Pion electromagnetic current in a light-front model

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The electromagnetic form factor of the pion is calculated in a pseudoscalar field theoretical model with constituent quarks. We extract the form factor using the ''+'' and ''−'' components of the electromagnetic current on the light front. For comparison, we also compute the form factor in the covariant framework. It is shown that the pair terms do not contribute to the ''+'' component of the electromagnetic current; however, they affect the ''−'' component of the electromagnetic current. [S0556-2813(99)05503-X]

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The pion, as quark-antiquark bound state, is an appropriate system to study aspects of QCD in low and intermediate energies regions. In the nonperturbative regime of QCD, the pion has indeed received much attention, e.g., using the constituent quark model on the light front [1–5]. In these studies, where the pion is described by light-front wave functions, the electromagnetic form factor has been calculated for low and high q^2 and a fairly good agreement with experiment has been obtained. However, in several light-front studies [6,7], the importance of the so-called pair terms, i.e., particle-antiparticle pair creation by the photon, has been extensively discussed for nonvanishing $q^+=q^0+q^3$. In Ref. [7], we studied pair terms in an explicit computation of the electromagnetic current of scalar and vector mesons. It was shown that these pair terms are essential for retaining full covariance. Recently [8], we also demonstrated in a boson model the relevance of pair terms for the Ward-Takahashi identity, which expresses local gauge invariance for (off-shell) Green's functions. The pair terms are the contribution of the zero-mode longitudinal momentum to the current.

In this Brief Report we calculate the pion form factor in a similar light-front model as in [3], however carefully adressing the issue of pair terms and zero modes in the on-shell matrix elements of $j^+=j^0+j^3$ and $j^-=j^0-j^3$ for $q^+=0$.

As in earlier applications [3], we use an effective Lagrangian approach with pion and quark degrees of freedom. We choose pseudoscalar coupling:

$$\mathcal{L}_I=-\imath\frac{m}{f_\pi}\vec{\pi}\cdot\overline{q}\gamma^5\vec{\tau}q, \tag{1}$$

where m denotes the constituent quark mass and f_π the pion decay constant. The electromagnetic field is coupled in the usual minimal way, ensuring gauge invariance. The light-front coordinates are defined as $k^+=k^0+k^3$, $k^-=k^0-k^3$, and $k_\perp=(k^1,k^2)$. For the electromagnetic current of the π^+, we get an expression corresponding to the Feynman triangle diagram:

$$j^\mu=-\imath 2e\frac{m^2}{f_\pi^2}N_c\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[S(k)\gamma^5S(k-P') \times\gamma^\mu S(k-P)\gamma^5\Lambda(k,P')\Lambda(k,P)], \tag{2}$$

with $S(p)=1/(\not{p}-m+\imath\epsilon)$. Here $N_c=3$ is the number of colors and the factor of 2 stems from isospin algebra. We will work in the Breit frame, where the momentum transfer $q^2=-(\vec{q}_\perp)^2, P^0=P'^0$ and $\vec{P}'_\perp=-\vec{P}_\perp=\vec{q}_\perp/2$. The function $\Lambda(k,p)=(2\pi)^2C/[(p-k)^2-m_R^2+\imath\epsilon]$ is our choice for regularizing the divergent integral. The normalization constant C is found by imposing the condition $F_\pi(0)=1$ on the pion form factor.

Rewriting Eq. (2) for the ''+'' component of the current in light-front coordinates yields

$$j^+=-2\imath e\frac{m^2C^2}{f_\pi^2}N_c\int\frac{d^2k_\perp dk^+dk^-}{2}\frac{-4k^-(k^+-P^+)^2+4(k_\perp^2+m^2)(k^+-2P^+)-k^+q^2}{k^+(P^+-k^+)^2(P'^+-k^+)^2\left(k^--\frac{f_1-\imath\epsilon}{k^+}\right)}$$
$$\times\frac{1}{\left(P^--k^--\frac{f_2-\imath\epsilon}{P^+-k^+}\right)\left(P'^--k^--\frac{f_3-\imath\epsilon}{P'^+-k^+}\right)\left(P^--k^--\frac{f_4-\imath\epsilon}{P^+-k^+}\right)\left(P'^--k^--\frac{f_5-\imath\epsilon}{P'^+-k^+}\right)}, \tag{3}$$

where $f_1=k_\perp^2+m^2$, $f_2=(P-k)_\perp^2+m^2$, $f_3=(P'-k)_\perp^2+m^2$, $f_4=(P-k)_\perp^2+m_R^2$, and $f_5=(P'-k)_\perp^2+m_R^2$.

We now separate the terms proportional to k^-, i.e., the light-front energy, in the loop integral. Therefore we define the trace

$$\mathrm{Tr}_\pi^{+\Delta} = -4k^-(k^+ - P^+)^2. \tag{4}$$

For the other terms, only momenta in the interval $0<k^+<P^+$ contribute to the Cauchy integration; this means that the spectator particle is on mass shell and the pole contribution stems from $k^- = (k_\perp + m^2)/k^+$. We also construct the Δ term for the pion current matrix elements:

$$\Delta^+(q^2) = -i\int \frac{d^2k_\perp dk^+ dk^-}{2} \frac{k^-(k^+-P^+)^2}{k^+(P^+-k^+)^2(P'^+-k^+)^2\left(k^- - \frac{f_1 - \imath\epsilon}{k^+}\right)}$$
$$\times \frac{1}{\left(P^- - k^- - \frac{f_2 - \imath\epsilon}{P^+ - k^+}\right)\left(P'^- - k^- - \frac{f_3 - \imath\epsilon}{P'^+ - k^+}\right)\left(P^- - k^- - \frac{f_4 - \imath\epsilon}{P^+ - k^+}\right)\left(P'^- - k^- - \frac{f_5 - \imath\epsilon}{P'^+ - k^+}\right)}. \tag{5}$$

This integral has contributions in two nonzero intervals:

$$(\mathrm{I})\, 0<k^+<P^+,$$

$$(\mathrm{II})\, P^+<k^+<P'^+ ,$$

where $P'^+ = P^+ + \delta$. Note that in the Breit frame $P^+ = P'^+$, which implies that there appear coinciding poles in Eq. (5). As in [8], we have dislocated them by shifting P'^+ with δ. The interval (I) corresponds to a spectator particle on mass shell. The other interval (II) corresponds to a pair term contribution Fig. (1). Eventually, we take the limit $\delta\to 0$, i.e., $P'^+ = P^+$, and the exact kinematics of the Breit frame is recovered.

Let us consider interval (II), $P^+<k^+<P'^+$; after integration in k^- we obtain

$$\Delta^{+(II)}(q^2) = \pi\int \frac{d^2k_\perp dk^+\left(P'^- - \frac{f_3}{P'^+ - k^+}\right)}{k^+(P'^+-k^+)^2\left(P'^- - \frac{f_3}{P'^+-k^+} - \frac{f_1}{k^+}\right)}$$
$$\times \frac{\theta(P'^+ - k^+)\,\theta(k^+ - P^+)}{\left(\frac{f_3}{P'^+-k^+} - \frac{f_2}{P^+-k^+}\right)\left(\frac{f_3}{P'^+-k^+} - \frac{f_4}{P^+-k^+}\right)\left(\frac{f_3}{P'^+-k^+} - \frac{f_5}{P^+-k^+}\right)}. \tag{6}$$

The limit to the Breit frame is performed, after the momentum fraction is used as integration variable, $x = (k^+ - P^+)/(P'^+ - P^+)$. The integration becomes

$$\Delta^{+(II)}(q^2) = \pi\frac{\delta}{P^+}\int d^2k_\perp dx \frac{\theta(x)\,\theta(1-x)}{(1-x)^2\left(\frac{f_3}{1-x} + \frac{f_2}{x}\right)\left(\frac{f_3}{1-x} + \frac{f_4}{x}\right)\left(\frac{f_3}{1-x} + \frac{f_5}{x}\right)}, \tag{7}$$

which vanishes linearly with δ when the Breit frame is recovered: $\Delta^{+(II)}(q^2)\to 0$. Thus we see that in this model, the pair terms in the ''+'' component of the electromagnetic current disappear. In other words, the zero modes do not contribute to Δ^+. As a consequence for j^+, one obtains agreement between naive light-front and covariant calculations.

In the same model, the ''−'' matrix element of the electromagnetic current is calculated from Eq. (2), where the trace reads

$$\mathrm{Tr}_\pi^- = -4k^{-2}k^+ - 4P^+(2k_\perp^2 + k^+P^+ + 2m^2) + k^-(4k_\perp^2 + 8k^+P^+ - q^2 + 4m^2). \tag{8}$$

In the same way as above, we separate the k^- terms in the trace (8) where the other terms do not get contributions of the pair terms. In this case, however, the Δ term acquires pole contributions in two k^+ intervals. The first one corresponds to a spectator particle on mass shell (I) and the other one to the pair term (II). The Δ term for this component of the electromagnetic current is

$$\Delta^-(q^2)=-i\int\frac{d^2k_\perp dk^+dk^-}{2}\frac{-4k^{-2}k^++k^-(4k_\perp^2+8k^+P^+-q^2+4m^2)}{k^+(P^+-k^+)^2(P'^+-k^+)^2\left(k^--\frac{f_1-\imath\epsilon}{k^+}\right)}$$
$$\times\frac{1}{\left(P^--k^--\frac{f_2-\imath\epsilon}{P^+-k^+}\right)\left(P'^--k^--\frac{f_3-\imath\epsilon}{P'^+-k^+}\right)\left(P^--k^--\frac{f_4-\imath\epsilon}{P^+-k^+}\right)\left(P'^--k^--\frac{f_5-\imath\epsilon}{P'^+-k^+}\right)}. \tag{9}$$

At this point we use the identity

$$\frac{(k^-)^2k^+}{k^2-m^2+i\epsilon}=(k^--P^-)+P^-+\frac{k_\perp^2+m^2}{k^+}+\frac{k^-_{(\mathrm{on})}(k_\perp^2+m^2)}{k^2-m^2+i\epsilon}, \tag{10}$$

in order to perform conveniently the k^+ and k^- integrations. The first term on the right-hand side of Eq. (10) is odd under the transformation $(k^--P^-)\to -(k^--P^-)$ and consequently its contribution to the integration in Eq. (9) vanishes. Taking the limit $\delta\to 0$ results in the following pole contribution in interval (II):

$$\Delta^{-(II)}(q^2)=4\pi\frac{m_\pi^2-\frac{q^2}{2}}{P^+}\int d^2k_\perp\sum_{i=2}^{5}\frac{\ln(f_i)}{\prod_{j=2,i\neq j}^{5}(f_i-f_j)}. \tag{11}$$

The sum of the contributions in intervals (I) and (II) yields the same result for this component of the pion current as the calculation in the covariant formalism. The residue associated with the virtual pair creation process by the photon in the triangle diagram is responsible for keeping covariance in the ''$-$'' component of the electromagnetic current and it corresponds to the contribution of a zero mode to j^-.

The issue of the covariance of light-front perturbation theory and the relation to the zero-mode contribution to the amplitudes has also been discussed in Ref. [9]. This result also confirms a conclusion in [7,10]; i.e., it is important to include pairs terms for maintaining full covariance of the light-front model.

In general the form factor is extracted from the covariant expression

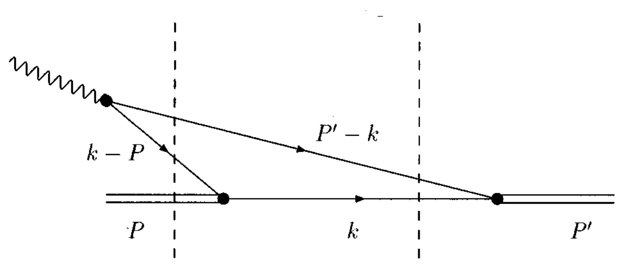

FIG. 1. Pair creation diagram for j^+. Note that for j^- other pair creation diagrams appear.

$$j^\mu=e(P^\mu+P'^\mu)F_\pi(q^2). \tag{12}$$

If covariance is respected in a model calculation of the current, one obviously can utilize each component to extract the electromagnetic form factor of the pion. We explicitly prove that, in the case of the ''$-$'' component of the electromagnetic current, it is necessary to include pair term contributions.

We only use the ''$+$'' and ''$-$'' components of the electromagnetic current. In the first case, the result for the form factor written in light-front coordinates for this model is obtained integrating Eq. (3) over k^- for $0<k^+<P^+$ [interval (I)]. In this way the null-plane (light-front) wave function for the π meson appears [3,5,11],

$$\Phi(x,k_\perp)=C\frac{m}{f_\pi}(8\pi N_c)^{1/2}\frac{1}{(1-x)(m_\pi^2-M_0^2)(m_\pi^2-M_R^2)}, \tag{13}$$

where $x=k^+/P^+$,

$$M_R^2=\mathcal{M}^2(m^2,m_R^2)=\frac{k_\perp^2+m^2}{x}+\frac{(P-k)_\perp^2+m_R^2}{1-x}-P_\perp^2,$$

and the free mass $M_0^2=\mathcal{M}^2(m^2,m^2)$. The form factor is

$$F_\pi^+(q^2)=\int\frac{d^2k_\perp dx}{2x(1-x)^2}\left(\frac{k_\perp^2+m^2}{x}+\frac{q^2}{4}\right)\Phi_f^*(x,k_\perp)\Phi_i(x,k_\perp), \tag{14}$$

where Φ_i is the initial pion wave function and Φ_f is the final one.

Extraction of the electromagnetic form factor of the pion via the ''$-$'' component of the current yields two contributions, corresponding to a spectator particle on mass shell [interval (I)] and the contribution of the pair terms [interval (II)]:

$$F_\pi^{-(I)}(q^2)=\int\frac{d^2k_\perp dx}{2x(1-x)^2}\left(P^{+2}x+\frac{q^2}{4}\frac{k_\perp^2+m^2}{xP^{+2}}\right)\times\Phi_f^*(x,k_\perp)\Phi_i(x,k_\perp), \tag{15}$$

$$F_\pi^{-(II)}(q^2)=\frac{C^2}{P^+}\frac{m^2}{f_\pi^2}N_c\Delta^{-(II)}(q^2). \tag{16}$$

Adding these results gives

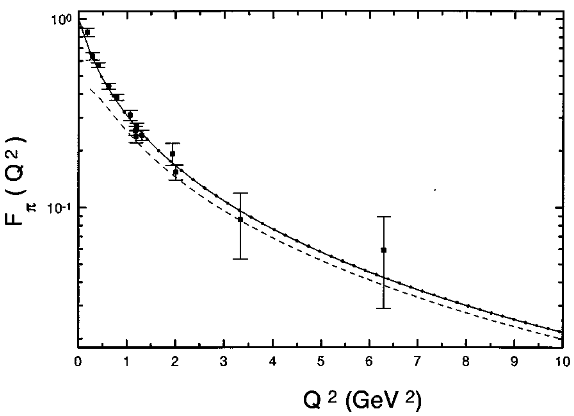

FIG. 2. Pion form factor as a function of $Q^2=-q^2$. Covariant results (solid curve), calculation with j^- without zero-mode contribution (dashed curve), and including it (dots); experimental data [12].

$$F_\pi^-(q^2)=F_\pi^{-(I)}(q^2)+F_\pi^{-(II)}(q^2)=F_\pi^+(q^2). \qquad (17)$$

This shows the fact that it is important to include pair terms in the light-front model for the extraction of the electromagnetic form factor of pseudoscalar particles (pion)—see also Fig. 2.

The remaining integrals are evaluated numerically and the results are presented in Fig. 2. The two free parameters in this model, the constituent quark mass m and the regulator mass m_R, were fixed as $m=0.220$ GeV [3] and $m_R=0.946$ GeV. The pion mass we take as $m_\pi=0.140$ GeV. The regulator mass is obtained by adjusting the pion electromagnetic radius. In this model the form factor calculated in the light-front framework agrees with the one obtained in the covariant formalism (see also Fig. 2). In the covariant calculation the energy integral, i.e., the k^0 integral, is obtained analytically via Cauchy's theorem. Again, the remaining part is computed numerically. Furthermore, in Fig. 2, we compare the calculated model pion form factors to experiment [12] and find good agreement.

The pion decay constant is measured in the weak leptonic decay of the charged pion and appears in the matrix element of the partially conserved axial vector current $P_\mu\langle 0|A_i^\mu|\pi_j\rangle=\imath m_\pi^2 f_\pi\delta_{ij}$. Following Ref. [3], we take $A_i^\mu=\bar{q}\gamma^\mu\gamma^5\tau_i q/2$ and use the interaction Lagrangian (1) for the pion-$\bar{q}q$ vertex function. In this way we obtain

$$\imath P^2 f_\pi=\frac{m}{f_\pi}N_c\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\not{P}\gamma^5 S(k)\gamma^5 S(k-P)\Lambda(k,P)], \qquad (18)$$

and in terms of the model light-front wave function we get

$$f_\pi=m\left(\frac{N_c}{(2\pi)^3}\right)^{1/2}\int\frac{d^2k_\perp dx}{x}\Phi(x,k_\perp). \qquad (19)$$

Numerically, this yields $f_\pi=101$ MeV to be compared with the experimental value $f_\pi=93$ MeV. Similar discrepancies were found in Refs. [1,3] and appear to be a property of these models, once the pion radius is fitted.

In summary, using a pseudoscalar constituent quark model, we calculated the pion form factor in light-front as well as in covariant field theory. The results are in perfect agreement with each other and also describe the experimental form factor well in the q^2 range considered. We have explicitly demonstrated that the contribution of the light-front pair terms vanishes for the "+" component of the electromagnetic current. The vanishing pair term contribution in the "+" component of the current is a peculiar property of the model under consideration, which does not justify omitting the pair terms in general. In the case of the "−"component of the electromagnetic current we find contributions of the pair term in the matrix elements. This is shown to be crucial for respecting full covariance in the light-front model.

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