Analysis of correlation data of $\pi^+\pi^-$ pair in the p+Ta reaction using Coulomb wave function including momentum resolution and the strong interaction effect

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We propose a new method for the Coulomb wave function correction which includes the momentum resolution for charged hadron pairs and apply it to the precise data on $\pi^+\pi^-$ correlations obtained in the p +Ta reaction at 70 GeV/c. We find that interaction regions of this reaction (assuming Gaussian source function) are 9.8 ± 5.8 and 7.7 ± 4.8 fm for the thicknesses of the target 8 and 1.4 μm, respectively. We also analyze the data by the formula including strong interaction effect. The physical picture of the source size obtained in this way is discussed. [S0556-2813(99)02604-7]

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I. INTRODUCTION

We have obtained recently new formulas for the Coulomb wave function corrections for charged hadron pairs [1,2]. Among other things we have applied them [2] to data on $\pi^+\pi^-$ correlation obtained in the p+Ta reaction at 70 GeV/c (and corrected by the usual Gamow factor only). However, as was pointed out to us by one of the authors [4], our approach does not account for the finite momentum resolution published in Ref. [3]. In fact, our formulas cannot be applied directly to experimental data in which such momentum resolution is accounted for. Therefore in the present work we would like to extend our method for the Coulomb wave correction provided in Refs. [1,2] in such a way as to also include the momentum resolution case and we would like to reanalyze data of Ref. [3] as well as the new data of Ref. [5] obtained with two kinds of thickness of the Ta target: 8 and 1.4 μm.

In the next section we provide, for the sake of completeness, the main points of the analysis method used in Ref. [3] whereas in Sec. III we outline our new method for the Coulomb wave correction including this time also the momentum resolution. In Sec. IV we consider also the effect of the strong final state interactions. The final part contains our concluding remarks.

II. DATA RECONSTRUCTION METHOD (USING THE GAMOW FACTOR WITH MOMENTUM RESOLUTION)

It has been stressed in Ref. [3] that relative momenta of $\pi^+\pi^-$ pairs observed by them have some finite resolutions. The averaged correlation function, defined as

$$R(\pi^+\pi^-)=\sigma\frac{d^2\sigma}{dp_1dp_2}\Bigg/\left(\frac{d\sigma}{dp_1}\frac{d\sigma}{dp_2}\right)=G_Cb+(1-b)\Phi(q) \tag{1}$$

depends therefore on this momentum resolution, where G_C denotes a correlation function of $\pi^+\pi^-$ pairs from decays of short-lived resonances (SLR's), and $\Phi(q)$ denotes a correlation function between pions from decays of the long-lived resonances (LLR's). The parameter b means the portion of the correlation function governed by the Coulomb wave function. For the sake of simplicity $\Phi(q)=1$. Figure 1(a) shows a physical picture of Eq. (1). To account for it the following random number method has been proposed in Ref. [3] in order to obtain the corresponding averaged quantities in analysis of the correlation data.

(i) The relative momentum of the measured pair $q=p_1-p_2$ is decomposed into its longitudinal and transverse components: q_L and q_T, respectively, by making use of the uniform random number $u\in(0,1)$ (it is worthwhile to notice at this point that the transverse components q_T in data of Ref. [3] are smaller than 10 MeV/c). The following scheme has been employed:

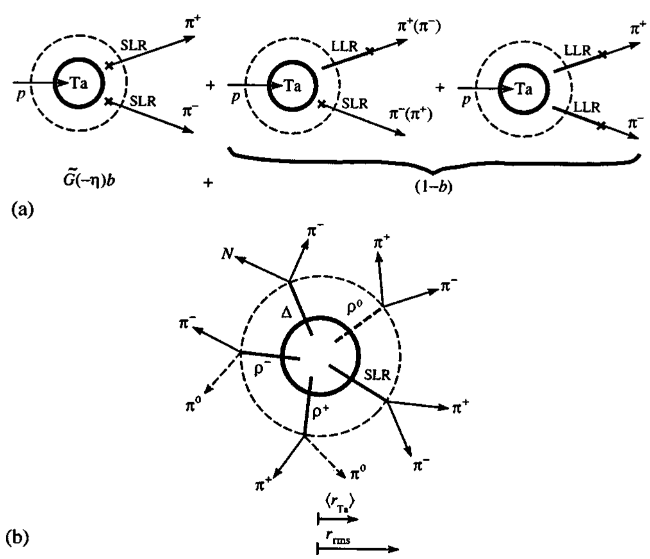

FIG. 1. (a) Physical picture of Eq. (1). (b) Enlarged physical picture of interaction region.

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TABLE I. Results of the χ^2 fits of $R(q)$ for Gaussian source by Eqs. (6) and (12).

Reaction	Formula	$1/2\beta$ [fm]	b	χ^2/N_{DF}
data of Ref. [3] (cf. Ref. [2])	Gamow factor			57.8/40
(without momentum resolution)	Eq. (9)	2.30 ± 0.88		51.0/39
data of Ref. [3]	Eq. (6)		1.0 (fixed)	53.6/37
(with momentum resolution)	Eq. (12) with (7)	2.93 ± 1.03	1.0 (fixed)	44.7/36
	Eq. (12) with (13)	4.63 ± 0.75	1.0 (fixed)	44.8/36
data of Ref. [5], 8 μm	Eq. (6)		0.43 ± 0.03	55.0/46
	Eq. (12) with (7)	5.63 ± 3.34	0.53 ± 0.07	51.5/45
	Eq. (12) with (13)	7.22 ± 2.84	0.54 ± 0.07	51.3/45
data of Ref. [5], 1.4 μm	Eq. (6)		0.51 ± 0.04	37.9/46
	Eq. (12) with (7)	4.44 ± 2.80	0.60 ± 0.07	35.1/45
	Eq. (12) with (13)	6.06 ± 2.23	0.61 ± 0.07	35.1/45

$$q_T=\begin{cases}10\sqrt{u} & \text{for} \quad q\geqslant 10 \ \ \mathrm{MeV}/c,\\ q\sqrt{u} & \text{for} \quad q\leqslant 10 \ \ \mathrm{MeV}/c,\end{cases} \tag{2a}$$

$$q_L=\sqrt{q^2-q_T^2}. \tag{2b}$$

(ii) The Gaussian random numbers for q_L and q_T are generated in the following way:

$$q_{L(\mathrm{random})}=\sigma_L X+q_L\,, \tag{3}$$

$$q_{T(\mathrm{random})}=\sigma_T X+q_T\,, \tag{4}$$

where X stands for the standard Gaussian random number [6] whereas σ_L and σ_T are longitudinal and transverse setup resolutions for the corresponding components, which are equal to (values used in Ref. [5]): $\sigma_L=1.3\mathrm{MeV}/c$; $\sigma_T=0.6$ MeV/c (for target of the thickness 8 μm); and $\sigma_T=0.4$ MeV/c (for the 1.4 μm target).

(iii) Using the randomized number $q_{\mathrm{random}}=\sqrt{q^2_{L(\mathrm{random})}+q^2_{T(\mathrm{random})}}$ one calculates the corresponding randomized Gamow factor correction:

$$G(-\eta_{\mathrm{random}})=\frac{-2\pi\eta_{\mathrm{random}}}{\exp(-2\pi\eta_{\mathrm{random}})-1}, \tag{5}$$

where $\eta_{\mathrm{random}}=m\alpha/q_{\mathrm{random}}$.

Calculating now the average value of $G(-\eta_{\mathrm{random}})$ in 10–100 k events one can estimate the Gamow factor with this finite momentum resolution,

$$R(q)=\tilde{G}(-\eta)b+(1-b), \tag{6}$$

where b is the portion from the SLR's. It is understood (or, rather, implicitly assumed) that the second term of light hand side $(1-b)$ originates from decays of LLR's such as η, K_0^S, Λ, and so on [7]. Table I shows the results of analysis of new data (for 8 μm target) [5] for region $q>3$ MeV/c using this method [9].

III. NEW METHOD OF COULOMB WAVE FUNCTION CORRECTIONS

We would like to propose the following new method of Coulomb wave function correction [with a source function $\rho(r)$] to be used instead of the Gamow factor and apply it to the same data as above. As usual we decompose the wave function of unlike charged bosons with momenta p_1 and p_2 into the wave function of the center-of-mass (c.m.) system with total momentum $P=\frac{1}{2}(p_1+p_2)$ and the inner wave function with relative momentum $q=(p_1-p_2)$. This allows us to express Coulomb wave function $\Psi_C(\mathbf{q},\mathbf{r})$ in terms of the confluent hypergeometric function F [10]:

$$\Psi_C(\mathbf{q},\mathbf{r})=\Gamma(1-i\eta)e^{\pi\eta/2}e^{i\mathbf{q}\cdot\mathbf{r}/2}F[i\eta;1;iqr(1-\cos\theta)/2], \tag{7}$$

where $r=r_1-r_2$ and the parameter $\eta=m\alpha/q$. Assuming factorization in the source functions $\rho(r_1)\rho(r_2)=\rho_R(R)\rho_r(r)$ [with $R=\frac{1}{2}(r_1+r_2)$, $\int\rho_R(R)d^3R=1$ is assumed], we obtain the expression for Coulomb correction for the system of $\pi^+\pi^-$ pairs identical (modulo the sign) as in Refs. [8,1,2]:

$$C_C(-\eta)=\int\rho_R(R)d^3R\int\rho_r(r)d^3r|\Psi_C(\mathbf{q},\mathbf{r})|^2 \tag{8}$$

$$=G(-\eta)\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-i)^n(i)^m}{n+m+1}q^{n+m}I(n,m)A_nA_m^*$$

$$=G(-\eta)[1+\Delta_{1C}(-\eta)], \tag{9}$$

where

$$I(n,m)=4\pi\int drr^{2+n+m}\rho(r), \quad A_n=\frac{\Gamma(-i\eta+n)}{\Gamma(-i\eta)}\frac{1}{(n!)^2}.$$

For the specific choice of Gaussian source distribution $\rho_r(r)=(\beta^3/\sqrt{\pi^3})\exp(-\beta^2r^2)$, we have

$$I^G(n,m)=\frac{2}{\sqrt{\pi}}\left(\frac{1}{\beta}\right)^{n+m}\Gamma\left(\frac{n+m+3}{2}\right), \tag{10}$$

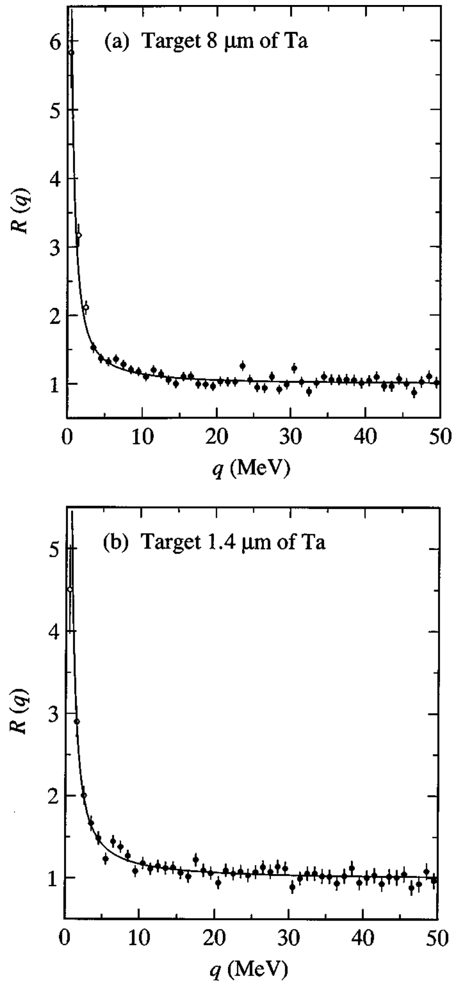

FIG. 2. Results of the χ^2 fits for $p+\mathrm{Ta}\to\pi^+\pi^-+X$ reactions with $q>3$ MeV/c by Eq. (12) with Eq. (7): (a) 8 μm target; (b) 1.4 μm target.

$$\Delta_{1C}(-\eta)=\frac{4\eta}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n(n+1)!}{(2n+2)!(2n+1)}\left(\frac{q}{\beta}\right)^{2n+1}. \quad (11)$$

Using now the same method of Gaussian random numbers as in a previous section, we can analyze the old and the new data on $\pi^+\pi^-$ pairs [3,5] using the following formula:

$$R(q)=\tilde{C}_C(-\eta)b+(1-b). \quad (12)$$

Figure 2 and Table I show our results obtained using Eq. (9) applied to old and new data with $q>3$ MeV/c. Notice that now we can estimate (albeit with large errors) also the dimension of the reaction region, not available when using only the Gamow factor.

IV. STRONG INTERACTION EFFECT

We shall consider now the possible effect of strong final state interactions, using a formulation proposed by Bowler in Ref. [11]. An extended formula for charged particles has been proposed in Ref. [12]. As far as we know data of the phase shift of the p wave *in the very small* q region have not reported. Thus we consider only the s wave. The wave function including Coulomb effect and strong interaction effect of the s wave is described by

$$\Psi_{\mathrm{total}}(\mathbf{q},\mathbf{r})=\Psi_C(\mathbf{q},\mathbf{r})+\Phi_{\mathrm{st}}(\mathbf{q},\mathbf{r}). \quad (13)$$

Here $\Phi_{\mathrm{st}}(\mathbf{q},\mathbf{r})$ stands for the wave function induced by the strong interactions [12] and is given by

$$\Phi_{st}(\mathbf{q},\mathbf{r})=\frac{\phi_0^{(2,0)}(\theta)[G_0(-\eta;qr/2)+iF_0(-\eta;qr/2)]}{r} \quad (14a)$$

$$\xrightarrow{\text{large } r}\frac{\phi_0^{(2,0)}(\theta)\exp[i(qr/2+\eta\ln(qr)+\sigma_0)]}{r}, \quad (14b)$$

where $\sigma_0=\arg\Gamma(1-i\eta)$, F_0 and G_0 are the regular and irregular solutions ($l=0$; s wave) of the radial coordinates including the Coulomb potential [13]. Equation (14) is the scattering amplitude and the asymptotic form.[1] We assume that an interpolation to the internal region $[0,r_c\approx(1-2)$ fm] is possible as in Refs. [11,12]. (In other words, we assume a small contribution from the internal region in the spatial integration. In the present calculation, we do not introduce the cutoff factor r_c.)

The amplitude $\phi_0^{(2,0)}$ is decomposed as

$$\phi_0^{(2,0)}=\frac{1}{3}f_0^{(2)}(\theta)+\frac{2}{3}f_0^{(0)}(\theta), \quad (15a)$$

$$f_0^{(2)}(\theta)=\frac{1}{iq}e^{i\sigma_0}[\exp(2i\delta_0^{(2)})-1], \quad (15b)$$

$$f_0^{(0)}(\theta)=\frac{1}{iq}e^{i\sigma_0}[\exp(2i\delta_0^{(0)})-1]. \quad (15c)$$

For the phase shift of $I=2$ and 0 channel $\pi\pi$ scattering, we use the following parametrization [14] :

$$\delta_0^{(2)}(q)=\frac{1}{2}\frac{a_0^{(2)}q}{1+0.5[\mathrm{GeV}^{-2}]q^2}, \quad a_0^{(2)}=-1.20\ \mathrm{GeV}^{-1}, \quad (16a)$$

$$\delta_0^{(0)}(q)=\frac{1}{2}a_0^{(0)}q, \quad a_0^{(0)}=1.50\ \mathrm{GeV}^{-1}. \quad (16b)$$

Figures 3 show results of our analysis of the new data. In Table I, we also show our results obtained using Eq. (13)

[1] Proof of Eq. (13) with Eq. (14) is given as follows:

$$e^{i\sigma_0}e^{i\delta_0}(\cos\delta_0F_0+\sin\delta_0G_0)=e^{i\sigma_0}[F_0+e^{i\delta_0}\sin\delta_0(G_0+iF_0)].$$

Using the above equality, and summing up all the partial waves with $\sigma_l=\arg\Gamma(l+1-i\eta)$, we have Eq. (13):

$$\left[e^{i\sigma_0}F_0+\sum_{l=1}^{\infty}i^l(2l+1)e^{i\sigma_l}F_l(-\eta;qr/2)P_l(\cos\theta)\right.$$
$$\left.+e^{i\sigma_0}e^{i\delta_0}\sin\delta_0(G_0+iF_0)\right]\bigg/(qr/2)$$
$$=\Psi_C(\mathbf{q},\mathbf{r})+\Phi_{\mathrm{st}}(\mathbf{q},\mathbf{r}).$$

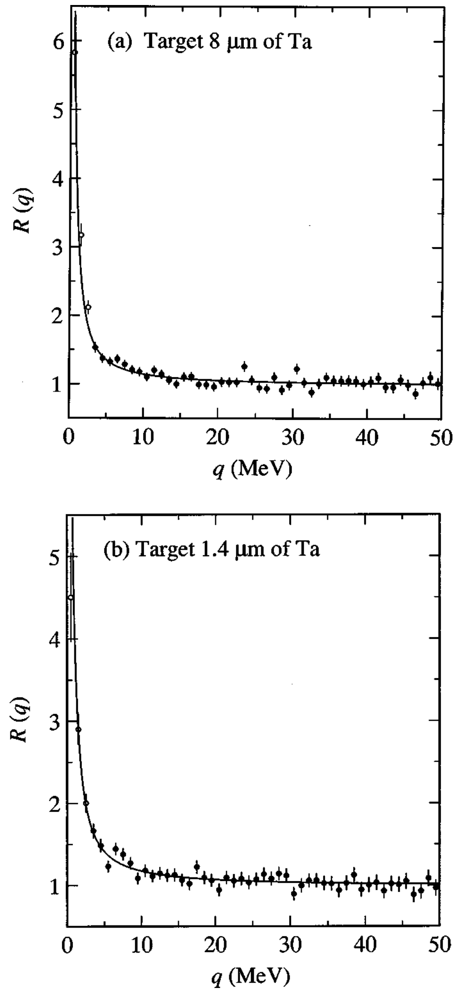

FIG. 3. Results of the χ^2 fits for $p+\mathrm{Ta}\rightarrow\pi^+\pi^-+X$ reactions with $q>3$ MeV/c by Eq. (12) with Eq. (13): (a) 8 μm target; (b) 1.4 μm target.

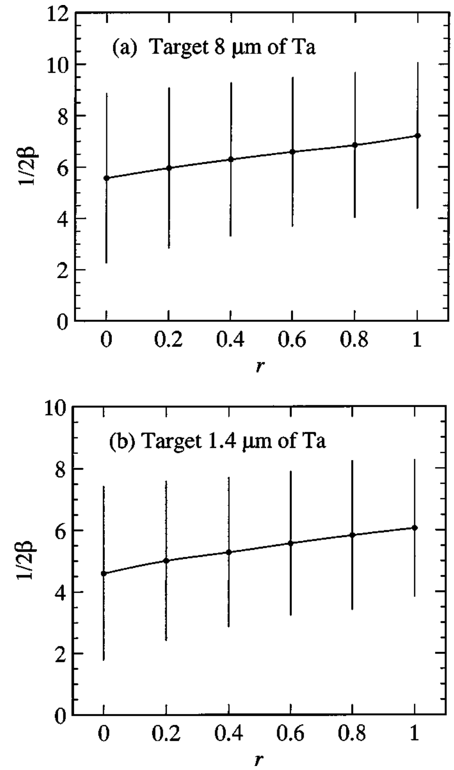

FIG. 4. Phase shift dependence of $1/2\beta$. The horizontal axis expresses strength of $a_0^{(2)}$ and $a_0^{(0)}$: (a) 8μm target; (b) 1.4 μm target, $ra_0^{(2)}$ and $ra_0^{(0)}$ $(0\leq r\leq 1.0)$.

instead of Eq. (7) in Eq. (9). As can be seen we observe systematically larger values of the interaction region with strong final state interactions included.

V. CONCLUDING REMARKS

We have proposed a new method for the Coulomb wave function correction with momentum resolution and applied it to the analysis of the precise data provided by Refs. [3,5]. Authors of Ref. [3] have analyzed their $\pi^+\pi^-$ correlation data using Gamow factor for Coulomb corrections together with the random numbers method to account for final momentum resolution. We have repeated this analysis replacing Gamow factor by the Coulomb wave function but following the same method for correction for the momentum resolution effect [cf. Eq. (9)]. As a result we were able to estimate the range of interaction for which (more precisely, for the root mean squared ranges of interaction r_{rms}) we have obtained the following values (for the Gaussian source function):

$$r_{\mathrm{rms}}=\frac{\sqrt{3}}{2\beta}=\begin{cases}9.8\pm5.8 \text{ fm} & \text{for 8 } \mu\text{m},\\ 7.7\pm4.8 \text{ fm} & \text{for 1.4 } \mu\text{m}.\end{cases} \tag{17}$$

From the formula which also includes a strong final state interaction effect [cf. Eq. (13)] we have obtained

$$r_{\mathrm{rms}}=\frac{\sqrt{3}}{2\beta}=\begin{cases}12.5\pm4.9 \text{ fm} & \text{for 8 } \mu\text{m},\\ 10.5\pm3.9 \text{ fm} & \text{for 1.4 } \mu\text{m}.\end{cases} \tag{18}$$

Values of Eq. (18) are fairly bigger than those of Eq. (17). This fact means that r_{rms} estimated by means of Eq. (13) depends strongly on phase shift values. The dependence is shown in Figs. 4. The results are also influenced by method of momentum resolution. To confirm Eq. (18), we need to investigate different approaches in the future, for example, Ref. [15]. Table II shows the results concerning cutoff dependences of momentum resolution.

The present study of $\pi^+\pi^-$ pair correlations has shown therefore that one can estimate the interaction region even from the $\pi^+\pi^-$ correlation data (although present data lead to large errors for r_{rms}). It can be compared with the size of the Ta nucleus, which is given by

$$\langle r_{Ta}\rangle=1.2\times A^{1/3}=1.2\times(181)^{1/3}=6.8 \text{ fm}. \tag{19}$$

As one can see, r_{rms} is significantly bigger than $\langle r_{Ta}\rangle$. We attribute this difference to a physical picture shown in Fig. 1(b), i.e., to the fact that unlike-sign pions are about 60% emerging from the SLR's $(\varphi,\Delta,\dots)$ shown there. In the future one should apply our theoretical formula to other data and estimate β's and b's in similar reactions, and also consider the possibility of a more direct estimation of the parameter b and its role in determining the source size parameter [16,17].

TABLE II. Momentum resolution and cutoff.

Data	Using Monte Carlo events	$1/2\beta$ [fm]	b	$\chi^2/N_{\rm DF}$
data of Ref. [5], 8 μm	without momentum resolution (central value is used)	7.95 ± 3.05	0.58 ± 0.08	50.5/45
	$\lvert q_{L(T)(\rm random)}-q_{L(T)}\rvert \leqslant \sigma_{L(T)}$	7.66 ± 2.96	0.56 ± 0.08	50.7/45
	$\lvert q_{L(T)(\rm random)}-q_{L(T)}\rvert \leqslant 2\sigma_{L(T)}$	7.48 ± 2.91	0.55 ± 0.08	51.1/45
	all	7.22 ± 2.84	0.54 ± 0.07	51.3/45
data of Ref. [5], 1.4 μm	without momentum resolution (central value is used)	6.45 ± 2.36	0.64 ± 0.08	34.7/45
	$\lvert q_{L(T)(\rm random)}-q_{L(T)}\rvert \leqslant \sigma_{L(T)}$	6.29 ± 2.30	0.63 ± 0.08	34.6/45
	$\lvert q_{L(T)(\rm random)}-q_{L(T)}\rvert \leqslant 2\sigma_{L(T)}$	6.18 ± 2.27	0.62 ± 0.08	35.3/45
	all	6.06 ± 2.23	0.61 ± 0.07	35.1/45

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[1] M. Biyajima, T. Mizoguchi, T. Osada, and G. Wilk, Phys. Lett. B **353**, 340 (1995).

[2] M. Biyajima, T. Mizoguchi, T. Osada, and G. Wilk, Phys. Lett. B **366**, 394 (1996).

[3] L. G. Afanas'ev *et al.*, Yad. Fiz. **52**, 1046 (1990) [Sov. J. Nucl. Phys. **52**, 666 (1990)].

[4] L. G. Afanas'ev (private communications).

[5] L. G. Afanas'ev *et al.*, Phys. At. Nucl. **60**, 938 (1997).

[6] The probability density of such random numbers is given by

$$f(x)=\frac{1}{\sqrt{2\pi}}\exp(-x^2/2) \quad (-\infty<x<\infty).$$

[7] This fact has some consequences on the determination of the radius of the interaction region as will be shown later on and makes it different from that obtained from the Bose-Einstein correlations.

[8] M. G. Bowler, Phys. Lett. B **270**, 69 (1991).

[9] In the region $q<3$ MeV/c Coulomb enhancement is influenced by Coulomb bound state of $\pi^+\pi^-$ pairs (see Ref. [5]). Then we use data with $q>3$ MeV/c in our analyses.

[10] L. I. Schiff, *Quantum Mechanics*, 2nd ed. (McGraw-Hill, New York, 1955), p. 117.

[11] M. G. Bowler, Z. Phys. C **39**, 81 (1988).

[12] T. Osada, S. Sano, and M. Biyajima, Z. Phys. C **72**, 285 (1996); see also, M. Biyajima, S. Sano, and T. Osada, Phys. Rev. D **58**, 114013 (1998).

[13] Authors of Ref. [12] used the form of Eq. (14b) with $\sqrt{G(-\eta)}$ factor. However, Eq. (14a) is more precise in the small q region than Eq. (14b).

[14] M. Suzuki, Phys. Rev. D **35**, 3359 (1987).

[15] S. Pratt, T. Csörgő, and J. Zimányi, Phys. Rev. C **42**, 2646 (1990).

[16] In the analysis of $\pi^0\pi^0$ correlation data, a similar function $f(q)$ is introduced:

$$R(\pi^0\pi^0)=f(q)+[1-f(q)][1+\lambda E_{2B}^2],$$

where λ and E_{2B} are the degree of coherence and an exchange function due to the Bose-Einstein effect. $f(q)$ is attributed to the resonances effect; $f(q)\approx 0.9\sim 0.7$ depends on the Monte Carlo programs used [17].

[17] E. A. De Wolf, Proceeding of the XXIVth International Symposium on Multiparticle Dynamics, Vietri sul Mare, 1994 (unpublished), p. 16.