Charge exchange reactions in the Glauber approximation

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(Received 17 July 1998)

A treatment of heavy ion charge exchange reactions based on the Glauber approximation is presented. It is applied to the analysis of the reactions ^{12}C(^{13}C,^{13}N)^{12}B, ^{12}C(^{13}C,^{13}B)^{12}N, and ^{12}C(^{12}C,^{12}N)^{12}B at intermediate energies. Overall agreement is found on the basis of a model whose ingredients are the experimental nucleon-nucleon scattering amplitudes, the phenomenological nuclear densities, and the transition densities derived from microscopic nuclear structure models. [S0556-2813(99)03604-3]

PACS number(s): 24.10.−i, 25.70.Kk

A considerable amount of experimental data on charge exchange processes in intermediate energy heavy ion collisions is now available and more is expected with the new facilities. By suitable choice of projectile and target, they can allow convenient tests of the isovector channel of the effective interaction useful for the analysis of Fermi and Gamow-Teller transitions. In the past years, several heavy-ion charge exchange reactions have been described within semimicroscopic approaches in which the charge-exchange form factors are calculated from microscopic interaction but the relative motion is described in terms of phenomenological optical potentials [1,2].

Since it is well known that high energy scattering processes are dominated by nucleon-nucleon collisions, in the present work we describe the charge exchange process on the line of the Glauber approximation [3] where the nucleus-nucleus collision is fully microscopically described. Successful applications of the model have been previously obtained for a variety of elastic and inelastic heavy ion data at intermediate energies [4]. Another encouraging aspect of the Glauber-like approaches is that their parameter-free nature makes them particularly suited to predict the gross features of reactions, also in regions where there is no previous knowledge of heavy ion scattering data, a fact which is important when planning new experiments.

In this paper a formalism to deal with charge exchange heavy-ion reactions based on the Glauber approximation is developed. We will assume that, at high energies, these reactions are dominated by one step processes [6]. The formalism is here applied to reactions where the projectile as well as the target can be described as *p*-shell nuclei. The choice of this region is made on the basis of the fact that the same nuclear structure (shell-) model can be used for projectile and target systems and that there exists a simple nuclear interaction [5] that has been extensively and successfully used to account for a large amount of spectroscopic data. The reactions studied cover a range of projectile, targets and final states that allow a good appraisal of some of the features of the charge exchange process.

In the one-step Glauber approximation, the scattering amplitudes of one-step charge exchange reactions ($Aa\rightarrow Bb$), for the momentum transfer Δ, can be expressed [7] in terms of an integral on the impact parameter b

$$f_{Aa\rightarrow Bb}(\Delta)=ik\sum_{LM}\int bdb\,\mu_{LM}^{Aa\rightarrow Bb}(b)J_M(\Delta b)\times e^{\lambda(b)+\chi(b)}e^{-iM\varphi_\Delta}. \tag{1}$$

In the above expression, $\chi(b)$ is the Coulomb phase shift, i.e., $\chi(b)=2i\eta\,\ln(kb)$, and $\lambda(b)$ is the elastic phase shift obtained from the central spin- and isospin-independent nucleon-nucleon scattering [see below, Eq. (6)].

The details of the charge exchange transition are contained in the matrix elements $\mu_{LM}^{Aa\rightarrow Bb}(b)$. By making explicit the nuclear spin and isospin of target (residual) and projectile (ejectile) systems these charge exchange matrix elements can be written in the form

$$\mu_{LM}^{Aa\rightarrow Bb}(b)=\frac{1}{ik_{NN}}\langle t_a n_a t_b-n_b|1n_a-n_b\rangle\langle t_A n_A t_B-n_B|1n_A-n_B\rangle\times\sum_{J_pJ_t}\hat{J}_p\hat{J}_t\langle j_a m_a j_b-m_b|J_p m_a-m_b\rangle\langle j_A m_A j_B-m_B|J_t m_A-m_B\rangle\times\sum_{KS}\hat{K}\sum_{l_pl_tL_{pt}}\hat{L}_{pt}\begin{Bmatrix} l_p & l_t & L_{pt}\\ S & S & K\\ J_p & J_t & L\end{Bmatrix}[[B_{l_p}B_{l_t}]^{L_{pt}}B_K]_M^L\int qdq\hat{\rho}_{ab}^{l_p,S,J_p}(q)\hat{\rho}_{AB}^{l_t,S,J_t}(q)f_{NN}^{1SK}(q), \tag{2}$$

where $\hat{\rho}^{l,S,J}(q)$ are the transition densities in the momentum space corresponding to the transfer of one unit of isospin in the $^{2S+1}l_J$ channel. The amplitudes f_{NN}^{1SK} stem either from the isovector central ($K=0$) or the tensor ($K=2$) parts of the nucleon-nucleon potential, as described below. The quantities B_{lm} arise from the projection of the spherical harmonics on the plane perpendicular to the trajectory $[Y_l^m(\pi/2,\varphi) = i^{l+m}B_{lm}e^{im\varphi}\delta_{l+m,even}]$. The squared brackets are used to indicate the couplings of the related angular momenta.

The nucleon-nucleon interaction. In the case of a nucleus-nucleus collision the general velocity-independent interaction between a target- and a projectile-nucleon can be written in the form

$$V(\vec{r}_{tp}) = \sum_{S,T,K} V_{ST}^K(\vec{r}_{tp})[[\sigma_t^S\sigma_p^S]^K Y_K^*(\hat{r}_{tp})]^0[\tau_t^T\tau_p^T]^0, \qquad (3)$$

where σ^S and τ^T are the spin and isospin Pauli operators when $S=1$ or $T=1$, and are the unity operators when $S=0$ or $T=0$.

The velocity independence of the interaction implies that in Eq. (3) the sum runs only on $K=0$ and 2. It can be shown that, as a consequence of the condition that $l+m$ must be even in the transition densities, no $L=0$ transfer is allowed when the parities of the entrance and exit channels are different, even if tensor contributions ($K=2$) are taken into account.

The scattering amplitude f_{NN}^{1SK} in Eq. (2) is related to the Fourier transform of the effective nucleon-nucleon potential, which has been parametrized by Love and Franey [8] to fit the free nucleon-nucleon scattering data. This effective potential is complex and energy dependent.

Transition densities. The transition densities appearing in Eq. (2) are, in the target-residual nuclei system, the Fourier transforms of the matrix elements

$$\rho_{AB}^{l_t,S,J_t}(r_t) = \frac{1}{\sqrt{3}\hat{J}_t}\langle j_B t_B||| T^{l_t S J_t}\tau|||j_A t_A\rangle, \qquad (4)$$

where the one-body transition operator is

$$T_{M_t}^{l_t S J_t} = \frac{\delta(r-r_t)}{r^2}[Y_{l_t}\sigma^S]_{M_t}^{J_t}. \qquad (5)$$

The states $|j_A t_A\rangle$ and $|j_B t_B\rangle$ are the many-body wave functions for the target and residual nucleus, respectively. Expressions similar to Eqs. (4) and (5) result for the projectile-ejectile system.

The calculation of the transition densities involves the choice of a model for the nuclear wave functions and of a suitable algorithm for the obtention of the matrix elements of $T_{M_t}^{l_t S J_t}$ and $T_{M_p}^{l_p S J_p}$. A specific choice for *p*-shell nuclei is explicitly described below.

The elastic phase shift. The elastic phase shifts

$$\lambda(b) = \frac{1}{2\pi i k_{NN}}\int \hat{\rho}_p(q)\hat{\rho}_t(q) f_{NN}^{000}(q) e^{-i\vec{q}\cdot\vec{b}} d\vec{q} \qquad (6)$$

have been evaluated by using nuclear densities $\rho(r)$ and the Love and Franey nucleon-nucleon interaction [8].

TABLE I. Transition density factors.

A	J_i	T_i	J_f	T_f	l	S	J	
12	0	0	1	1	0	1	1	-0.13381×10^{-1}
					2	1	1	0.32225×10^{-2}
12	0	0	2	1	2	0	2	-0.19949×10^{-1}
					2	1	2	0.33413×10^{-2}
13	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	-0.62647×10^{-1}
					0	1	1	0.56849×10^{-2}
					2	1	1	0.48795×10^{-1}
13	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	0	1	1	-0.13303×10^{-1}
					2	1	1	0.27070×10^{-1}
					2	0	2	0.24213×10^{-1}
					2	1	2	-0.32506×10^{-1}

The above microscopic method has proved to be reliable in many applications. It is known, however, that exceedingly diffractive elastic cross sections result when very transparent nuclear systems are involved, as for example, in the case of the $^{12}C+^{12}C$ reactions. In this case better fits are obtained by resorting to optical model approaches, where the elastic phase shift $\lambda(b)$ is obtained from the macroscopic ion-ion potential. In the high energy range one can use the eikonal approximation starting from an optical potential. The price to be paid is the need of *ad hoc* fits of optical parameters for each reaction.

Normal parity states in nuclei around ^{12}C can be satisfactorily described in terms of many nucleon configurations in the $0p$ shell. For the states that are considered below, a complete basis (see, e.g., Ref. [9]) is provided by the $L-S$ coupled wave functions $|p^n[f]TSLJ\rangle$, $(n=A-4)$. Then, the nuclear states are written as

$$|A,J,T,\gamma\rangle = \sum_{[f]SL} c_{[f]SL}^{\gamma}|p^n[f]TSLJ\rangle, \qquad (7)$$

where γ identifies one specific J,T state. The coefficients $c_{[f]SL}^{\gamma}$ are obtained by diagonalizing a nuclear interaction, in the present case the (8-16)POT of Cohen and Kurath [5]. The matrix elements of the charge exchange transition operators (5) are calculated in Ref. [10]. In this scheme the transition densities of Eq. (4) result

$$\rho_{if}^{l,S,J}(r) = \frac{\varphi_{0p}^i(r)\varphi_{0p}^f(r)}{r^2}\mathcal{K}_{if}. \qquad (8)$$

In the above formula i stands for the labels corresponding to projectile or target (a or A) and f for those of ejectile or residual nucleus (b or B), respectively. All the Hamiltonian eigenvectors, angular momentum and charge-spin fractional parentage and spherical tensor recoupling coefficients are contained in $\mathcal{K}_{if}$. The Table I gives the values of the factor $\mathcal{K}_{if}$ for the lSJ charge-exchange transitions under consideration.

The single-nucleon wave functions $\varphi_{nl}(r)$ are the solutions of a Woods-Saxon potential, which depth is adjusted to

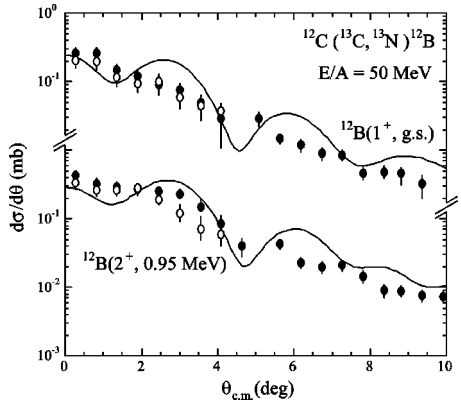

FIG. 1. Calculated and experimental angular distributions for the reaction ^{12}C(^{13}C,^{13}N)^{12}B.

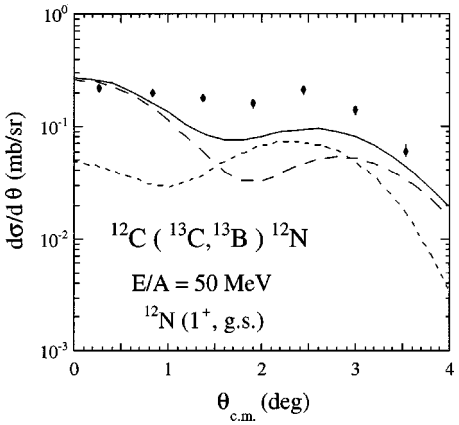

FIG. 2. Calculated and experimental angular distributions for the reaction ($\Delta S=1,\Delta T=1$)^{12}C(^{13}C,^{13}B)^{12}N(1$^+$, g.s.). Contributions from the central (dashed-line) and tensor (dotted-line) are displayed separately.

get the experimental binding energies of the corresponding nucleon in each of the four nuclear systems (projectile, target, ejectile, and residual nucleus).

The ^{12}C(^{13}C,^{13}N)^{12}B *reaction*. This reaction changes a proton into a neutron in the target and conversely a neutron into a proton in the projectile. Table I shows that the projectile can go to the mirror $T=\frac{1}{2}$, $J^\pi=\frac{1}{2}^-$ ground state of ^{13}N via the $^1S_0$, $^3S_1$ and $^3D_1$ transition channels. The reaction populates the $T=1$, $J^\pi=1^+$ ground state in ^{12}B (analogue of the level at 15.11 MeV in ^{12}C) through transitions in the channels $^3S_1$, $^3D_1$. Although a non-spin-flip channel ($^1S_0$) is possible in the projectile, Eq. (2) implies a similar transition in the target and therefore only $\Delta S=1$ terms contribute to the cross section.

This reaction populates more strongly the 2^+, 0.95 MeV state in ^{12}B with the target undergoing $^1D_2$ and $^3D_2$ transitions. In this case both, spin-flip and non-spin-flip, channels are allowed, but the factors coming from Eqs. (2) and (8) predict at least three orders of magnitude larger contributions to the cross section for $\Delta S=0$ than for $\Delta S=1$.

In Fig. 1 we show the results of the present calculations compared to the experimental data [11,12].

The ^{12}C(^{13}C,^{13}B)^{12}N *reaction*. Although from the target point of view this reaction is mirror to that treated previously, differences arise from the projectile, that in this case goes to a $T=\frac{3}{2}$ state (the $J^\pi=\frac{3}{2}^-$ ground state of ^{13}B). As seen in Table I, possible transition channels would be $^1D_2$, $^3S_1$, $^3D_1$ and $^3D_2$. Once again the non-spin-flip channels are removed by the absence of matches in the target.

In an extreme single-particle picture, with ^{12}C acting as the core, the transition in the projectile in the previous reaction corresponds to the flip of the spin and isospin in the $p_{1/2}$ state. On the contrary, the present case leads to more complicated $[p_{1/2,\nu}p^{-1}_{3/2,\pi}]^{\Delta T=1,\Delta S=1}$ particle-hole configurations. This fact does not seem to reflect in a particular way in the cross sections. Figure 2 shows the data [11] and the calculation results.

The ^{12}C(^{12}C,^{12}N)^{12}B *reaction*. In this case there are measurements at $E/A=70$ MeV (Refs. [6,13,14]) and 135 MeV (Refs. [15,16]). For the theoretical analysis, the isovector nucleon-nucleon interaction of Love and Franey [8] has to be interpolated since for those energies there is no available parametrization tabulated.

Being the final nuclei the mirrors ^{12}N and ^{12}B, the same spin-flip transition channels $^3S_1$ and $^3D_1$ are involved, both in the projectile and target systems.

As stated earlier the Glauber approximation gives too diffractive cross sections already for the elastic ^{12}C+^{12}C collision. Therefore, for this case the elastic phase shift entering into Eq. (1) has been obtained in the eikonal approximation starting from the optical potential obtained in Ref. [15] for the elastic scattering (best fit). The results of the present calculations are compared with the experimental data in Fig. 3 for $E/A=70$ MeV and in Fig. 4 for $E/A=135$ MeV.

In the case of $E/A=70$ MeV there is a not solved discrepancy between the data of Ref. [13] and those of Ref. [14] and both are plotted in Fig. 3. The selectivity of the reaction (^{12}C,^{12}N) for spin-flip transitions, in contrast with the (n,p) reaction, allows one to test the isovector ($\Delta S=1$) components of the effective interaction. In Fig. 3, the contributions arising from the central and the tensor parts are displayed. The central component carries mainly the $L=0$ transition terms while the tensor component is mainly associated to the $L=2$ transitions. Although $L=2$ ($L=0$) transitions are not absent in the cross section associated to the central (tensor)

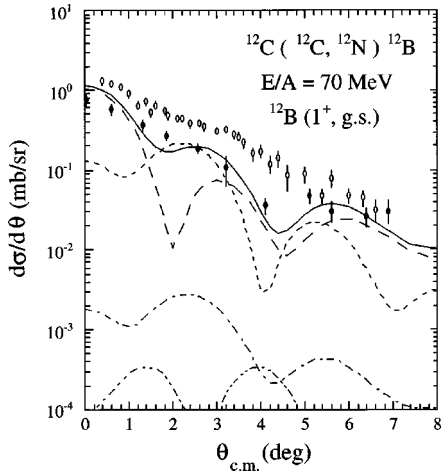

FIG. 3. Calculated and experimental angular distributions for the reaction ^{12}C(^{12}C,^{12}N)^{12}B (1$^+$, g.s.).

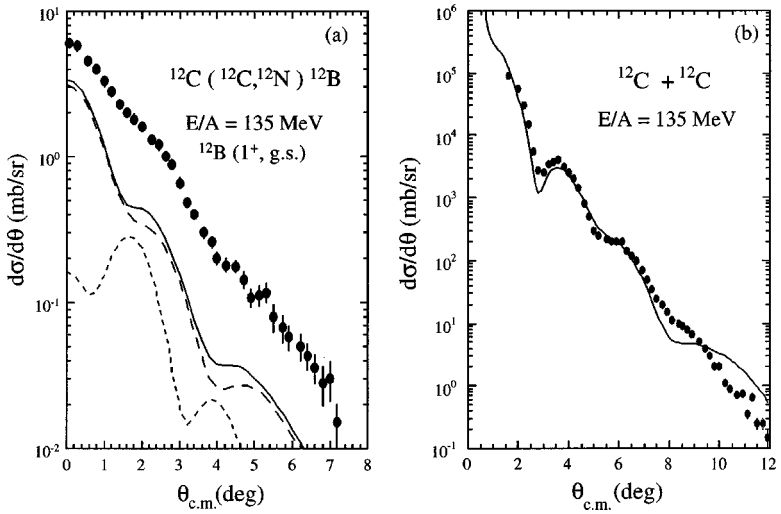

FIG. 4. Calculated and experimental angular distributions for (a) the charge-exchange reaction $^{12}C(^{12}C,^{12}N)^{12}B$ (1^+, g.s.); (b) elastic scattering for the reaction $^{12}C+^{12}C$.

term, their contributions, shown as dot-dashed lines (dot-dot-dashed lines), are very small. However to get a good description of the experimental angular distributions, both terms have to be included. Our results are consistent with the findings of Ref. [6], where microscopically constructed form factors were used in a DWBA formalism.

For the case of $E/A=135$ MeV, the elastic differential cross section is in very good agreement with the experimental data as shown in Fig. 4(b). However, for the charge-exchange reaction, the quality of the calculation is not so good. To reproduce the experimental data one should multiply the central interaction by a factor 1.4 and the tensor term by a factor 2. Since the parametrizations for all other cases studied in this work seem to reproduce well the data, we report the theoretical predictions without any normalization as done for the other reactions. Further experimental data around this or at higher energies could help to understand this discrepancy for which we have no explanation.

We have also plotted in Fig. 4(a) the separate contributions for the central and tensor spin-flip isovector components. By comparing with Fig. 3, it can be seen that the tensor term becomes less important as the energy increases. These reactions could therefore be very useful to reproduce in a heavy-ion reaction the conditions of vanishing angular momentum transfer occurring in a beta decay of the Fermi type.

In conclusion, as a general remark the study of the charge exchange heavy ion reactions at intermediate energies within the Glauber approximation shows an overall agreement with the experimental data, both in magnitude and general trends. It is worthwhile to point out that only one, and the same, structure model has been used for all the nuclear systems involved, and that the reaction and transition-producing ingredients are those coming from the parameterization of the free nucleon-nucleon scattering data of Love and Franey [8], without any *ad hoc* modification of the relative weights of the central and tensor components. Such an agreement exists in spite of the fact that for the various reactions and energies the cross sections arise from the coherent contribution of many different components, both in the nuclear structure transition densities and in the nucleon-nucleon interaction (Ref. [8]). This agreement makes one confident for applications of the present treatment to unexplored regions of energies or masses.

This work was supported in part by the Argentine Antorchas Foundation, the Agencia de Promoción Científica y Tecnológica under Contract PMT-PICT0190 and the CONICET under Contract PIP 4486/96.

[1] H. Lenske, Nucl. Phys. **A495**, 127c (1989).
[2] C. Bertulani, Nucl. Phys. **A554**, 493 (1993).
[3] R.J. Glauber, *High-energy Collisions Theory, Lectures in Theoretical Physics* (Interscience, New York, 1959), p. 315.
[4] S.M. Lenzi, A. Vitturi, and F. Zardi, Phys. Rev. C **40**, 2114 (1989); **49**, 1635 (1994).
[5] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).
[6] H. Lenske, H.H. Wolter, and H.G. Bohlen, Phys. Rev. Lett. **62**, 1457 (1989).
[7] S.M. Lenzi, F. Zardi, and A. Vitturi, Phys. Rev. C **42**, 2079 (1990).
[8] W.G. Love and M.A. Franey, Phys. Rev. C **24**, 1073 (1981); **31**, 488 (1985).
[9] J.P. Elliott and A.M. Lane, *Handbuch der Physik, Vol. XXXIX* (Springer-Verlag, Berlin, 1957).
[10] H.A. Jahn and H. van Wieringen, Proc. R. Soc. London, Ser. A **209**, 502 (1951).
[11] C. Bérat, thesis, Université de Grenoble, 1988.
[12] C. Bérat *et al.*, Phys. Lett. B **218**, 299 (1989).
[13] H.G. Bohlen *et al.*, Nucl. Phys. **A488**, 89c (1988).
[14] N. Anantamaran *et al.*, Phys. Rev. C **44**, 398 (1991).
[15] T. Ichihara *et al.*, Phys. Lett. B **323**, 278 (1994).
[16] T. Ichihara *et al.*, Nucl. Phys. **A583**, 109c (1995).