Pion single-charge-exchange reaction on ^{7}Li

Naoko Nose-Togawa and Kenji Kume
Department of Physics, Nara Women's University, Nara 630-8506, Japan
(Received 6 July 1998)

We have studied the pion single-charge-exchange reaction on ^{7}Li leading to isobaric analog state under distorted-wave impulse approximation. A cluster-model wave function is used to describe the ground state of ^{7}Li and ^{7}Be, and the results are compared with those of the shell-model calculation. The forward cross section above 50 MeV is well explained but the theoretical value is larger than the experimental cross section about a factor 2–2.5 at lower energies. The shell-model and the cluster-model wave functions predict quite different second rank tensor polarization. [S0556-2813(99)01804-X]

PACS number(s): 25.80.Gn, 21.60.Gx, 27.20.+n

The pion single- and double-charge-exchange reactions on complex nuclei have been studied extensively both theoretically and experimentally [1–5]. The pion single-charge-exchange (SCX) reaction is expected to provide us with information about the isovector component of the nucleus. Recently, the experiments of SCX reactions on polarized targets have been done for light p-shell nuclei [2]. For the spin-$\frac{1}{2}$ nuclei, the right-left asymmetry comes from the interference between spin-dependent and independent isovector amplitudes. Thus, the right-left asymmetry is sensitive to the $\sigma\cdot\tau$ component of the nuclear density which is largely influenced by the nuclear core polarization. In this context, the effects of the higher configurations in some p-shell nuclei have been studied [6–9]. On the other hand, for the case of a polarized nucleus with spin larger than one, the multipole contribution affects the polarization observables and the quadrupole contributions are important for deformed nuclei. In the elastic and inelastic scattering of pion from polarized ^{7}Li, we have shown that the nuclear quadrupole deformation affects appreciably the second-rank tensor polarization [10]. In the pion SCX reaction, we can study the isovector quadrupole component of the target nucleus.

In the present paper, we have studied the pion SCX reaction on ^{7}Li leading to the isobaric analog state $^7\mathrm{Li}(\pi^+,\pi^0)^7\mathrm{Be}_{\mathrm{gs}}$. It is well known that the ground state of ^{7}Li and ^{7}Be nuclei are known to have cluster structure and they are well described with the resonating-group wave function [11–17]. The electromagnetic properties at $q \leqslant 2\ \mathrm{fm}^{-1}$ are explained well with the cluster-model wave function [17]. A more elaborate calculation has been done [18], but the additional components of the wave function give an important contribution only to excited states. A number of works have been done for the (p,γ) reactions on ^{7}Li to extract the astrophysical S factor [19]. Since we have fairly good wave function for these nuclei, we could minimize the ambiguity coming from the nuclear wave function. We have calculated the pion SCX reaction cross section $^7\mathrm{Li}(\pi^+,\pi^0)^7\mathrm{Be}_{\mathrm{gs}}$ in the low-energy region and compared it with the available experimental data. We have also studied the polarization observables to see the sensitivity to the isovector quadrupole densities. We could consider this as a typical example of the pion SCX reaction for the deformed nuclei. The above calculations with the cluster-model wave function are compared with those by the shell-model wave function with effective charge. As will be shown, the cluster-model and the shell-model wave functions give similar results for the forward cross section, while these models predict considerably different second-rank tensor polarization. The second-rank tensor polarization T_{20} takes a nonzero value in the forward direction and is shown to be quite sensitive to the quadrupole deformation of the isovector type.

We calculate the SCX reaction cross section for $^7\mathrm{Li}\ (\pi^+,\pi^0)^7\mathrm{Be}$ under the distorted-wave impulse approximation (DWIA). The reaction amplitude can be written as

$$f(\mathbf{k}_f,\mathbf{k}_i)=\int \chi_f^{(-)\dagger}(\mathbf{k}_f,\mathbf{r})\left\langle \Phi_f\left|\sum_j^A t(\mathbf{r}_j,\mathbf{r})\right|\Phi_i\right\rangle \times\chi_i^{(+)}(\mathbf{k}_i,\mathbf{r})d\mathbf{r}, \tag{1}$$

where Φ_i and Φ_f are the initial and the final nulcear states. The incoming and outgoing pion wave functions are denoted as $\chi_i^{(+)}(\mathbf{k}_i,\mathbf{r})$ and $\chi_f^{(-)}(\mathbf{k}_f,\mathbf{r})$, respectively. The scattering amplitude between pion and jth nucleon can be written up to p wave as

$$t(\mathbf{r}_j,\mathbf{r})=[b_0+c_0\mathbf{k}_f\cdot\mathbf{k}_i+id_0\boldsymbol{\sigma}_j\cdot\mathbf{k}_f\times\mathbf{k}_i+(b_1+c_1\mathbf{k}_f\cdot\mathbf{k}_i +id_1\boldsymbol{\sigma}_j\cdot\mathbf{k}_f\times\mathbf{k}_i)\boldsymbol{\tau}_j\cdot\mathbf{I}_\pi]\delta(\mathbf{r}-\mathbf{r}_j), \tag{2}$$

where $\boldsymbol{\sigma}_j$ and $\boldsymbol{\tau}_j$ are the spin and the isospin operators for the jth nucleon and $\mathbf{I}_\pi$ is the pion isospin operator. The relevant coefficients of the isovector terms b_1, c_1, and d_1 are given by the pion-nucleon phase shifts as

$$b_1=\frac{1}{\kappa}\frac{\alpha_s^{3/2}-\alpha_s^{1/2}}{3},$$

$$c_1=\frac{1}{3\kappa^3}(2\alpha_{p_+}^{3/2}-\alpha_{p_+}^{1/2}+\alpha_{p_-}^{3/2}-\alpha_{p_-}^{1/2}), \tag{3}$$

$$d_1=-\frac{1}{3\kappa^3}(\alpha_{p_+}^{3/2}-\alpha_{p_+}^{1/2}-\alpha_{p_-}^{3/2}+\alpha_{p_-}^{1/2}),$$

with

$$\alpha=e^{i\delta}\sin\delta,\ \ \text{etc.} \tag{4}$$

The upper indices represent the pion-nucleon isospin channel and $p\pm$ corresponds to $j=l\pm\frac{1}{2}$ for the p-wave terms. The pion-nucleon relative momentum in the pion-nucleon center of mass system is denoted as κ. The pion-nucleus SCX amplitude is written as

$$f(\mathbf{k}_f,\mathbf{k}_i)=\int \chi_f^{(-)\dagger}(\mathbf{k}_f,\mathbf{r})[b_1\rho_T(\mathbf{r})+c_1(\mathbf{k}_f\cdot\mathbf{k}_i)\rho_T(\mathbf{r}) + id_1\boldsymbol{\rho}_{ST}(\mathbf{r})\cdot(\mathbf{k}_f\times\mathbf{k}_i)]\chi_i^{(+)}(\mathbf{k}_i,\mathbf{r})d\mathbf{r} \quad (5)$$

with the transition densities

$$\rho_T(\mathbf{r})=\left\langle \Phi_f\left|\sum_j \delta(\mathbf{r}-\mathbf{r}_j)\boldsymbol{\tau}_j\cdot\mathbf{I}_\pi\right|\Phi_i\right\rangle, \quad (6)$$

$$\boldsymbol{\rho}_{ST}(\mathbf{r})=\left\langle \Phi_f\left|\sum_j \delta(\mathbf{r}-\mathbf{r}_j)\boldsymbol{\sigma}_j(\boldsymbol{\tau}_j\cdot\mathbf{I}_\pi)\right|\Phi_i\right\rangle. \quad (7)$$

We expand the pion wave function into the multipole series

$$\chi_i^{(+)}(\mathbf{k}_i,\mathbf{r})=4\pi\sum_{l_im_i} i^{l_i}u_{l_i}^{(i)}(r)Y_{l_im_i}^*(\hat{\mathbf{k}}_i)Y_{l_im_i}(\hat{\mathbf{r}})\xi_i, \quad (8)$$

$$\chi_f^{(-)}(\mathbf{k}_f,\mathbf{r})=4\pi\sum_{l_fm_f} i^{l_f}u_{l_f}^{(f)}(r)Y_{l_fm_f}^*(\hat{\mathbf{k}}_f)Y_{l_fm_f}(\hat{\mathbf{r}})\xi_f, \quad (9)$$

where the ξ's are the pion isospin wave functions. We replace the momentum variables $\mathbf{k}_i$ and $\mathbf{k}_f$ to the derivative operator acting on the pion wave functions. Then the DWIA amplitude $f(\mathbf{k}_f,\mathbf{k}_i)$ can be written as [20]

$$f(\mathbf{k}_f,\mathbf{k}_i)=(4\pi)^{3/2}\sum_{LM}(I_iLI_{iz}M|I_fI_{fz}) \times\sum_{l_il_f} i^{l_i-l_f}\sqrt{\frac{(2l_i+1)(2l_f+1)}{(2I_f+1)(2L+1)}}[Y_{l_f}(\hat{\mathbf{k}}_f) \otimes Y_{l_i}(\hat{\mathbf{k}}_i)]_{LM}^*[\Xi_1(l_i,l_f;L)+\Xi_2(l_i,l_f;L)]. \quad (10)$$

Here, we define the following matrix elements:

$$\Xi_1(l_i,l_f;L)=\frac{1}{\sqrt{3}}\int_0^\infty r^2dr\ \ (l_il_f00|L0)\Bigg[\Bigg\{b_1u_{l_f}^{(f)}(r)u_{l_i}^{(i)}(r)+c_1\Bigg(\frac{du_{l_f}^{(f)}(r)}{dr}\cdot\frac{du_{l_i}^{(i)}(r)}{dr}+\frac{\Delta_1(l_f,l_i;L)}{r^2} \times u_{l_f}^{(f)}(r)u_{l_i}^{(i)}(r)\Bigg)\Bigg\}F_{L0L}(r)-d_1\Bigg\{\frac{\Delta_2(l_f,l_i;L)}{r}\frac{d}{dr}[u_{l_f}^{(f)}(r)u_{l_i}^{(i)}(r)]-\frac{\sqrt{L(L+1)}}{2r}\Bigg(u_{l_f}^{(f)}(r)\frac{du_{l_i}^{(i)}(r)}{dr} -\frac{du_{l_f}^{(f)}(r)}{dr}u_{l_i}^{(i)}(r)\Bigg)\Bigg\}F_{L1L}(r)\Bigg], \quad (11)$$

and

$$\Xi_2(l_i,l_f;L)=\frac{1}{\sqrt{3}}\int_0^\infty r^2dr\ \ d_1\Bigg[\Delta^{(-)}(l_f,l_i;L-1)\frac{1}{r}\Bigg(-\frac{d}{dr}-\frac{L}{r}\Bigg)u_{l_f}^{(f)}(r)u_{l_i}^{(i)}(r)(l_il_f00|L-10)F_{L-11L}(r) +\Delta^{(+)}(l_f,l_i;L+1)\frac{1}{r}\Bigg(-\frac{d}{dr}+\frac{L+1}{r}\Bigg)u_{l_f}^{(f)}(r)u_{l_i}^{(i)}(r)(l_il_f00|L+10)F_{L+11L}(r)\Bigg], \quad (12)$$

with kinematical factors defined by

$$\Delta_1(l_f,l_i;L)\equiv\frac{1}{2}[l_i(l_i+1)+l_f(l_f+1)-L(L+1)], \quad (13)$$

$$\Delta_2(l_f,l_i;L)\equiv\begin{cases}0, & \text{for } l_i=l_f=L=0\\ \frac{1}{2}[l_i(l_i+1)-l_f(l_f+1)]/\sqrt{L(L+1)}, & \text{otherwise,}\end{cases} \quad (14)$$

$$\Delta^{(-)}(l_fl_i;L)\equiv-\frac{1}{2}\sqrt{\frac{(l_f+l_i+L+1)(l_i+L-l_f)(l_f+L-l_i)(l_f+l_i-L+1)}{L(2L-1)}}, \quad (15)$$

$$\Delta^{(+)}(l_fl_i;L)\equiv\frac{1}{2}\sqrt{\frac{(l_f+l_i+L+2)(l_i+L-l_f+1)(l_f+L-l_i+1)(l_f+l_i-L)}{(L+1)(2L+3)}}. \quad (16)$$

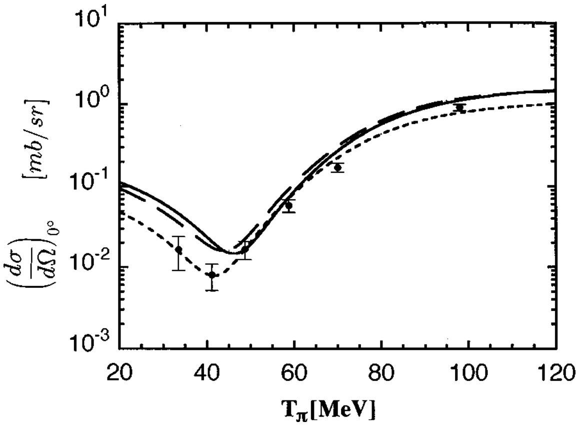

FIG. 1. Pion SCX reaction cross section for ^{7}Li(π^+,π^0)^{7}Be$_{gs}$ in the forward direction. The solid and the long dashed lines are the results with the cluster and the shell-model wave functions, respectively. The renormalization factors determined by Glover *et al.* [27] are used for the shell-model wave function. The dotted line corresponds to the result of the cluster-model calculation with the modification of *s*- and *p*-wave isovector pion-nucleon amplitudes b_1 and c_1 to $0.7b_1$ and $0.8c_1$. The experimental data are taken from Ref. [28].

The nuclear form factors of the isovector type are defined by

$$F_{LSJ}(r)=\left\langle \Phi_f \left|\left| \sum_j \frac{\delta(r-r_j)}{r^2}[Y_L(\hat{\mathbf{r}}_j)\otimes\sigma_j^{(S)}]^J\tau_j \right|\right| \Phi_i \right\rangle, \tag{17}$$

where the double bar denotes the spin and the isospin reduced matrix element. To describe the polarization of the target nucleus with spin *I*, the tensor operator τ_{kq} of rank *k* is introduced as

$$\langle I\mu'|\tau_{kq}|I\mu\rangle=\sqrt{2I+1}(-1)^{I-\mu}(II\mu'-\mu|kq), \tag{18}$$

then the density matrix describing the target polarization is written as

$$\rho(I)=\sum_{kq}\frac{\tau_{kq}^\dagger t_{kq}}{2I+1} \tag{19}$$

with t_{kq} being the tensor polarization for the target nuclei. The cross section for pion SCX reaction is given as

$$\frac{d\sigma}{d\Omega}=\mathrm{Tr}(f\rho f^\dagger), \tag{20}$$

and the tensor polarization

$$T_{kq}=\frac{\mathrm{Tr}(f\tau_{kq}f^\dagger)}{\mathrm{Tr}(ff^\dagger)}. \tag{21}$$

For the ground states of ^{7}Li and ^{7}Be, we employ the resonating-group-method (RGM) wave function. We assumed the $\alpha-t$ and $\alpha-{}^3$He cluster states for these nuclei. The internal wave functions are assumed to be those of the harmonic-oscillator shell model. The oscillator-size parameters for these clusters are determined so as to reproduce the experimental values of the rms charge radii. For an effective

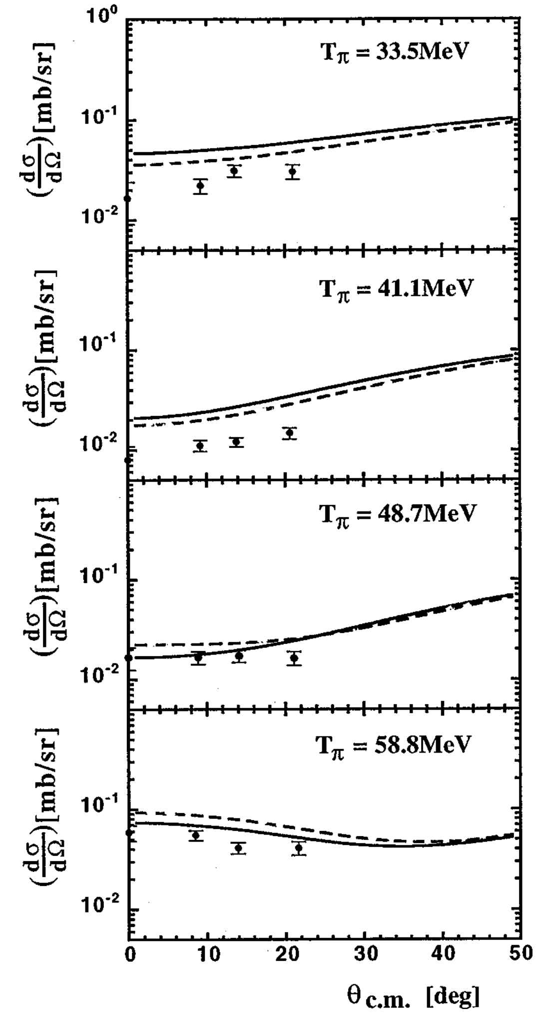

FIG. 2. The differential cross section for the pion SCX reaction ^{7}Li(π^+,π^0)^{7}Be$_{gs}$ for the low-energy region. The experimental data are taken from Ref. [29].

interaction, we use the Volkov No. 2 force [21] with the Majorana exchange parameter $m=0.585$ which reproduces the experimental rms charge radius of the ^{7}Li. The RGM wave function gives the quadrupole moment $Q=-4.45$ fm^2 for ^{7}Li. We have also examined the Hasegawa-Nagata effective interaction [22]. Since it gives similar wave functions, we use the Volkov-2 interaction throughout. The center-of-mass motion of the two clusters is eliminated. The details of the calculations for the nuclear multipole densities are found in Ref. [10]. Using the nuclear wave function described above, we have carried out the DWIA calculation of pion SCX reactions from polarized ^{7}Li at the low-energy region.

We have adopted the pion-nucleus optical potential by the Michigan group [23] which has been extensively applied to calculate the low-energy pion-nucleus scattering. We have used the impulse values for the potential parameters. We also calculated the SCX cross section with the potential parameters given by the Michigan group [23]. The results for the cross section and the polarization observables are almost the same and hence we use the impulse values throughout. For the absorption parameters B_0 and C_0, we adopted the value determined phenomenologically by Gmitro *et al.* [24]. For the pion-nucleon phase shifts, we have used the parameters

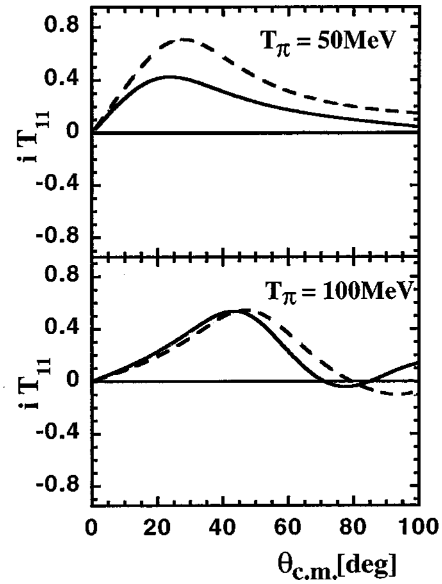

FIG. 3. The asymmetries T_{1k} for the reaction $^7\mathrm{Li}(\pi^+,\pi^0)^7\mathrm{Be}_{gs}$ at $T_\pi = 50$ and 100 MeV. The cluster-model (solid) and the shell-model (dashed) wave functions are used.

in Ref. [25]. Since we are mainly concerned with the cross section and the polarization observables around the forward direction, we have neglected the contribution from the quadrupole term in the optical potential.

First, we have calculated the SCX forward cross section and these are compared with the experimental data in Fig. 1. For comparison, we also show the results calculated with the Cohen-Kurath $0p$ shell-model wave function [26] with the oscillator parameter $b = 1.76$ fm. In this case, we have renormalized the isovector-type nuclear form factors using the

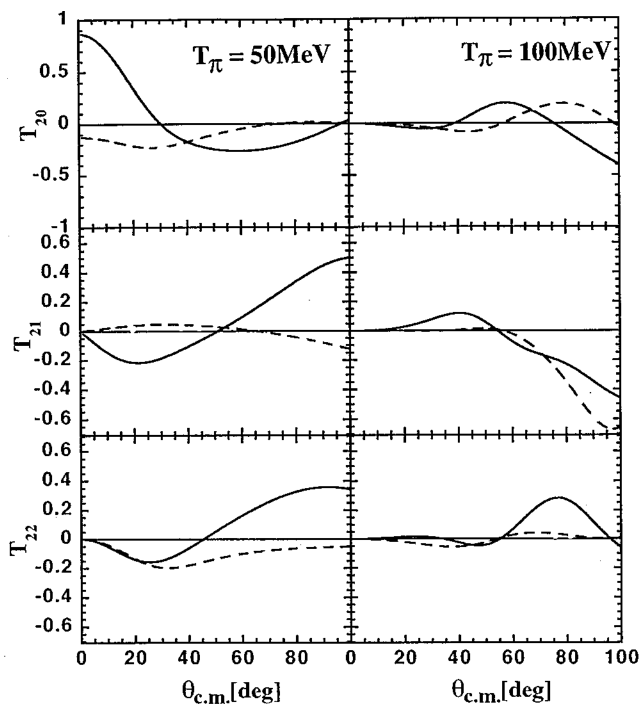

FIG. 4. The same as in Fig. 3, but for the second-rank tensor polarization.

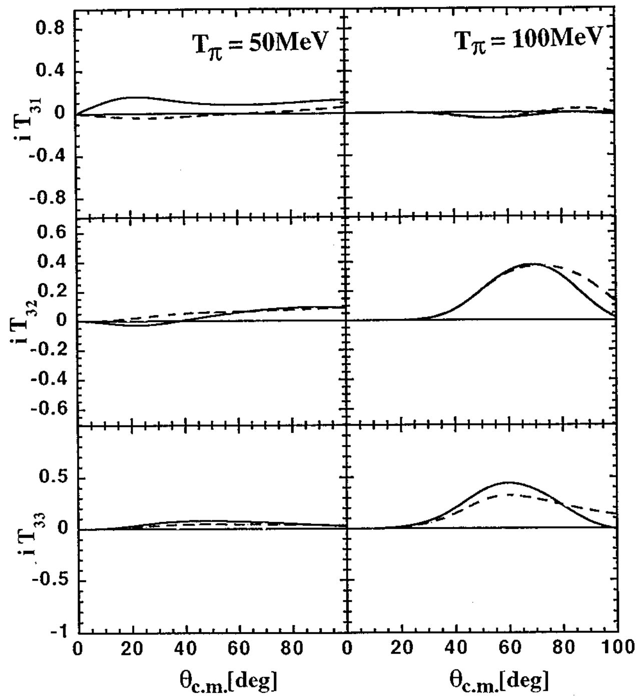

FIG. 5. The same as in Fig. 3, but for the third-rank tensor polarization.

scale factors by Glover *et al.* [27] which were determined from the the proton elastic scattering at 200 MeV. Then the $0p$ shell-model wave function gives the quadrupole moment $Q = -3.68\ \mathrm{fm}^2$ for ^{7}Li. As seen in Fig. 1, both of the theoretical values are close with each other and agree with the experiment around $T_\pi \geqslant 50$ MeV. In the forward SCX cross section, there appears a dip structure around 50 MeV which is due to the interference between *s*- and *p*-wave contributions. Because of this, the results are sensitive to the *s*- and *p*-wave amplitude of the pion-nucleon *t* matrix. In order to see the sensitivity of the forward cross section to the pion-nucleon amplitude, we have slightly modified the *s*- and the *p*-wave isovector parameters b_1 and c_1 to $0.7b_1$ and $0.8c_1$. The results are shown as the dotted line in Fig. 1. This shows the sensitivity of the forward cross sections to the pion-nucleon amplitude in the low-energy region. In Fig. 2, the angular distributions are shown and are compared with the experimental data. In these figures, the experimental cross section exhibits rather flat structure and the theory reproduces these features except for the overall normalization at energies lower than 40 MeV.

Next, we have calculated the polarization observables. In the previous work, we have shown that the semi-inclusive transitions in $^7\mathrm{Li}(\pi^\pm,\pi^\pm)$, leading to the ground plus first-excited states, the quadrupole effects are considerably large for the second-rank tensor polarization [10] due to the large quadrupole deformation of ^{7}Li. In the case of SCX reactions, we can study the quadrupole deformation of the isovector type. The calculated polarization observables are shown in Figs. 3–5. In the pion elastic scattering, the isoscalar spin-nonflip amplitude dominates and the first-rank tensor polarization (asymmetry) is fairly small [30], especially in the forward direction. Contrary to this, in the pion SCX reactions, the isoscalar part does not contribute to the DWIA amplitude and the resulting asymmetry takes fairly large values as seen in Fig. 3. The shell-model and the cluster-model wave fuctions give somewhat different results for the asymmetry but the overall features are almost the same. On the other hand, as seen in Fig. 4, the second-rank tensor polarizations are large and, in addition, the cluster and the shell models predict considerably different results. For the cluster wave function, the effect of the higher configuration is automatically incorporated which affects the radial dependence of the multipole densities. The cross sections of the SCX reaction are not sensitive to the detailed behavior of the multipole densities. If we use the appropriate effective charges for the shell-model wave function, the results are almost the same as those of the cluster wave function. On the other hand, the polarization observables come from the interference between various multipole matrix elements, and because of this, the results are sensitive to the details of the multipole densities. In particular, the second-rank tensor polarization T_{20} takes large values even in the forward direction and the cluster model predicts large positive value while the shell model gives a small negative value at $T_\pi = 50$ MeV. In the present case, the second-rank tensor polarization comes from the interference between the isovector monopole and quadrupole density terms F_{000} and F_{202}. These densities calculated with the shell and cluster models are fairly different resulting in the different second-rank tensor polarization. As seen in Fig. 5, the third-rank tensor polarizations are small in the forward direction for almost all energy regions considered here. The polarization T_{20} accidentally vanishes in the forward direction at $T_\pi = 100$ MeV.

In conclusion, we have calculated the cross section and the polarization observables for the SCX reactions $^7\mathrm{Li}(\pi^+,\pi^0)^7\mathrm{Be}_{\mathrm{gs}}$ in the low-energy region. The results for the cross section are compared with the available experimental data. The present DWIA calculation reproduce the forward cross section well at $T_\pi \geqslant 50$ MeV. Since the forward SCX cross section comes from the interference between the *s*- and *p*-wave interactions, it is sensitive to the relative strengths of the isovector pion-nucleon interactions. For lower energies, the theoretical value overestimates the forward cross section about a factor 2–2.5. The cluster- and shell-model wave functions give similar results for the reaction cross section, the asymmetry, and the third-rank tensor polarization if we use the appropriate renormalization factor for the shell-model wave function. On the other hand, the cluster- and shell-model wave functions predict quite different results for the second-rank tensor polarization at low energy. Even in the forward direction, the second-rank polarization T_{20} takes large values and the cluster and the shell models give considerably different results. This shows that the polarization observables in the SCX reaction are sensitive to the isovector part of the various multipole densities and this could be used to study the details of the nuclear structure.

---

[1] A. Doron *et al.*, Phys. Rev. C **26**, 189 (1982).
[2] J. J. Görgen *et al.*, Phys. Rev. Lett. **66**, 2193 (1991).
[3] R. Mach and S. S. Kamalov, Nucl. Phys. **A511**, 601 (1990).
[4] P. B. Siegel and W. R. Gibbs, Phys. Rev. C **48**, 1939 (1993).
[5] W. B. Kaufmann and W. R. Gibbs, Phys. Rev. C **28**, 1286 (1983).
[6] S. S. Kamalov, C. Bennhold, and R. Mach, Phys. Lett. B **259**, 410 (1991).
[7] C. Bennhold, L. Tiator, S. S. Kamalov, and R. Mach, Phys. Rev. C **46**, 2456 (1992).
[8] P. Bydzovsky, R. Mach, and S. S. Kamalov, Nucl. Phys. **A574**, 685 (1994).
[9] N. Nose and K. Kume, Phys. Rev. C **54**, 432 (1996).
[10] N. Nose, K. Kume, and S. Yamaguchi, Phys. Rev. C **50**, 321 (1994).
[11] M. Bouten, M. C. Bouten, and P. van Leuven, Nucl. Phys. **A102**, 322 (1967); **A111**, 385 (1968).
[12] J. Kruger and P. van Leuven, Nucl. Phys. **A139**, 418 (1969).
[13] C. M. Chesterfield and B. M. Spicer, Nucl. Phys. **41**, 675 (1963).
[14] D. Kurath, Nucl. Phys. **14**, 398 (1960); **140**, B1190 (1965).
[15] H. Kanada, Q. K. K. Liu, and Y. C. Tang, Phys. Rev. C **22**, 813 (1980).
[16] H. Walliser, Q. K. K. Liu, H. Kanada, and Y. C. Tang, Phys. Rev. C **28**, 57 (1983).
[17] T. Kajino, T. Matsuse, and A. Arima, Nucl. Phys. **A413**, 323 (1984); **A414**, 185 (1984).
[18] M. Unkelbach and H. M. Hofmann, Phys. Lett. B **261**, 211 (1991).
[19] M. A. Godwin, C. M. Laymon, R. M. Proor, D. R. Tilley, and H. R. Weller, Phys. Rev. C **56**, 1605 (1997), and references therein.
[20] T. Nishiyama and H. Ohtsubo, Prog. Theor. Phys. **52**, 952 (1974).
[21] A. B. Volkov, Nucl. Phys. **74**, 33 (1965).
[22] A. Hasegawa and S. Nagata, Prog. Theor. Phys. **45**, 1786 (1971).
[23] J. A. Carr, H. McManus, and K. Stricker-Bauer, Phys. Rev. C **25**, 952 (1982).
[24] M. Gmitro, S. S. Kamalov, and R. Mach, Phys. Rev. C **36**, 1105 (1987).
[25] A. A. Ebrahim and R. J. Peterson, Phys. Rev. C **54**, 2499 (1996).
[26] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).
[27] C. W. Glover *et al.*, Phys. Rev. C **43**, 1664 (1991).
[28] F. Irom, M. J. Leitch, H. W. Baer, J. D. Bowman, M. D. Cooper, B. J. Dropesky, E. Piasetzky, and J. N. Knudson, Phys. Rev. Lett. **55**, 1862 (1985).
[29] F. Irom, H. W. Baer, J. D. Bowman, M. D. Cooper, E. Piasetzky, U. Sennhauser, H. J. Ziock, M. J. Leitch, A. Erell, M. A. Moinester, and J. R. Comfort, Phys. Rev. C **31**, 1464 (1985).
[30] K. Kume and N. Nose, Phys. Rev. C **51**, 2006 (1995).