IV. CONCLUSIONS

The results described above clearly indicate a sensitivity of the cross section in certain kinematical regions to the input nucleon-nucleus interaction and to the input NN potential. In our model, the various nucleon-nucleus $(3+1)$ interactions employed for this investigation were phase equivalent up to approximately 300 MeV. They all reproduced the correct binding energy for the four-nucleon bound state. However, beyond 300 MeV, the potentials produced phase shifts that differed slightly. The corresponding nucleon-nucleus potentials provided scattering states that were not always orthogonal to the initial bound state. This in turn is translated into large differences in the cross sections in certain kinematical regions. This result was not surprising, as already in 1970, Fiedeldey [38] investigated the dependence of the triton binding energy on the high-energy part of the phase shift. He concluded that arbitrary variations of the NN phase shift at high energies ($E_{\rm lab}>300$ MeV) can produce large differences in the triton binding energy. This sensitivity is also manifested in the electro-disintegration cross sections which implies that we need a much clearer idea of what constitutes a physically acceptable extension of the phase shift to higher energies. This sounds a warning that in model calculations care must be taken when using effective interactions, the most important constraint being the condition that the initial bound state and the final scattering state be orthogonal.

The dip around 450 MeV/c, which is present in all model calculations, is something of a surprise. At missing momenta less than 300 MeV/c all calculations show a good agreement with the experimental data, i.e., the PWIA performs reasonably well in a region where the FSI could be expected to be more important. At missing momenta beyond the dip area, where the MEC become important, the agreement with the data is fair. Thus the zero in the PWIA cross section is not necessarily a manifestation of strong FSI effects or MEC. It can simply be attributed to the vanishing Born term which contains only genuine $3+1$ components. The Born term should, however, be treated in a more rigorous way using the AGS formalism which allows for the effects coming from the $2+2$ channel. The inclusion of this channel should result in an improvement of the the bound-state wave functions and of the Born term. This in turn is expected to remove the apparent zero in the PWIA cross section. Investigations in this regard are currently under way.

[1] T. A. Griffy and R. J. Oakes, Phys. Rev. **135**, B1161 (1964).
[2] D. R. Lehman, Phys. Rev. Lett. **23**, 1339 (1969); Phys. Rev. C **3**, 1827 (1971).
[3] C. R. Heimbach, D. R. Lehman, and J. S. O'Connell, Phys. Rev. C **16**, 2135 (1977).
[4] J. M. Barbour and A. C. Phillips, Phys. Rev. C **1**, 165 (1970).
[5] F. B. Gibson and D. R. Lehman, Phys. Rev. C **11**, 29 (1975).
[6] E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. **B2**, 167 (1967).
[7] E. van Meijgaard and J. A. Tjon, Phys. Rev. C **42**, 74 (1990); **42**, 96 (1990); **45**, 1463 (1992).
[8] R. A. Malfliet and J. A. Tjon, Nucl. Phys. **A127**, 161 (1969).
[9] R. Schiavilla and V. R. Pandharipande, Phys. Rev. C **36**, 2221 (1987).
[10] I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A359**, 331 (1989).
[11] R. B. Wiringa, R. A. Smith, and T. A. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[12] J. M. Laget, Phys. Lett. **151B**, 325 (1985).
[13] S. Ishikawa, H. Kamada, W. Glöcle, J. Golak, and H. Witala,

Nuovo Cimento A **107**, 305 (1994).
[14] M. Lacombe *et al.*, Phys. Rev. C **21**, 861 (1980).
[15] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987); R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[16] J. Golak, H. Kamada, H. Witala, W. Glöcle, and S. Ishikawa, Phys. Rev. C **51**, 1638 (1995).
[17] S. A. Sofianos, H. Fiedeldey, and W. Sandhas, Phys. Rev. C **48**, 2285 (1993).
[18] R. Schiavilla, Phys. Rev. Lett. **65**, 835 (1990).
[19] R. Schiavilla, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A449**, 219 (1986).
[20] W. T. H. van Oers *et al.*, Phys. Rev. C **25**, 390 (1982).
[21] J. M. Laget, Nucl. Phys. **A579**, 333 (1994).
[22] S. I. Nagorny, Yu. A. Kasatkin, E. V. Inopin, and I. K. Kirichenko, Yad. Fiz. **49**, 749 (1989) [Sov. J. Nucl. Phys. **49**, 465 (1989)]; **53**, 365 (1991) [**53**, 228 (1991)].
[23] A. A. Zayatz, V. A. Zolenko, Yu. A. Kasatkin, and S. I. Nagorny, Yad. Fiz. **55**, 325 (1992) [Sov. J. Nucl. Phys. **55**, 178 (1992)].
[24] J. J. van Leeuwe *et al.*, Phys. Rev. Lett. **80**, 2543 (1998).
[25] J. J. van Leeuwe, Ph.D. thesis, Rijks Universiteit Utrecht, 1996.
[26] Z. S. Agranovich and V. A. Marchenko, *The Inverse Problem of the Scattering Theory* (Gordon and Breach, New York, 1963).
[27] K. Chadan and P. C. Sabatier, *Inverse Problems in Quantum Scattering Theory* (Springer, New York, 1977).
[28] E. O. Alt, L. L. Howell, M. Rauh, and S. A. Sofianos, Phys. Rev. C **49**, 176 (1994).
[29] M. Fabre de la Ripelle, H. Fiedeldey, and S. A. Sofianos, Phys. Rev. C **38**, 449 (1988).
[30] W. Oehm, S. A. Sofianos, H. Fiedeldey, and M. Fabre de la Ripelle, Phys. Rev. C **42**, 2322 (1990); **43**, 25 (1991).
[31] W. Oehm, H. Fiedeldey, S. A. Sofianos, and M. Fabre de la Ripelle, Phys. Rev. C **44**, 81 (1991).
[32] K. W. McVoy and L. Van Hove, Phys. Rev. **125**, 1034 (1962).
[33] C. D. Epp and T. A. Griffy, Phys. Rev. C **1**, 1633 (1970).
[34] T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. **142**, 922 (1966).
[35] H. Fiedeldey, S. A. Sofianos, and G. Ellerkmann, Few-Body Syst. **18**, 173 (1995).
[36] W. Sandhas, Acta Phys. Austriaca, Suppl. **IX**, 57 (1972).
[37] J. M. Le Goff *et al.*, Phys. Rev. C **50**, 2278 (1994).
[38] H. Fiedeldey, Nucl. Phys. **A156**, 242 (1970).