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Time development of vacuum structure in chiral phase transitions

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The conditions for the restoration of chiral symmetry and the subsequent formation of the disoriented chiral condensates in ultrarelativistic nucleus-nucleus collisions are studied by using the linear σ model. A rapid increase of temperature in the initial thermalization stage is parametrized so as to simulate the result of the parton cascade model. The subsequent decrease of temperature in the cooling stage is described in terms of the one- or three-dimensional scaling hydrodynamics. The effective potential at each temperature is calculated in the massless free particle approximation. Those ingredients are used to solve numerically the equation of motion for the chiral condensates. In general, solutions exhibit characteristic damped oscillations of which patterns are very sensitive to the maximum temperature T_m and the time when the maximum temperature is attained. In particular, it is found that the maximum temperature must be much higher than the critical temperature T_c in order that the chiral symmetry is restored temporarily, e.g., $T_m \geqslant 175$ MeV for $T_c \sim 123$ MeV with longitudinal expansion. Moreover, it is suggested that the disoriented chiral condensates will be formed most easily in the $-\sigma$ direction. [S0556-2813(99)02404-8]

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I. INTRODUCTION

Quantum chromodynamics predicts that a hadronic system will undergo a phase transition into quark gluon plasma (QGP) when the temperature T is larger than a critical value T_c. Ultrarelativistic nucleus-nucleus collisions have thus attracted much attention because the colliding system may evolve into a hot matter with the maximum temperature T_m which is larger than T_c. However, the condition $T_m \geqslant T_c$ does not necessarily guarantee the occurrence of phase transition in a time-dependent collision process because the change of the vacuum structure that characterizes the phase transition (e.g., melting of the chiral condensate) takes a finite relaxation time. The system cannot undergo a phase transition instantaneously even if T exceeds T_c. It could happen that the system does not undergo the phase transition even if the temperature becomes much higher than T_c if the system expands and cools down very rapidly. In such a case, there might not be enough time for the vacuum to rearrange itself. Nevertheless, little attention has been given to the behavior of chiral condensates in the initial thermalization stage. It is thus necessary to study the time development of the vacuum structure under the environment which may be realized in the entire stage of ultrarelativistic heavy-ion collisions

It is expected that the chiral symmetry of the vacuum will be restored in the QGP phase. It is signaled by the vanishing chiral condensates. If the vacuum stays near the origin of the chiral space for a while and then starts to roll down toward a point on the chiral circle regenerated due to the cooling, the location of the point will distribute at random event by event or domain by domain. The location of the regenerated condensate will in general be different from that of the physical vacuum at $T=0$. Condensates at any point on the chiral circle would be realized with equal probabilities if the effect due to the explicit symmetry breaking is ignored. This has been an explicit or an implicit assumption made often in previous works on the disoriented chiral condensates (DCC) [1,2]. It is, however, very questionable if such a supposition is realistic. An initial condition for the rolldown which carries information on the earlier stage of the collision process may easily change the scenario of DCC formation. Again it is necessary to study the time development of the vacuum structure using a plausible equation of motion for the chiral condensates with plausible initial conditions appropriate for ultrarelativistic heavy-ion collisions.

The purpose of this paper [3] is to study the time development of the chiral condensates in ultrarelativistic nucleus-nucleus collisions by using the linear σ model and the effective potential evaluated within the massless free particle approximation [1]. The maximum temperature for which the chiral symmetry is restored is estimated. The possibility of DCC formation subsequent to the chiral symmetry restoration is also discussed.

In Sec. II, an equation of motion for the chiral condensate is derived and is applied to the case where the system heats up with the time-dependent temperature which simulates the result of the parton-cascade model and then cools down following the one- or three-dimensional scaling solution of relativistic hydrodynamics. Numerical results for some typical choices of the initial conditions are given in Sec. III. The possibilities of the chiral symmetry restoration and of the DCC formation are examined. Conclusions and discussions are given in Sec. IV.

II. THE LINEAR σ MODEL AND EQUATION OF MOTION

The linear σ model Lagrangian is given by

$$\mathcal{L}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{\lambda}{4}(\phi^2-v^2)^2+H\sigma, \tag{1}$$

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where $H\sigma$ is the explicit breaking term and $\phi=(\sigma,\vec{\pi})=(\phi_0,\phi_1,\phi_2,\phi_3)$. Since we are interested in the condensation of the field ϕ, it is divided into the condensation part $\Phi=(\Phi_0,\Phi_1,\Phi_2,\Phi_3)$ and the fluctuation part $\tilde{\phi}=(\tilde{\phi}_0,\tilde{\phi}_1,\tilde{\phi}_2,\tilde{\phi}_3)$. The tilde fields have the property, $\langle\tilde{\phi}_\mu\rangle=0$, where the expectation values refer to the vacuum with the condensation Φ. The normal ordered Lagrangian becomes

$$:\mathcal{L}:=:\mathcal{L}_{\mathrm{cl}}:+:\mathcal{L}_{\mathrm{linear}}:+:\mathcal{L}_{\mathrm{int}}^{\mathrm{dep}}:+:\mathcal{L}_{\mathrm{int}}^{\mathrm{indep}}:, \tag{2a}$$

where

$$:\mathcal{L}_{\mathrm{cl}}:=\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi-\frac{\lambda}{4}(\Phi^2-v^2)^2+H\Phi_0, \tag{2b}$$

$$:\mathcal{L}_{\mathrm{linear}}:=\partial_\mu\Phi\partial^\mu\tilde{\phi}+H\tilde{\sigma}-\lambda(\Phi^2-v^2)\left(\sum_i\Phi_i\tilde{\phi}_i\right), \tag{2c}$$

$$:\mathcal{L}_{\mathrm{int}}^{\mathrm{dep}}:=-\lambda\left\{:\left(\sum_i\Phi_i\tilde{\phi}_i\right)^2:+:\left(\sum_i\Phi_i\tilde{\phi}_i\right)\tilde{\phi}^2:+\frac{1}{2}\Phi^2:\tilde{\phi}^2:\right\}, \tag{2d}$$

$$:\mathcal{L}_{\mathrm{int}}^{\mathrm{indep}}:=:\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}-\frac{\lambda}{4}\{(\tilde{\phi}^2)^2-2v^2\tilde{\phi}^2\}:. \tag{2e}$$

Here, the normal ordering is defined for the vacuum with condensation Φ. Note that $:\mathcal{L}_{\mathrm{int}}^{\mathrm{indep}}:$ does not depend on Φ explicitly.

Taking the thermal average for $\tilde{\phi}$, we have

$$\langle:\mathcal{L}_{\mathrm{linear}}:\rangle=0, \tag{3}$$

$$\langle:\mathcal{L}_{\mathrm{int}}^{\mathrm{dep}}:\rangle=-\lambda\left\{\sum_i\Phi_i^2\langle:\tilde{\phi}_i^2:\rangle+\frac{1}{2}\Phi^2\sum_i\langle:\tilde{\phi}_i^2:\rangle\right\}, \tag{4}$$

$$\langle:\mathcal{L}_{\mathrm{int}}^{\mathrm{indep}}:\rangle=\left\langle:\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}:\right\rangle-\frac{\lambda}{4}\{\langle:(\tilde{\phi}^2)^2:\rangle-2v^2\langle:\tilde{\phi}^2:\rangle\}. \tag{5}$$

The expectaion values of the cubic terms in $\tilde{\phi}$ vanish automatically in the massless free particle approximation. This approximation for the fluctuation fields $\tilde{\phi}_j$ gives [1]

$$\langle:\tilde{\phi}_j^2:\rangle=\mathrm{Tr}\{\exp(-H_0/T):\tilde{\phi}_j^2:\}=\frac{T^2}{12}, \tag{6}$$

where T is the temperature and H_0 is the free part of Hamiltonian. It should be noted that Φ-dependent normal ordering may affect the equation of motion through Φ-dependent mass. However, the effects can be neglected in the massless free particle approximation. The term $\langle:\mathcal{L}_{\mathrm{int}}^{\mathrm{indep}}:\rangle$ is independent of Φ because of the above approximation. One can write the effective potential as

$$V_{\mathrm{eff}}(\Phi,\Phi_0)=\frac{\lambda}{4}(\Phi^2-v^2)^2+\frac{1}{4}\lambda T^2\Phi^2-H\Phi_0. \tag{7}$$

The equation of motion for Φ_μ becomes

$$\Box\Phi_\mu+(A_{\mathrm{cl}}+A_{\mathrm{th},\mu})\Phi_\mu-H\delta_{\mu,0}=0, \tag{8}$$

where

$$A_{\mathrm{cl}}(\Phi)=\lambda(\Phi^2-v^2), \tag{9a}$$

$$A_{\mathrm{th},\mu}=\lambda(\langle:\tilde{\phi}^2:\rangle+2\langle:\tilde{\phi}_\mu{}^2:\rangle)=\frac{1}{2}\lambda T^2,\quad \tilde{\phi}^2=\sum_{\nu=0}^{3}\tilde{\phi}_\nu{}^2. \tag{9b}$$

The suffix cl means the classical contribution and th implies the thermal contribution.[1]

The next task is to solve Eq. (8) in A-A collision cases. In A-A collisions, the system expands initially along the collision axis and then the expansion along the transverse direction also becomes appreciable afterwards. We consider first the one-dimensional expansion. Convenient variables are the proper time $\tau=(t^2-z^2)^{1/2}$ and the rapidity $\eta=0.5\ln[(t+z)/(t-z)]$. For simplicity, the scaling case (η-independent case) is considered here [4]. Then the equation of motion depends on only the variable τ. The temperature T as a function of τ is taken as follows.

The parameter τ_{th} is introduced to represent the time dependence of the temperature. The temperature becomes maximum when $\tau=\tau_{\mathrm{th}}$ (thermalization time). Then we use a scaled proper time $x=\tau/\tau_{\mathrm{th}}$. The time dependence of the temperature after $x=1$ is determined by scaling hydrodynamics. Since the system expands one dimensionally, $T(x)$ is proportional to $x^{-1/3}$ for $x\geqslant1$. The temperature may not be well-defined for the nonequilibrium thermalization stage where $0\leqslant x\leqslant1$. Thermalization may be achieved at $x=x_t$ where $0\leqslant x_t\leqslant1$. Nevertheless, we treat the system as if it is in a thermal equilibrium for $0\leqslant x\leqslant1$ in order to parametrize conveniently the time development of the quantity $A_{\mathrm{th},\mu}$. Some trial functions of $T(x)$ are considered to check the sensitivity of the results on the choice of $T(x)$ for $x<1$.

First, we assume an exponential behavior for $0\leqslant x\leqslant1$:

$$T(x)=T_i\left(\frac{T_m}{T_i}\right)^x\theta(1-x)+T_m x^{-1/3}\theta(x-1), \tag{10a}$$

where θ is the step function. This form for $0\leqslant x\leqslant1$ is chosen to simulate the result of the parton-cascade model [5]. For comparison, we consider also two other choices for $0\leqslant x\leqslant1$,

$$T(x)=(T_m-T_i)x+T_i, \tag{10b}$$

$$T(x)=\frac{(T_m-T_i)}{\ln 2}\ln(x+1)+T_{\mathrm{i}}, \tag{10c}$$

and Eq. (8) is now rewritten as

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[1]The effects of back reaction are ignored in our calculation. However, it is expected that the effects do not change the main results because the energy of the environment is much larger than that of the condensates in the thermalization stage and also in the cooling stage before freeze-out.

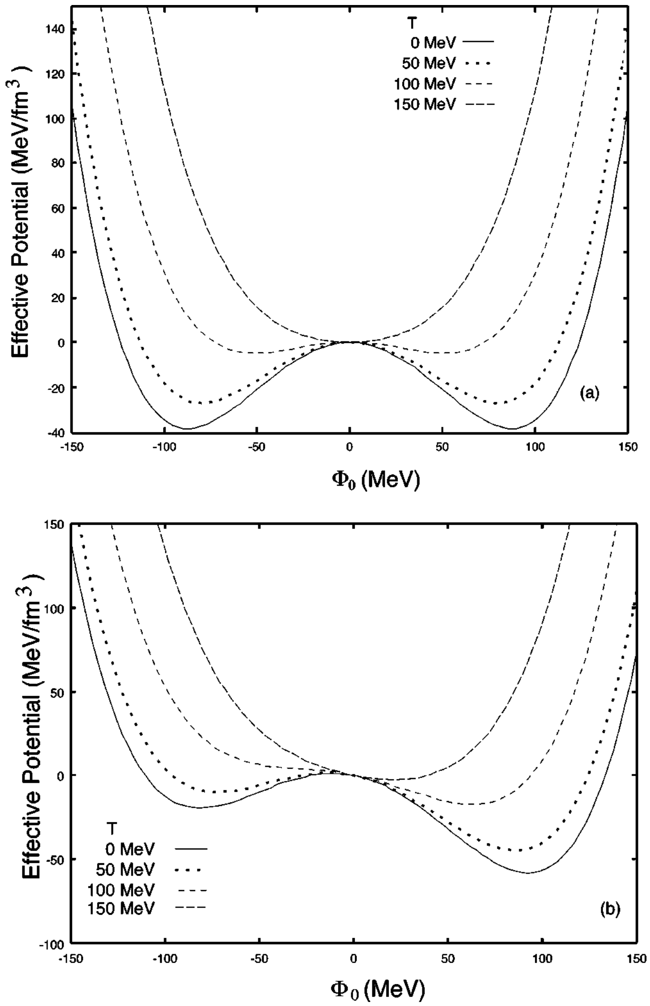

FIG. 1. Finite-temperature effective potential without (a) and with (b) the explicit breaking term at various temperatures.

$$\left(\frac{\partial^2}{\partial\tau^2}+\frac{1}{\tau}\frac{\partial}{\partial\tau}+A_{\rm cl}+A_{\rm th}\right)\Phi_\mu-H\delta_{\mu,0}=0, \tag{11}$$

where

$$A_{\rm cl}=\lambda(\Phi^2-v^2)-\Delta_{\rm tr}, \quad A_{\rm th}=\frac{1}{2}\lambda T^2(\tau).$$

The factor $\Delta_{\rm tr}$ comes from the derivatives $\partial^2/\partial x^2$ and $\partial^2/\partial y^2$. However, we neglect $\Delta_{\rm tr}$ since we consider the case where the system is homogeneous along the transverse direction, that is, we neglect the effect of a finite transverse size of the colliding system.

For comparison, we consider also the three-dimensional expansion [1]. The equation of motion is

$$\left(\frac{\partial^2}{\partial\tau^2}+\frac{3}{\tau}\frac{\partial}{\partial\tau}+\lambda(\Phi^2-v^2)+\frac{1}{2}\lambda T^2\right)\Phi_\mu-H\delta_{\mu,0}=0, \tag{12}$$

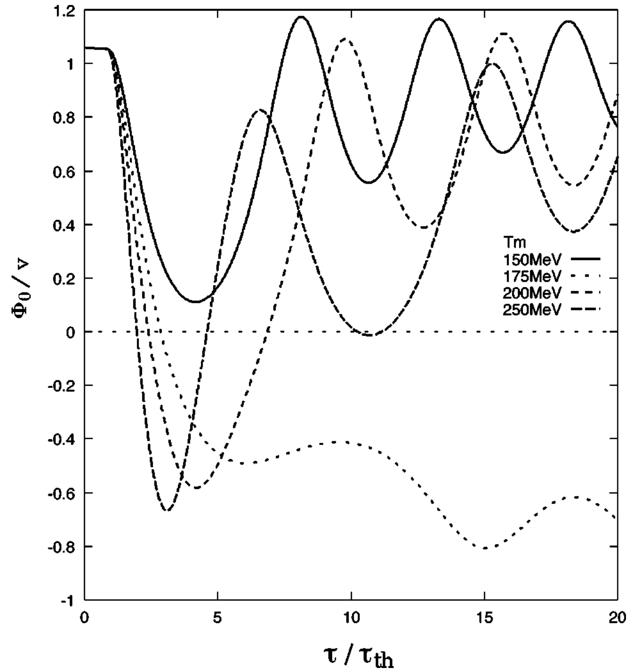

FIG. 2. Time evolution of chiral condensate for various maximum temperatures with $\tau_{\rm th}=0.5$ fm and $T_i=1.0$ MeV in the one-dimensional expansion.

where $\tau=(t^2-x^2-y^2-z^2)^{1/2}$. The time dependence of temperature is taken as

$$T(x)=T_{\rm i}\left(\frac{T_m}{T_i}\right)^x\theta(1-x)+\frac{T_m}{x}\theta(x-1). \tag{13}$$

III. NUMERICAL RESULTS

In this section, the results of numerical calculations are presented. Equations of motion (11) and (12) are solved numerically for given initial conditions. The linear σ model has three parameters λ, v, and H which are determined by the pion mass, the σ mass, and the pion decay constant $f_\pi \sim 92.5$ MeV. The physical vacuum is located at the minimum of the potential, $(\Phi_0/v,\vec{\Phi}/v)=(f_\pi/v,\vec{0})$.

Pion and σ meson masses are taken as 135 and 600 MeV, respectively. Then, one has $\lambda=20.0$, $v=87.4$ MeV, and $H=(119\ \text{MeV})^3$. These values of λ and v with $H=0$ yield the critical temperature for chiral symmetry restoration $T_c \sim 123$ MeV [see Fig. 1(a)]. In the realistic case, the critical temperature cannot be determined exactly since H is nonzero. However, one can estimate the temperature T_c^* at which one (false) minimum disappears. It is given by

$$T_c^*=\sqrt{2}\left[v^2-\frac{3}{2^{2/3}}\left(\frac{H}{\lambda}\right)^{2/3}\right]^{1/2}. \tag{14}$$

This temperature is about 90 MeV [see Fig. 1(b)], which is considerably smaller than T_c.

The initial condition is taken to be $\Phi=(f_\pi,\vec{0})$ and $d\Phi/d\tau\simeq 0$. As an order of magnitude guess, we assume that $(d\Phi_\mu/d\tau)^2\simeq(d\Phi_\mu/dt)^2\simeq\langle:(d\tilde{\phi}_\mu/dt)^2:\rangle$:

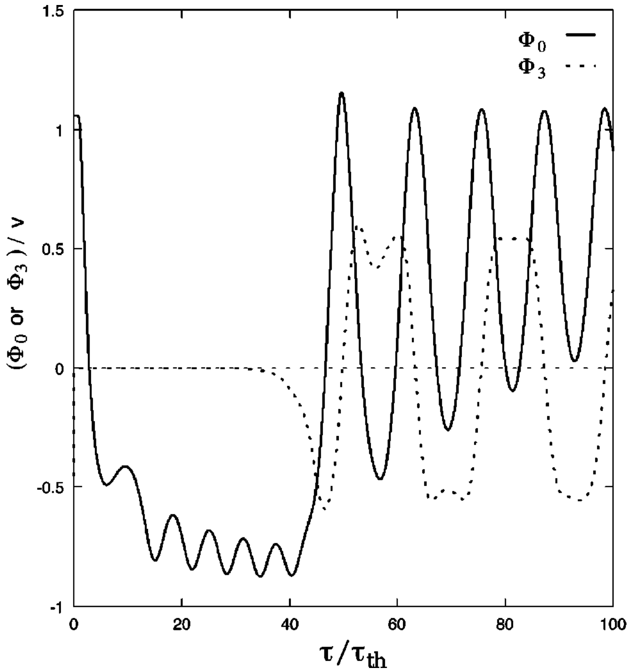

FIG. 3. Time evolution of sigma and pion (π^0) condensates in one-dimensional expansion for $T_m = 175$ MeV, $\tau_{th} = 0.5$ fm, and $T_i = 1$ MeV.

$$\left(\frac{d\Phi_0}{d\tau}, \frac{d\Phi_1}{d\tau}, \frac{d\Phi_2}{d\tau}, \frac{d\Phi_3}{d\tau}\right)\bigg|_{\tau=0} = \left(-\frac{\pi T_i^2}{\sqrt{30}}, \frac{\pi T_i^2}{\sqrt{30}}, \frac{\pi T_i^2}{\sqrt{30}}, \frac{\pi T_i^2}{\sqrt{30}}\right). \tag{15}$$

Here, the sign is chosen arbitrarily.

The time dependence of the condensation for various T_m in the one-dimensional expansion case is shown in Fig. 2, where we take $T_i = 1$ MeV and $\tau_{th} = 0.5$ fm. The condensation value gives us the information on chiral symmetry restoration. Vanishing condensation implies chiral symmetry restoration. It is found that the condensation of σ cannot reach zero even when $T_m = 160$ MeV$> T_c$. Temporal restoration occurs for $T_m \geqslant 175$ MeV. It should be noted that the condensation does not exactly follow the minimum of the effective potential. This is due to a dynamical retardation effect. In the case when $T_m = 175$ MeV, the vacuum stays at

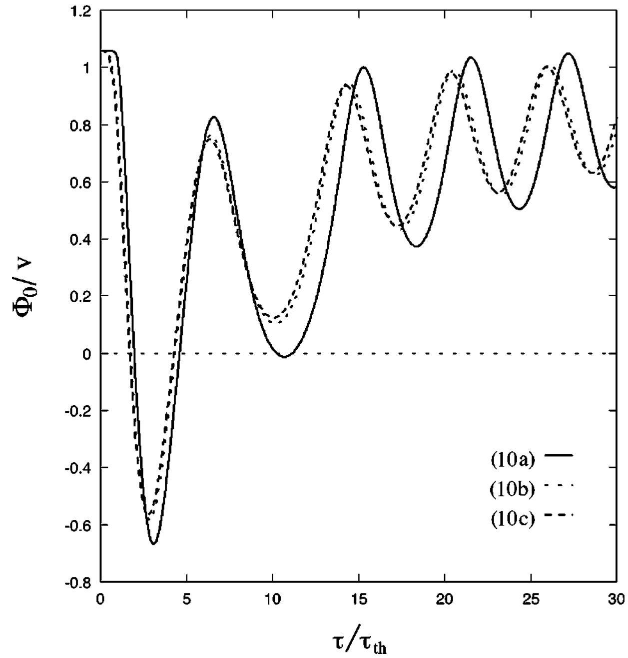

FIG. 5. Time development of the condensate for various thermalization functions with $\tau_{th} = 0.5$ fm and $T_m = 250$ MeV.

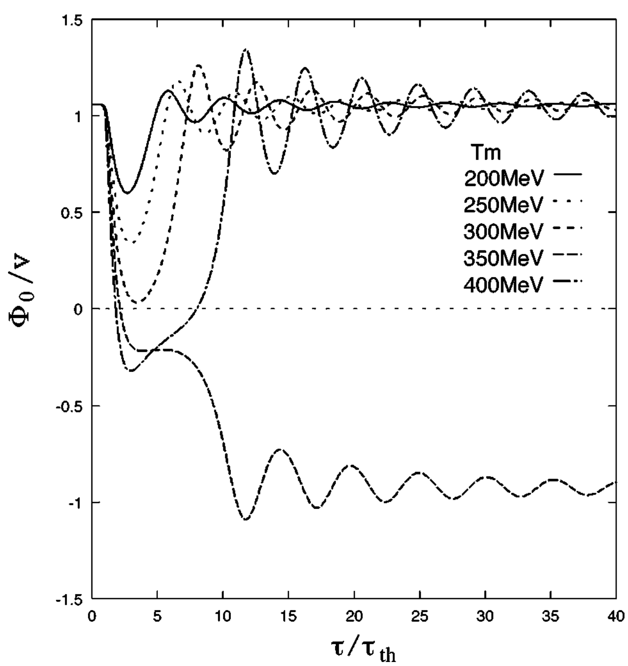

FIG. 4. Time evolution of chiral condensate for various maximum temperatures with $\tau_{th} = 0.5$ fm and $T_i = 1.0$ MeV in three-dimensional expansion.

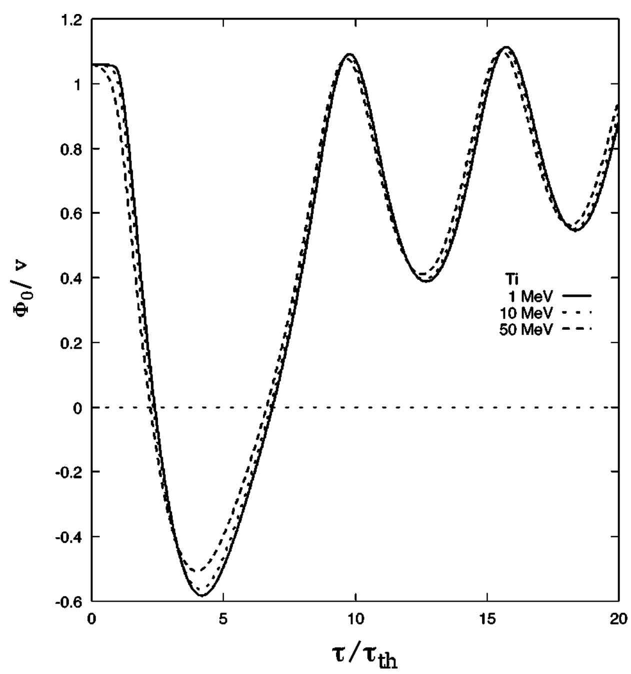

FIG. 6. Time development of the condensate for various initial temperatures with $\tau_{th} = 0.5$ fm and $T_m = 200$ MeV.

the minimum (really a saddle point) $\Phi_\mu = (-f_\pi, \vec{0})$ for a while and then comes back to the true minimum. This behavior is distinctly different from the other cases. The long stay at the false minimum implies just DCC formation.

These behaviors of condensation are understood as follows. One is the case with relatively low $T_m \geqslant T_c$ (the case with $T_m = 150$ MeV in Fig. 2). In this case, condensation cannot reach zero since the system cools rapidly. The potential becomes a double-well type rapidly enough before condensation goes through zero and hence the chiral symmetry is not restored. It is restored if T_m is much higher. In this case (the cases with $T_m = 200$ MeV and 250 MeV in Fig. 2), the condensation can go through zero and reach near the opposite side of the chiral circle. Afterwards, it comes back to zero again and finally stays at the true minimum of the potential. The last case is DCC formation case. In this case (the case with $T_m = 175$ MeV in Fig. 2), condensation cannot cross zero again because the system cools quickly. The vacuum stays at the other minimum of the potential for a long time. It is just DCC. Of course, the vacuum moves along the chiral circle and eventually comes back to the true minimum as shown in Fig. 3. The π^0 condensation stays almost zero and only σ condensation varies at an early stage. After the vacuum stays at the false minimum (really a saddle point) for a while, both the pion and the σ condensations start to vary. The vacuum moves along the chiral circle and makes a damped oscillation.

In the same way, the time development of vacuum condensate was also investigated for the three-dimensional expansion. The result is shown in Fig. 4. Since the decrease of temperature is too rapid, there is not enough time for the chiral symmetry to be restored unless T_m is very high. The chiral symmetry is restored only when $T_m \geqslant 300$ MeV $\geqslant T_c \sim 123$ MeV. Hereafter, we focus on the one-dimensional expansion case which is more realistic at very high energies.

We found that, in most cases, the time development of the condensation is insensitive to the choice of $T(x)$ for $0 \leqslant x \leqslant 1$. An example for Φ_0 is shown in Fig. 5. We also found that the result is insensitive to T_i as shown in Fig. 6. Finally, we study the dependence on the thermalization time. An example is shown in Fig. 7. Now the result is sensitive to $\tau_{\rm th}$. The rapid thermalization (small $\tau_{\rm th}$) implies that the system starts to cool at an early time. Therefore, a decreasing $\tau_{\rm th}$ with a fixed T_m is effectively and approximately equivalent to a decreasing T_m with a fixed $\tau_{\rm th}$, as is seen in Fig. 7. (Compare it with Fig. 2.)

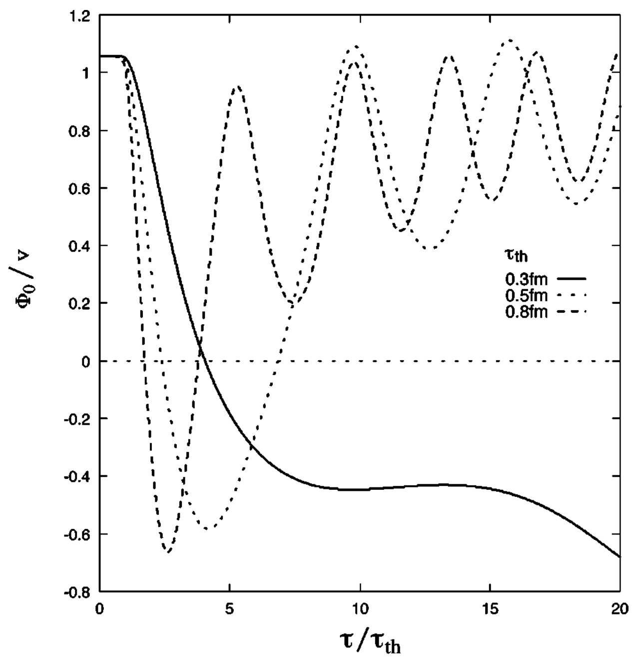

FIG. 7. Time development of the condensate for various thermalization times with $T_m = 200$ MeV and $T_i = 1$ MeV.

IV. CONCLUSIONS AND DISCUSSIONS

We have investigated the time development of the chiral condensates which may take place in ultrarelativistic nucleus-nucleus collisions. We have used the linear σ model with the temperature-dependent effective potential as a framework to describe effectively the chiral phase transition in both the thermalization and the cooling stages. The time dependence of temperature in the thermalization stage was parametrized to simulate the result of the parton-cascade model and that in the cooling stage was described in terms of the one- and three-dimensional hydrodynamical scaling. It was found that (a) in general the condensates exhibit characteristic damped oscillations of which patterns are sensitive to both the maximum temperature and the time when the maximum temperature is attained, (b) temporal restoration of chiral symmetry can take place only when T_m is much larger than T_c, and (c) a large domain of DCC may be formed most easily in the $-\sigma$ direction. The oscillations damp toward the asymptotic values given by the physical vacuum at zero temperature. The points (a) and (b) thus imply that the condensates do not stay near the origin of the chiral space for a long time even when the symmetry is restored. It is highly unlikely that the condensates are nearly at rest near the origin. Therefore, it is not expected that the rolldown towards every direction takes place with equal probability. Our result suggests that the rolldown toward the $-\sigma$ direction is most probable. These phenomenon are caused by the initial condition and the change of environment expected in ultrarelativistic heavy-ion collisions.

In our model, the effective potential changes continuously as time goes on while the condensates do not follow the minimum of the potential because the potential changes rapidly. In this sense, our scenario is in between a quenching scenario and an annealing one.

We are aware that the massless free particle approximation is not necessarily reliable at low temperature. The effects of finite mass and the friction [6] have to be taken into account in order to improve the theory. The finite volume of the colliding system may also give a considerable effect. However, we expect that the qualitative features of our results will be preserved even after such an improvement provided the friction is not very strong. A crucial point is that the time scale of the change of the condensates is comparable to that of the change of temperature. This is the reason why our picture is in between quenching and annealing. We expect that this feature will be retained in an improved theory which is under investigation.

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