Gauge invariance in quantum hadrodynamics

Gary Prézeau
Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795
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This paper describes the derivation of a new representation of an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ locally invariant model of the strong interactions called quantum hadrodynamics 3 (QHD-III). QHD-III is a gauge-invariant theory based on the linear σ model, the gauge bosons being the $\boldsymbol{\rho}$ and the $\boldsymbol{a}_1$. The new representation considerably simplifies the Lagrangian. To derive the new representation, the gauge invariance of the model is exploited. The role of the gauge boson masses in gauge-invariant models of the strong interactions based on the σ model is also discussed, and it is shown that these masses are necessary if the pion is to survive as a physical field. [S0556-2813(99)04104-7]

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Hadronic models have had significant phenomenological success in describing the many-body strongly interacting system at low energies [1]. These models take hadrons as their effective degrees of freedom. The Lagrangians of hadronic models are constructed in such a way that they reflect the symmetries of QCD, while incorporating low-energy phenomenology. In particular, these models must give rise to conserved vector and partially conserved axial-vector currents, and they should include the exchange of mesons which are known to carry the strong force at low energies. The mesons should be introduced such that corrections coming from meson loops are consistently calculable within the model. Since there is a large number of mesons, any model based on meson exchange must find a consistent way to choose the relevant mesons; this can be achieved by introducing them as gauge bosons.

Briefly, quantum hydrodynamics 3 (QHD-III) [2] is a gauge-invariant hadronic quantum field theory based on the gauged σ-ω model with pions. The σ-ω model is built from the linear σ model with the ω introduced as a massive U(1) gauge boson (see below). When the global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry of the σ-ω model is made local and parity conservation is imposed, the $\boldsymbol{\rho}$ and its chiral partner, the $\boldsymbol{a}_1$, appear as the gauge bosons. The $\boldsymbol{\rho}$ and the $\boldsymbol{a}_1$ are made massive by the inclusion of a Higgs sector composed of two complex doublets: one transforming under $\mathrm{SU}(2)_L$ and the other under $\mathrm{SU}(2)_R$; the doublets couple to the gauge bosons through their covariant derivatives. This procedure for giving mass to the $\boldsymbol{\rho}$ and the $\boldsymbol{a}_1$ is very similar to the one used to give mass to weak bosons in the standard model, and preserves the gauge invariance of the model. Keeping the gauge invariance allows the unambiguous derivation of the strong conserved currents. A small symmetry-breaking term is included to yield massive pions. The resulting Lagrangian has a minimal number of massive mesons which couple to conserved vector and axial-vector currents; in this model, the $\boldsymbol{a}_1$ naturally comes out heavier than the $\boldsymbol{\rho}$. The Lagrangian is also renormalizable [2].

As shown below, the physical pion disappears in an hadronic model based on the gauged σ model.[1] To retain the pion, the gauge bosons must be given mass. In contrast to QHD-III, the usual procedure [4–7] is to put in the same mass m_ρ by hand for both the $\boldsymbol{\rho}$ and the $\boldsymbol{a}_1$. The spontaneous symmetry breaking (SSB) that occurs in the σ sector provides an extra contribution to the mass of the $\boldsymbol{a}_1$, making the $\boldsymbol{a}_1$ mass m_a larger than the $\boldsymbol{\rho}$ mass m_ρ. However, introducing mass terms for the gauge bosons by hand violates current conservation, and loops can no longer be calculated unambiguously. Furthermore, because of the SSB in the σ sector, the $\boldsymbol{a}_1$ and the gradient of the pion mix and the resulting Lagrangian must be diagonalized. The diagonalization of the lagrangian is carried out by making a change of variables involving the $\boldsymbol{a}_1$ and the gradient of the pion. The final Lagrangian is complicated because of the introduction of momentum-dependent vertices due to the gradient of the pion.

In a gauge-invariant quantum field theory such as QHD-III [2], one can make a gauge transformation to diagonalize the Lagrangian instead of making the above-mentioned change of variables. The result is a considerably simpler Lagrangian where no new momentum-dependent vertices appear. In particular, to $\mathcal{O}(g_\rho^2)$, the diagonalization of the Lagrangian is equivalent to the rescaling of the pion field by the ratio m_ρ/m_a. This work details the derivation of this new representation of the QHD-III Lagrangian.

This paper begins by discussing the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ locally invariant σ-ω model with pions. The gauge bosons, the $\boldsymbol{a}_1$ and the $\boldsymbol{\rho}$, are originally massless. It is shown that SSB gives a mass to the $\boldsymbol{a}_1$. Working in an arbitrary ξ gauge, it is shown that the field originally identified with the pion acquires a ξ-dependent mass, which identifies it as a *fictitious particle*; what looks like a pion is labeled as $\boldsymbol{\pi}'$ to distinguish it from the physical pion. By looking at nucleon-nucleon scattering, it is shown that the $\boldsymbol{\pi}'$ exchange diagram is always *canceled* by the ξ-dependent part of the $\boldsymbol{a}_1$ exchange diagram. The disappearance of the real pion is forced by gauge invariance and demonstrates the need for massive gauge bosons with mass provided from outside this sector of the theory.

QHD-III is then reviewed. In this locally gauge-invariant model, pions appear as physical Goldstone bosons. Here, the vector meson masses are generated from a Higgs sector, as in the σ model. Gauge invariance is used to diagonalize the Lagrangian in an arbitrary ξ gauge so as to avoid the origi-

[1] This point has also been previously independently noted in [3].

nal, momentum-dependent, diagonalization procedure used in [2]. This new diagonalization scheme produces a considerably simpler representation of QHD-III than the representation given in [2]. It is shown how, to $\mathcal{O}(g_\rho^2)$, the new diagonalization procedure is equivalent to simply rescaling the pion field by the ratio m_ρ/m_a.

To demonstrate the need for massive vector mesons, consider the σ-ω model with pions (QHD-II) [2]:

$$\mathcal{L}_{\sigma\text{-}\omega}=\bar{\psi}[i\gamma^\mu(\partial_\mu+ig_vV_\mu)-g_\pi(s+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]\psi+\frac{1}{2}(\partial_\mu s\partial^\mu s+\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})-\frac{1}{4}\lambda(s^2+\boldsymbol{\pi}^2-v^2)^2$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_v^2V_\mu V^\mu+\epsilon s. \tag{1}$$

In Eq. (1), V^μ represents the ω field, and ϵs is the chiral-symmetry-violating term that gives a mass to the pion. The global $SU(2)_L\times SU(2)_R$ symmetry of $\mathcal{L}_{\sigma\text{-}\omega}$ is now made local, and the scalar field is given a vacuum expectation value ($s=\sigma_0-\sigma$ with $\sigma_0\equiv M/g_\pi$). This yields for the Lagrangian $\mathcal{L}_g$ of the gauged σ-ω model with pions:

$$\mathcal{L}_g=\bar{\psi}\left\{i\gamma^\mu\left[\partial_\mu+ig_vV_\mu+\frac{i}{2}g_\rho\boldsymbol{\tau}\cdot(\boldsymbol{\rho}_\mu+\gamma_5\boldsymbol{a}_\mu)\right]-(M-g_\pi\sigma)-ig_\pi\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}'\right\}\psi+\frac{1}{2}[(\partial_\mu\boldsymbol{\pi}'+g_\rho\sigma\boldsymbol{a}_\mu+g_\rho\boldsymbol{\pi}'\times\boldsymbol{\rho}_\mu)^2$$
$$-m_{\pi'}^2\boldsymbol{\pi}'\cdot\boldsymbol{\pi}']+\frac{1}{2}[(\partial_\mu\sigma-g_\rho\boldsymbol{\pi}'\cdot\boldsymbol{a}_\mu)^2-m_\sigma^2\sigma^2]-g_\rho\sigma_0\boldsymbol{a}^\mu\cdot(\partial_\mu\boldsymbol{\pi}'+g_\rho\sigma\boldsymbol{a}_\mu+g_\rho\boldsymbol{\pi}'\times\boldsymbol{\rho}_\mu)+\frac{m_\sigma^2-m_{\pi'}^2}{2\sigma_0}\sigma(\sigma^2+\boldsymbol{\pi}'^2)$$
$$-\frac{m_\sigma^2-m_{\pi'}^2}{8\sigma_0^2}(\sigma^2+\boldsymbol{\pi}'^2)^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_v^2V_\mu V^\mu-\frac{1}{4}\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu}-\frac{1}{4}\boldsymbol{A}_{\mu\nu}\cdot\boldsymbol{A}^{\mu\nu}+\frac{1}{2}g_\rho^2\sigma_0^2\boldsymbol{a}_\mu\cdot\boldsymbol{a}^\mu. \tag{2}$$

From Eq. (2), we see that SSB in the σ sector has given a mass M to the nucleon and a mass $g_\rho\sigma_0$ to the $\boldsymbol{a}_1$. The σ mass is m_σ, and the $\boldsymbol{\rho}$ remains massless. We also note the presence of a bilinear term $-g_\rho\sigma_0\boldsymbol{a}^\mu\cdot\partial_\mu\boldsymbol{\pi}'$, and thus the need for diagonalization. Most importantly, the pion has disappeared and has been replaced by the auxiliary field, $\boldsymbol{\pi}'$. The fact that $\boldsymbol{\pi}'$ is an auxiliary field can be seen either by counting the degrees of freedom before and after SSB or by making the change of variables

$$\boldsymbol{a}_\mu\rightarrow\boldsymbol{a}_\mu+\frac{1}{g_\rho\sigma_0}\partial_\mu\boldsymbol{\pi}'. \tag{3}$$

This change of variables both diagonalizes the Lagrangian and forces a cancellation of the kinetic energy term $\partial_\mu\boldsymbol{\pi}'\cdot\partial^\mu\boldsymbol{\pi}'$, clearly identifying $\boldsymbol{\pi}'$ as an auxiliary field. The initial pion was ‘‘eaten’’ by the $\boldsymbol{a}_1$. $\mathcal{L}_g$ can also be diagonalized using a gauge-fixing function and working in a ξ gauge [8]. For simplicity, take the chiral limit ($m_{\pi'}=0$) and add to the Lagrangian the gauge-fixing function $-\frac{1}{2}G^2$, where

$$G=\frac{1}{\sqrt{\xi}}(\partial_\mu\boldsymbol{a}^\mu+\xi g_\rho\sigma_0\boldsymbol{\pi}'). \tag{4}$$

The gauge-fixed Lagrangian $\mathcal{L}_g^{gf}$ becomes

$$\mathcal{L}_g^{\rm gf}=\mathcal{L}_g-g_\rho\sigma_0\boldsymbol{\pi}'\cdot\partial_\mu\boldsymbol{a}^\mu-\frac{1}{2\xi}(\partial_\mu\boldsymbol{a}^\mu)^2-\frac{1}{2}\xi g_\rho^2\sigma_0^2\boldsymbol{\pi}'^2. \tag{5}$$

It is noted that the second term in Eq. (5) cancels the bilinear term in Eq. (2) after a partial integration, and that the $\boldsymbol{\pi}'$ has acquired a mass that depends on the gauge parameter ξ. From Eq. (5), the propagators in momentum space of the $\boldsymbol{a}_\mu$ and the $\boldsymbol{\pi}'$ are found to be, respectively,

$$\Delta_{\mu\nu}^{ij}=\frac{-i\delta^{ij}}{q^2-m_a^2}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{m_a^2}\right]-\frac{i\delta^{ij}}{q^2-\xi m_a^2}\frac{q_\mu q_\nu}{m_a^2}, \tag{6}$$

$$\Delta^{ij}=\frac{i\delta^{ij}}{q^2-\xi m_a^2}. \tag{7}$$

Here, $m_a=g_\rho\sigma_0$ and we have separated out the ξ-dependent part of the $\boldsymbol{a}_\mu$ propagator in the last term of Eq. (6). In the limit $\xi\rightarrow\infty$, the $\boldsymbol{\pi}'$ decouples from the problem, while the $\boldsymbol{a}_\mu$ propagator goes into the unitary gauge. In an arbitrary ξ gauge, the ξ-dependent part of the $\boldsymbol{a}_\mu$ propagator always cancels the contribution coming from $\boldsymbol{\pi}'$ exchange. This can be seen in nucleon-nucleon scattering at the tree level in Fig. 1 [8]. The $\boldsymbol{\pi}'$ exchange diagram is precisely canceled by the ξ-dependent part of the $\boldsymbol{a}_1$ propagator as is easily verified. The fact that the $\boldsymbol{\pi}'$ does not contribute to physical processes and the fact that the gauge invariance is preserved are visibly true because the $\boldsymbol{\pi}'$ has the correct mass: $m_{\pi'}^2=\xi m_a^2$.[2]

To retain the physical pion in the model, the gauge bosons must develop a mass from *outside* the σ sector. In QHD-III

[2]Since the nonlinear σ model is the limit of the linear σ model as $m_\sigma\rightarrow\infty$, the pion also disappears in the gauged nonlinear σ model.

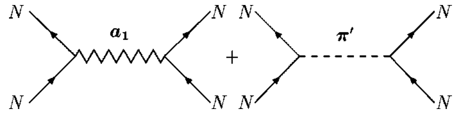

FIG. 1. $\boldsymbol{a}_1$ and $\boldsymbol{\pi}'$ exchange diagrams in NN scattering.

[2], a Higgs sector composed of left and right complex doublets is included to preserve the gauge invariance of the model:

$$\mathcal{L}_H = \partial_\mu \phi_R^\dagger \partial^\mu \phi_R + \partial_\mu \phi_L^\dagger \partial^\mu \phi_L + \mu_H^2 (\phi_R^\dagger \phi_R + \phi_L^\dagger \phi_L) - \frac{\lambda_H}{4}[(\phi_R^\dagger \phi_R)^2 + (\phi_L^\dagger \phi_L)^2]. \tag{8}$$

As detailed in [2], $\mathcal{L}_H$ is then made locally invariant by minimal substitution. After the SSB in the full theory and the elimination of Goldstone bosons from the Higgs sector by going into the unitary gauge, the Higgs Lagrangian becomes

$$\begin{aligned}\mathcal{L}_H =& \frac{1}{2}(\partial_\mu \eta \partial^\mu \eta - m_H^2 \eta^2) + \frac{1}{2}(\partial_\mu \zeta \partial^\mu \zeta - m_H^2 \zeta^2) \\ &\times \frac{1}{2}\left[g_\rho m_\rho \eta + \frac{1}{4} g_\rho^2 (\eta^2 + \zeta^2) \right] (\boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu + \boldsymbol{a}_\mu \cdot \boldsymbol{a}^\mu) \\ &+ \left(g_\rho m_\rho \zeta + \frac{1}{2} g_\rho^2 \eta \zeta \right) \boldsymbol{\rho}_\mu \cdot \boldsymbol{a}^\mu \\ &- \left(\frac{3 m_H^2 g_\rho}{4 m_\rho} \eta + \frac{3 m_H^2 g_\rho^2}{16 m_\rho^2} \eta^2 \right) \zeta^2 - \frac{m_H^2 g_\rho}{4 m_\rho} \eta^3 \\ &- \frac{m_H^2 g_\rho^2}{32 m_\rho^2}(\eta^4 + \zeta^4) + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu + \frac{1}{2} m_\rho^2 \boldsymbol{a}_\mu \cdot \boldsymbol{a}^\mu, \end{aligned} \tag{9}$$

where

$$\mu_H^2 = \frac{1}{2} m_H^2, \quad u^2 = \frac{8 \mu_H^2}{\lambda_H} = \frac{4 m_\rho^2}{g_\rho^2}, \quad \lambda_H = \frac{m_H^2 g_\rho^2}{m_\rho^2}. \tag{10}$$

In the equation above, u is twice the vacuum expectation value of the scalar fields in the Higgs sector. The Higgs fields η and ζ are, respectively, scalar and pseudoscalar fields. The QHD-III Lagrangian $\mathcal{L}_{\mathrm{III}}$ is given by

$$\mathcal{L}_{\mathrm{III}} = \mathcal{L}_g + \mathcal{L}_H. \tag{11}$$

It is seen that the $\boldsymbol{a}_1$ obtains contributions to its mass m_a from both the σ sector and from the Higgs sector. Thus, the $\boldsymbol{a}_1$ comes out naturally more massive than the $\boldsymbol{\rho}$ with

$$m_a^2 = m_\rho^2 + g_\rho^2 \sigma_0^2 > m_\rho^2. \tag{12}$$

The pion in QHD-III is now *real* and $\mathcal{L}_{\mathrm{III}}$ must be diagonalized further to remove the term $-g_\rho \sigma_0 \boldsymbol{a}^\mu \cdot \partial_\mu \boldsymbol{\pi}'$ in $\mathcal{L}_g$. This is achieved by performing the change of variables

$$\boldsymbol{a}_\mu \to \boldsymbol{a}_\mu + \frac{g_\rho \sigma_0}{m_a^2} \partial_\mu \boldsymbol{\pi}', \quad \boldsymbol{\pi}' = \frac{m_a}{m_\rho} \boldsymbol{\pi}, \quad m_{\pi'} = \frac{m_\rho}{m_a} m_\pi. \tag{13}$$

$\boldsymbol{\pi}$ is now the physical pion field. Since the $\boldsymbol{a}_1$ appears in many places in $\mathcal{L}_{\mathrm{III}}$ and the change of variables involves the gradient of the pion, the diagonalized Lagrangian is quite complicated with momentum-dependent vertices showing up everywhere.

In contrast, the gauge invariance of QHD-III allows another, simpler diagonalization procedure to work. Consider Eq. (9): to obtain $\mathcal{L}_H$, the Goldstone bosons of the Higgs sector were eliminated from the theory by going into the unitary gauge, as is done in the standard model. These Goldstone bosons are fictitious since they can be removed by a gauge transformation. This can be seen by counting the degrees of freedom: initially, before SSB in the Higgs sector, there are two complex doublets (the Higgs fields), which yield eight degrees of freedom, and two massless isovector fields (the $\boldsymbol{\rho}$ and the $\boldsymbol{a}_1$), which add 12 degrees of freedom; this yields a total of 20 degrees of freedom. After SSB, there are two isoscalar Higgs fields (the η and the ζ) and two massive isovector fields for a total 20 degrees of freedom. Thus, two isovector fields ''disappeared'' from the Higgs sector to become the longitudinal polarization states of the $\boldsymbol{\rho}$ and the $\boldsymbol{a}_1$; these are the Goldstone bosons. If one does not work in the unitary gauge, the Goldstone bosons must still couple to the other fields to maintain gauge invariance. One of the isovector fields must couple directly to the $\boldsymbol{\rho}$ and must therefore be a scalar field because of parity conservation, while the other isovector field which we denote as $\boldsymbol{\chi}'$ must couple directly to the $\boldsymbol{a}_1$ and must be a pseudoscalar field. The scalar isovector field that was ''eaten'' by the $\boldsymbol{\rho}$ is not needed for this discussion, and can be decoupled from the problem independently of the other Goldstone bosons. As for the pseudoscalar Goldstone bosons, the contributions to $\mathcal{L}_H$ stemming from working in an arbitrary gauge, and therefore keeping the $\boldsymbol{\chi}'$, are

$$\mathcal{L}_H' = \mathcal{L}_H + \partial_\mu \boldsymbol{\chi}' \cdot \partial^\mu \boldsymbol{\chi}' + m_\rho \boldsymbol{a}_\mu \cdot \partial^\mu \boldsymbol{\chi}' + \Delta \mathcal{L}_H'. \tag{14}$$

$\Delta \mathcal{L}_H'$ represents terms given in the Appendix for completeness. The presence of the bilinear term in Eq. (14) is noted and is typically removed with a gauge-fixing function similar to the one given in Eq. (4). The $\boldsymbol{\chi}'$ is then decoupled by taking ξ to infinity as discussed in the case of the $\boldsymbol{\pi}'$ below Eq. (7). This is what was done in [2].

Consider instead the following gauge-fixing function:

$$G_a = \frac{1}{\sqrt{\xi}}(\partial_\mu \boldsymbol{a}^\mu - \xi m_\rho \boldsymbol{\chi}' + \xi g_\rho \sigma_0 \boldsymbol{\pi}'). \tag{15}$$

Adding $-\frac{1}{2}G_a^2$ to $\mathcal{L}_{\mathrm{III}} = \mathcal{L}_H' + \mathcal{L}_g$ cancels the bilinear terms $-g_\rho \sigma_0 \boldsymbol{a}^\mu \cdot \partial_\mu \boldsymbol{\pi}'$ and $m_\rho \boldsymbol{a}_\mu \cdot \partial^\mu \boldsymbol{\chi}'$. The propagator of the $\boldsymbol{a}_1$ is exactly as in Eq. (6) with m_a now given by Eq. (12). What is left from the gauge-fixing function is

$$-\frac{1}{2} G_a^2 \doteq -\frac{1}{2} \xi m_\rho^2 \boldsymbol{\chi}' \cdot \boldsymbol{\chi}' - \frac{1}{2} \xi g_\rho^2 \sigma_0^2 \boldsymbol{\pi}' \cdot \boldsymbol{\pi}' + \xi g_\rho \sigma_0 m_\rho \boldsymbol{\pi}' \cdot \boldsymbol{\chi}'. \tag{16}$$

Equation (16) needs a further diagonalization to cancel the last term; this can be achieved by making the following change of variables:

$$\boldsymbol{\chi}' = a \boldsymbol{\pi} + b \boldsymbol{\chi}, \quad \boldsymbol{\pi}' = c \boldsymbol{\pi} + d \boldsymbol{\chi}. \tag{17}$$

The parameters $\{a,b,c,d\}$ are constrained by the requirements that all bilinear terms that couple the $\boldsymbol{\pi}$ and the $\boldsymbol{\chi}$ be canceled and that the kinetic energies be normalized:

$$\partial_\mu \boldsymbol{\pi}' \cdot \partial^\mu \boldsymbol{\pi}' + \partial_\mu \boldsymbol{\chi}' \cdot \partial^\mu \boldsymbol{\chi}' = \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} + \partial_\mu \boldsymbol{\chi} \cdot \partial^\mu \boldsymbol{\chi}. \tag{18}$$

These equations have the solution

$$a = \frac{g_\rho \sigma_0}{m_a}, \quad b = \frac{m_\rho}{m_a}, \quad c = \frac{m_\rho}{m_a}, \quad d = -\frac{g_\rho \sigma_0}{m_a}. \tag{19}$$

It is found that the masses of the $\boldsymbol{\chi},\boldsymbol{\pi}$ fields are, respectively,

$$m_\chi^2 = \xi(m_\rho^2 + g_\rho^2 \sigma_0^2) = \xi m_a^2, \quad m_\pi = \frac{m_\rho}{m_a} m_{\pi'}. \tag{20}$$

The mass of the $\boldsymbol{\chi}$ is exactly the mass needed to cancel the ξ-dependent part of the $\boldsymbol{a}_1$ propagator given in Eq. (6), as discussed below Eq. (7). The $\boldsymbol{\chi}$ is the field that provided the longitudinal polarization states of the $\boldsymbol{a}_1$ after SSB.

$\boldsymbol{\chi}$ can be decoupled by taking $\xi \to \infty$ and Eq. (17) becomes

$$\boldsymbol{\chi}' = \frac{g_\rho \sigma_0}{m_a} \boldsymbol{\pi}, \quad \boldsymbol{\pi}' = \frac{m_\rho}{m_a} \boldsymbol{\pi}. \tag{21}$$

It is thus seen that diagonalization is achieved by a *constant rescaling* of the pion field. Because of the constraint (18), the pion kinetic energy is not rescaled. This completely avoids the introduction of new momentum-dependent vertices.

Through the first equation of Eq. (21), the pion couples directly to the Higgs fields.[3] The pion and Higgs vertices generally involve a high power of g_ρ as seen in the Appendix. By inspection, it is seen that any amplitude that does not involve both a Higgs field and a gauge boson as external legs will not contribute to $\mathcal{O}(g_\rho^2)$. Hence, to order $\mathcal{O}(g_\rho^2)$, all S-matrix elements that do not involve a Higgs field as either an incoming or an outgoing field can be calculated by ignoring the Higgs sector and rescaling the pion field by the constant factor m_ρ / m_a. In the chiral limit, when $m_\pi = 0$, this new, simpler representation of the QHD-III Lagrangian is completely equivalent to the one given in [2]; when $m_\pi = 0$, the two representations lead to exactly the same physical predictions. In particular, a calculation in QHD-III of an amplitude such as $\boldsymbol{\pi}\boldsymbol{\pi}$ scattering to one loop in the chiral limit [9] is considerably simplified.

When $m_\pi \neq 0$, the two representations will differ slightly in their physical predictions since the pion mass term violates chiral symmetry; i.e., the contribution of the chiral-symmetry-breaking term is gauge dependent. Although the pion mass term is gauge dependent, the symmetry-breaking term in Eq. (1) is always chosen so as to result in the physical pion mass once the gauge has been picked. This is what was done in Eq. (20).

In summary, it is shown that a quantum field model of the strong interactions, based on an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ locally invariant σ model, is not realistic if the corresponding gauge bosons are massless before SSB; in such a model, the pion disappears. We saw how putting masses in by hand violates gauge invariance and forces a complicated diagonalization procedure on the model. QHD-III is reviewed, and the gauge invariance of the model is exploited to provide a *new*, considerably *simpler* representation of the model that makes it more accessible. The pion here appears as a physical degree of freedom.

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APPENDIX

When retaining the Goldstone boson $\boldsymbol{\chi}'$ in the model, the following extra terms appear:

$$\begin{aligned}\Delta \mathcal{L}'_H = & \frac{g_\rho}{2}(\eta \boldsymbol{a}_\mu \cdot \partial^\mu \boldsymbol{\chi}' + \zeta \boldsymbol{\rho}_\mu \cdot \boldsymbol{\chi}' - \boldsymbol{\chi}' \cdot \boldsymbol{a}_\mu \partial^\mu \eta - \boldsymbol{\chi}' \cdot \boldsymbol{\rho}_\mu \partial^\mu \zeta) \\ & + \frac{g_\rho}{2} \boldsymbol{\chi}' \times \boldsymbol{\rho}_\mu \cdot \partial^\mu \boldsymbol{\chi}' - \frac{g_\rho m_H^2}{4 m_\rho} \eta \boldsymbol{\chi}'^2 + \frac{g_\rho^2}{8} \boldsymbol{\chi}'^2 (\boldsymbol{a}_\mu^2 + \boldsymbol{\rho}_\mu^2) \\ & - \frac{g_\rho^2 m_H^2}{16 m_\rho^2} \boldsymbol{\chi}'^2 (\zeta^2 + \eta^2) - \frac{g_\rho^2 m_H^2}{32 m_\rho^2} \boldsymbol{\chi}'^4. \end{aligned} \tag{A1}$$

When $(g_\rho \sigma_0 / m_a) \boldsymbol{\pi}$ is substituted for the $\boldsymbol{\chi}'$ in Eq. (A1), it is seen that the interactions in Eq. (A1) are at least of order g_ρ^2. In particular, the terms in parentheses of the first line of Eq. (A1) are the only ones of $\mathcal{O}(g_\rho^2)$, and they all involve the Higgs fields; the contributions of these terms can be minimized by making the Higgs fields very heavy.

[3]This was also the case in the change of variable (13) as is seen by substituting Eq. (13) into Eq. (9).

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