Combined effects of nuclear Coulomb field, radial flow, and opaqueness on two-pion correlations

H. W. Barz
Institut für Kern- und Hadronenphysik, Forschungszentrum Rossendorf, Postfach 510119, D-01314 Dresden, Germany

(Received 25 June 1998)

Correlations of two like charged pions emitted from a hot and charged spherically expanding nuclear system are investigated. The motion of the pions is described quantum mechanically using the Klein-Gordon equation which includes Coulomb field and pion absorption. Flow modifies the radial distribution of the source function and rescales the pion wave functions. The radii extracted from the correlation functions are calculated in sideward and outward direction as a function of the pair momentum. Comparison with recent measurements at bombarding energies of 1 GeV and 11 GeV per nucleon is made. [S0556-2813(99)02504-2]

PACS number(s): 25.75.Gz, 25.75.Ld

I. INTRODUCTION

Measurements of correlations of pions and kaons as a function of their relative momenta are used to determine the source radius in heavy-ion reactions at intermediate and relativistic energies. On the basis of the Hanbury-Brown and Twiss effect (see [1,2]) the correlation function is related to the source radius via $R_0=\hbar c/\Delta q$, where Δq is the width of the correlation function. However it is well known [3,4] that the correlation is a complicated function not only of the relative momentum but also of the total momentum of the pion pair. The correlation is determined by the size of the region from which pions are emitted with roughly the same momenta. This has the consequence that for collectively streaming matter this region is smaller than the total source due to the strong correlation between the momenta and the emission points of the particles. A further modification of the apparent source size arises if pions are often rescattered within the source. Thus, for pions the source is opaque [5], and their emission points lie within a thin surface layer.

On the other hand, the central Coulomb force changes the momenta of the particles while moving towards the detector. This latter effect was investigated in Refs. [6,7]. Pions with small momenta are mostly influenced by the Coulomb forces which act quite differently on positively and negatively charged pions. Indeed quite different radii have been observed recently in collisions of Au on Au at bombarding energies of 1 GeV [8] and 11 GeV per nucleon [9] at the SIS and AGS accelerators, respectively.

The aim of this work is to study the combined effects of central charge, opaqueness, and flow on the extracted radii. As a model we consider the emission of pions from a hot spherical nucleonic system, the expansion of which can described by radial hydrodynamical flow. To ease the necessary numerical calculations we use a spherical spatial distribution while for the momentum distribution a relativistic Boltzmann distribution is used. Starting from a covariant formalism for two particle emission (Sec. II) we derive the source function in Sec. III and obtain a concise expression for the matrix element for the emission of two particles. Once we have obtained the matrix element the standard technique is applied to calculate the correlation function. Coulomb field and opaqueness are included via mean fields in calculating the distorted waves for the pions. Numerical results are finally discussed in Sec. VI.

II. BASIC EQUATIONS

Here, we briefly review the formulation of the Hanbury-Brown and Twiss effect which allows us to incorporate the mean fields between the source and the two emitted mesons. At asymptotic distance the two mesons move with momenta $\mathbf{p}$ and $\mathbf{p}'$. Each meson is described by a wave function $\psi_p(x)$ which satisfies an equation of motion which contains the mean field, and $p=(\omega,\mathbf{p})$ and $x=(t,\mathbf{r})$ denote[1] the four-momentum and the position in time and space, respectively. It is customary to introduce a source term $J(x)$ from which the wave function ψ_p is generated. With these definitions one can express [1,10] the probability for the emission of two meson in terms of the source operators $J^+(x),J(x)$ as

$$\omega\omega'\frac{dN}{d\mathbf{p}d\mathbf{p}'}\sim\int d^4x_1d^4x_2d^4x_3d^4x_4\psi_p^*(x_1)\times\psi_{p'}^*(x_2)\psi_{p'}(x_3)\psi_p(x_4)\times\langle J^+(x_3)J^+(x_4)J(x_1)J(x_2)\rangle. \tag{1}$$

The basic assumption for applying interferometry to nuclei is the chaoticity of the source i.e., the absence of initial correlations between the two emitted pions except those correlations coming from the Bose-Einstein statistics. Thus, one writes

$$\langle J^+(x_3)J^+(x_4)J(x_1)J(x_2)\rangle=\langle J^+(x_4)J(x_1)\rangle\langle J^+(x_3)J(x_2)\rangle+\langle J^+(x_3)J(x_1)\rangle\times\langle J^+(x_4)J(x_2)\rangle. \tag{2}$$

This allows one to express Eq. (1) as products of density matrices

$$\omega\omega'\frac{dN}{d\mathbf{p}d\mathbf{p}'}\sim\rho(p,p)\rho(p',p')+|\rho(p,p')|^2 \tag{3}$$

[1]The convention $\hbar=c=k_{\mathrm{Boltzmann}}=1$ is used.

with

$$\rho(p,p')=\int\int d^4x d^4x' \psi_p^*(x)\psi_{p'}(x')\langle J^+(x')J(x)\rangle, \tag{4}$$

which contains the one-body source term. This term can be connected to a classical source function $S(k,x)$ via a Wigner transformation

$$S(k,x)=\int d^4x' e^{ikx'}\langle J^+(x-x'/2)J(x+x'/2)\rangle, \tag{5}$$

which describes the creation of a meson at spacetime x with four-momentum k.

The problem is further essentially reduced by assuming that the interactions do not depend on time. This assumption allows the use of stationary solutions for $\psi_p(x) =\exp(-i\omega t)\psi_{\mathbf{p}}(\mathbf{r})$ with $\omega=p^0=\sqrt{m^2+\mathbf{p}^2}$. Inserting these functions into Eq. (4), using definition (5) and integrating over the variable $t-t'$ we obtain

$$\rho(p,p')=\int\int d\mathbf{r}d\mathbf{r}' \psi_{\mathbf{p}}^*(\mathbf{r})\psi_{\mathbf{p}'}(\mathbf{r}') \times\int\frac{d\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}\int dt e^{i(\omega-\omega')t}S \times\left[\left(\frac{\omega+\omega'}{2},\mathbf{k}\right),\left(t,\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\right]. \tag{6}$$

In the interference term for $p\neq p'$ the energy of the pion in the source function is fixed to the mean energy of the observed pions. If plane waves are used the integration over the difference $\mathbf{r}-\mathbf{r}'$ can be carried out which fixes the $\mathbf{k}$ momentum to half the pair momentum leading to the standard form [3] of the matrix element.

Now, the correlation function C_2 is defined as the two-particle emission function normalized to the product of the one-particle emission functions which is given by the first part in Eq. (3). It is convenient to introduce the average pair momentum $\mathbf{K}$ and the relative momentum $\mathbf{q}$ via

$$\mathbf{K}=\frac{1}{2}(\mathbf{p}+\mathbf{p}'),\quad \mathbf{q}=\mathbf{p}-\mathbf{p}'. \tag{7}$$

Then, the correlation function reads

$$C_2(\mathbf{K},\mathbf{q})=1+\frac{|\rho(K-q/2,K+q/2)|^2}{\rho(K-q/2,K-q/2)\rho(K+q/2,K+q/2)}. \tag{8}$$

For completeness we mention that the final-state interaction [11,12] between the two outgoing mesons has not been considered. Inclusion of this interaction would require replacing the product of the two outgoing waves $\psi_{\mathbf{p}}(x)\psi_{\mathbf{p}'}(x')$ in Eq. (1) by a correlated wave function $\Psi(x,x')$. Including the interaction with the source the function $\Psi(x,x')$ is, however, the complicated solution of a genuine three-body problem. As a first approximation one could assume that the source interacts mainly with the center-of-mass of the pair while the final-state interaction is a function of the relative distance between the outgoing particles only. As a consequence the function Ψ could be split up into a product leading to the standard expression

$$C_2(\mathbf{K},\mathbf{q})_{\text{final state}}=P(q)C_2(\mathbf{K},\mathbf{q}), \tag{9}$$

where the factor $P(q)$ is in the simplest case the Coulomb penetrability. In the case of the Coulomb interaction such an approximation means that the quadrupole and higher momenta which influence the relative motion of the pair are neglected. Alternatively, one can try to factorize the wave function Ψ as it has been done in calculating proton-proton correlations [13].

III. HYDRODYNAMICAL PICTURE

Now we restrict ourselves to a simple situation. We consider a hot spherical source from which pions are emitted. Since the source expands the problem is not stationary. However, we circumvent this difficulty by considering only mesons which move with sufficiently large velocities. Those mesons are essentially outside the source and always feel a time independent Coulomb potential. Having in mind a collision of Au on Au nuclei at a few A GeV generating a source of a temperature of about 100 MeV, the thermal velocity of the protons in the source is about 0.4 c embedded in a flow field of an average velocity of about 0.3 c. Thus, the model may be applicable for pion velocities larger than 0.7 c corresponding to pion momenta above 100 MeV/c or kaon momenta above 300 MeV/c. However, if the particles are released in the center of the source they stay a while within the decreasing part of the Coulomb potential. Within our model the only way to treat this problem is diminishing the central charge in a heuristic manner. Such modifications of the effective charge have been observed [14] in describing the π^-/π^+ ratio.

For ultrarelativistic energies the system does not expand spherically but is preferentially stretched in the longitudinal direction. Since our treatment neglects this dimension we do not consider the longitudinal correlation function. Due to the longitudinal expansion the Coulomb force decreases with time. As an approximate measure of the Coulomb action one can replace the central charge Z with twice the rapidity density $2(dN^+/dy-dN^-/dy)$ of the net charge as was shown in Ref. [15].

In the hydrodynamical approach the momentum distribution is defined by a local temperature T and a velocity field given by the four-vector u^μ with $u^\mu u_\mu=1$. Here we use typical parameters for a heavy-ion reaction: constant temperature T, radial mean velocity $\langle\beta\rangle$, and radial size R_0. Thus, we start with the source function

$$S(k,x)=\frac{1}{4\pi^2R_0^3\tau}e^{-ku/T-\mathbf{r}^2/2R_0^2-t^2/2\tau^2}. \tag{10}$$

The pions are radiated off during the emission time τ. The thermal distribution of the momenta within a fluid cell is coupled via $ku=k^0u^0-\mathbf{ku}$ to the four-velocity u^μ of the cell. We assume that the source function is obtained from Eq. (5) and can be taken off-shell. For large pion density one should replace the Jüttner or relativistic Boltzmann distribution with a Bose distribution. A spherically expanding system can described by the flow velocity field

$$u^{\mu}=(u^0,\mathbf{u}), \quad u^0=\sqrt{1+\mathbf{u}^2}, \quad \mathbf{u}=\beta_0\mathbf{r}/R_0, \tag{11}$$

where the four-velocity scales with the distance from the center. The parameter β_0 is related to the mean flow velocity $\langle\beta\rangle=\langle|\mathbf{u}|/u^0\rangle$ averaged over the density (10) which leads to $\langle\beta\rangle=\sqrt{8/\pi}\beta_0$ in the limit of small β_0. Since β_0 characterizes a four-velocity it does not have an upper bound, contrary to $\langle\beta\rangle<1$. In the Appendix we discuss the case of a hard-sphere density distribution together with a linearly increasing velocity field as a function of the radius which is often used to describe transverse momentum spectra.

Inserting the source function (10) into Eq. (6) and integrating over the time we obtain

$$\rho(p,p')=\frac{1}{(2\pi)^{9/2}R_0^3}e^{-(\omega-\omega')^2\tau^2/2} \times\int d\mathbf{r}d\mathbf{r}'\,\psi^*_{\mathbf{p}}(\mathbf{r})\psi_{\mathbf{p}'}(\mathbf{r}') \times\int d\mathbf{k}e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}e^{\mathbf{ku}/T} \times e^{-(\mathbf{r}+\mathbf{r}')^2/(8R_0^2)-(\omega+\omega')\sqrt{1+\mathbf{u}^2}/(2T)}, \tag{12}$$

where the velocity field $\mathbf{u}$ is proportional to $(\mathbf{r}+\mathbf{r}')/2$. In the range of the convergence of this integral the integration over $\mathbf{k}$ leads to a δ function $\delta(\mathbf{r}-\mathbf{r}'-i\mathbf{u}/T)$ which allows one to carry out the integration over $(\mathbf{r}-\mathbf{r}')$. Thus, the final expression reads

$$\rho(p,p')=\frac{1}{(2\pi)^{3/2}R_0^3}e^{-(\omega-\omega')^2\tau^2/2} \times\int d\mathbf{r}[\psi_{\mathbf{p}}(\alpha\mathbf{r})]^*\psi_{\mathbf{p}'}(\alpha\mathbf{r}) \times e^{-\mathbf{r}^2/(2R_0^2)-(\omega+\omega')\sqrt{1+(\beta_0\mathbf{r}/R_0)^2}/(2T)}. \tag{13}$$

The factor $\alpha=1-i\beta_0/(2TR_0)$ rotates the integration path for the distorted-wave functions $\psi_{\mathbf{p}}$ into the complex plane.

Using Eq. (13) we obtain the correlation function from Eq. (8). Since the system is spherically symmetric we investigate the correlation as a function of the the momentum-difference vector $\mathbf{q}_{\text{out}}$ pointing in the direction of the average pair momentum $\mathbf{K}$ and $\mathbf{q}_{\text{side}}$ being orthogonal to $\mathbf{K}$. It turned out that the correlation function C_2 can well be approximated by

$$C_2=1+\exp(-\mathbf{q}^2_{\text{side}}R^2_{\text{side}}-\mathbf{q}^2_{\text{out}}R^2_{\text{out}}), \tag{14}$$

where the Hanbury-Brown and Twiss (HBT) radii R_{side} and R_{out} characterize the extension of the source in and perpendicular to the direction of the pair momentum $\mathbf{K}$.

IV. CASE OF ZERO CHARGE

Before we turn to the full problem let us discuss the case of a negligible potential U. The distorted waves in Eq. (13) can be replaced by plane waves

$$[\psi_{\mathbf{p}}(\alpha\mathbf{r})]^*\psi_{\mathbf{p}'}(\alpha\mathbf{r})=\exp[-i\,\text{Re}(\alpha)\mathbf{qr}+2\,\text{Im}(\alpha)\mathbf{Kr}] \tag{15}$$

using the definitions (7). To discuss the main effect it is instructive to find an analytical approximation. Expanding the integrand in Eq. (13) for small flow velocities up to order β_0^2 and using $\omega-\omega'=\mathbf{Kq}/\tilde{m}$ for small momenta $\mathbf{q}$ one obtains from Eq. (8) the result

$$C_2=1+\exp\left\{-q^2\left[\left(|\alpha|^2\frac{R_0^2}{1+\beta_0^2\tilde{m}/T}\right)+\left(\frac{\mathbf{qK}}{q\tilde{m}}\right)^2\tau^2\right]\right\} \tag{16}$$

with $\tilde{m}=\sqrt{m^2+\mathbf{K}^2}$.

Comparing the last equation with Eq. (14) one identifies the first expression in the round bracket with the sideward radius while the whole square bracket is the outward radius with $|\mathbf{K}|/\tilde{m}$ being the pair velocity. The value of $|\alpha|^2$ exceeds unity by at most a few percent for realistic values of flow velocities. Equation (16) contains the well-known result [3] that the radial flow reduces the observed radius R_{side} with increasing pair momentum and decreasing temperature. From the approximate behavior of the radius $R_0/\sqrt{1+\beta_0^2\tilde{m}/T}$ one recognizes that the dependence on the pair momentum is essentially a relativistic effect that is caused by the change of the relativistic pion mass $\tilde{m}$. Equation (16) also contains the fact that the outward radius is larger for a finite pion emission time τ. However, the effect of mean fields could violate this statement as shown in the following.

V. NUMERICAL TREATMENT

To incorporate the mean field for the pion with energy $\omega=\sqrt{m^2+\mathbf{p}^2}$ we solve the Klein-Gordon equation

$$\left[-\frac{\partial^2}{\partial\mathbf{r}^2}-(\omega-U)^2+m^2\right]\psi^{(-)}_{\mathbf{p}}(\mathbf{r})=0. \tag{17}$$

The boundary conditions are chosen such that ψ behaves asymptotically like an outgoing wave in the direction of $\mathbf{p}$ with incoming spherical waves. This is indicated by the upper index $(-)$. We mention that functions with outgoing spherical waves can also be used to calculate the matrix element (4) applying the relation $\psi^{(-)}_{\mathbf{p}}=\psi^{(+)*}_{-\mathbf{p}}$.

The potential U

$$U=\pm Z\frac{e^2}{r}\Phi\left(\frac{r}{\sqrt{2}R_0}\right)+i\frac{p}{\omega}\frac{1}{2\lambda}, \tag{18}$$

contains the Coulomb potential of the Gaussian source (10) with charge number Z, and the quantity Φ denotes the error function. Further we include the possibility that the pions might be absorbed within the source. For this purpose we have also introduced an imaginary part which depends on the mean free path λ. The positive imaginary part ensures that the wave function $\psi^{(-)}_{\mathbf{p}}$ increases in the direction of the outgoing momentum $\mathbf{p}$. The potential (18) is only a rudiment of the standard pion potentials used in calculating pion-nucleus scattering and is usually derived as part in a Schrödinger equivalent equation, see e.g., Refs. [16,17]. However, the simple form of Eq. (18) suffices for the study of the effect of opaqueness which arises from the absorption and reemission of pions in the matter.

The solution is numerically obtained by expanding the distorted wave into partial waves f_l

$$\psi_{\mathbf{p}}^{(-)}(\mathbf{r})=\frac{4\pi}{pr}\sum_{lm} i^l e^{-i\sigma_l} f_l(r) Y_{lm}^*(\mathbf{p}) Y_{lm}(\mathbf{r}), \qquad (19)$$

where the quantities Y_{lm} denote the spherical harmonics and the symbols σ_l are the Coulomb scattering phases. In order to use the nonrelativistic standard method known from optical model calculations the numerical integration was extended to large radii $R_{\max}$ to render the term U^2 in Eq. (17) negligible. Once the correct radial function has been obtained it is analytically continued by integrating the radial differential equation from $r=R_{\max}$ to $r=\alpha R_{\max}$. The value obtained at this point is used to normalize the function $\psi(\alpha r)$ obtained by integrating the radial equation along the path αr.

VI. RESULTS AND DISCUSSION

We study the model for a situation which is typical for a collision of Au on Au nuclei at bombarding energies of 1 GeV per nucleon. In nearly central collisions a system of charge $Z=120$ is formed which a temperature of about $T=80$ MeV for pions and a flow velocity of about $\langle\beta\rangle=0.32$ [18,19]. We use a source radius of $R_h=10$ fm which corresponds to the parameter $R_0=R_h/\sqrt{5}$.

The correlation function (8) has been calculated for various pair momenta $\mathbf{K}$ as a function of the relative momentum q. In all cases the obtained correlation function has nearly a Gaussian shape and is fitted to Eq. (14) in the region of $C_2=1.5$ to obtain the HBT radii R_{out} and R_{side}.

In Fig. 1 the ratios of the fitted source radii to the true radius are displayed as a function of the averaged pair momentum $|\mathbf{K}|$ for two extreme values of the flow velocity $\langle\beta\rangle=0$ and $\langle\beta\rangle=0.5$. Compared to R_0 the observed radii R_{side} are increased for negatively charged pions and diminished for positively charged pions in comparison to the true radius. An opposite but smaller effect is seen for the radii R_{out}. These changes are significant for pions with momenta below 300 MeV/c. Results for pair momenta smaller than 100 MeV are not shown since the stationary approach is not justified. Below the Coulomb threshold the behavior of the radius changes drastically, see Ref. [6] for details. For comparison we have inserted into the right-hand panel of Fig. 1 the sideward radius extracted for π^0 mesons. This curve nearly averages the lines for the charged pions. Comparing the curves for the two flow velocities one recognizes that the corrections arising from the Coulomb field and the flow field add up nearly independently. Figure 2 shows the ratio R_{HBT}/R_0 for a duration of the emission of $\tau=4$ fm/c which leads to the expected increase of the outward radius.

Differences between extracted HBT radii for positively and negatively pions have been measured at AGS [9] in the projectile rapidity region. A ratio of $R_{\text{side}}(\pi^-\pi^-)$ to $R_{\text{side}}(\pi^+\pi^+)$ of (5.6 ± 0.7) fm/(3.9 ± 0.8) fm $=1.4\pm0.3$ has been found which agrees with our predictions. The ratio of the outward radii of (5.8 ± 0.5) fm/(6.5 ± 0.5) fm$=0.9\pm0.15$ is smaller than unity although the relatively large error bars do not allow a definite comparison. These measurements also qualitatively agree with the ratio of 1.2 ± 0.4 of the sideward radii observed by Pelte *et al.* [8]. In those measurements the same increase was found for the radius R_{out} contradicting our predictions.

Now we investigate the effect of opaqueness of the source. In Ref. [5] a drastic change of the HBT radii was

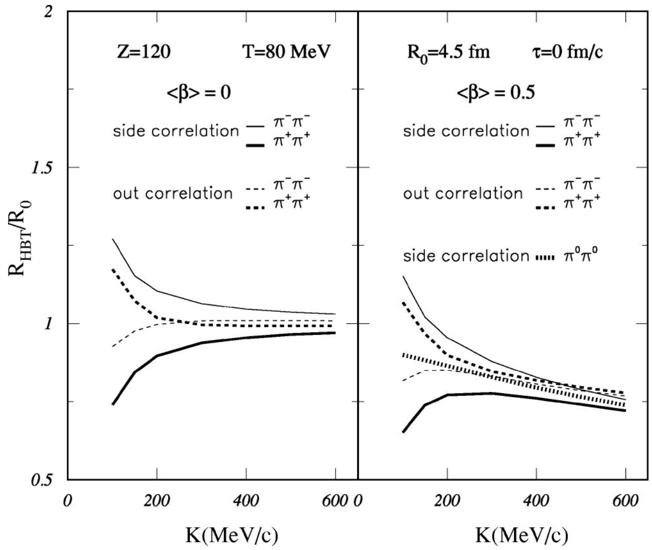

FIG. 1. Ratios of sideward and outward HBT radii to the true radius R_0 of a Gaussian-shaped source as a function of half the pair momentum K. The ratios have been calculated without (left panel) and with (right panel) radial flow of mean velocity $\langle\beta\rangle$.

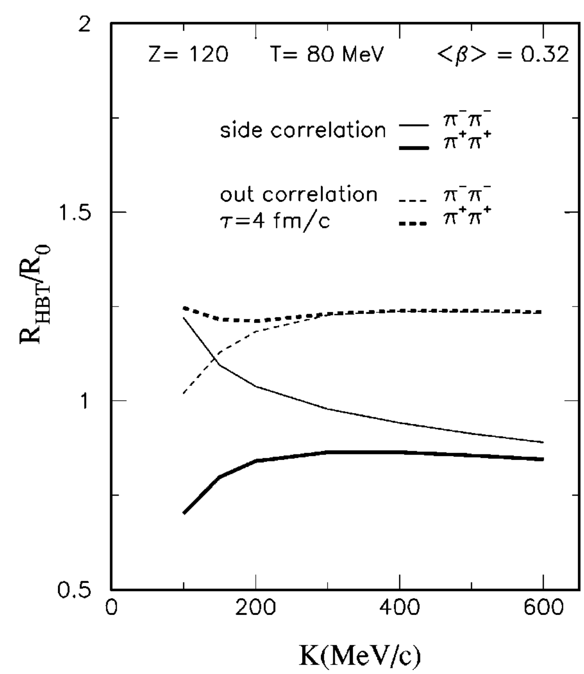

FIG. 2. Ratios of sideward and outward HBT radii to the radius R_0 of a Gaussian-shaped source with radial flow for an emission time of 4 fm/c.

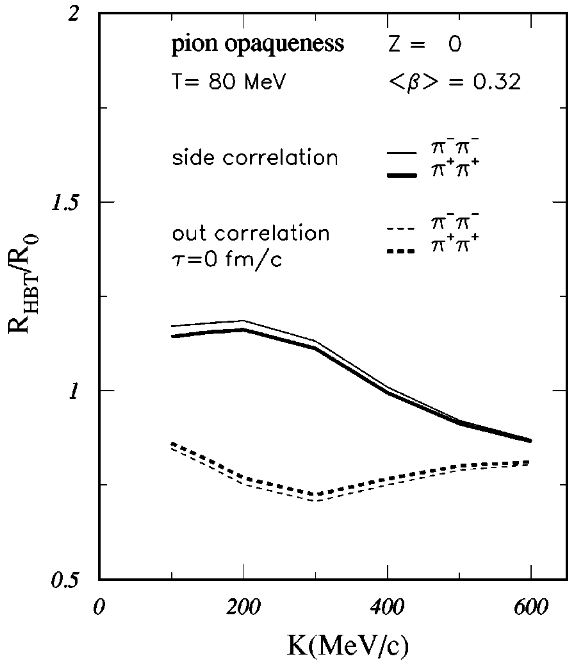

FIG. 3. Ratios of sideward and outward HBT radii to the true radius R_0 affected by flow and opaqueness of the source as a consequence of the small mean free path of pions within the source.

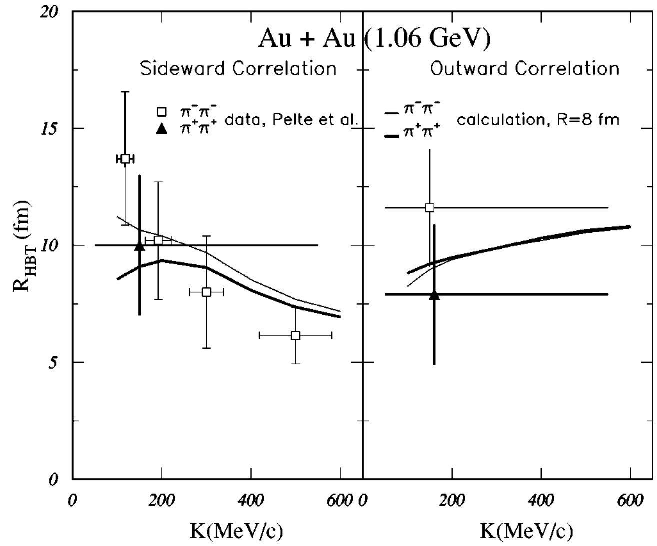

FIG. 4. Comparison of measured sideward and outward radii [8] to calculated HBT radii affected by flow, central Coulomb field, and opaqueness as a function of half the pair momentum. The large extension of some of the horizontal error bars indicates that the full range of pair momenta has been used to extract the radius. The calculations are carried out with a fixed source radius of $R_h=8$ fm which cannot fully explain the momentum dependence of the measured sideward radius.

predicted. Pions with momenta around $k_0=270$ MeV/c have a large total cross section with nucleons exciting strongly the Δ resonance. Therefore, those pions have their last interaction points within a thin surface zone near the direction of their momenta. The thickness of this zone is determined by the mean free path λ of the pions. The inverse path length is estimated to be

$$\frac{1}{\lambda_{\pi^\pm}}=n\left(\sigma_n^{\pi^\pm}\frac{N}{A}+\sigma_p^{\pi^\pm}\frac{Z}{A}\right), \tag{20}$$

which is proportional to the baryonic density n and the total cross sections $\sigma_N^{\pi^\pm}$ of pion-nucleon collisions averaged over the thermal motion of the nucleons. Due to isospin coupling the cross sections $\sigma_n^{\pi^-}=\sigma_p^{\pi^+}$ are by a factor of 3 larger than the remaining two. This creates an isospin asymmetry of the mean free path in neutron-rich matter. The thermal motion widens the Δ resonance to $\Gamma=240$ MeV and reduces the maximum cross section by about a factor of 2 resulting in a value of 100 mb for $\sigma_n^{\pi^-}$. At normal nuclear matter density of $n_0=0.16$ fm^{-3} one obtains for heavy nuclei like Au values of $\lambda^0_{\pi^+}=1.05$ fm and $\lambda^0_{\pi^-}=0.85$ fm. Similar values are known from Boltzmann-Uehling-Uhlenbeck calculations [20]. Now we can simulate the opaque source by introducing the momentum and density-dependent mean free path $\lambda=\lambda^0 n_0/n[1+(2(p-k_0)/\Gamma)^2]$ into the potential (18).

Figure 3 shows the effect of the opaqueness for the Gaussian density distribution (10) with $R_h=10$ fm for positive and negative pions without considering the electric charge. The opaqueness increases the radius R_{side} while the radius R_{out} decreases as a consequence of the relatively thin middle part of the half-moon shaped source region [5]. The curves reflect the resonance shape of the absorption. An essential effect of the opaqueness is that the difference $R_{\text{out}}-R_{\text{side}}$ could become negative which may compensate the positive contribution from the emission time. In the ultrarelativistic regime one should also add the effect of the $\pi^-\pi^+$ scattering since the pion density is large and the cross section could reach values up to 15 mb for pion momenta around 200 MeV/c.

A strong dependence of the side correlation on the pion-pair momentum has been found for the collision of Au on Au at a bombarding energy of 1.06 GeV per nucleon in Ref. [8]. A comparison to that data give us a good opportunity to illustrate how the different effects discussed so far change the true radius. We employ the parameters $T=80$ MeV and $\langle\beta\rangle=0.35$ as before, however, we reduce the central charge to $Z_{\text{eff}}=60$ to diminish the Coulomb effect trying to correct for the expansion during the pion emission. Using a hard-sphere radius of $R_h=8$ fm and an emission time of $\tau=4$ fm/c the obtained sideward and outward radii are shown together with the measurements [8] in Fig. 4.

We do not intend to fit the data to our parameters since our simple model lacks essential features, especially the time evolution of the collision treated in recent dynamical models. Figure 4 shows however that essential deviations from the true radius are to be expected and high precision measurements are needed to gain insight in the dynamics of nuclear collisions. For negative pions it is clearly seen that the measured sideward radius depends stronger on the pair momentum than one would expect from calculations using a fixed radius. This means that indeed the fast pions come from an earlier more compressed stage of the matter while the low-energy pions are emitted from a zone with a larger size.

VII. CONCLUSIONS

The nuclear Coulomb field increases (decreases) the observed HBT radii extracted from sideward correlations for negatively (positively) charged pions. The influence is opposite and smaller for the outward correlation. This effect is the largest for small momenta and is superimposed on the overall reduction caused by the radial flow. The opaqueness due to pion rescattering leads to a decrease of the outward radius and an increase of the sideward radius. The decrease could compensate the general increase of the outward radius due to the duration of the emission process.

ACKNOWLEDGMENTS

Support by the German ministry BMBF under Contract No. 06DR829 is acknowledged.

APPENDIX: EFFECT OF A HARD-SPHERE DENSITY DISTRIBUTION WITH A LINEAR VELOCITY PROFILE

Here we investigate the consequences of using a different breakup profile with a sharp surface at $r=R_h$

$$S(k,x)\sim e^{-(ku/T)-(t^2/2\tau^2)}\Theta(R_h-r) \tag{A1}$$

instead of Eq. (10). Now the three-velocity $\mathbf{v}=\beta_h\mathbf{r}/R_h$ scales with the radius while $\mathbf{u}=u^0\mathbf{v}$ with $u^0=1/\sqrt{1-v^2}$. The parameter β_h denotes the surface velocity and which is related to the average velocity via $\beta_h=4/3\langle\beta\rangle$. In the absence of any distortion as flow or potential the square-well profile leads to the correlation function $C_2-1=9[\sin(x)-x\cos(x)]^2/x^6$, $x=qR_h$ which behaves very similarly to the Gaussian profile as it has a width at half maximum of $qR_h=1.815$ compared to the value of 1.862 for the Gaussian profile with $R_0=R_h/\sqrt{5}$.

Since the step function Θ is not an analytical function the potential U in Eq. (17) resulting from Eq. (A1) cannot be continued into the complex plane. To overcome this difficulty we replace the step function by

$$\Theta(R_h-r)\rightarrow\Theta_a(r)=\frac{1}{1+\exp[(r^2-R_h^2)/2R_h a])}, \tag{A2}$$

which behaves analytically for any finite value of the diffuseness parameter a and tends to the step function in the limit $a=0$. (In the following calculations we use a value of $a=0.1$ fm.) The special choice (A2) allows us to calculate the Coulomb potential as

$$U_{\mathrm{Coul}}(r)=\pm Ze^2\frac{aR_h}{\int dr'r'^2\Theta_a}\frac{1}{r}\int_0^r dr' \times\ln\left[1+\exp\left(\frac{R_h^2-r'^2}{2R_h a}\right)\right], \tag{A3}$$

and Eq. (13) takes the form

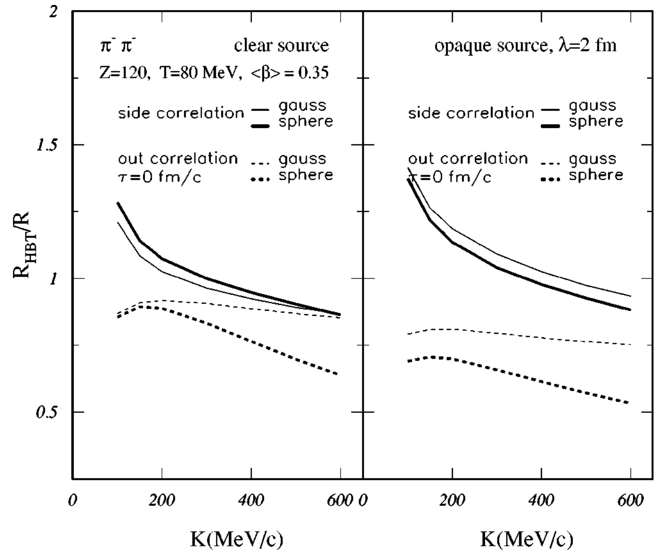

FIG. 5. Comparison of extracted radii calculated with two different density and velocity profiles for negatively charged pions. Thin lines are calculated with the Gaussian profile (10) while thick lines are obtained using a hard-sphere density-distribution (A1) with a velocity linearly increasing as a function of the radius.

$$\rho(p,p')\sim e^{-(\omega-\omega')^2\tau^2/2}\int d\mathbf{r}[\psi_{\mathbf{p}}(\alpha\mathbf{r})]^* \times\psi_{\mathbf{p}'}(\alpha\mathbf{r})\Theta_a(r)e^{-(\omega+\omega')/[2T\sqrt{1-(\beta_h\mathbf{r}/R_h)^2}]}. \tag{A4}$$

Now the factor $\alpha=1-i\beta_h/[2T\sqrt{1-(\beta_h\mathbf{r}/R_h)^2}]$ depends itself on the distance r which bends the trajectory in the complex plane. We have found that this effect is not very important as the imaginary part of α is usually smaller than 0.1. Therefore, the main effect results from the radial profile function (A2).

In Fig. 5 we compare the extracted HBT radii obtained with the two different profiles (10) and (A1) for negative pions with a flow velocity of $\langle\beta\rangle=0.35$ and $R_h=10$ fm. Comparing the thick and thin lines one recognizes the large difference for the outward correlation. An important feature of a Gaussian profile is that it is separable within the Cartesian coordinates. This fact has the result that in the plane-wave limit the sideward and outward radius do not differ as a function of the mean pion momentum for $\tau=0$ and small flow velocities, see Eq. (16). The profile with a sharp surface, however, decreases the outward radius stronger than the sideward radius as a function of the mean momentum. Also, for an opaque source essentially the outward correlation is sensitive to the shape of the profile. The effective thickness of the source appears relatively smaller for the hard-sphere distribution due to its more compact surface. These calculations show that a quantitative analysis depends to a certain degree on the breakup profile.

[1] M. Gyulassy, S.K. Kaufmann, and L.W. Wilson, Phys. Rev. C **20**, 2267 (1979).
[2] W. Bauer, C.-K. Gelbke, and S. Pratt, Annu. Rev. Nucl. Part. Sci. **42**, 77 (1992).
[3] S. Pratt, Phys. Rev. Lett. **53**, 1219 (1984); S. V. Akkelin and Yu. M. Sinyukov, Phys. Lett. B **356**, 525 (1995); U.A. Wiedemann, P. Scotto, and U. Heinz, Phys. Rev. C **53**, 918 (1996); J. Helgesen, T. Csörgö, M. Asakawa, and B. Lörstad, *ibid.* **56**, 2626 (1997).
[4] S. Herrmann and G.F. Bertsch, Phys. Rev. C **51**, 328 (1995).

[5] H. Heiselberg and A.P. Vischer, Eur. Phys. J. C **1**, 593 (1998).
[6] H. W. Barz, Phys. Rev. C **53**, 2536 (1996).
[7] G. Baym and P. Braun-Munzinger, Nucl. Phys. **A610**, 286c (1996).
[8] D. Pelte, FOPI Collaboration, *Proceedings of the International Research Workshop on Heavy Ion Physics at Low, Intermediate and Relativistic Energies Using* 4π *Detectors*, Poiana Brasov, Romania, 1996, edited by M. Petrovici *et al.* (World Scientific, Singapore, 1997), p. 271.
[9] J. Barette *et al.*, Phys. Rev. Lett. **78**, 2916 (1997).
[10] P. Danielewicz and P. Schuck, Phys. Lett. B **274**, 268 (1992).
[11] S. Pratt, Phys. Rev. D **33**, 72 (1986).
[12] D. Anchishkin, U. Heinz, and P. Renk, Phys. Rev. C **57**, 1428 (1998).
[13] L. Martin, C.K. Gelbke, B. Erazmus, and R. Lednitzky, Nucl. Phys. **A604**, 69 (1996).
[14] A. Wagner *et al.*, Phys. Lett. B **420**, 20 (1998).
[15] H.W. Barz, J.P. Bondorf, J.J. Gaardhøje, and H. Heiselberg, Phys. Rev. C **57**, 2536 (1998).
[16] L.S. Kisslinger and F. Tabakin, Phys. Rev. C **9**, 188 (1974).
[17] E.R. Siciliano and R.M. Thaler, Phys. Rev. Lett. **41**, 927 (1978).
[18] D. Pelte *et al.*, Z. Phys. A **357**, 215 (1997).
[19] C. Müntz *et al.*, Z. Phys. A **357**, 399 (1997).
[20] Gy. Wolf, W. Cassing, and U. Mosel, Nucl. Phys. **A552**, 549 (1993).