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γ transitions in $A=7$ hypernuclei and a possible derivation of hypernuclear size

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On the basis of the ${}^{5}_{\Lambda}\mathrm{He}+N+N$ three-body model which has successfully been applied to a systematic study of the energy and nucleon halo structure of the $A=7$ isotriplet hypernuclei, strengths of γ transitions in ${}^{7}_{\Lambda}\mathrm{Li}$ and ${}^{7}_{\Lambda}\mathrm{He}$ are calculated. The new model confirms that the $B(E2;5/2^{+}\rightarrow 1/2^{+})$ value in ${}^{7}_{\Lambda}\mathrm{Li}$ is reduced remarkably in comparison with the corresponding $B(E2;3^{+}\rightarrow 1^{+})$ in the core nucleus ${}^{6}\mathrm{Li}$. This is due to the gluelike role of the Λ particle which induces a contraction of the core nuclear size. It is suggested that a measurement of the $5/2^{+}\rightarrow 1/2^{+} E2$ transition rate in ${}^{7}_{\Lambda}\mathrm{Li}$ (ongoing at KEK as E419) provides a unique opportunity to derive the hypernuclear size and hence to confirm the size contraction experimentally. The $E2$ and $M1$ transition strengths are also predicted for low-lying states in the hypernucleus ${}^{7}_{\Lambda}\mathrm{He}$ whose core nucleus ${}^{6}\mathrm{He}$ is known to have a neutron halo. Another prediction is made of much enhanced $E2$ transitions in ${}^{7}_{\Lambda}\mathrm{Li}$ from the $5/2^{+}$ and $3/2^{+}(T=1)$ states which are expected to have a proton halo structure. [S0556-2813(99)00705-0]

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I. INTRODUCTION

One of the interesting aspects in hypernuclear physics is to investigate nuclear responses to an added Λ hyperon which is free from the nuclear Pauli principle. Since nuclear incompressibility is high in general, one might expect only little changes in the nuclear core due to the Λ particle [1]. However, this is not necessarily the case in light nuclei where cluster structure prevails and the constituent clusters are loosely combined and hence can be readily changed by even a weak influence. Theoretical studies of light hypernuclei based on the cluster model have predicted a sizable dynamical contraction of hypernuclear systems induced by the gluelike role of the Λ particle [2]. These studies were made mostly of systems composed of a stable nucleus and a Λ particle.

On the other hand, a new type of Λ addition to an *unstable* nucleus having a neutron or proton halo was investigated by the present authors and their collaborators in the case of the $A=7$ isotriplet hypernuclei [3]. Instead of the models ${}^{7}_{\Lambda}\mathrm{Li}=\alpha+d+\Lambda$ and ${}^{6}\mathrm{Li}=\alpha+d$ employed in Ref. [2], Hiyama *et al.* [3] proposed a ${}^{5}_{\Lambda}\mathrm{He}+N+N$ three-body model for the $A=7$ hypernuclei (${}^{7}_{\Lambda}\mathrm{He}$, ${}^{7}_{\Lambda}\mathrm{Li}$, and ${}^{7}_{\Lambda}\mathrm{Be}$) together with an $\alpha+N+N$ model for the $A=6$ nuclei (${}^{6}\mathrm{He}$, ${}^{6}\mathrm{Li}$, and ${}^{6}\mathrm{Be}$) and made a unified study of those systems successfully. It was also found that the deuteron-cluster approximation is broken by $\sim$40% in ${}^{6}\mathrm{Li}$ and ${}^{7}_{\Lambda}\mathrm{Li}$. The usefulness of the new ${}^{5}_{\Lambda}\mathrm{He}+N+N$ model was demonstrated by the fact that all the existing experimental data for the $A=7$ hypernuclei (the binding energies of ${}^{7}_{\Lambda}\mathrm{Li}$ and ${}^{7}_{\Lambda}\mathrm{Be}$, and the excitation energy of the $5/2^{+}$ state of ${}^{7}_{\Lambda}\mathrm{Li}$) were well reproduced and the existence of nucleon halo states was predicted in these hypernuclei.

In order to investigate the structure of the $A=7$ hypernuclei and extract information on the ΛN interaction, an experimental project (E419) with high-resolution γ spectroscopy is under way at KEK [4]. The first purpose of this paper is to predict $E2$ and $M1$ γ-transition probabilities of $A=7$ hypernuclei with the improved wave functions [3] before experimental results are reported. The calculated results not only update the strengths for electromagnetic decay of low-lying states in ${}^{7}_{\Lambda}\mathrm{Li}$ given in Ref. [2] but also give essentially new estimates associated with the hypernuclear halo states. It is indeed interesting to expect enhanced $E2$ transition rates in proton-halo states in ${}^{7}_{\Lambda}\mathrm{Li}$ $(T=1)$.

It is of particular interest to experimentally measure the size of any hypernucleus for the first time and compare it with that of the core nucleus. In the work of Ref. [2] based on the microscopic $\alpha+x+\Lambda$ three-cluster model ($x=d,t,{}^{3}\mathrm{He}$) for light p-shell hypernuclei together with the $\alpha+x$ two-cluster model for the nuclear core, it was pointed out that such contraction of the hypernuclear size could be recognized from a reduction of the $B(E2)$ strength since it is proportional to the fourth power of the distance between the α and x clusters; for example, a (10–18)% reduction of the $\alpha-x$ distance due to the Λ participation leads to a (35–

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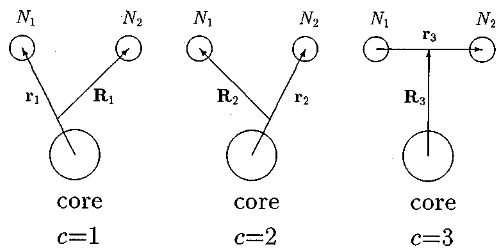

FIG. 1. Jacobian coordinates of the three rearrangement channels adopted for the ''core''$+N+N$ model of $A=6$ nuclei and $A=7$ hypernuclei for which ''core'' denotes α and ${}^5_\Lambda$He clusters, respectively.

55)% reduction in the $B(E2)$ values [2].

In general the electric quadrupole moment $Q(J_{\rm g.s.})$ should provide direct information on the size of a hypernucleus if it is measurable. However, the possibility might be very low in the near future. In addition it is hard to know the intrinsic quadrupole deformation of the ground state when $J=1/2^+$. We think about measurement of the hypernuclear $B(E2)$ values, instead of $Q(J_{\rm g.s.})$, as a unique indication of the size. In order to realize this possibility, we have to find a nucleus-hypernucleus combination such that both of them have at least two bound states which are stable to particle-emission decays and are connected by the $E2$ transition. In the light p-shell region where the cluster structure dominates, we see two candidates for such a combination: ^{6}Li-${}^7_\Lambda$Li and ^{7}Li-${}^8_\Lambda$Li. In view of the easier production of ${}^7_\Lambda$Li through the (K^-,π^-) or (π^+,K^+) reaction, we have nominated the first combination. In fact, a γ ray from ${}^7_\Lambda$Li with an energy of 2.034 MeV has been already observed at BNL [5], so that a measurement of the absolute $E2$ transition probability for $5/2^+\rightarrow 1/2^+$ in the ${}^7_\Lambda$Li ground band seems most promising. It should be noted that the empirical $B(E2;3^+\rightarrow 1^+)$ value of the corresponding transition in the core nucleus, ^{6}Li, is known already from an inelastic electron scattering experiment.

The second aim of this paper is therefore to encourage the measurement of $B(E2;5/2^+\rightarrow 1/2^+)$ in ${}^7_\Lambda$Li and to propose a prescription to derive the size of the ground state of ${}^7_\Lambda$Li from that $B(E2)$ value with the aid of the empirical $B(E2;3^+\rightarrow 1^+)$ and the size of the ground state of ^{6}Li. We examine seriously the usefulness of this prescription by checking the consistency between the theoretical values of those physical quantities calculated with the ${}^5_\Lambda$He$+N+N$ model and the $\alpha+N+N$ model of Ref. [3]. In Sec. II, we briefly recapitulate the model and interaction of Ref. [3] in order to show that the wave functions are general enough to describe the systems. In Sec. III, $B(E2)$ and $B(M1)$ values are calculated for the low-lying states of ${}^7_\Lambda$He and ${}^7_\Lambda$Li. In Sec. IV, we propose a prescription to estimate the size of hypernucleus ${}^7_\Lambda$Li from the $B(E2;5/2^+\rightarrow 1/2^+)$ value to be measured experimentally. A summary is given in Sec. V. An appendix shows distorted-wave impulse approximation (DWIA) predictions of differential cross sections for the ^{7}Li$(\pi^+,K^+){}^7_\Lambda$Li reaction at $p_\pi=1.05$ GeV/c.

II. MODEL AND INTERACTION

The model and Hamiltonian used to generate the wave functions employed in this paper are the same as those in our preceding work [3]. Namely, we employ a ${}^5_\Lambda$He$+N+N$ model for the $A=7$ hypernuclei and an $\alpha+N+N$ model for the A=6 nuclei (Fig. 1). The interactions between the constituent particles are taken as follows: We fully take into account the NN correlations between the two valence nucleons by employing a realistic NN interaction of the Bonn-A type [6]. As for the αN interaction, the one proposed in Ref. [7] is adopted; it reproduces precisely the αN scattering phase shifts at low energies. The Pauli principle between the valence nucleons and the α-core nucleons is taken into consideration by introducing a projection operator which rules out the amplitude of the Pauli-forbidden αN state $(0s)$ from the total wave function. As for the ΛN interaction, we employ a one-range Gaussian (ORG) potential proposed in Ref. [2] which has been useful in the systematic study of the structure of light hypernuclei based on the cluster model [2]. The $\alpha\Lambda$ interaction is constructed by folding the ΛN interaction into the nucleon density of the α particle. Similarly, the interaction between the ${}^5_\Lambda$He cluster and a valence nucleon is obtained by folding the αN and ΛN interactions into the α and Λ densities of ${}^5_\Lambda$He. We note that the use of another ΛN interaction such as YNG (effective YNG-matrix interaction by Yamamoto and Bandō [8]) may lead to a little different results [9]. Here, however, we focus our attention on the dynamical contraction of the relative motion among the core and two valence nucleons in the $A=7$ hypernuclei without assuming a ''deuteron'' cluster in the case of ${}^7_\Lambda$Li. One may refer to Ref. [10] for results of the contraction with ORG and YNG by assuming a deuteron cluster in ${}^7_\Lambda$Li.

The three-body Schrödinger equation with the interactions mentioned above is solved accurately with the use of the coupled-rearrangement-channel Gaussian basis variational method which has been developed by two of the present authors (E.H. and M.K.) and their collaborators, and has successfully been applied to a variety of three- and four-body systems [3,11–15]. According to this method, the total wave function of the $A=7$ hypernucleus ${}^7_\Lambda$Z is described as a sum of the amplitudes of the three rearrangement channels $c=1\sim 3$ in Fig. 1 [see Eq. (3.4) of Ref. [3]]:

$$\Psi_{JM}({}^7_\Lambda Z)=\sum_{c=1}^{3}\sum_{I,S}\big(\Phi_{1/2}({}^5_\Lambda\mathrm{He})\{\phi_I^{(c)}(\mathbf{r}_c,\mathbf{R}_c)\times[\chi_{1/2}(N_1)\chi_{1/2}(N_2)]_S\}_{J_0}\big)_{JM}. \tag{2.1}$$

Here, $\phi_I^{(c)}(\mathbf{r}_c,\mathbf{R}_c)$ is the spatial coordinate amplitude with angular momentum I, and $\chi_{1/2}(N_1)$ and $\chi_{1/2}(N_2)$ are the two-nucleon spin wave functions coupled to spin S, with I and S being coupled to J_0. $\Phi_{1/2}({}^5_\Lambda\mathrm{He})$ denotes the wave function of ${}^5_\Lambda$He with spin 1/2 which couples with J_0 to the total angular momentum $J=J_0\pm 1/2$. Similar wave functions are adopted for ^{6}Li [see Eq. (3.2) of Ref. [3]]. Each $\phi_I(\mathbf{r},\mathbf{R})$ is expanded in terms of Jacobian-coordinate Gaussian basis functions associated with $\mathbf{r}$ and $\mathbf{R}$:

$$\phi_{IM}(\mathbf{r},\mathbf{R})=\sum_{l,L}\sum_{n=1}^{n_{\rm max}}\sum_{N=1}^{N_{\rm max}}C_{nlNL}^{(I)}r^lR^Le^{-(r/r_n)^2}e^{-(R/R_N)^2}\times[Y_l(\hat{\mathbf{r}})\otimes Y_L(\hat{\mathbf{R}})]_{IM}. \tag{2.2}$$

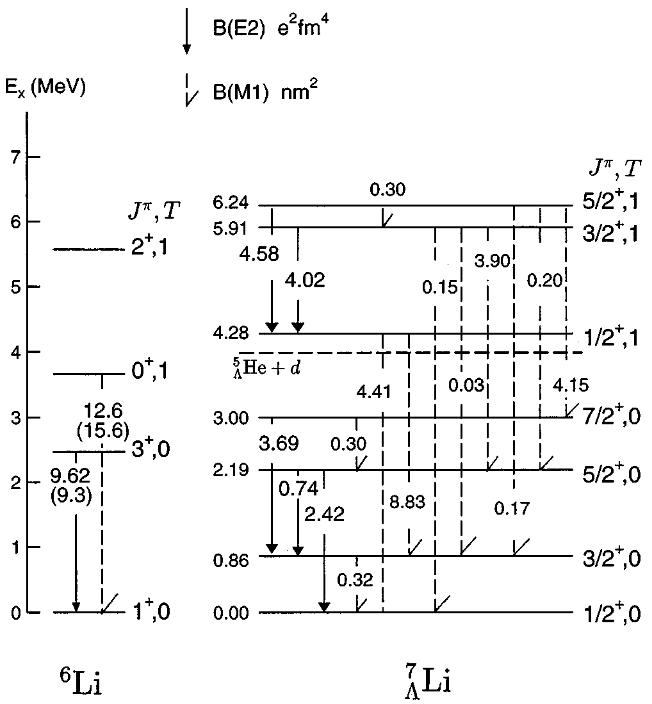

FIG. 2. Calculated $B(E2)$ and $B(M1)$ values for ^{6}Li and $^7_\Lambda$Li. The observed values for ^{6}Li are in parentheses. The particle-decay thresholds are 3.94 MeV ($^5_\Lambda$He$+d$), 5.58 MeV (^{6}Li$+\Lambda$), 5.99 MeV ($^6_\Lambda$He$+p$), and 6.16 MeV ($^5_\Lambda$He$+p+n$).

Here, the Gaussian range parameters are chosen to lie in a geometric progression; this choice is known to be suited for describing both short-range correlations and long-range tail behavior of three-body systems [11–15]. Examples of the angular momentum space and the Gaussian range parameters are given in Ref. [3].

III. CALCULATED $E2$ AND $M1$ STRENGTHS IN $^7_\Lambda$He AND $^7_\Lambda$Li

For the $A=6$ nuclei and $A=7$ hypernuclei, the calculated energies and intercluster distances are all listed in the tables of Ref. [3]. In the present paper, we newly calculate the γ transition strengths for these systems. The calculated $B(E2)$ and $B(M1)$ values are summarized in Fig. 2 for $^7_\Lambda$Li$=(\alpha+\Lambda)+n+p$ and in Fig. 3 for $^7_\Lambda$He$=(\alpha+\Lambda)+n+n$, respectively, together with the energy spectra. The transition rates $T(E2)$ and $T(M1)$ are listed in Tables I and II as well as the γ-ray energies. In the following subsections, we discuss the results for $E2$ transitions between $T=1$ states, those for $M1$ transitions, and those for $E2$ transitions between $T=0$ states, respectively.

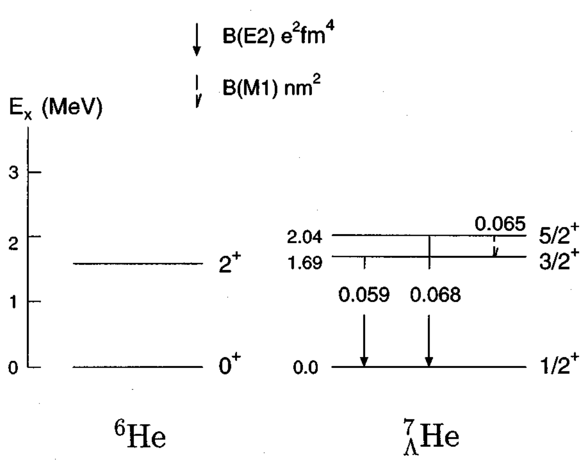

FIG. 3. Calculated $B(E2)$ and $B(M1)$ values for ^{6}He and $^7_\Lambda$He.

From an experimental point of view, the yield of each γ transition to be observed is closely related to the formation rates of the states in the production reaction. Thus we show in the Appendix DWIA estimates of typical differential cross sections for the ^{7}Li$(\pi^+,K^+)^7_\Lambda$Li reaction at $p_\pi=1.05$ GeV/c. In fact, these predictions have been referenced in the proposal of the KEK-E419 experiment [4]. A minor remark is that shell-model wave functions are employed in these cross section estimates.

Before discussing the results, we note the experimental particle-decay thresholds which are relevant to Fig. 2. The thresholds in ^{6}Li are at $E_x=1.48$ MeV and 3.70 MeV for $\alpha+d(T=0)$ and $\alpha+p+n$, respectively. The lowest particle threshold in $^7_\Lambda$Li is for $^5_\Lambda$He$+d(T=0)$ at 3.94 MeV. The ^{6}Li$+\Lambda(T=0)$, $^6_\Lambda$He$+p$, and $^5_\Lambda$He$+p+n$ thresholds are located at 5.58 MeV, 5.99 MeV, and 6.16 MeV, respectively. It is to be noted that, within the present $^5_\Lambda$He$+N+N$ three-body model, the calculated $1/2^+(T=1)$ state is obtained slightly (by a few parts of 1 MeV) above the $^5_\Lambda$He$+d(T=0)$ threshold and the $5/2^+(T=1)$ state is above the $^6_\Lambda$He$+p$ and $^5_\Lambda$He$+p+n$ thresholds. Here we like to remark that, when a more sophisticated calculation within the $\alpha+N+N+\Lambda$ four-body model is performed [9], these states go down further under the respective thresholds. Therefore, even in the present framework, we list the calculated $B(E2)$ and $B(M1)$ values for the γ transitions from the $T=1$ states in $^7_\Lambda$Li, assuming that the values will not change significantly in the four-body model calculation.

A. $E2$ transitions involving neutron- and proton-halo states with $T=1$

The $0^+(T=1)$ ground state of ^{6}He and the $0^+(T=1)$ state at $E_x=3.56$ MeV in ^{6}Li are known to have a neutron halo [16] and a proton halo [17], respectively. In our preceding work [3], it was shown that the addition of a Λ particle to these $A=6$ nuclear halo states stabilizes the system remarkably; the Λ particle even makes the next excited $2^+(T=1)$ states come down below the nucleon-emission threshold to form new hypernuclear nucleon-halo states in the $A=7$ hypernuclei with the spin $3/2^+$ and $5/2^+$. Thus the $E2$ transition probabilities from these $3/2^+$ and $5/2^+(T=1)$ states in $^7_\Lambda$Li are expected to be enhanced because of the extended tail of a valence proton. In fact we obtain $B(E2;\ 5/2^+(T=1)\to 1/2^+(T=1))=4.58\ e^2\,\mathrm{fm}^4$ which is much larger than the ''normal'' transition rate between the corresponding $T=0$ states: $B(E2;\ 5/2^+(T=0)\to 1/2^+(T=0))=2.42\ e^2\,\mathrm{fm}^4$.

The calculated $3/2^+(T=1)$ and $5/2^+(T=1)$ states are located above the $^5_\Lambda$He$+d(T=0)$ threshold. The decay from these $T=1$ states into the $^5_\Lambda$He$+d$ channel requires some isospin mixing, as discussed many years ago by Dalitz and Gal [18]. In their shell-model study of $^7_\Lambda$Li, they concluded that such isospin mixing would be sufficient to prevent the γ

TABLE I. Calculated $E2$ transition rates $T(E2)$ and $B(E2)$ values of (a) ${}^{7}_{\Lambda}$Li and (b) ${}^{7}_{\Lambda}$He. E_γ is the γ-ray energy. The shell-model estimate is denoted as S.M.

(a) ${}^{7}_{\Lambda}$Li

$J_i,T_i \rightarrow J_f,T_f$	$B(E2)$ (e^2 fm^4)	E_γ (MeV)	$T(E2)$ (sec^{-1})	
$3/2^+,0 \rightarrow 1/2^+,0$	0.15	0.86	8.6×10^7	Present
$5/2^+,0 \rightarrow 1/2^+,0$	2.42	2.19	1.5×10^{11}	Present
	2.46	1.99	9.5×10^{10}	$\alpha+d+\Lambda$ [2]
$\rightarrow 3/2^+,0$	0.74	1.33	3.8×10^9	Present
	0.40	0.89	2.7×10^8	$\alpha+d+\Lambda$ [2]
$7/2^+,0 \rightarrow 3/2^+,0$	3.69	2.14	2.0×10^{11}	Present
	3.04	1.82	7.4×10^{10}	$\alpha+d+\Lambda$ [2]
$\rightarrow 5/2^+,0$	0.38	0.81	1.6×10^8	Present
	0.13	0.93	1.1×10^8	$\alpha+d+\Lambda$ [2]
$5/2^+,1 \rightarrow 1/2^+,1$	4.58	1.96	1.6×10^{11}	Present
$3/2^+,1 \rightarrow 1/2^+,1$	4.02	1.63	5.7×10^{10}	Present

(b) ${}^{7}_{\Lambda}$He

$J_i,T_i \rightarrow J_f,T_f$	$B(E2)$ (e^2 fm^4)	E_γ (MeV)	$T(E2)$ (sec^{-1})	
$3/2^+,1 \rightarrow 1/2^+,1$	0.059	1.69	1.0×10^9	Present
	0.034	(1.69)	5.7×10^8	S.M. [20]
$5/2^+,1 \rightarrow 1/2^+,1$	0.068	2.04	2.9×10^9	Present
	0.032	(2.04)	1.4×10^9	S.M. [20]

rays from the $T=1$ states from being observed. In this connection, however, we note that, as far as the ^{6}Li$0^+(T=1)$ state ($E_x=3.56$ MeV) lying above the $\alpha+d$ threshold is concerned, no particle decay has been reported experimentally [19], suggesting that there is almost no isospin mixing in the $1/2^+(T=1)$ state in ${}^{7}_{\Lambda}$Li and that the state lying even above the ${}^{5}_{\Lambda}$He$+d$ threshold will decay only through γ transitions. Although the situation for the $3/2^+(T=1)$ and $5/2^+(T=1)$ states in ${}^{7}_{\Lambda}$Li may be different due to the higher-energy position of the $2^+(T=1)$ core state in ^{6}Li, we list in Table I the theoretical γ-decay rates to be referenced in future study.

Contrary to the case of ${}^{7}_{\Lambda}$Li, $E2$-decay rates of the $3/2^+$ and $5/2^+$ states in ${}^{7}_{\Lambda}$He$=(\alpha+\Lambda)+n+n$ are estimated to be very small; there is no contribution coming from the motion of halo neutrons with respect to the c.m. of ${}^{7}_{\Lambda}$He but there is a small contribution from the recoil motion of the $\alpha+\Lambda$ part. It is interesting, however, to point out that in the present estimate the $E2$-decay rates of the $5/2^+$ and $3/2^+$ states going down to the $1/2^+$ ground state are both the same order of magnitude as the free Λ particle's decay rate (3.8×10^9 s^{-1}, or the lifetime is 260 ps), while a shell-model estimate suggests it qualitatively to be less than 10^8 s^{-1} [18], although the numbers are rather sensitive to the wave functions employed [20]. Note that, according to the theoretical estimate, the $5/2^+$ state itself will decay predominantly to the $3/2^+$ state by the $M1$ transition. Thus we remark that observation of the $E2\gamma$ rays from the $3/2^+$ ($3/2^+$, $5/2^+$; $T=1$) halo state in ${}^{7}_{\Lambda}$He (${}^{7}_{\Lambda}$Li) would contribute to the study of not only the hypernuclear halo structure but also the excitation mechanism of neutron/proton halo of the ordinary core nucleus ^{6}He and ^{6}Li ($T=1$) in which no γ transition can be seen due to the prompt particle decays from the 2^+ state.

B. $M1$ transitions in ${}^{7}_{\Lambda}$He and ${}^{7}_{\Lambda}$Li

As seen in Fig. 2, the large $B(M1)$ value of the decay from the $0^+(T=1)$ proton halo state in ^{6}Li to the $1^+(T=0)$ ground state is known experimentally by the inverse inelastic electron scattering. The value is well reproduced by the present $\alpha+n+p$ three-body model, and the large $M1$ transition probability is attributed to the fact that spin of the (np) pair changes from $S_{np}=0$ to $S_{np}=1$ with the total orbital angular momentum unchanged. The addition of Λ particle to the $0^+(T=1)$ and $1^+(T=0$, g.s.) states of $A=6$ nucleus leads to the $1/2^+(T=1)$ state and the $1/2^+-3/2^+$ doublet ($T=0$) of states in ${}^{7}_{\Lambda}$Li, respectively. Thus it is interesting to see whether the corresponding $M1$ decay in ${}^{7}_{\Lambda}$Li [$1/2^+(T=1)\rightarrow1/2^+(T=0)$ and $3/2^+(T=0)$] undergoes any change due to the added Λ particle. We find that sum of $B(M1;\ 1/2^+(T=1)\rightarrow1/2^+(T=0))$ and $B(M1;\ 1/2^+(T=1)\rightarrow3/2^+(T=0))$ in ${}^{7}_{\Lambda}$Li, with the ratio 1 : 2, is nearly equal to $B(M1;\ 0^+(T=1)\rightarrow1^+(T=0))$ in ^{6}Li, showing no reduction of the former sum in comparison with latter. This is rather natural since the $B(M1)$ strength merely reflects the radial overlap between the initial- and final-state wave functions, and in fact the relevant overlap value in the former is similar to the one in the latter. One should note, however, that shapes of the wave functions themselves are substantially different between ${}^{7}_{\Lambda}$Li and ^{6}Li.

In the $M1$ transitions for $5/2^+(T=1)\rightarrow7/2^+(T=0)$ and $3/2^+(T=1)\rightarrow5/2^+(T=0)$, both process involve the (np) pair-spin changes from $S_{np}=0$ to $S_{np}=1$, since the $5/2^+(T=1)$ and $3/2^+(T=1)$ have the structure $[2^+(T=1,{}^6\text{Li})\otimes1/2(\Lambda)]$. Therefore, we have as large $B(M1)$ values as in the $M1$ transitions $1/2^+(T=1)\rightarrow1/2^+,3/2^+(T=0)$ mentioned above. On the other hand, we see that $B(\text{M1};5/2^+(T=1)\rightarrow5/2^+(T=0))$ is much smaller since the change of the

TABLE II. (a) Calculated $M1$ transition probabilities $T(M1)$ and $B(M1)$ values for ${}^{7}_{\Lambda}$He and ${}^{7}_{\Lambda}$Li($T=0$) together with the results given in Refs. [2], [18], and [20]. E_γ is the γ-ray energy. The shell-model estimate is denoted as S.M. (b) The same values for the $T=1\rightarrow T=0$ and $T=1\rightarrow T=1$ transitions in ${}^{7}_{\Lambda}$Li.

(a)

	$J_i,T_i \rightarrow J_j,T_f$	$B(M1)$ (μ_N^2)	E_γ (MeV)	$T(M1)$ (sec^{-1})	
${}^{7}_{\Lambda}$He	$3/2^+,1\rightarrow 1/2^+,1$	1.5×10^{-5}	(1.69)	1.3×10^{9}	S.M. [20]
	$5/2^+,1\rightarrow 3/2^+,1$	0.065	0.35	4.9×10^{10}	Present
		0.053	1.22	1.7×10^{12}	S.M. [18]
		0.098	0.20	1.5×10^{10}	S.M. [20]
${}^{7}_{\Lambda}$Li	$3/2^+,0\rightarrow 1/2^+,0$	0.322	0.86	3.6×10^{12}	Present
		0.352	1.10	8.2×10^{12}	$\alpha+d+\Lambda$ [2]
		0.364	0.25	1.0×10^{11}	S.M. [18]
		0.309	0.43	4.5×10^{11}	S.M. [20]
	$5/2^+,0\rightarrow 3/2^+,0$	1.2×10^{-5}	1.05	2.5×10^{8}	S.M. [20]
	$7/2^+,0\rightarrow 5/2^+,0$	0.299	0.81	2.8×10^{12}	Present
		0.365	0.93	5.2×10^{12}	$\alpha+d+\Lambda$ [2]
		0.352	0.40	4.0×10^{11}	S.M. [20]

(b)

$J_i,T_i \rightarrow J_j,T_f$	$B(M1)$ (μ_N^2)	E_γ (MeV)	$T(M1)$ (sec^{-1})
$1/2^+,1\rightarrow 1/2^+,0$	4.41	4.28	6.1×10^{15}
$\rightarrow 3/2^+,0$	8.83	3.42	6.2×10^{15}
$3/2^+,1\rightarrow 1/2^+,0$	0.15	5.91	5.4×10^{15}
$\rightarrow 3/2^+,0$	0.03	5.05	7.5×10^{13}
$\rightarrow 5/2^+,0$	3.90	3.72	3.6×10^{15}
$5/2^+,1\rightarrow 3/2^+,0$	0.17	5.38	4.7×10^{14}
$\rightarrow 5/2^+,0$	0.20	4.05	2.4×10^{14}
$\rightarrow 7/2^+,0$	4.15	3.28	2.6×10^{15}
$\rightarrow 3/2^+,1$	0.30	0.33	1.9×10^{11}

(np) spin and the Λ-spin flip are both necessary in this transition. Also, $B(M1)$ values of $5/2^+(T=1)\rightarrow 3/2^+(T=0)$ and $3/2^+(T=1)\rightarrow 1/2^+,3/2^+(T=0)$ are small. This is simply because the dominant components of the wave functions require the change of the total orbital angular momentum from $I=2$ to $I=0$.

It is well known that Λ spin-doublet states in ${}^{7}_{\Lambda}$Li provide us with very useful information on the spin-spin component of ΛN interaction. The calculated $B(M1)$ values for the transitions $7/2^+(T=0)\rightarrow 5/2^+(T=0)$ and $3/2^+(T=0)\rightarrow 1/2^+(T=0)$ in ${}^{7}_{\Lambda}$Li turn out to be almost the same as those given by the $\alpha+d+\Lambda$ cluster model [2] and by the shell model [18]. In ${}^{7}_{\Lambda}$He, $B(M1;\ 5/2^+(T=1)\rightarrow 3/2^+(T=1))$ based on the present model is nearly the same as that with the shell model [18]. This is because the Λ spin flip is essential, and the radial overlap between the initial- and final-state wave functions within each model is similar in spite of the fact that the shapes of these model wave functions are different from each other.

It is interesting to remark that the $5/2^+(T=0)\rightarrow 3/2^+(T=0)$ $M1$ transition in ${}^{7}_{\Lambda}$Li should be very small because the $M1$ matrix element connecting the dominant components of the states is both L forbidden and isoscalar. However, the $B(M1)$ values of this type of transition have not been estimated with the present wave functions, since the estimate of very small transition rates should be calculated more reliably with the use of an $\alpha+N+N+\Lambda$ four-body model which provides an extended model space [9]. In order to get a simple idea for the order of magnitude, we list instead the shell-model value; the estimate [20] gives $B(M1)=1.22\times10^{-5}\mu_N^2$ which corresponds to $T(M1)=5.1\times10^{8}$ s^{-1} if our $E_\gamma=1.33$ MeV is assumed. This rate is one order of magnitude smaller than $T(E2:\ 5/2^+\rightarrow 3/2^+)$ in Table I. The $\alpha+d+\Lambda$ cluster model [2] leads to a far smaller value (we will make the estimate within the $\alpha+N+N+\Lambda$ model in a forthcoming paper).

It is also interesting to see another L-forbidden $M1$ transition for $3/2^+$ going to $1/2^+$ in ${}^{7}_{\Lambda}$He. Again the shell-model value is very small: $B(M1)=1.5\times10^{-5}\mu_N^2$, which corresponds to $T(M1)=1.3\times10^{9}$ s^{-1}. As the $E2$ transition probability $T(E2;\ 3/2+\rightarrow 1/2^+)=1.0\times10^{9}$ s^{-1} is similarly small [cf. Table I(b)], the decay of the $3/2^+$ in ${}^{7}_{\Lambda}$He occurs mostly via the weak decay channel and the electromagnetic decay should be the minor channel. This is one of the characteristic features of the hypernuclear decays involving neutron-halo structure together with L-forbidden $M1\gamma$ transition.

C. $E2$ transitions in ${}^{7}_{\Lambda}$Li ($T=0$)

In Table III(a), calculated $B(E2)$ strengths in ${}^{7}_{\Lambda}$Li are listed together with those given by the $\alpha+d+\Lambda$ cluster

TABLE III. Calculated $B(E2)$ values for (a) ${}^{7}_{\Lambda}$Li and (b) ^{6}Li in units of e^2 fm^4 together with the experimental value for ^{6}Li.

(a) ${}^{7}_{\Lambda}$Li			
Transition	Present	Ref. [2]	Ref. [18]
$5/2^+ \to 1/2^+$	2.42	2.46	8.6
$\to 3/2^+$	0.74	0.40	3.1
$7/2^+ \to 3/2^+$	3.69	3.04	—

(b) ^{6}Li			
Transition	Present	Ref. [2]	Exp. [19]
$3^+ \to 1^+$	9.62	6.6	9.3 ± 2.1

model [2] and by the shell model [18]. The $B(E2)$ value for the corresponding transition $3^+ \to 1^+$ in the core nucleus ^{6}Li is shown in Table III(b) together with the experimental value [19]. In the calculations of the present model and the $\alpha + d + \Lambda$ model, no additional effective charge is assumed. The $B(E2)$ values of ${}^{7}_{\Lambda}$Li from the shell-model calculation [18] are much larger than those given by the two types of cluster-model calculations. This is due to the *a priori* assumption [18] that no dynamical change of the hypernuclear size occurs and also that the sum of the two transitions from the $5/2^+$ state is normalized to the observed $B(E2)$ value (at that time) of the core-nucleus transition $3^+ \to 1^+$.

On the other hand, the results from the two types of cluster model exhibit a strong reduction of the $B(E2)$ strength due to the shrinkage of the nuclear size in ${}^{7}_{\Lambda}$Li by the glue-like role of the Λ particle. The dynamical change in structure is naturally taken into account in these cluster models. In Table IV, which lists the ratio $B(E2;\ {}^{7}_{\Lambda}\mathrm{Li})/B(E2;\ {}^{6}\mathrm{Li})$, one sees how large the shrinkage effect is. The present calculation updates the reduction of $B(E2)$ obtained previously by the $\alpha + d + \Lambda$ model [2]. The reason for a little underestimate in the previous calculation is that the basis functions adopted in Ref. [2] are not quite adequate to describe the spatial extension of the 3^+ state wave function of ^{6}Li and therefore the calculated $B(E2;\ 3^+ \to 1^+)$ value is smaller by $\sim 30\%$ in comparison with the observed value. It is to be stressed that the present $\alpha + N + N$ model including sufficiently long-ranged basis functions reproduces the observed value.

TABLE IV. Ratio of the $B(E2)$ strength in ${}^{7}_{\Lambda}$Li to the corresponding $B(E2; 3^+ \to 1^+)$ in the core nucleus ^{6}Li and the ratio of the core-(np) mean distances.

Reduction factor	Present	Ref. [2]	Ref. [18]
$\dfrac{B(E2;\ 5/2^+ \to 1/2^+,\ 3/2^+)}{B(E2;\ 3^+ \to 1^+)}$	0.33	0.44	1.0 (assumed)
$\dfrac{B(E2;\ 7/2^+ \to 3/2^+)}{B(E2;\ 3^+ \to 1^+)}$	0.38	0.46	-
$\Gamma_{\rm B} \equiv \dfrac{B(E2;5/2^+ \to 1/2^+)}{\frac{7}{9} B(E2;\ 3^+ \to 1^+)}$	0.32	0.49	-
$\dfrac{\overline{R}_{c-d}({}^{7}_{\Lambda}\mathrm{Li})}{\overline{R}_{\alpha-d}({}^{6}\mathrm{Li})}$	0.75 [a]	0.83	-

[a]This value corresponds to $\overline{R}_{c-d}({}^{7}_{\Lambda}\mathrm{Li})^{\rm cal} = 2.94$ fm and $\overline{R}_{\alpha-d}({}^{6}\mathrm{Li}) = 3.95$ fm.

IV. POSSIBLE DERIVATION OF HYPERNUCLEAR SIZE FROM $B(E2)$ STRENGTH

We now discuss in more detail the shrinkage of the nuclear size which is reflected in the reduction of the $B(E2)$ strengths estimated here. Since the $5/2^+ \to 1/2^+$ transition is the most probable of the three $E2$ transitions in the ground-state band of ${}^{7}_{\Lambda}$Li to be measured experimentally [4], we consider only this case in the following.

In order to see how the $E2(5/2^+ \to 1/2^+)$ transition probability in ${}^{7}_{\Lambda}$Li is reduced with respect to the $E2(3^+ \to 1^+)$ one in ^{6}Li, and also to see how its reduction is related to the change of size, we define the following ratio:

$$\Gamma_B \equiv \frac{B(E2;5/2^+ \to 1/2^+)}{\frac{7}{9} B(E2;3^+ \to 1^+)}. \tag{4.1}$$

Here the factor of $\frac{7}{9}$ in the denominator is introduced to take into account the branching relation that the $B(E2)$ value of the $3^+ \to 1^+$ "core transition" is shared as $\frac{7}{9} B(E2;5/2^+ \to 1/2^+) + \frac{2}{9} B(\mathrm{E}2;5/2^+ \to 3/2^+)$ if the coupling of Λ particle's spin ($S_\Lambda = 1/2$) works only kinematically with no dynamical effect; the branching relation between the hypernuclear transition $J_i = J'_c \pm S_\Lambda \to J_f = J_c \pm S_\Lambda$ and the "core transition" $J'_c \to J_c$ is described by [2]

$$B(E2;J_i \to J_f) = (2J_f + 1)(2J'_c + 1) \times W(J_c S_\Lambda 2 J_i;\ J_f J'_c)^2 \cdot B(E2;\ J'_c \to J_c)_H, \tag{4.2}$$

where the subscripts c and H emphasize the "core transition" in the hypernucleus (H). Then Eq. (4.1) is expressed alternatively as

$$\Gamma_B = \frac{B(E2;3_c^+ \to 1_c^+)_H}{B(E2;3^+ \to 1^+)}. \tag{4.3}$$

We can obtain guidance to the relationship between this Γ_B value and the ratio of hypernuclear and nuclear sizes by considering a simple minded $\alpha + d$ cluster model for ^{6}Li in which the deuteron is tentatively assumed to have no internal structure except the spin $S_d = 1$. Denoting the $\alpha - d$ relative angular momentum as $l_{\alpha-d}$, the 3^+ and 1^+ states of ^{6}Li have the stretched angular momentum coupling $[l_{\alpha-d} \otimes S_d]_{J=3^+,1^+}$ with $l_{\alpha-d} = 2$ and 0, respectively. Therefore the $3^+ \to 1^+ E2$ transition probability can be related to the "rotational deexcitation" corresponding to the $l_{\alpha-d} = 2^+ \to 0^+$ transition as far as the angular momentum is concerned. By applying also the relation of Eq. (4.2) with $S_d = 1$ instead of $S_\Lambda = 1/2$, we obtain $B(E2;\ 3^+ \to 1^+) = B(E2;\ l_{\alpha-d} = 2^+ \to 0^+)$. If one simply assumes the $\alpha + d$ dicluster system to be a rigid rotor for which the deformation is represented by the intrinsic quadrupole moment

Q_0, then one gets $B(E2;\ l_{\alpha-d}=2^+\rightarrow 0^+)=(1/16\pi)e^2Q_0^2$. By inserting these two relations into Eq. (4.3), the ratio Γ_B can be reexpressed within this approximation as

$$\Gamma_B=\frac{Q_0(c-d,\ {}^7_\Lambda\mathrm{Li})^2}{Q_0(\alpha-d,\ {}^6\mathrm{Li})^2}, \tag{4.4}$$

where c denotes ${}^5_\Lambda$He, with the structure of ${}^7_\Lambda$Li similarly accounted for by a ${}^5_\Lambda\mathrm{He}+d$ dicluster. Because the intrinsic quadrupole moment $Q_0(\alpha-d,\ {}^6\mathrm{Li})$ is proportional to the square of the $\alpha-d$ mean distance $\overline{R}_{\alpha-d}$ and similarly $Q_0(c-d,\ {}^7_\Lambda\mathrm{Li})$ to the square of $\overline{R}_{c-d}$, we finally obtain the approximate relation $\Gamma_B=\overline{R}^4_{c-d}/\overline{R}^4_{\alpha-d}$ or

$$\frac{\overline{R}_{c-d}({}^7_\Lambda\mathrm{Li})}{\overline{R}_{\alpha-d}({}^6\mathrm{Li})}=\left[\frac{B(E2;\ 5/2^+\rightarrow 1/2^+)}{\frac{7}{9}B(E2;\ 3^+\rightarrow 1^+)}\right]^{1/4}. \tag{4.5}$$

In the present paper, we do not employ the rigid deuteron model but allow the free motion of n and p with respect to the core (α or $c={}^5_\Lambda$He). In the following, the relation of Eq. (4.5) obtained above in an approximate way is not assumed *a priori* but it should be tested with the wave functions calculated within the sufficiently large model space and wider degrees of freedom. If the addition of the Λ particle does not change the internal motion of the $n-p$ pair along $\mathbf{r}_{n-p}$ (namely, $\mathbf{r}_3$ in Fig. 1) but contracts only the core$-(np)$ relative motion along $\mathbf{R}_{\mathrm{core}-(np)}$ (namely, $\mathbf{R}_3$ in Fig. 1), then the expectation value of the angle part of the $E2$ operator, $Y_2(\hat{\mathbf{R}}_{\mathrm{core}-(np)})$, is not affected by the contraction of the core$-(np)$ distance. In this case, we can safely assume the $B(E2)$ value to be propotional to the fourth power of $R_{\mathrm{core}-(np)}$ and use Eq. (4.5).

First, we examine the consistency of Eq. (4.5) by calculating the left-hand side (LHS) and (RHS) separately with the wave functions obtained with the preceding $\alpha+d+\Lambda$ model [2]. Then we find that the RHS of Eq. (4.5) is estimated to be $[2.46/(\frac{7}{9}\times 6.6)]^{1/4}=0.83$, which is revealed to be very close to the LHS value of (3.13 fm)/(3.80 fm) $=0.82$. This consistency in calculation guarantees the validity of the use of Eq. (4.5) in the following.

As mentioned before, the observed $B(E2;\ 3^+\rightarrow 1^+)$ strength is underestimated in Ref. [2], but is reproduced by the present work. In the present treatment with the $\alpha+n+p$ model (^{6}Li) and ${}^5_\Lambda\mathrm{He}+n+p$ model (${}^7_\Lambda$Li), a ''deuteron cluster'' is not assumed for the valence neutron and proton, since the deuteron-cluster approximation turned out to be broken by $\sim$40% in ^{6}Li and ${}^7_\Lambda$Li [3]. In other words, here we have all three-body degrees of freedom in our wave functions on an equal footing. We first examine whether the shrinkage of ${}^7_\Lambda$Li occurs along the $n-p$ relative distance $\mathbf{r}_{n-p}$ or along the distance between the core and the c.m. of (np) pair, $\mathbf{R}_{\mathrm{core}-(np)}$. We introduce the $n-p$ relative density $\rho(r_{n-p})$ which is given by integrating the three-body density over $\mathbf{R}_{\mathrm{core}-(np)}$ and the angle $\hat{\mathbf{r}}_{n-p}$:

$$\rho(r_{n-p})=\int|\Psi({}^6\mathrm{Li}\ \mathrm{or}\ {}^7_\Lambda\mathrm{Li,g.s.})|^2 d\mathbf{R}_{\mathrm{core}-(np)}d\hat{\mathbf{r}}_{n-p}/4\pi. \tag{4.6}$$

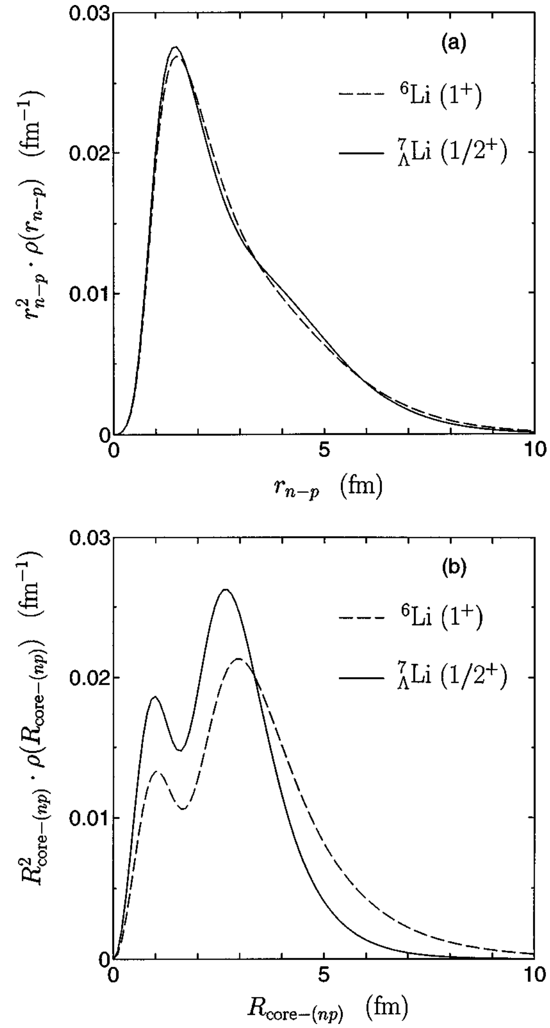

FIG. 4. (a) The $n-p$ relative density distribution $\rho(r_{n-p})$ defined by Eq. (4.6) multiplied by r^2_{n-p}. (b) The (np) pair c.m. density distribution $\rho(R_{\mathrm{core}-(np)})$ defined by Eq. (4.7) multiplied by $R^2_{\mathrm{core}-(np)}$. Both are for the ground states of ^{6}Li and ${}^7_\Lambda$Li.

As shown in Fig. 4(a), the $n-p$ relative density $\rho(r_{n-p})$ exhibits almost the same shape for the ground state of ^{6}Li and that of ${}^7_\Lambda$Li; namely, the shrinkage of the $n-p$ distance due to the Λ participation is found to be negligibly small.

On the other hand, in order to see the degree of shrinkage in the motion of the c.m. of the (np) pair with respect to the α (${}^5_\Lambda$He) core, we introduce the (np) c.m. density as a function of $R_{\mathrm{core}-(np)}$ by

$$\hat{\rho}(R_{\mathrm{core}-(np)})=\int|\Psi({}^6\mathrm{Li}\ \mathrm{or}\ {}^7_\Lambda\mathrm{Li,g.s.})|^2 \times d\mathbf{r}_{n-p}d\hat{\mathbf{R}}_{\mathrm{core}-(np)}/4\pi. \tag{4.7}$$

As illustrated in Fig. 4(b), this density distribution $\hat{\rho}(R_{\mathrm{core}-(np)})$ of ${}^7_\Lambda$Li is remarkably different from that of ^{6}Li, showing a significant contraction along the $\mathbf{R}_{\mathrm{core}-(np)}$ coordinate due to the Λ addition. In fact, the rms distance $\overline{R}_{\mathrm{core}-(np)}$ is estimated as 2.94 fm for ${}^7_\Lambda$Li ($1/2^+$) vs 3.85 fm for ^{6}Li (1^+).

We then conclude that, by the addition of the Λ particle to ^{6}Li (1^+), contraction of ${}^7_\Lambda$Li occurs between the c.m. of the

(np) pair and the core whereas the $n-p$ relative motion remains almost unchanged. It is interesting to point out that, in spite of considerable breaking of the ''deuteron cluster,'' the present result assures the validity of Eq. (4.5) as in the case of $\alpha+d+\Lambda$ model. Now we again make a test of Eq. (4.5) based on our model. Using the present wave functions, we find that the LHS of Eq. (4.5) is (2.94 fm)/(3.85 fm) $=0.76$, while the RHS equals to $[2.42/(\frac{7}{9}\times 9.62)]^{1/4}=0.75$. This consistency again demonstrates the usefulness of Eq. (4.5).

Therefore, we consider that, if the observation of $B(E2;\ 5/2^+\to 1/2^+)$ is realized in the near future, a reasonable estimation of the size of the ground state of the hypernucleus ${}^{7}_{\Lambda}$Li will be possible by means of Eq. (4.5) using the experimental value of the RHS and that of $\overline{R}_{\alpha-(np)}(^6\mathrm{Li})$ in the LHS denominator. The experimental value of $\overline{R}_{\mathrm{core}-(np)}(^6\mathrm{Li})$ may be estimated by the often-used relation

$$6\overline{r^2}(^6\mathrm{Li})=4\overline{r^2}(\alpha)+2(\overline{r}_{n-p}/2)^2+\frac{4\cdot 2}{4+2}\overline{R}^2_{\mathrm{core}-(np)}(^6\mathrm{Li}), \tag{4.8}$$

where $\overline{r}(^6\mathrm{Li})=2.44$ fm and $\overline{r}(\alpha)=1.50$ fm are the observed rms radii of the mass distributions of ^{6}Li and the α particle, respectively [21]. Here, $\overline{r}_{n-p}$ is the r.m.s. distance between the valence neutron and proton in the ground state of ^{6}Li and is estimated as $\overline{r}_{n-p}=3.42$ fm [3]. From these we have $\overline{R}_{\mathrm{core}-(np)}(^6\mathrm{Li})=3.95$ fm which is close to the our theoretical value of 3.85 fm based on the wave functions. Now we come to an expression to estimate an empirical value for $\overline{R}_{\mathrm{core}-(np)}({}^{7}_{\Lambda}\mathrm{Li})$ from the $B(E2)$:

$$\overline{R}_{\mathrm{core}-(np)}({}^{7}_{\Lambda}\mathrm{Li})=2.4[B(E2,\ 5/2^+\to 1/2^+)/e^2\ \mathrm{fm}^4]^{(1/4)}\ \mathrm{fm}. \tag{4.9}$$

The factor of 2.4 is derived by using Eq. (4.5), namely, $\overline{r}_{\mathrm{core}-(np)}(^6\mathrm{Li})/[\frac{7}{9}B(\mathrm{E2};\ 3^+\to 1^+)]^{1/4}=3.95/[\frac{7}{9}\times 9.3]^{1/4}=2.4$, which is consistent with our theoretical value, $3.85/[\frac{7}{9}\times 9.62]^{1/4}=2.33$. We consider the relative error of this expression to be within a few percent, whereas the expected reduction in $\overline{R}_{\mathrm{core}-(np)}$ due to the Λ participation will be some 25%. We note again that our prediction is $\overline{R}_{\mathrm{core}-(np)}({}^{7}_{\Lambda}\mathrm{Li})=2.94$ fm.

The $B(E2;\ 5/2^+\to 1/2^+)$ strength is expected to be measured by the E419 experiment in progress at KEK-PS [4] with Ge detectors having a few keV resolution. In order to obtain the $B(E2)$, they plan to measure the lifetime of the ${}^{7}_{\Lambda}\mathrm{Li}(5/2^+)$ state using the Doppler-shift attenuation method. It is to be noted that the state has another $E2$-decay branch to the $3/2^+$ state. As seen in Table I(a) from the partial transition probabilities of $T(E2;5/2^+\to 1/2^+)=1.5\times 10^{11}\mathrm{sec}^{-1}$ and $T(E2;5/2^+\to 3/2^+)=3.8\times 10^9\ \mathrm{sec}^{-1}$, the effect of the $E2$ decay to the $3/2^+$ state on the lifetime can be neglected within an error of a few percent. It was already mentioned in the preceding section that the $M1$ decay to the $3/2^+$ state was estimated [20] to be an order of magnitude smaller than the $E2$ rate. Therefore, the lifetime of the $5/2^+$ state directly provides us with the $B(E2;\ 5/2^+\to 1/2^+)$ value and hence the size of ${}^{7}_{\Lambda}$Li can be derived along the prescription proposed above.

V. SUMMARY

We have calculated the strengths of γ transitions in ${}^{7}_{\Lambda}$Li and ${}^{7}_{\Lambda}$He on the basis of the ${}^{5}_{\Lambda}\mathrm{He}+N+N$ three-body model which was used in our preceding work on the energy spectra and halo structure of the isotriplet $A=7$ hypernuclei. Most of the transition calculations are essentially new on account of the wide applicability of the present model. Major points to be emphasized are as follows.

(1) The calculation makes firm that the $B(E2;\ 5/2^+\to 1/2^+)$ value of the ground-state band in ${}^{7}_{\Lambda}$Li is remarkably reduced in comparison with $B(E2;\ 3^+\to 1^+)$ of the corresponding transition in the core nucleus ^{6}Li. This is due to the nuclear size contraction by the gluelike role of the Λ particle [2], and the shrinkage is found to occur along the distance between the (np) pair and the α core with the $n-p$ internal motion unchanged.

(2) We encourage a measurement of this $5/2^+\to 1/2^+ E2$ transition rate to confirm the size contraction experimentally. This measurement is already underway at KEK (E419). We further proposed a prescription for how to derive the size of the ground state of the hypernucleus ${}^{7}_{\Lambda}$Li using the empirical value of $B(E2;\ 5/2^+\to 1/2^+)$ together with that of $B(E2;\ 3^+\to 1^+)$ in ^{6}Li and the size of the ground state of ^{6}Li; see Eq. (4.5). We have examined the consistency of Eq. (4.5) by evaluating the ratios on both sides using the three-body wave functions obtained for ^{6}Li and ${}^{7}_{\Lambda}$Li.

(3) In addition to the $B(E2)$ of ${}^{7}_{\Lambda}$Li $(T=0)$, a number of $E2$ and $M1$ transitions in ${}^{7}_{\Lambda}$Li $(T=1)$ and ${}^{7}_{\Lambda}$He have been estimated. Remarkably enhanced $B(E2)$ values are predicted for the decay from the $5/2^+(T=1)$ and $3/2^+(T=1)$ states in ${}^{7}_{\Lambda}$Li; the proton-halo part in the states dominates in this enhancement. The $E2$ transitions from the neutron-halo states in ${}^{7}_{\Lambda}$He are found to be rather weak but still within a measurable range. Therefore, observation of these $E2$ transitions in ${}^{7}_{\Lambda}\mathrm{Li}(T=1)$ and ${}^{7}_{\Lambda}$He, as well as the $M1$ transitions, would give helpful information not only on the hypernuclear structure but also on the excitation mechanism of the neutron and proton halos of the core nuclei. We emphasize that the present examples demonstrate the importance of hypernuclear γ decay measurements, since in ordinary nuclei it is often impossible to observe γ decays from the halo excited states (the 2^+ state in the present case) because in general the prompt nucleon emission prevails.

Finally, we note that ${}^{7}_{\Lambda}$Li and ${}^{7}_{\Lambda}$He hypernuclei are important systems to obtain information on the spin-spin component of the ΛN interaction through the energy splitting of the spin-doublet states. In this respect, it is highly desirable to perform a more extended study of the $A=7$ hypernuclei on the basis of an $\alpha+N+N+\Lambda$ four-body model with more realistic ΛN interactions. This calculation is in progress, and a preliminary result is reported in Ref. [9] together with a similar study of ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He based on a $3N+\Lambda$ model.

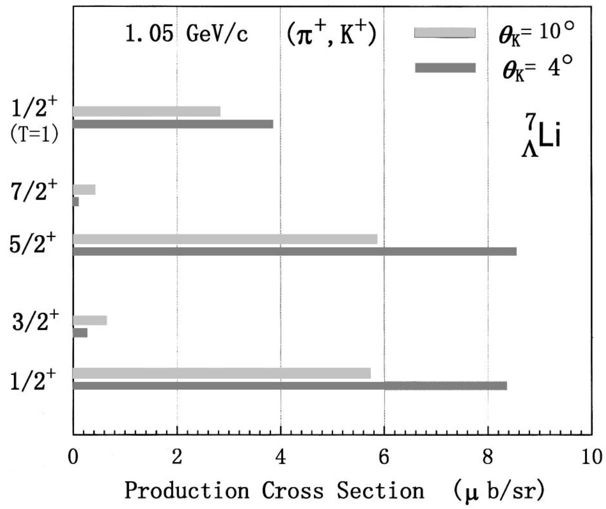

FIG. 5. Differential cross sections calculated in the DWIA for the $^7\mathrm{Li}(\pi^+,K^+)^7_\Lambda\mathrm{Li}$ reaction at $p_\pi = 1.05$ GeV/c and the laboratory scattering angles $\theta_K = 4°$ and $10°$.

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APPENDIX: (π^+,K^+) FORMATION RATES AND RELATIVE γ-RAY YIELDS

In this appendix we show in Fig. 5 the calculated cross sections for producing low-lying hypernuclear states in the $^7\mathrm{Li}(\pi^+,K^+)^7_\Lambda\mathrm{Li}$ reaction at $p_\pi = 1.05$ GeV/c and typical K^+ scattering angles. We confine ourselves to the five low-lying states which are connected by γ cascades as discussed in the text. Here, for simplicity, shell-model wave functions generated with the modified Nijmegen model D are employed and the cross sections are calculated in the DWIA framework described in Ref. [22]. The numbers of particular γ quanta to be observed in a $(\pi^+,K^+\gamma)$ coincidence measurement are related to the population probabilities of the states relevant to the γ decays. They are also restricted, for example, by actual experimental setup such as acceptances of energy and detection angle of the outgoing K^+.

As shown in Fig. 5, the $J=1/2^+_{\mathrm{g.s.}}(T=0)$ and $5/2^+(T=0)$ states in $^7_\Lambda$Li are equally strongly excited and the $1/2^+(T=1)$ state is also pronouncedly excited with (45–50)% strength of the former states. The selective excitation is atributed to the fact that all three wave functions have a dominant spin-1/2 character, like the target wave function of $^7\mathrm{Li}(J_i=3/2^-)$. On the other hand, the $3/2^+$ and $7/2^+$ states, both of which have dominant spin-3/2 components in their wave functions, are only weakly excited through the minor spin-flip component of the (π^+,K^+) reaction operator. The role of the spin-flip component increases gradually as the K^+ scattering angle increases. However the relative formation rate for the $3/2^+$ state remains as small as 3% ($\theta_K = 4°$)–11% (10°) with respect to the $5/2^+$ formation rate, and the $7/2^+$ formation rate is only 1.2% ($\theta_K = 4°$) to 7% (10°).

If the K^+ detection is performed over some range of scattering angle in the experiment, then the integrated cross sections for those states should be relevant to the relative formation rates. As an example for such a case, we list the cross sections integrated over $\theta_K = 0-15°$ (in units of μb):

$$1.21(1/2^+_{\mathrm{g.s.}}),\quad 0.13(3/2^+),\quad 1.23(5/2^+),$$

$$0.08(7/2^+),\quad 0.60(1/2^+_{T=1}).$$

These values seem consistent with the analysis of the KEK experiment [23] as far as the relative formation rates for the three pronounced peaks are concerned.

On the other hand, according to the calculated $E2$ and $M1$ transition probabilities listed in Tables I and II, the γ-decay branching ratios concerned here are summarized as

$$1/2^+_{T=1}\ \text{decay:}\ 50.4\%-3/2^+(M1)\ \text{and}\ 49.6\%-1/2^+_{\mathrm{g.s.}}(M1);$$

$$7/2^+\ \text{decay:}\ 93.3\%-5/2^+(M1)\ \text{and}\ 6.7\%-3/2^+(E2);$$

$$5/2^+\ \text{decay:}\ 2.5\%-3/2^+(E2)\ \text{and}\ 97.5\%-1/2^+_{\mathrm{g.s.}}(E2);$$

$$3/2^+\ \text{decay:}\ 100\%-1/2^+_{\mathrm{g.s.}}(M1).$$

By combining the integrated cross sections with these γ-decay branching ratios, we obtain the modified formation rates of the low-lying states. Then the relative γ-ray yields for the interesting three transitions are finally obtained as[1] (in arbitrary units)

$$\Gamma(E2;5/2^+\to 1/2^+_{\mathrm{g.s.}}):\ \Gamma(M1;3/2^+\to 1/2^+_{\mathrm{g.s.}}):$$

$$\Gamma(M1;\ 7/2^+\to 5/2^+) = 1.27{:}0.47{:}0.07\quad (1.45{:}0.52{:}0.23).$$

In parentheses, we also list another prediction obtained under the assumption that the higher $3/2^+(T=1)$ and $5/2^+(T=1)$ states also have influence on the lower-state populations through γ cascades. In both cases the yield of the $E2(5/2^+ \to 1/2^+_{\mathrm{g.s.}})$ transition is about 3 times larger than that of the

[1]According to the recent preliminary report of E419 by Tamura [24], the theoretical prediction that the first γ-ray yield is about 3 times larger than the second one (1.27 : 0.47) is in good agreement with the experiment, as discussed in [25]. This means that the $1/2^+(T=1)$ level surely γ decays even if the state is above the $^5_\Lambda\mathrm{He}+d$ threshold, since the theory indicates the contribution of 0.30 out of 0.47 comes from the γ decay of the $M1$ cascade: $1/2^+(T=1)\to 3/2^+(T=0)$.

$M1(3/2^+ \rightarrow 1/2^+_{\mathrm{g.s.}})$ transition. It will be quite interesting to compare this theoretical yield ratio with the experimental count rates of the corresponding γ quanta.

For further comparison with the experiment for ${}^{7}_{\Lambda}$Li, it is also remarked on the basis of Tables I and II that the theoretical lifetime of the $5/2^+$ state is much longer than the $3/2^+$ state: $\tau(5/2^+)=6.67$ ps (or 9.73 ps and 9.57 ps [2] if one uses $E_\gamma^{\mathrm{expt}}=2.034$ MeV [5]), while $\tau(3/2^+)=0.28$ ps (0.12 ps [2]). On the other hand, just for reference, we express the nuclear core transition rate in the form of ''lifetime'' as $\tau(3^+;\ {}^6\mathrm{Li})^{\mathrm{expt}}=1.77^{+0.51}_{-0.68}$ ps which is deduced from $B(E2;\ 3^+ \rightarrow 1^+)=9.3\pm2.1e^2\ \mathrm{fm}^4$.

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